

18CSC301T / FORMAL LANGUAGE AND AUTOMATA

Multiple Choice Questions/ Question Bank

UNIT I

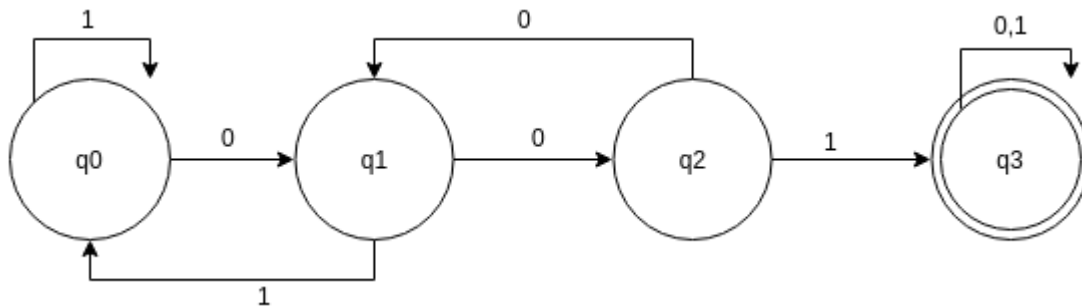
Syllabus: *Introduction to Automaton Mathematical concepts, Formal Languages: Strings, Languages, Properties, Finite Representation : Regular Expressions, Problems related to regular expressions, Finite Automata :Deterministic Finite Automata, Nondeterministic Finite Automata, Finite Automaton with ϵ - moves, Problems related to Deterministic and Nondeterministic Finite Automata, Problems related to Finite Automaton with ϵ - moves, Equivalence of NFA and DFA, Heuristics to Convert NFA to DFA, Equivalence of NDFA's with and without ϵ - moves, Problems related Equivalence of NDFA's with and without ϵ -moves, Minimization of DFA, Problems related to Minimization of DFA, Regular Languages : Equivalence of Finite Automata and Regular Languages, Equivalence of Finite Automata and Regular Grammars, Variants of Finite Automata: Two-way Finite Automaton Mealy Machines, Properties of Regular Languages: Closure Properties, Set Theoretic Properties & Other Properties, Pumping Lemma*

PART A

1. A regular language over an alphabet Σ is one that cannot be obtained from the basic languages using the operation
 - a) Union
 - b) Concatenation
 - c) Kleene*
 - d) All of the mentioned**
2. The number of elements in the set for the Language $L = \{x \in (\Sigma^r)^* \mid \text{length of } x \text{ is at most } 2\}$ and $\Sigma = \{0,1\}$ is _____
 - a) 7**
 - b) 6
 - c) 8
 - d) 5
3. Given: $\Sigma = \{a, b\}$ $L = \{x \in \Sigma^* \mid x \text{ is a string combination}\}$ Σ^4 represents which among the following

- a) {aa, ab, ba, bb}
 - b) {aaaa, abab, ε, abaa, aabb}**
 - c) {aaa, aab, aba, bbb}
 - d) {ε}
4. Mealy and Moore machine can be categorized as:
- a) Inducers
 - b) Transducers**
 - c) Turing Machines
 - d) Linearly Bounded Automata
5. A Language for which no DFA exist is a_____
- a) Regular Language
 - b) Non-Regular Language**
 - c) May be Regular
 - d) Cannot be said
6. Which of the following is not an example of finite state machine system?
- a) Control Mechanism of an elevator
 - b) Combinational Locks
 - c) Traffic Lights
 - d) Digital Watches**
7. Numbers of states require to accept string ends with 10.
- a) 3**
 - b) 2
 - c) 1
 - d) 0
8. $L_1 = \{w \mid w \text{ does not contain the string } tr\}$
 $L_2 = \{w \mid w \text{ does contain the string } tr\}$
 Given $\Sigma = \{t, r\}$, The difference of the minimum number of states required to form L_1 and L_2 ?
- a) 0**
 - b) 1

- c) 2
d) Infinite
9. The total number of states to build the given language using DFA:
 $L = \{w \mid w \text{ has exactly 2 a's and at least 2 b's}\}$
 a) 10
 b) 11
 c) 12
 d) 13
10. Given Language: $\{x \mid \text{it is divisible by 3}\}$
 The total number of final states to be assumed in order to pass the number constituting $\{0, 1\}$ is
 a) 0
 b) 1
 c) 2
 d) 3
11. Which of the following is same as the given DFA?



- a) $(0+1)^*001(0+1)^*$
 b) $1^*001(0+1)^*$
 c) $(01)^*(0+0+1)(01)^*$
 d) $(0+1)^*$
12. Which among the following cannot be accepted by a regular grammar?
- a) L is a set of numbers divisible by 2
 b) L is a set of binary complement
 c) L is a set of string with odd number of 0
 d) **L is a set of 0^n1^n**

13. Consider the languages $L_1 = \emptyset$ and $L_2 = \{1\}$. Which one of the following represents

$L_1^* \cup L_1^* L_2^*$?

(A) $\{\epsilon\}$

(B) $\{\epsilon, 1\}$

(C) \emptyset

(D) 1^*

14. The non- Kleene Star operation accepts the following string of finite length over set $A =$

$\{0,1\}$ | where string s contains even number of 0 and 1

a) 01,0011,010101

b) 0011,11001100

c) ϵ ,0011,11001100

d) ϵ ,0011,11001100

15. Transition function maps.

a) $\Sigma * Q \rightarrow \Sigma$

b) $Q * Q \rightarrow \Sigma$

c) $\Sigma * \Sigma \rightarrow Q$

d) $Q * \Sigma \rightarrow Q$

16. $\delta^*(q, ya)$ is equivalent to .

a) $\delta((q, y), a)$

b) $\delta(\delta^*(q, y), a)$

c) $\delta(q, ya)$

d) independent from δ notation

17. If we select a string w such that $w \in L$, and $w = xyz$. Which of the following portions cannot be an empty string?

a) x

b) y

c) z

d) xz

18. Fill in the blank in terms of p , where p is the maximum string length in L .

Statement: Finite languages trivially satisfy the pumping lemma by having $n = \underline{\hspace{2cm}}$

a) $p+1$

b) $p+1$

c) $p-1$

d) p

19. Predict the analogous operation for the given language:

A: $\{[p, q] \mid p \in A1, q \text{ does not belong to } A2\}$

a) $A1-A2$

b) $A2-A1$

c) $A1.A2$

d) $A1+A2$

20. e-transitions are

a) conditional

b) unconditional

c) input dependent

d) independent

21. RR^* can be expressed in which of the forms:

a) R^+

b) R^-

c) $R^+ \cup R^-$

d) R

22. Simplify the following regular expression:

$\epsilon+1^*(011)^*(1^*(011)^*)^*$

a) $(1+011)^*$

b) $(1^*(011)^*)^*$

c) $(1+(011)^*)^*$

d) $(1011)^*$

23. Precedence of regular expression in decreasing order is

a) $^*, \cdot, +$

b) $\cdot, ^*, +$

c) $\cdot, +, ^*$

d) $+, \cdot, ^*$

24. L and $\sim L$ are recursive enumerable then L is

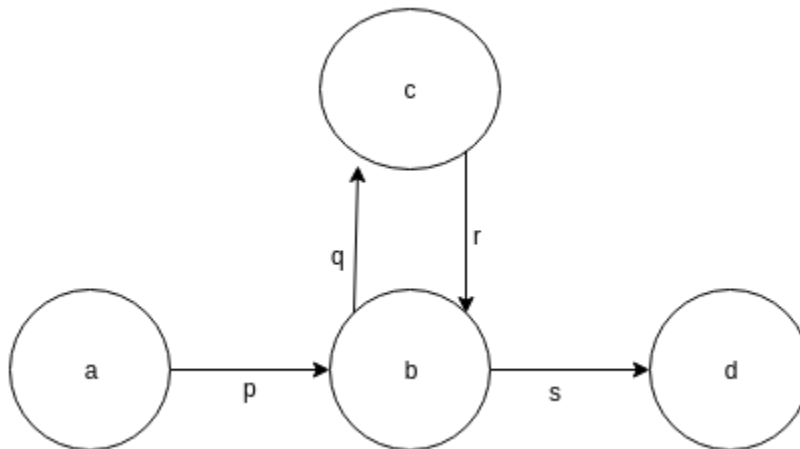
- a) Regular
- b) Context free
- c) Context sensitive
- d) Recursive**

25. Which of the following statements is false?

- (A) Every NFA can be converted to an equivalent DFA
- (B) Every non-deterministic Turing machine can be converted to an equivalent deterministic Turing machine
- (C) Every regular language is also a context-free language
- (D) Every subset of a recursively enumerable set is recursive**

PART B

1. If d is a final state, which of the following is correct according to the given diagram?



- a) $x=p, y=qr, z=s$**
- b) $x=p, z=qrs$
- c) $x=pr, y=r, z=s$
- d) All of the mentioned

Answer: A

Explanation: The FSA accepts the string $pqrs$. In terms of pumping lemma, the string $pqrs$ is broken into an x portion a , a y portion qr and a z portion s .

2. Which of the following one can relate to the given statement:

Statement: If n items are put into m containers, with $n > m$, then atleast one container must contain more than one item.

- a) Pumping lemma
- b) Pigeon Hole principle**
- c) Count principle
- d) None of the mentioned

Answer: B

Explanation: Pigeon hole principle states the following example: If there exists $n=10$ pigeons in $m=9$ holes, then since $10 > 9$, the pigeonhole principle says that at least one hole has more than one pigeon.

3. Statement 1: NFA computes the string along parallel paths.

Statement 2: An input can be accepted at more than one place in an NFA.

Which among the following options are most appropriate?

- a) Statement 1 is true while 2 is not
- b) Statement 1 is false while is not
- c) Statement 1 and 2, both are true**
- d) Statement 1 and 2, both are false

Answer: C

Explanation: While the machine runs on some input string, if it has the choice to split, it goes in all possible way and each one is different copy of the machine. The machine takes subsequent choice to split further giving rise to more copies of the machine getting each copy run parallel. If any one copy of the machine accepts the strings, then NFA accepts, otherwise it rejects.

4. Statement 1: ϵ - transition can be called as hidden non-determinism.

Statement 2: $\delta(q, \epsilon) = p$ means from q it can jump to p with a shift in read head.

Which among the following options is correct?

- a) Statement 1 and 2, both are correct
- b) Statement 1 and 2, both are wrong
- c) Statement 1 is correct while Statement 2 is wrong**
- d) Statement 1 is wrong while Statement 2 is correct

Answer: C

Explanation: The transition with ϵ leads to a jump but without any shift in read head. Further, the method can be called one to introduce hidden non-determinism.

5. Reverse of a DFA can be formed by
- a) using PDA
 - b) making final state as non-final
 - c) making final as starting state and starting state as final state**
 - d) Not possible

Answer: C

Explanation: By making final state as starting state string starting from end will be accepted.

