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1.   1.

$$
\begin{array}{llr}
(1) & \{\neg A \to A, A\} \vdash A & (\in) \\
(2) & \{\neg A \to A, \neg A\} \vdash \neg A \to A & (\in) \\
(3) & \{\neg A \to A, \neg A\} \vdash \neg A & (\in) \\
(4) & \{\neg A \to A, \neg A\} \vdash A & (\to -) \\
(5) & \{\neg A \to A\} \vdash A & ((1)(4) -)
\end{array}
$$

2. $((A \vee B) \to C) \to (A \to C) \wedge (B \to C)$

$$
\begin{array}{llr}
(1) & \{(A \vee B) \to C, A\} \vdash (A \vee B) \to C & (\in) \\
(2) & \{(A \vee B) \to C, A\} \vdash A \vee B & (\vee +) \\
(3) & \{(A \vee B) \to C, A\} \vdash C & (\to -) \\
(4) & \{(A \vee B) \to C, B\} \vdash (A \vee B) \to C & (\in) \\
(5) & \{(A \vee B) \to C, B\} \vdash A \vee B & (\vee +) \\
(6) & \{(A \vee B) \to C, B\} \vdash C & (\to -) \\
(7) & ((A \vee B) \to C) \vdash (A \to C) \wedge (B \to C) &
\end{array}
$$

$$
\begin{array}{llr}
(1) & \{(A \to C) \wedge (B \to C), (A \vee B), A\} \vdash A \to C & (\wedge -) \\
(2) & \{(A \to C) \wedge (B \to C), (A \vee B), \neg A\} \vdash B \to C & (\wedge -) \\
(3) & \{(A \to C) \wedge (B \to C), (A \vee B), A\} \vdash A & (\in) \\
(4) & \{(A \to C) \wedge (B \to C), (A \vee B), A\} \vdash C & (\to -) \\
(5) & \{(A \to C) \wedge (B \to C), (A \vee B), \neg A\} \vdash A \vee B & (\in) \\
(6) & \{(A \to C) \wedge (B \to C), (A \vee B), \neg A, A\} \vdash B & (\neg -) \\
(7) & \{(A \to C) \wedge (B \to C), (A \vee B), \neg A, B\} \vdash B & (\in) \\
(8) & \{(A \to C) \wedge (B \to C), (A \vee B), \neg A\} \vdash B & (\vee -) \\
(9) & \{(A \to C) \wedge (B \to C), (A \vee B), \neg A\} \vdash C & ((2)(8) \to -) \\
(10) & \{(A \to C) \wedge (B \to C), (A \vee B)\} \vdash C & ((4)(9) \neg -) \\
(11) & (A \to C) \wedge (B \to C) \vdash (A \vee B) \to C & (演绎)
\end{array}
$$

3.

(1) $\{\neg(A \to B), \neg A\} \vdash \neg A \to (\neg(A \to B) \to \neg A)$ (定理 7)

(2) $\{\neg(A \to B), \neg A\} \vdash \neg A$ ($\in$)

(3) $\{\neg(A \to B), \neg A\} \vdash (\neg(A \to B) \to \neg A)$ ($\to -$)

(4) $\{\neg(A \to B), \neg A\} \vdash (\neg(A \to B) \to \neg A) \to (A \to (A \to B))$

(定理 7)

(5) $\{\neg(A \to B), \neg A\} \vdash A \to B$ ($\to -$)

(6) $\{\neg(A \to B), \neg A\} \vdash \neg(A \to B)$ ($\in$)

(7) $\{\neg(A \to B)\} \vdash A$ ($(5)(6)\neg-$)

(8) $\neg\neg B \vdash B$ ($\neg\neg-$)

(9) $\neg\neg B \vdash B \to (A \to B)$ (定理 7)

(10) $\neg\neg B \vdash A \to B$ ($\to -$)

(11) $\neg\neg B \vdash \neg\neg(A \to B)$ ($\neg\neg+$)

(12) $\vdash \neg\neg B \to \neg\neg(A \to B)$ (演绎)

(13) $\vdash (\neg\neg B \to \neg\neg(A \to B)) \to (\neg(A \to B) \to \neg B)$ (定理 7)

(14) $\vdash \neg(A \to B) \to \neg B$ ($\to -$)

(15) $\neg(A \to B) \vdash \neg B$ (演绎)

(16) $\neg(A \to B) \vdash A \wedge \neg B$ ($(7)(15)\wedge+$)

(1) $\{A \wedge \neg B, (A \to B)\} \vdash A$ ($\wedge-$)

(2) $\{A \wedge \neg B, A \to B\} \vdash A \to B$ ($\in$)

(3) $\{A \wedge \neg B, A \to B\} \vdash B$ ($\to -$)

(4) $\{A \wedge \neg B, A \to B\} \vdash \neg B$ ($\wedge-$)

(5) $A \wedge \neg B \vdash \neg(A \to B)$ ($\neg+$)

4.

(1) $A \wedge B \vdash A$ ($\wedge-$)

(2) $A \wedge B \vdash B$ ($\wedge-$)

(3) $A \wedge B \vdash \neg A \vee B$ ($\vee+$)

(4) $A \wedge B \vdash A \wedge (\neg A \vee B)$ ($\wedge+, (1), (3)$)

$$
\begin{array}{lll}
(1) & A \wedge (\neg A \vee B) \vdash A & (\wedge-) \\
(2) & A \wedge (\neg A \vee B) \vdash (\neg A \vee B) & (\wedge-) \\
(3) & \{A \wedge (\neg A \vee B), \neg A\} \vdash \neg A & (\in) \\
(4) & \{A \wedge (\neg A \vee B), \neg A\} \vdash B & ((1)\,(3)\,\neg-) \\
(5) & \{A \wedge (\neg A \vee B), B\} \vdash B & (\in) \\
(6) & \{A \wedge (\neg A \vee B)\} \vdash B & ((2)(4)(5)\,\vee-) \\
(7) & A \wedge (\neg A \vee B) \vdash A \wedge B & ((1)(6), \wedge+) \\
\end{array}
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