prove that $\sum_{0 \leq i \leq n} x_i^k l_i(x) = x^k$

It is known that $\sum_{0 \leq i \leq n} l_i(x) = 1$

For that the function $F(x) = \sum_{0 \leq i \leq n} l_i(x) - 1$ is polynomial and have $n+1$ roots.

So the function must be a constant.

We need to prove that

$$\sum_{0 \leq i \leq n} x_i^k l_i(x) = x^k$$

that is prove that

$$\left(\sum_{0 \leq i \leq n} x_i^k l_i(x) \right) \Big/ x^k = 1$$

Similarily,

$$F(x) = \left(\sum_{0 \leq i \leq n} x_i^k l_i(x) \right) \Big/ x^k - 1$$

have $n+1$ roots thus that $F(x)$ is a constant.