6. Which among the following looks similar to the given expression?
 $((0+1).(0+1))^*$
- a) $\{x \in \{0,1\}^* | x \text{ is all binary number with even length}\}$
 - b) $\{x \in \{0,1\} | x \text{ is all binary number with even length}\}$
 - c) $\{x \in \{0,1\}^* | x \text{ is all binary number with odd length}\}$
 - d) $\{x \in \{0,1\} | x \text{ is all binary number with odd length}\}$

Answer: A

Explanation: The given regular expression corresponds to a language of binary strings which is of even length including a length of 0.

7. Generate a regular expression for the following problem statement:
Password Validation: String should be 8-15 characters long. String must contain a number, an Uppercase letter and a Lower case letter.
- a) $^(?=.*[a-z])(?=.*[A-Z])(?=.*\d).\{8,15\}$**
 - b) $^(?=.*[a-z])(?=.*[A-Z])(?=.*\d).\{9,16\}$
 - c) $^(?=.*[a-z])(?=.*[A-Z])(?=.*\d).\{8,15\}$
 - d) None of the mentioned

Answer: A

Explanation: Passwords like abc123, 123XYZ, should not be accepted. If one also wants to include special characters as one of the constraint, one can use the following

regular expression:

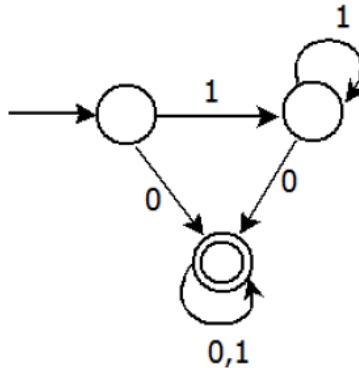
$^(?=.*[a-z])(?=.*[A-Z])(?=.*\d)(?=.*[^\da-zA-Z]).\{8,15\}$

8. Let S and T be language over $=\{a,b\}$ represented by the regular expressions $(a+b^*)^*$ and $(a+b)^*$, respectively. Which of the following is true? (GATE CS 2000)

- (a) $S \subset T$ (S is a subset of T)
- (b) $T \subset S$ (T is a subset of S)
- (c) $S=T$
- (d) $S \cap T = \emptyset$

Answer: C

9. Consider the DFA given.



Which of the following are FALSE?

- 1. Complement of $L(A)$ is context free
- 2. $L(A) = L((11^*0+0)(0+1)^*0^*1^*)$
- 3. For the language accepted by A, A is the minimal DFA
- 4. A accepts all strings $\{0,1\}$ of length at least 2

- (A) 1 and 3 only
- (B) 2 and 4 only
- (C) 2 and 3 only
- (D) 3 and 4 only**

Answer: (D)

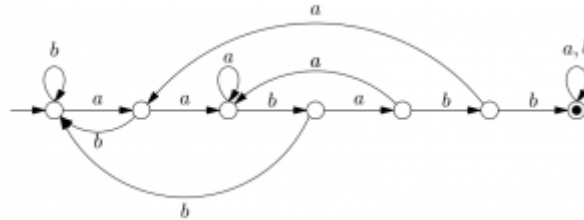
Explanation: 1 is true. $L(A)$ is regular, its complement would also be regular. A regular language is also context free.

2 is true.

3 is false, the DFA can be minimized to two states. Where the second state is final state and we reach second state after a 0.

4 is clearly false as the DFA accepts a single 0.

10. Consider the following Deterministic Finite Automata



Which of the following is true?

- (A) It only accepts strings with prefix as “aababb”
- (B) It only accepts strings with substring as “aababb”**
- (C) It only accepts strings with suffix as “aababb”
- (D) None of the above

Answer: (B)

Explanation:

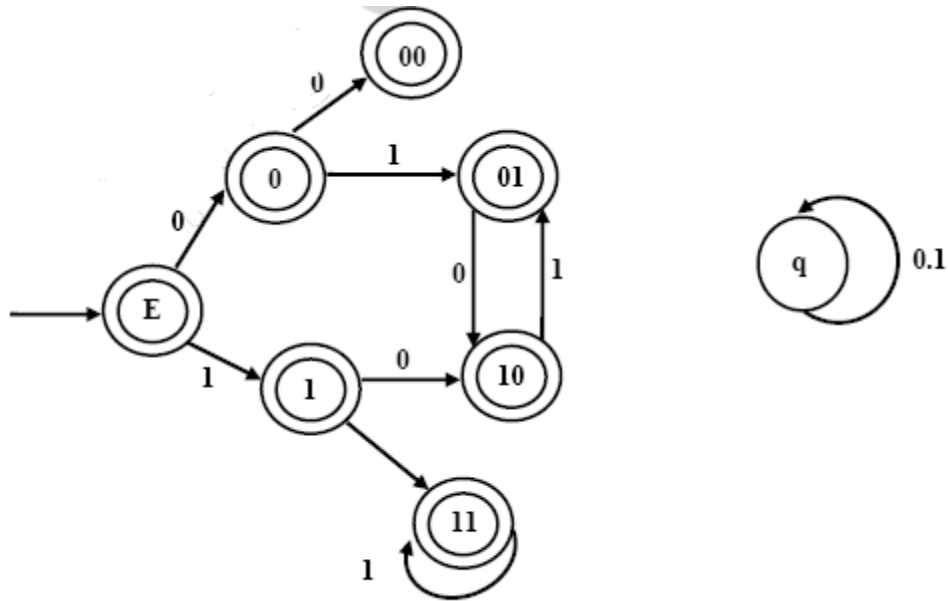
To reach the accepting state, any string will have to go through edges having aababb as labels in order. Though it might not be a continuous substring, but it sure will be a substring. There might be some cases where same substring always exists as a prefix or suffix for some DFA, but in this situation we don't have to consider those cases, given this question has single choice answer.

→ O – a –> O – a –> O – b –> O – a –> O – b –> O – b –> O

Hence, correct answer should be (B).

PART C

1. Consider the set of strings on {0,1} in which, every substring of 3 symbols has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially completed DFA that accepts this language is shown below.



The missing arcs in the DFA are

(A)

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11			0		

(B)

	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

(C)

	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

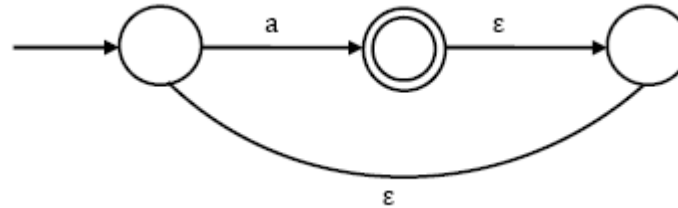
(D)

	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

- a. A
- b. B
- c. C
- d. D**

State 'q' is trap state. All other states are accept states. In state 00, DFA must move to 'q' for input symbol 0. All (non-trap) states indicate names indicate the characters seen before reaching that particular state. Option (D) is the only option that follow these rules.

2. What is the complement of the language accepted by the NFA shown below? Assume $\Sigma = \{a\}$ and ϵ is the empty string

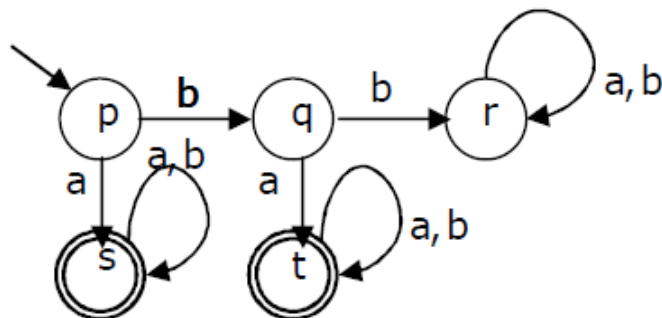


- (A) Φ
 (B) ϵ
 (C) a
 (D) $\{a, \epsilon\}$

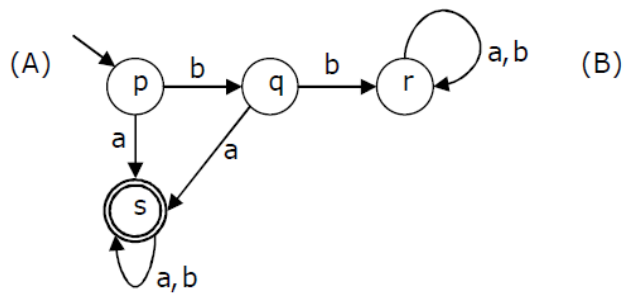
Answer (B)

Explanation: The given alphabet Σ contains only one symbol $\{a\}$ and the given NFA accepts all strings with any number of occurrences of 'a'. In other words, the NFA accepts a^+ . Therefore complement of the language accepted by automata is empty string.

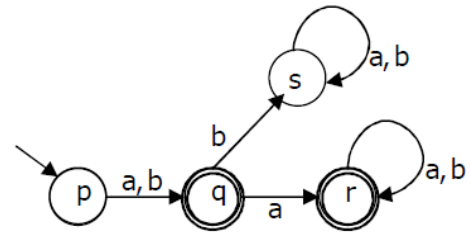
3. A deterministic finite automation (DFA) D with alphabet $\{a,b\}$ is given below



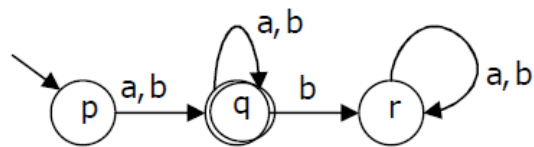
Which of the following finite state machines is a valid minimal DFA which accepts the same language as D?



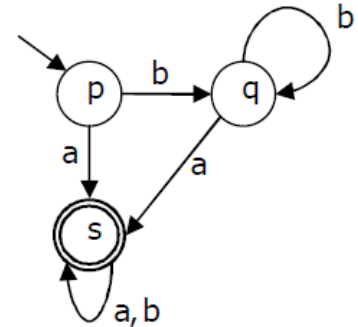
(B)



(C)



(D)



(A) A

(B) B

(C) C

(D) D

Answer: (A)

Explanation: Options (B) and (C) are invalid because they both accept 'b' as a string which is not accepted by given DFA. (D) is invalid because it accepts "bba" which are not accepted by given DFA.

4. Which one of the following is FALSE?

(A) There is unique minimal DFA for every regular language

(B) Every NFA can be converted to an equivalent PDA.

(C) Complement of every context-free language is recursive.

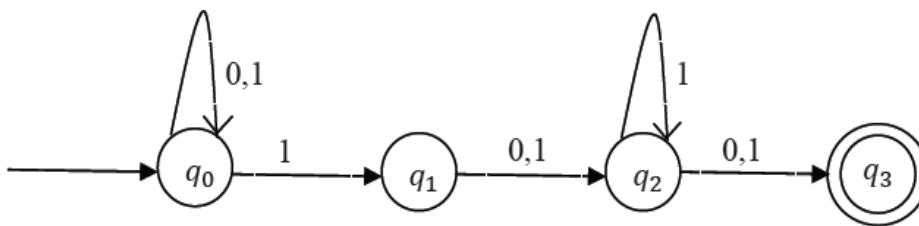
(D) Every nondeterministic PDA can be converted to an equivalent deterministic PDA.

