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We have our six numbers, $x, y, z, x-y, y-z, x-z$, all of which are prime. Firstly, x, y, and z must be distinct. If they are not distinct, then one of $x-y, y-z$, or $x-z$ must not be zero, which is not a postive integer. Let us then split this problem into two cases:

1. None of x, y, z are 2

If this is true, then x, y and z must be odd, as there are no even primes other than 2. However, this means that $x-y, y-z, x-z$ must be even, as they are the result of subtracting two odd numbers. Therefore, $x-y$ and $y-z$ must both be 2, since it is the only even prime. However, if

$$\begin{cases} x-y=2 \\ y-z=2 \end{cases}$$

Then

$$\begin{aligned} x-z &= (x-y)+(y-z) \\ &= 2+2 \\ &= 4 \end{aligned}$$

Therefore, $x-z$ is not prime, and therefore, if x, y and z are not 2, then at least one of $x-y, y-z$ or $x-z$ is not prime.

2. one of x, y, z is 2 We know that z must be 2, as $x-z>0$ and $y-z>0$, $x>z$ and $y>z$, and 2 is the smallest prime. Because we know $z=2$, $x-2$ and $y-2$ must be prime. Additionally, x and y are odd, as they are distinct from z and 2 is the only even prime. Therefore, $x-y$ is even, as it is the result of subtracting two odd numbers. The only solution for $x-y$ is therefore, 2, and therefore, $x=y+2$

Since $x=y+2$ and $y-2$ must be prime, $x-4$ must be prime.

$x, x-2$ and $x-4$ must all be prime. But this is impossible, as $x, x-2$ and $x-4$ are all distinct mod3, and therefore by pidgeonhole principle, one of them is divisible by 3

Therefore, there are no triplets (x, y, z) such that $x, y, z, x-y, y-z, x-z$ are all prime.