Answer: (D)

Explanation: Power of Deterministic PDA is not same as the power of Non-deterministic PDA. Deterministic PDA cannot handle languages or grammars with ambiguity, but NDPDA can handle languages with ambiguity and any context-free grammar. So every non-deterministic PDA can not be converted to an equivalent deterministic PDA.

5.

Consider the finite automaton in the following figure.

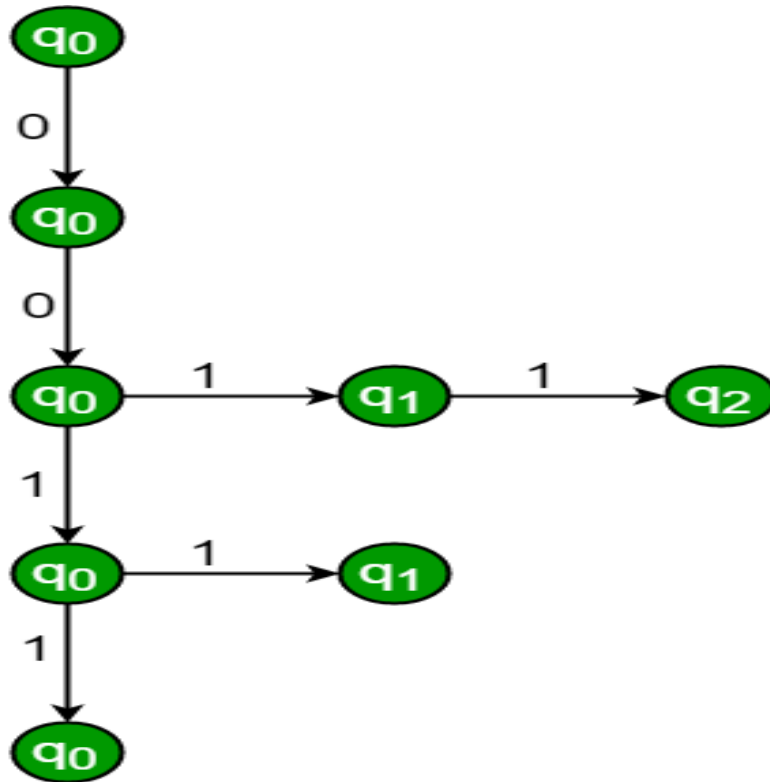


What is the set of reachable states for the input string 0011?

- (A) {q0, q1, q2}
- (B) {q0, q1}
- (C) {q0, q1, q2, q3}
- (D) {q3}

Answer: (A)

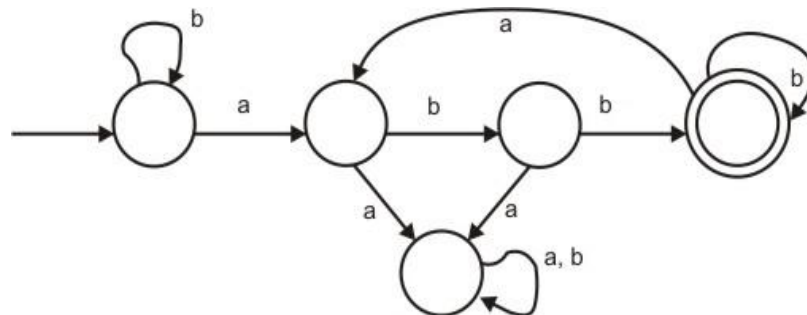
Explanation:



So, q_0 , q_1 and q_2 are reachable states for the input string 0011, but q_3 is not.

So, option (A) is answer.

6. Consider the machine M:



The language recognized by M is :

- (A) $\{w \in \{a, b\}^* \mid \text{every } a \text{ in } w \text{ is followed by exactly two } b\text{'s}\}$
- (B) $\{w \in \{a, b\}^* \mid \text{every } a \text{ in } w \text{ is followed by at least two } b\text{'s}\}$
- (C) $\{w \in \{a, b\}^* \mid w \text{ contains the substring 'abb'}\}$
- (D) $\{w \in \{a, b\}^* \mid w \text{ does not contain 'aa' as a substring}\}$

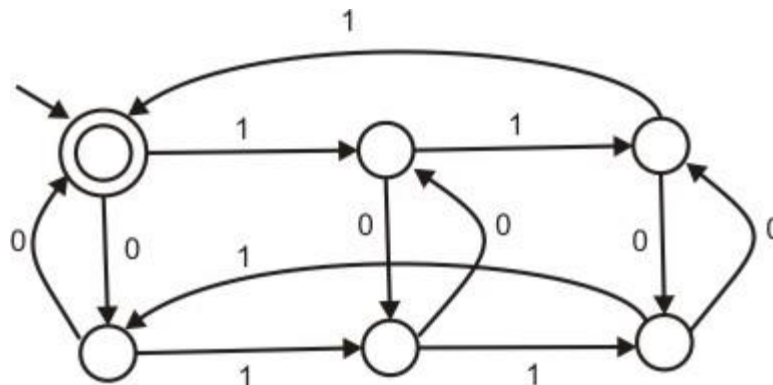
Answer: (B)

Explanation:

Here $w \in \{a, b\}^*$ means w can be any string from the set of $\{a, b\}^*$ and $\{a, b\}^*$ is set of all strings composed of a and b (any string of a and b that you can think of) like null, a , b , aaa , $abbaaa$, $bbbbbb$, $aaaaa$, $aaaabbbbbaabbababab$ etc.

These type of questions are frequently asked in GATE, where it is asked to choose best fit language among the options. To solve the question like this, there is a better way, we try to eliminate wrong options by choosing testing strings intelligently until we are left with one right option. As given in question, let's we try to eliminate option (A), it recognizes **only those string** (composed of a and b) in which every a in w is followed by exactly two b 's, so if we take string $abbb$ (three b 's), then it is accepted by machine, so this options is wrong. Now we try to eliminate option (C), it recognizes only those strings (composed of a and b) in which w contains the substring 'abb', so if we take string $abbaa$ (has substring abb), then it is not accepted by machine, so this options is also wrong. Now we try to eliminate option (D), it recognizes only those string (composed of a and b) in which w does not contains 'aa' as a substring, so if we take string $abbaba$ ('aa' not as a substring), then it is not accepted by machine, so this options is also wrong. Only option with which we are left, is option (b) in which every a in w is followed by at least two b ', is correct. So answer is option (B).

7. The following finite state machine accepts all those binary strings in which the number of 1's and 0's are respectively.



- (A) divisible by 3 and 2
- (B) odd and even
- (C) even and odd
- (D) divisible by 2 and 3

Answer: (A)

Explanation:

Option (B) is eliminated because string 100 contains odd number of 1s and even number of 0s but is not accepted by the DFA.

Option(C) is eliminated because string 011 contains even number of 1s and odd number of 0s but is not accepted by the DFA.

Option (D) is eliminated because string 11000 has number of 1s divisible by 2 and number of 0s divisible by 3 but still not accepted by the DFA.

Option (A) accepts all strings with number of 1s divisible by 3 and number of 0s divisible by 2.

UNIT-2

Syllabus: *Grammars: Introduction: Types of Grammar, Context Free Grammars and Languages, Derivations, Ambiguity, Relationship between derivation and derivation trees, Problems related to Context free Grammar, Simplification of CFG : Elimination of Useless Symbols, Simplification of CFG : Unit productions, Simplification of CFG : Null productions, Problems related to Simplification of CFG, Chomsky normal form, Problems related to CNF, Greiback Normal form, Problems related to GNF*

PART A

1. Push down automata accepts which language

- a) Context sensitive language
- b) Context free language
- c) Recursive language

Answer: d)

2. A context free grammar G is in Chomsky normal form if every production is of the form

- a) $A \rightarrow BC$ or $A \rightarrow A$
- b) $A \rightarrow BC$ or $A \rightarrow a$
- c) $A \rightarrow BCa$ or $B \rightarrow b$
- d) None of these

Answer: b)

3. Which of the following statement is false?

- a) A recursive language is also a regular language
- b) A context free language is also a regular language
- c) A context free language is also recursive enumerable language
- d) Both (a) and (b)

Answer: b)

4. A context free language is called ambiguous if

- a) It has two or more leftmost derivations for some terminal string $w \in L(G)$

- b) It has two or more leftmost derivations for some terminal string $w \in L(G)$
- c) Both (a) and (b)
- d) None of these

Answer:c)

5. Which of the following statement is false?

- a) The context free language can be converted into Chomsky normal form
- b) The context free language can be converted into Greibach normal form
- c) The context free language is accepted by pushdown automata
- d) None of these

Answer:d)

6. The language $L = \{0^m 1^m 0^m \mid m \geq 1\}$ is a

- a) Regular language
- b) Context free language
- c) Both (a) and (b)
- d) None of these

Answer:d)

7. While converting the context free grammar into Greibach normal form, which of the following is not necessary

- a) Elimination of null production
- b) Elimination of unit production
- c) Converting given grammar in Chomsky normal form
- d) None of these

Answer:d)

8. The context free grammar $S \rightarrow A111|S1, A \rightarrow A0 \mid 00$

is equivalent to a) $\{0^n 1^m \mid n=2, m=3\}$

- b) $\{0^n 1^m \mid n=1, m=5\}$
- c) $\{0^n 1^m \mid n \text{ should be greater than two and } m \text{ should be greater than four}\}$
- d) None of these

Answer:a)

9. The context free grammar $S \rightarrow SS \mid 0S1 \mid 1S0 \mid \epsilon$ generates

- a) Equal number of 0's and 1's
- b) Unequal number of 0's and 1's
- c) Any number of 0's followed by any number of 1's
- d) None of these

Answer:a)

10. Which of the following statement is false?

- a) In derivation tree, the label of each leaf node is terminal
- b) In derivation tree, the label of all nodes except leaf nodes is a variable
- c) In derivation tree, if the root of a sub tree is X then it is called –tree
- d) None of these

Answer:d)

11. Push down automata accepts which language

- a) Context sensitive language
- b) Context free language
- c) Recursive language
- d) None of these

Answer: b)

12. A context free grammar G is in Chomsky normal form if every production is of the form

- a) $A \rightarrow BC$ or $A \rightarrow A$
- b) $A \rightarrow BC$ or $A \rightarrow a$
- c) $A \rightarrow BCa$ or $B \rightarrow b$
- d) None of these

Answer:b)

13. Which of the following statement is false?

- a) A recursive language is also a regular language
- b) A context free language is also a regular language
- c) A context free language is also recursive enumerable language
- d) Both (a) and (b)

Answer: b)

14. A context free language is called ambiguous if

- a) It has two or more leftmost derivations for some terminal string $w \in L(G)$
- b) It has two or more leftmost derivations for some terminal string $w \in L(G)$
- c) Both (a) and (b)
- d) None of these

Answer: c)

15. Which of the following statement is false?

- a) The context free language can be converted into Chomsky normal form
- b) The context free language can be converted into Greibach normal form
- c) The context free language is accepted by pushdown automata
- d) None of these

Answer: d)

16. The language $L = \{0^m 1^m 0^m \mid m \geq 1\}$ is a

- a) Regular language
- b) Context free language
- c) Both (a) and (b)
- d) None of these

Answer: d)

17. While converting the context free grammar into Greibach normal form, which of the following is not necessary

- a) Elimination of null production

- b) Elimination of unit production
- c) Converting given grammar in Chomsky normal form
- d) None of these

Answer:d)

18. The context free grammar $S \rightarrow A111|S1$, $A \rightarrow A0 | 00$

is equivalent to a) $\{0^n 1^m \mid n=2, m=3\}$

b) $\{0^n 1^m \mid n=1, m=5\}$

c) $\{0^n 1^m \mid n \text{ should be greater than two and } m \text{ should be greater than four}\}$

d) None of these

Answer:a)

19. The context free grammar $S \rightarrow SS \mid 0S1 \mid 1S0 \mid \epsilon$ generates

a) Equal number of 0's and 1's

b) Unequal number of 0's and 1's

c) Any number of 0's followed by any number of 1's

d) None of these

Answer:a)

20. Which of the following statement is false?

a) In derivation tree, the label of each leaf node is terminal

b) In derivation tree, the label of all nodes except leaf nodes is a variable

c) In derivation tree, if the root of a sub tree is X then it is called –tree

d) None of these

Answer:d)

PART B

1. Consider the following statements about the context free grammar

$G = \{S \rightarrow SS, S \rightarrow ab, S \rightarrow ba, S \rightarrow ?\}$

- I. G is ambiguous
- II. G produces all strings with equal number of a's and b's
- III. G can be accepted by a deterministic PDA

which combination below expresses all the true statements about G?

- A. I only
- B. I and III only
- C. II and III only
- D. I, II and III

Answer : B. I and III only

2. Which one of the following statements is FALSE?

- A. There exist context-free languages such that all the context-free grammars generating them are ambiguous.
- B. An unambiguous context free grammar always has a unique parse tree for each string of the language generated by it.
- C. Both deterministic and non-deterministic pushdown automata always accept the same set of languages.
- D. A finite set of string from one alphabet is always a regular language

Answer: C. Both deterministic and non-deterministic pushdown automata always accept the same set of languages.

3. Let us now consider the following grammar:

Set of alphabets $\Sigma = \{0, \dots, 9, +, *, (,)\}$

$E \rightarrow I$

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow (E)$

$I \rightarrow \epsilon \mid 0 \mid 1 \mid \dots \mid 9$

From the above grammar String $3*2+5$ can be derived in 2 ways:

I) First leftmost derivation

II) Second leftmost derivation

$$E \Rightarrow E * E$$

$$E \Rightarrow E + E$$

$$\Rightarrow I * E$$

$$\Rightarrow E * E + E$$

$$\Rightarrow 3 * E + E$$

$$\Rightarrow I * E + E$$

$$\Rightarrow 3 * I + E$$

$$\Rightarrow 3 * E + E$$

$$\Rightarrow 3 * 2 + E$$

$$\Rightarrow 3 * I + E$$

$$\Rightarrow 3 * 2 + I$$

$$\Rightarrow 3 * 2 + I$$

$$\Rightarrow 3 * 2 + 5$$

$$\Rightarrow 3 * 2 + 5$$

Which combination below expresses all the true statements about G?

- A. I only
- B. Both I and II
- C. II only
- D. Neither I nor II

Answer: B. Both I and II

4. Consider the following statements about the context free grammar

$$G = \{S \rightarrow SS, S \rightarrow ab, S \rightarrow ba, S \rightarrow \epsilon\}$$

- **I** G is ambiguous
- **II** G produces all strings with equal number of a's and b's
- **III** G can be accepted by a deterministic PDA

Which combination below expresses all the true statements about G?

- A. I only
- B. I and III only
- C. II and III only

D. I, II and III

Answer: B I and III only

5. Identify and remove the unit productions from the following CFG

$S \rightarrow S + T / T$

$T \rightarrow T * F / F$

$F \rightarrow (S)/a$

Consider

i. $S \rightarrow S + T / T * F / (S) / a$

$T \rightarrow T * F / F$

$F \rightarrow (S) / a$

ii. $S \rightarrow S + T / T + F / (S) / a$

$T \rightarrow T * F / F$

$F \rightarrow (S)$

Which combination below expresses all the true statements about G?

A. I only

B. II only

C. Both I and II

D. Both I and II

Answer: A. I only

6. What are the three ways to simplify a context free grammar?

i. By removing the useless symbols from the set of productions.

ii. By eliminating the empty productions.

ii. By eliminating the unit productions.

Which of the above statements are true

a. i true

b. ii true

c. i and ii true

d. i , ii and iii true

Answer d. i , ii and iii true

7. Which of the following CFG's can't be simulated by an FSM ?

i. $S \rightarrow Sa \mid b$

ii. $S \rightarrow aSb \mid ab$

iii. $S \rightarrow abX, X \rightarrow cY, Y \rightarrow d \mid aX$

iv. None of these

Which of the following option justifies the above question

A. Option (ii) generates the set $\{a^n b^n, n=1,2,3, \dots\}$ which is not regular ,

B. Option (i) is left linear

C. Option (iii) is right linear .

Answer Option (ii) generates the set $\{a^n b^n, n=1,2,3, \dots\}$ which is not regular

8. Correct hierarchical relationship among context- free, right-linear, and context-sensitive language is

A. Context-free \subset right-linear \subset context-sensitive

- B. context-free \subset context-sensitive \subset right-linear
- C. Context-sensitive \subset right-linear \subset context-free
- D. Right-linear \subset context-free \subset context-sensitive

Answer: Right-linear \subset context-free \subset context-sensitive

9. For the language $L = \{x^n y^n z^n \mid n \geq 1\}$

Let L is context free. Then, L must satisfy pumping lemma.

At first, choose a number n of the pumping lemma. Then, take z as $0^n 1^n 2^n$.

Break z into $uvwxy$, where

$|vwx| \leq n$ and $vx \neq \epsilon$.

Hence vwx cannot involve both 0s and 2s, since the last 0 and the first 2 are at least $(n+1)$ positions apart.

There are two cases –

Case 1 – vwx has no 2s. Then vx has only 0s and 1s. Then uwy , which would have to be in L , has n 2s, but fewer than n 0s or 1s.

Case 2 – vwx has no 0s.

From the above statements

- i. L is not a context-free language
- ii L is a context-free language
- iii. L is not a PDA
- iv L is a PDA

Which of the above statements are true?

- A. I only
- B. I and II only
- C. II only
- D. II and III only

Answer A. I only

10. From the following – $S \rightarrow ASA \mid aB \mid b$, $A \rightarrow B$, $B \rightarrow b \mid \epsilon$ which is the resultant production set after removing null production

- i. $S \rightarrow ASA \mid aB \mid b \mid a \mid SA \mid AS \mid S$, $A \rightarrow B \mid b$, $B \rightarrow b$
- ii. $A \rightarrow B$, $B \rightarrow b$
- iii. $S \rightarrow ASA \mid aB \mid b$
- iv. $S \rightarrow ASA \mid aB \mid b$, $A \rightarrow B$

- A. I only
- B. i and ii only
- C. i , ii , iii only
- D. ii, iii, iv only

Answer A. I only

PART C

1. Consider the given grammar G1:

$S \rightarrow XA \mid BB$

$B \rightarrow b \mid SB$

$X \rightarrow b$

$A \rightarrow a$

Consider:

1. $S \rightarrow bA \mid bCB \mid bABCB \mid bB \mid bABB$

$B \rightarrow bC \mid bABC \mid b \mid bAB$

$C \rightarrow BBC$

$C \rightarrow bCB \mid bABCB \mid bB \mid bABB$ 6

$X \rightarrow b$

$A \rightarrow a$

$$2. S \rightarrow bA \mid bCB \mid bABCB \mid bB \mid bABB$$

$$B \rightarrow bC \mid bABC \mid b \mid bAB5$$

$$C \rightarrow BBC \mid BB$$

$$X \rightarrow b$$

$$A \rightarrow a$$

$$3. S \rightarrow XA \mid BB$$

$$B \rightarrow b \mid XAB \mid BBB \ 1$$

$$X \rightarrow b$$

$$A \rightarrow a$$

$$4. S \rightarrow bA \mid BB$$

$$B \rightarrow bC \mid bABC \mid b \mid bAB4$$

$$C \rightarrow BBC \mid BB$$

$$X \rightarrow b$$

$$A \rightarrow a$$

$$5. S \rightarrow bA \mid BB$$

$$B \rightarrow b \mid bAB \mid BBB \ 2$$

$$X \rightarrow b$$

$$A \rightarrow a$$

$$6. S \rightarrow bA \mid BB$$

$$B \rightarrow bC \mid bABC \ 3$$

$$C \rightarrow BBC \mid \varepsilon$$

$$X \rightarrow b$$

$$A \rightarrow a$$

7. $S \rightarrow bA \mid bCB \mid bABCB \mid bB \mid bABB$
 $B \rightarrow bC \mid bABC \mid b \mid bAB$
 $C \rightarrow bCBC \mid bABCB \mid bBC \mid bABBC$ 7
 $C \rightarrow bCB \mid bABCB \mid bB \mid bABB$
 $X \rightarrow b$
 $A \rightarrow a$

Under the following options which are the steps to be followed to achieve the GNF 7

- A. 3, 5, 6, 4, 2, 1, 7
 B. 1, 2, 3, 4, 5, 6, 7
 C. 5, 6, 3, 4, 1, 2, 7
 D. 3, 6, 4, 5, 1, 2, 7

Answer: 3, 5, 6, 4, 2, 1, 7

2. Let us take an example to convert CFG to CNF. Consider the given grammar G1:

- $S \rightarrow ASB$
 $A \rightarrow aAS \mid a \mid \epsilon$
 $B \rightarrow SbS \mid A \mid bb$

Consider:

1. $S_0 \rightarrow S$
 $S \rightarrow ASB \mid SB$
 $A \rightarrow aAS \mid aS \mid a$
 $B \rightarrow SbS \mid A \mid \epsilon \mid bb$
2. $S_0 \rightarrow AS \mid ASB \mid SB$
 $S \rightarrow AS \mid ASB \mid SB$
 $A \rightarrow XAS \mid XS \mid a$

$B \rightarrow SYS|bb|XAS|XS|a$

$X \rightarrow a$

$Y \rightarrow b$

3. $S_0 \rightarrow AS|ASB| SB$

$S \rightarrow AS|ASB| SB$

$A \rightarrow XAS|XS|a$

$B \rightarrow SYS|VV|XAS|XS|a$

$X \rightarrow a$

$Y \rightarrow b$

$V \rightarrow b$

4. $S_0 \rightarrow S$

$S \rightarrow AS|ASB| SB| S$

$A \rightarrow aAS|aS|a$

$B \rightarrow SbS|bb|aAS|aS|a$

5. $S_0 \rightarrow AS|ASB| SB$

$S \rightarrow AS|ASB| SB$

$A \rightarrow aAS|aS|a$

$B \rightarrow SbS|bb|aAS|aS|a$

6. $S_0 \rightarrow S$

$S \rightarrow AS|ASB| SB| S$

$A \rightarrow aAS|aS|a$

$B \rightarrow SbS| A|bb$

7. $S \rightarrow AS|ASB|SB|S$

$S \rightarrow AS|ASB|SB|S$

$A \rightarrow aAS|aS|a$

$B \rightarrow SbS|bb|aAS|aS|a$

8. $S \rightarrow AS|PB|SB$

$S \rightarrow AS|ASB|SB$

$A \rightarrow XAS|XS|a$

$B \rightarrow SYS|VV|XAS|XS|a$

$X \rightarrow a$

$Y \rightarrow b$

$V \rightarrow b$

$P \rightarrow AS$

9. $S \rightarrow S$

$S \rightarrow AS|ASB|SB|S$

$A \rightarrow aAS|aS|a$

$B \rightarrow SbS|bb|aAS|aS|a$

10. $S \rightarrow AS|PB|SB$

$S \rightarrow AS|QB|SB$

$A \rightarrow XAS|XS|a$

$B \rightarrow SYS|VV|XAS|XS|a$

$X \rightarrow a$

$Y \rightarrow b$

$V \rightarrow b$

$P \rightarrow AS$

$Q \rightarrow AS$

11. $S_0 \rightarrow AS|PB| SB$

$S \rightarrow AS|QB| SB$

$A \rightarrow RS|XS|a$

$B \rightarrow SYS|VV|XAS|XS|a$

$X \rightarrow a$

$Y \rightarrow b$

$V \rightarrow b$

$P \rightarrow AS$

$Q \rightarrow AS$

$R \rightarrow XA$

12. $S_0 \rightarrow AS|PB| SB$

$S \rightarrow AS|QB| SB$

$A \rightarrow RS|XS|a$

$B \rightarrow TS|VV|XAS|XS|a$

$X \rightarrow a$

$Y \rightarrow b$

$V \rightarrow b$

$P \rightarrow AS$

$Q \rightarrow AS$

$R \rightarrow XA$

$T \rightarrow SY$

13. $S_0 \rightarrow AS|PB| SB$

$S \rightarrow AS|QB| SB$

$A \rightarrow RS|XS|a$

$B \rightarrow TS|VV|US|XS|a$

$X \rightarrow a$
 $Y \rightarrow b$
 $V \rightarrow b$
 $P \rightarrow AS$
 $Q \rightarrow AS$
 $R \rightarrow XA$
 $T \rightarrow SY$
 $U \rightarrow XA$

Under the following options which are the steps are correct to achieve the CNF

- a. 1,4,6,7,9,2,5,8,3,12,11,13,10
- b. 2,3,1,5,7,4,12,10,8,9,13,6,11
- c. 1,6,4,9,7,5,2,3,8,10,11,12,13
- d. 3, 6, 4, 5, 1, 2, 7

Answer: 1,6,4,9,7,5,2,3,8,10,11,12,13

3. Remove the unit productions from the following grammar results in which of the following answers

S -> AB

A -> a

B -> C / b

C -> D

D -> E

E -> a

Answer:

i. S -> AB

A -> a

B -> a / b

ii. $S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow a / b$

$C \rightarrow a$

$D \rightarrow a$

$E \rightarrow a$

Solutions:

- a. i only true
- b. i and ii true
- c. ii only true
- d. neither I nor ii true

Answer: a. i only true

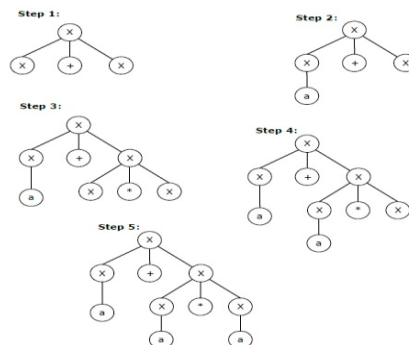
4. Let any set of production rules in a CFG be

$$X \rightarrow X+X \mid X*X \mid X \mid a$$

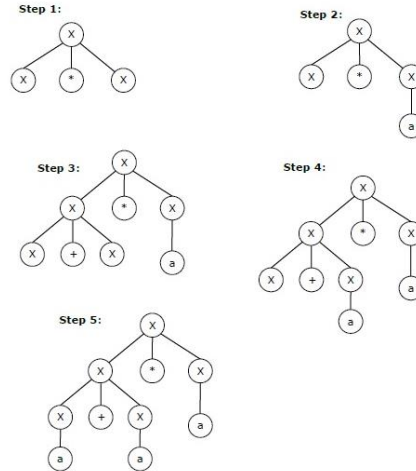
over an alphabet $\{a\}$.

The leftmost derivation for the string "**a+a*a**" may be

i



ii



- A. i only
- B. ii only
- C. Both i and ii
- D. Neither i nor ii

Answer A i only

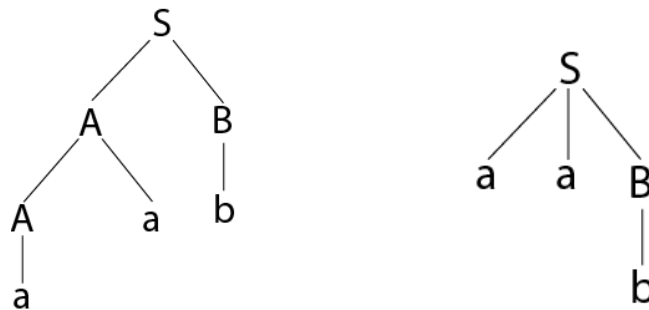
5. Consider a grammar G is given as follows:

$S \rightarrow AB \mid aaB$

$A \rightarrow a \mid Aa$

$B \rightarrow b$

State which option is right based on the following grammar G



- i. Grammar is ambiguous
- ii. Grammar is Unambiguous

- iii. Ambiguity cant be determined
- iv. None of these

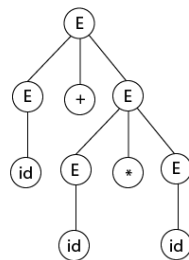
Answer : i. Grammar is ambiguous

6. Consider

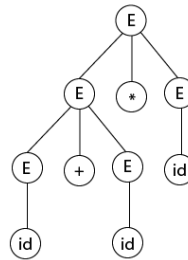
$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow id$



Parse tree 1



Parse tree 2

Based on the above parse trees state which of the following is unambiguous grammar

A. $E \rightarrow E + T$

$T \rightarrow T * F$

$F \rightarrow id$

B. $E \rightarrow E + T$

$T \rightarrow F$

$F \rightarrow id$

C. $E \rightarrow E + T$

$E \rightarrow T$

$T \rightarrow T * F$

$T \rightarrow F$

$$F \rightarrow \text{id}$$

D.

$$E \rightarrow E + T$$

$$E \rightarrow T$$

$$T \rightarrow T * F$$

Answer C: $E \rightarrow E + T$

$$E \rightarrow T$$

$$T \rightarrow T * F$$

$$T \rightarrow F$$

$$F \rightarrow \text{id}$$

UNIT-3

Syllabus: *Pushdown Automata: Definitions Moves, Instantaneous descriptions, Deterministic pushdown automata, Problems related to DPDA, Non - Deterministic pushdown automata, Problems related to NDPDA, Problems related to DPDA and NDPDA, Pushdown automata to CFL Equivalence, Problems related to Equivalence of PDA to CFG, CFL to Pushdown automata Equivalence, Problems related to Equivalence of CFG to PDA, Pumping lemma for CFL*

PART-A

1. Which one of the following pairs have different expressive power?
 - a) Deterministic Finite Automata AND Non Deterministic Finite Automata
 - b) Deterministic Push Down Automata AND Non deterministic Push Down Automata**
 - c) Deterministic single tape turing machine and Non-Deterministic single tape turing machine
 - d) single tape turing machine and multiple tape turing machine
2. The lexical analysis for a modern computer language such as java needs the power of which one of the following models in a necessary and sufficient sense?
 - a) Finite state automata**
 - b) Deterministic pushdown automata
 - c) Non-deterministic pushdown automata
 - d) Turing machine
3. Which one of the following problem is undecidable?
 - a) Membership problem of CFG
 - b) Ambiguity problem for CFG**

c) Finiteness problem for FSA

d) Equivalence problem for FSA

4. The recognizing capability of Non Deterministic Finite State Machine and Deterministic Finite State Machine

a) is different

b) sometimes different

c) is the same

d) none of these

5. Finite State Machine can recognize

a) any grammar

b) only CFG

c) any unambiguous grammar

d) only regular grammar

6. Pumping lemma is generally used for proving

a) a given grammar is regular

b) a given grammar is not regular

c) whether two given regular expressions are not equivalent

d) none of above

7. Which one of following is regular?

a) Strings of 0's whose length is a perfect square

b) Set of all palindromes made up of 0's and 1's

c) String of 0's, whose length is a prime number

d) String of odd numbers of zeros

8. Which one of the following pairs of regular expression are not equivalent?

a) $1(01)^*$ and $(10)^*1$

b) $x(xx)^*$ and $(xx)^*x$

c) $(ab)^*$ and a^*b^*

d) x^+ and x^*x^+

9. Assuming $P \neq NP$, which of the following is TRUE?

- a) $NP\text{-complete} = NP$
- b) **$NP\text{-complete intersection } P = \emptyset$**
- c) $NP\text{-hard} = NP$
- d) $P = NP\text{-complete}$

10. The lexical analysis for a modern language such as Java needs the power of which one of the following machine models in a necessary and sufficient sense?

- a) **Finite state automata**
- b) Deterministic pushdown automata
- c) Non-deterministic pushdown automata
- d) Turing machine

11. Let L denote the language generated by the grammar $S \rightarrow OSO/00$. Which of the following is true?

- (a) $L = O$
- (b) **L is regular but not O**
- (c) L is context free but not regular
- (d) L is not context free

12. Consider the following two statements:

S1: $\{ 0^{2n} \mid n \geq 1 \}$ is a regular language

S2: $\{ 0^m 0^n 0^{(m+n)} \mid m \geq 1 \text{ and } n \geq 2 \}$ is a regular language

Which of the following statements is correct?

- a) Only S1 is correct
- b) Only S2 is correct
- c) **Both S1 and S2 are correct**
- d) None of S1 and S2 is correct

13. Which of the following statements is true? (GATE CS 2001)

- (a) If a language is context free it can always be accepted by a deterministic push-down automaton
- (b) **The union of two context free languages is context free**

- (c) The intersection of two context free languages is context free
- (d) The complement of a context free language is context free

14. Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA is at least. (GATE CS 2001)

- (a) N^2
- (b) 2^N**
- (c) $2N$
- (d) $N!$

15. Let S and T be language over $\Sigma = \{a, b\}$ represented by the regular expressions $(a+b^*)^*$ and $(a+b)^*$, respectively. Which of the following is true? (GATE CS 2000)

- (a) $S \subset T$ (S is a subset of T)
- (b) $T \subset S$ (T is a subset of S)
- (c) $S = T$**
- (d) $S \cap T = \emptyset$

16. What is the complement of the language accepted by the NFA shown below? Assume $\Sigma = \{a\}$ and ϵ is the empty string

- (A) Φ
- (B) ϵ**
- (C) a
- (D) $\{a, \epsilon\}$

17. Given the language $L = \{ab, aa, baa\}$, which of the following strings are in L^* ?

- 1) abaabaaabaa
- 2) aaaabaaaa
- 3) baaaaabaaaab
- 4) baaaaabaa

- (A) 1, 2 and 3
- (B) 2, 3 and 4
- (C) 1, 2 and 4**
- (D) 1, 3 and 4

18. Which of the following problems are decidable?

- 1) Does a given program ever produce an output?
- 2) If L is a context-free language, then is L' (complement of L) also context-free?
- 3) If L is a regular language, then is L' also regular?
- 4) If L is a recursive language, then, is L' also recursive?

(A) 1, 2, 3, 4

(B) 1, 2,

(C) 2, 3, 4

(D) 3, 4

19. Which of the following strings do not belong the given regular expression?

$(a)^*(a+cba)$

a) aa

b) aaa

c) acba

d) acbacba

Answer (d)

20. Which of the following regular expression allows strings on $\{a,b\}^*$ with length n where n is a multiple of 4.

a) $(a+b+ab+ba+aa+bb+aba+bab+abab+baba)^*$

b) $(bbbb+aaaa)^*$

c) $((a+b)(a+b)(a+b)(a+b))^*$

d) $((a+b)(a+b)(a+b)(a+b))$

Answer (c)

Part- B

1. For $\Sigma = \{a, b\}$, let us consider the regular language $L = \{x \mid x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$. Which one of the following can be a pumping length (the constant guaranteed by the pumping lemma) for L ?

A. 3 B. 5 C. 9 **D. 24**

Answer D

2. Which one of the following statements is FALSE?

A. Context-free grammar can be used to specify both lexical and syntax rules.

B. Type checking is done before parsing.

C. High-level language programs can be translated to different Intermediate Representations.

D. Arguments to a function can be passed using the program stack.

Answer (B)

3. Consider the grammar: $S \rightarrow aSa|bSb|a|b$

The language generated by the above grammar over the alphabet $\{a,b\}$ is the set of:

(A) All palindromes

(B) All odd length palindromes.

(C) Strings that begin and end with the same symbol

(D) All even length palindromes

4. Which one of the following grammar generates the language $L = \{a^i b^j \mid i \neq j\}$?

(A)

$S \rightarrow AC|CB$
 $C \rightarrow aC b|a|b$
 $A \rightarrow aA| \epsilon$
 $B \rightarrow Bb| \epsilon$

(B) $S \rightarrow aS|Sb|a|b$

(C)

$S \rightarrow AC|CB$
 $C \rightarrow aC b| \epsilon$
 $A \rightarrow aA| \epsilon$
 $B \rightarrow Bb| \epsilon$

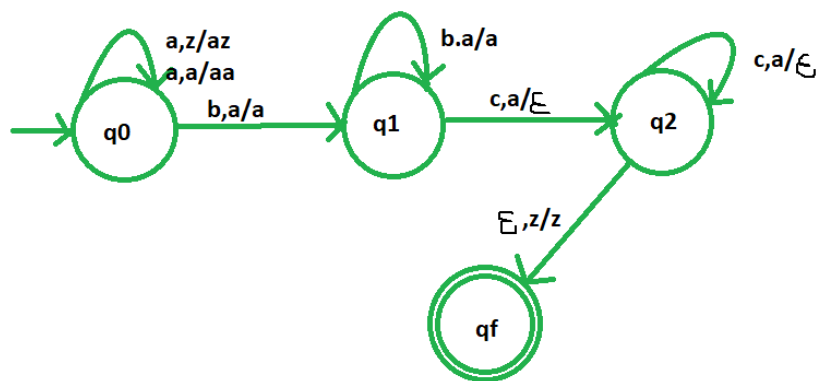
(D)

$S \rightarrow AC|CB$
 $C \rightarrow aC b| \epsilon$
 $A \rightarrow aA|a$
 $B \rightarrow Bb|b$

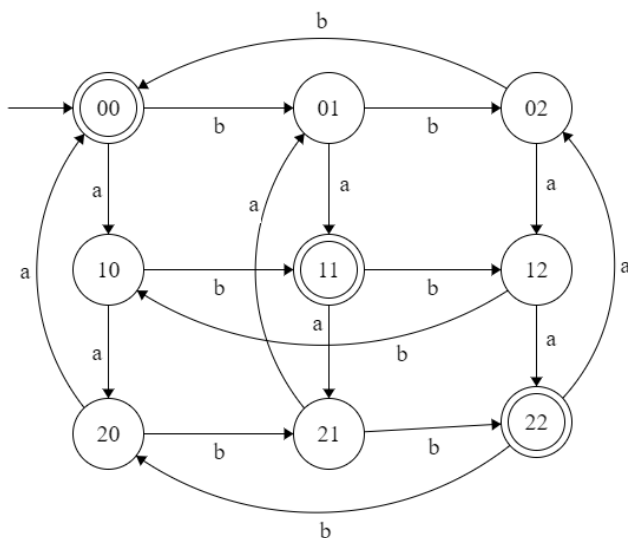
Answer : D

5. Design a non deterministic PDA for accepting the language $L = \{a^n b^m c^n \mid m, n \geq 1\}$, i.e. $L = \{abc, abbc, abbbc, aabbcc, aaabccc, aaaabbbccccc, \dots\}$

Solution:



6. Construct a deterministic finite automata (DFA) for accepting the language $L = \{w \mid w \in \{a,b\}^* \text{ and } Na(w) \bmod 3 = Nb(w) \bmod 3\}$.



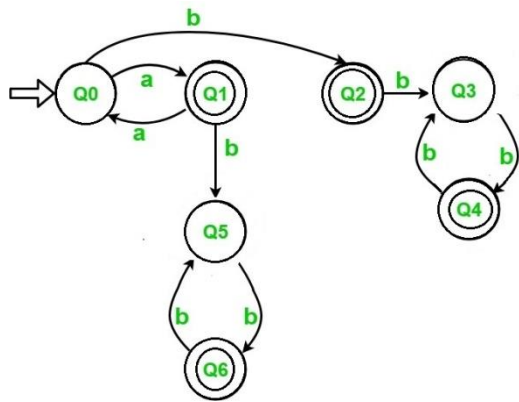
7. Design a deterministic finite automata (DFA) for accepting the language $L = \{a^n b^m \mid n+m=\text{odd}\}$

Approaches:

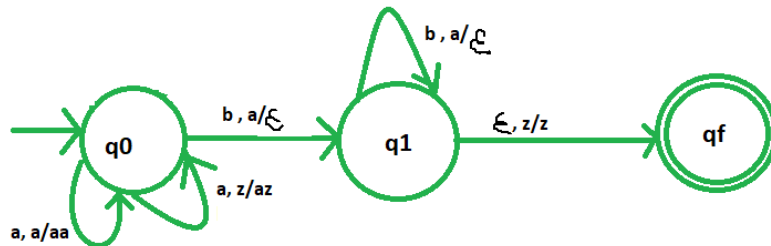
There is 2 cases which results in acceptance of string:

1. If n is odd and m is even then their sum will be odd
2. If n is even and m is odd then their sum will be odd

Any other combination result is the rejection of the input string.

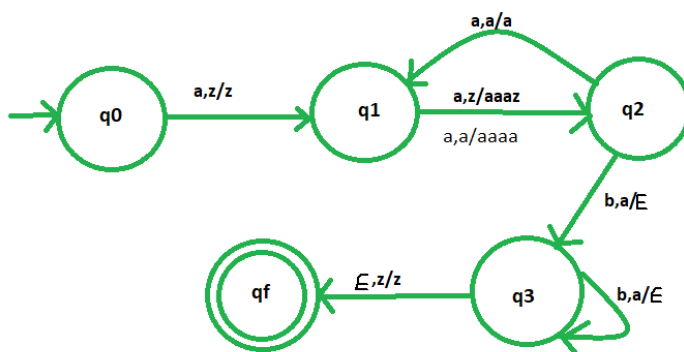


8. Design a non deterministic PDA for accepting the language $L = \{a^n b^n \mid n \geq 1\}$



Required PDA

9. Design a non deterministic PDA for accepting the language $L = \{a^{2m}b^{3m} \mid m \geq 1\}$



Required PDA

10. Design NPDA for $L = \{0^i 1^j 2^k \mid i=j \text{ or } j=k; i, j, k \geq 1\}$

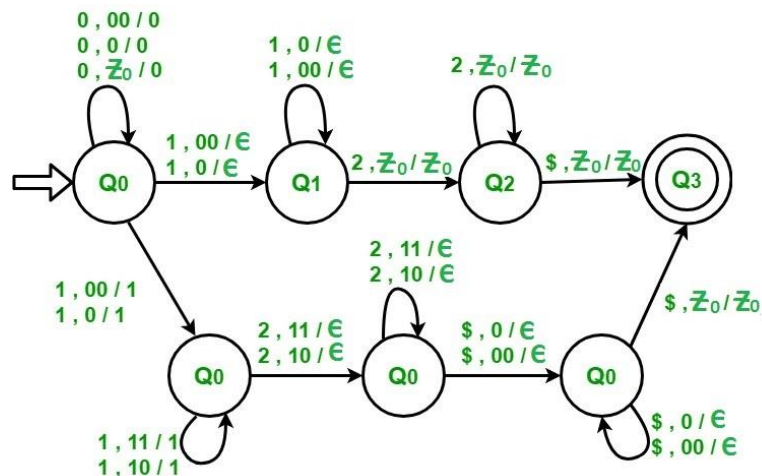
There are 2 approaches for the solution. First is for $i=j$ and second is for $j=k$. These are:

Steps for $i == j$:

1. Input all 0's in the stack
2. When we get 1 as input pop a 0 from stack and goto next state.
3. If input is 1 then pop 0 from stack.
4. If stack becomes empty (i.e., every 0 corresponding to a 1 has been popped so $i = j$) and input is 2 then ignore it and goto next state.
5. If input is 2 then ignore it . If input is finished and \$ is received then goto final state.

Steps for $j == k$:

1. Input all 0's in the stack
2. When we get 1 as input push it onto stack and goto next state.
3. If input is 1 then push it onto stack.
4. If input is 2 pop a 1 from stack and goto next state.
5. If input is 2 then pop 1 from stack. If input is finished and \$ is received then pop a 0 from stack.

**Part-C**

1. Design a non deterministic PDA for accepting the language $L = \{a^i b^j c^k d^l : i == k \text{ or } j == l, i \geq 1\}$, ie.,

$L = \{abcd, aabccd, aaabcccd, abbcdd, aabbccdd, aabbbccddd, \dots\}$ In each string, the number of a's are followed by any number of b's and b's are followed by the number of c's equal to the number of a's and c's are followed by number of d's equal number of b's.

Explanation –

Here, we need to maintain the order of a's, b's, c's and d's. That is, all the a's are coming first then all the b's are coming and then all the c's are coming then all the d's are coming. Thus, we need a stack along with the state diagram. The count of a's and b's is maintained by the stack. We will take 2 stack alphabets:

Approach used in the construction of PDA –

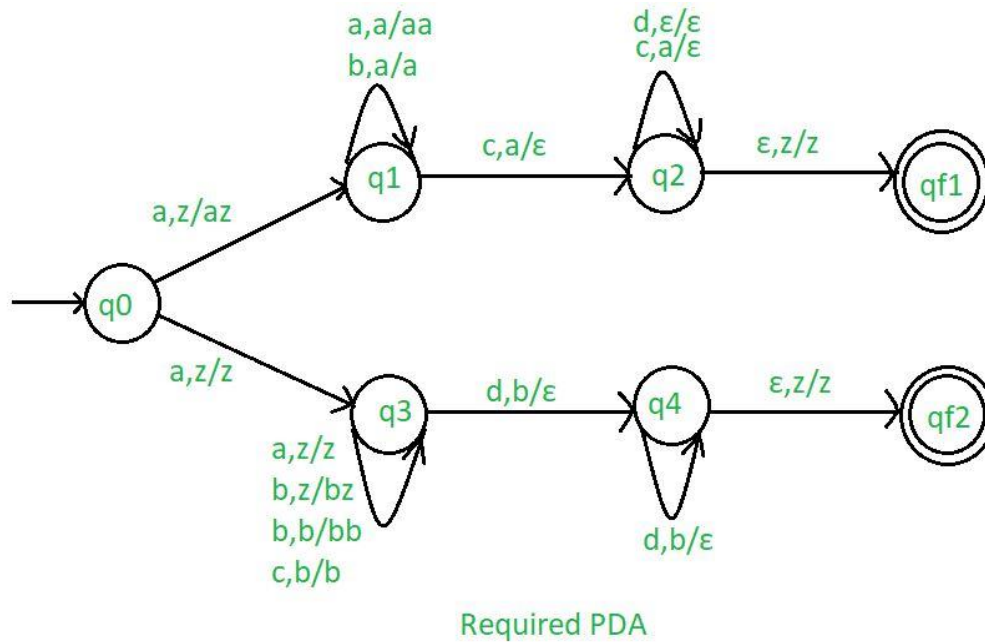
In designing a NPDA, for every a', 'b', 'c' and 'd' will comes in proper order.

- For $i=k$: Whenever 'a' comes, push it in stack and if 'a' comes again then also push it in the stack. After that, if 'b' comes not do any operation. After that, when 'c' comes then pop 'a' from the stack each time. After that, if 'd' comes not do any operation.
- For $j=l$: Whenever 'a' comes, not do any operation. After that, if 'b' comes push it in stack and if 'b' comes again then also push it in the stack. After that, when 'c' comes not do any operation. After that, if 'd' comes then pop 'b' from the stack each time.

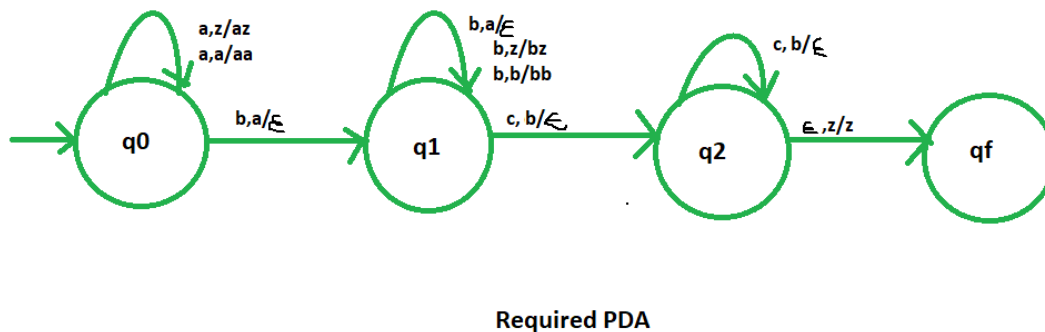
So that the stack becomes empty. If stack is empty then we can say that the string is accepted by the PDA.

Construct Stack transition functions

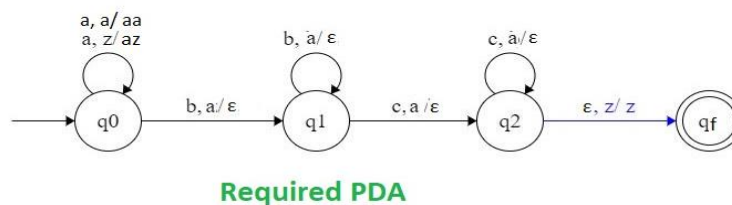
Required NPDA



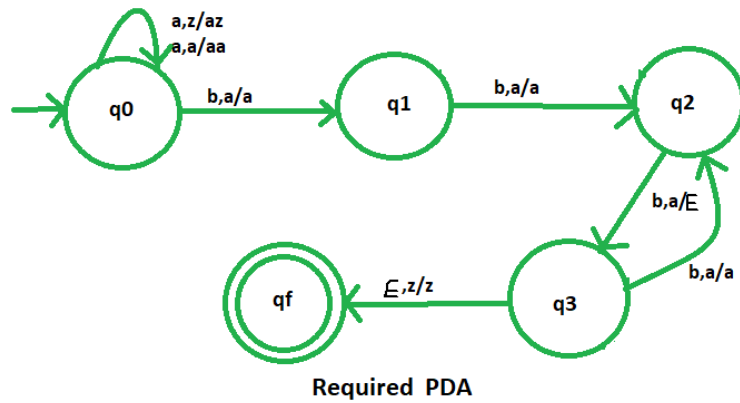
2. Design a non deterministic PDA for accepting the language $L = \{a^m b^{(m+n)} c^n \mid m, n \geq 1\}$



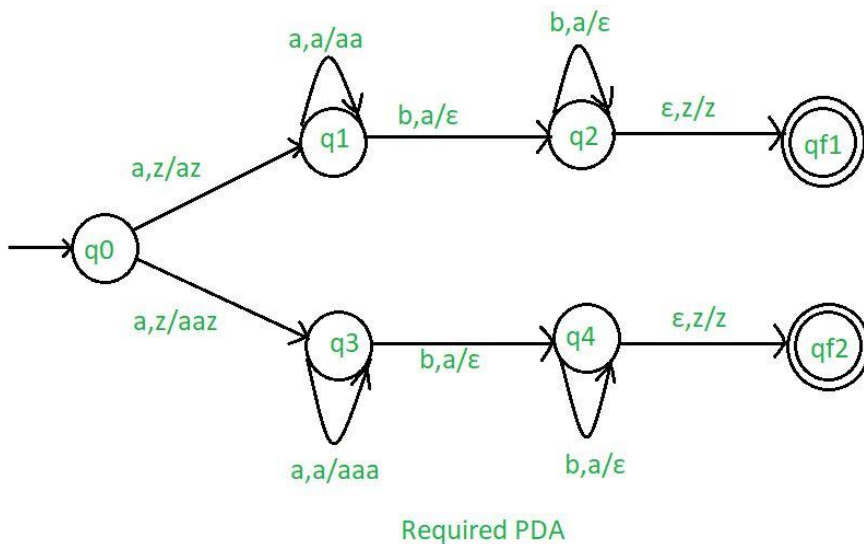
3. Design a non deterministic PDA for accepting the language $L = a^{(m+n)} b^m c^n, m, n \geq 1$



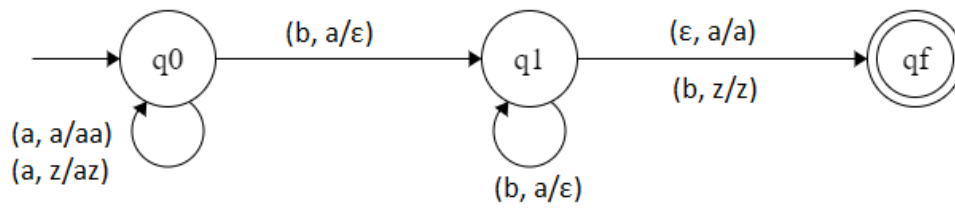
4. NPDA for accepting the language $L = \{a^m b^{(2m+1)} \mid m \geq 1\}$ Design a non deterministic PDA for accepting the language



5. Design a non deterministic PDA for accepting the language NPDA for accepting the language $L = \{a^n b^{(2^n)} \mid n \geq 1\} \cup \{a^n b^n \mid n \geq 1\}$

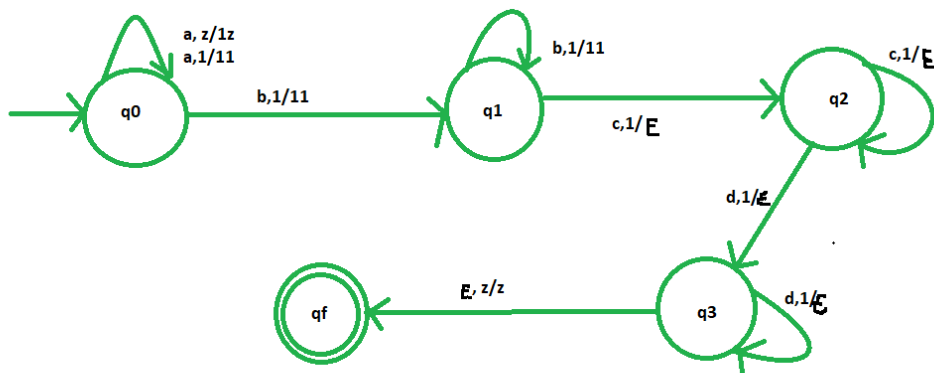


6. Design a non deterministic PDA for accepting the language $L = \{a^n b^m \mid n, m \geq 1 \text{ and } n \neq m\}$



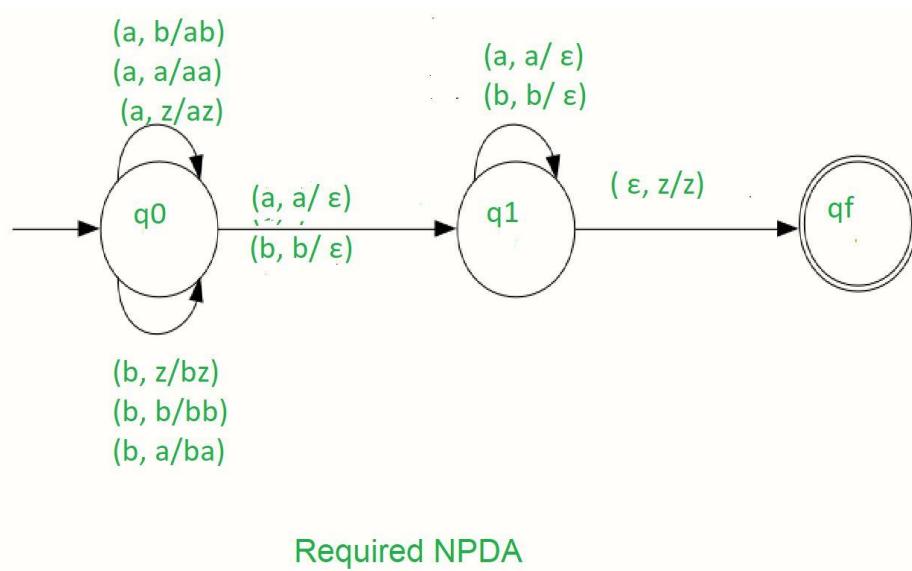
Required PDA

7. Design a non deterministic PDA for accepting the language $L = \{a^m b^n c^p d^q \mid m+n=p+q ; m,n,p,q \geq 1\}$



Required PDA

8. Design a non deterministic PDA for accepting the language $L = \{ww^R \mid w \in (a,b)^*\}$



UNIT-4

Syllabus: Turing Machines: Introduction, Formal definition of Turing machines, Instantaneous descriptions, Turing Machine as Acceptors, Problems related to turning machine as Acceptors, Turing Machine as a Computing Device, Problems related to turning Turing Machine as a Computing Device, Techniques for Turing Machine Construction, Considering the state as a tuple Considering the tape symbol as a tuple, Checking off symbols, Modifications of Turing Machine, Multi-tape Turing Machine, Non-Deterministic Turing Machine, Semi-Infinite Tape Turing Machine

1. The estimation of n if Turing machine is characterized utilizing n-tuples:

- a) 6 b) 8 c) 5 **d) 7**

2 Which of the statement is valid for the language $\{a^p \mid p \text{ is a prime}\}?$

- (a) It is not accepted by a Turing Machine
(b) It is regular but not context-free
(c) It is context-free but not regular
(d) It is neither regular nor context-free, but accepted by a Turing machine

3. Which of the following isn't right regarding potential results while executing a Turing Machine for a given input?

- a) it may halt and accept the input
b) it may halt by changing the input
c) it may halt and reject the input
d) It may never halt.

4. What is the purpose for a Turing machine is more impressive than FSM

- a) Turing machine head movement is continued to one direction.
b) Turing machine head moment is in both directions i.e. left moment and right moment as well.
c) Turing machine has capability remember arbitrary long sequence

of input string.

d) All are correct.

5. A Turing machine with a few tapes is known as

a) Multi-tape Turing machine

b) Poly-tape Turing machine

c) Universal Turing machine

d) Non Deterministic Turing Machine

6. A Multitape Turing machine is _____ powerful than a single tape Turing machine.

a) More b) less c) equal d) none of the mentioned.

7. Which of the following is not a Non deterministic Turing machine?

a) Alternating Turing machine

b) Probabilistic Turing machine

c) Read only Turing machine.

d) None of the mentioned.

8. Which of the following is true about Turing's a-machine?

a) a stands for automatic

b) left ended, right end-infinite

c) finite number of tape symbols were allowed

d) all of the mentioned.

9. According to Chomsky hierarchy, which of the following is adopted by Recursively Enumerable language?

a) Type 0 b) Type 1 c) Type 2 d) Type 3

10. An instantaneous description of Turing machine consists of

- a) Present state and input to be processed
- b) Present state and entire input to be processed**
- c) Present input only
- d) Previous State and input to be processed.

11. Let L_1 be a recursive language, and let L_2 be a recursively enumerable but not a recursive language. Which one of the following is TRUE?

L_1' --> Complement of L_1

L_2' --> Complement of L_2

- (A) L_1' is recursive and L_2' is recursively enumerable
- (B) L_1' is recursive and L_2' is not recursively enumerable**
- (C) L_1' and L_2' are recursively enumerable
- (D) L_1' is recursively enumerable and L_2' is recursive

12. Every language accepted by a k -tape TM is _____ by a single-tape TM.

- a) accepted**
- b) not accepted
- c) generated
- d) not generated

13. State true or false:

Statement: We can use the finite control of turing machine to hold a finite amount of data.

- | | | | |
|---------------|---------|----------|-------------|
| a.true | b.false | c.may be | d.can't say |
|---------------|---------|----------|-------------|

14. Which of the following statements is/are true?

- a) Every multitape Turing machine has its equivalent single tape Turing machine
- b) Every multitape Turing machine is an abstract machine

c) **Both a and b**

d) Multitape Turing machine is more powerful than Single tape Turing machine

15. In multi head Turing machine there are

a) **More than one heads of the Turing machine**

b) More than one input tapes of Turing machine

c) Similar to the basic model of Turing machine

d) All of these.

16. Which of the following is the restricted model of Turing machines

a) Turing machine with semi-infinite tape

b) Multi stack machine

c) Offline Turing machine

d) Both (a) and (b)

17. A pushdown automata behaves like a Turing machine, when it has number of memory.

a)0

b)1

c)exactly 2

d)2 or more.

18.A FSM can be converted to be a _____ of finite tape length without rewinding capability and unidirectional tape movement

a)**Turing Machine**

b)Push Down Automata

c)Context Free Language

d)Regular Language

19. Which of the accompanying proclamation isn't right?

a) **An Finite state Machine can be converted to be a Turing of finite tape length , rewinding capability and unidirectional tape movement.**

b) Palindromes can't be recognised by any FSM because an FSM can't deterministically fix the mid point.

- d) Palindromes can't be recognised by any FSM because even if the midpoint is known, an FSM can't find whether the second half of the string matches the first half.
- d) Palindromes can't be recognised by any FSM because an FSM can't remember arbitrarily large amount of information.

20.If Turing machine accepts all the words of the languages L and rejects or loops for other words which are not in L , then L is said to be

- a) **Recursive enumerable**
- b) Recursive
- c) Context Free Language
- d) Context Sensitive Language

PART-B

1.A single tape Turing Machine M has two states q_0 and q_1 , of which q_0 is the starting state. The tape alphabet of M is $\{0, 1, B\}$ and its input alphabet is $\{0, 1\}$. The symbol B is the blank symbol used to indicate end of an input string. The transition function of M is described in the following table

	0	1	B
q_0	$q_1, 1, R$	$q_1, 1, R$	Halt
q_1	$q_1, 1, R$	$q_0, 1, L$	q_0, B, L

. Which of the following statements is true about M ?

- a) **M does not halt on any string in $(0 + 1)^+$**
- b) M does not halt on any string in $(00 + 1)^*$
- c) M halts on all string ending in a 0
- d) M halts on all string ending in a 1

2.Given a TM $M = (\{q_0, q_1\}, \{0, 1\}, \{0, 1, B\}, \delta, B, \{q_1\})$

Where δ is a transition function defined as

$$\delta(q_0, 0) = (q_0, 0, R)$$

$$\delta(q_0, B) = (q_1, B, R)$$

which of the following string is accepted by this Turing Machine

- (i)001 (ii) 010 **(iii)000** (iv) None of these

3. Given a Turing Machine

$M = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \{a, b, B\}, \delta, B, \{q_3\})$

Where δ is a transition function defined as

$$\delta(q_0, a) = (q_1, a, R)$$

$$\delta(q_1, b) = (q_2, b, R)$$

$$\delta(q_2, a) = (q_2, a, R)$$

$$\delta(q_3, b) = (q_3, b, R)$$

The language $L(M)$ accepted by the Turing Machine is given as

(i) aa^*b

(ii) $abab$

(iii) aba^*b

(iv) aba^*

4. Given a TM $M = (\{q_0, q_1\}, \{0, 1\}, \{0, 1, B\}, \delta, B, \{q_1\})$

Where δ is a transition function defined as

$$\delta(q_0, 0) = (q_0, 0, R)$$

$$\delta(q_0, B) = (q_1, B, R)$$

which are of the following Regular Expression accepted by this TM represent the strings

(i) 01^* (ii) 000^* (iii) 10^* **(iv) 00^***

5. Given a Turing Machine $M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}, \{a, b\}, \{a, b, B\}, \delta, B, \{q_7\})$

Where δ is a transition function defined as

$$\delta(q_0, a) = (q_1, B, R)$$

$$\delta(q_0, b) = (q_2, B, R)$$

$$\delta(q_3, b) = (q_3, b, R)$$

$$\delta(q_0, B) = (q_7, B, R)$$

$$\delta(q_5, a) = (q_5, a, L)$$

$$\delta(q_5, b) = (q_5, b, L)$$

$$\delta(q_5, B) = (q_0, B, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, b) = (q_1, b, R)$$

$$\delta(q_1, B) = (q_3, B, L)$$

$$\delta(q_4, a) = (q_6, B, L)$$

$$\delta(q_6, a) = (q_6, a, L)$$

$$\delta(q_6, b) = (q_6, b, L)$$

$$\delta(q_6, B) = (q_0, B, R)$$

$$\delta(q_2, a) = (q_2, a, R)$$

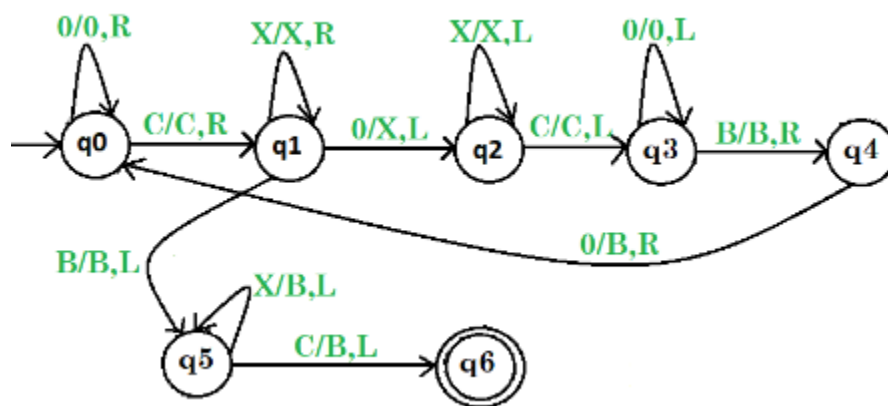
$$\delta(q_2, b) = (q_2, b, R)$$

$$\delta(q_2, B) = (q_4, B, L)$$

Which of the following language accepts?

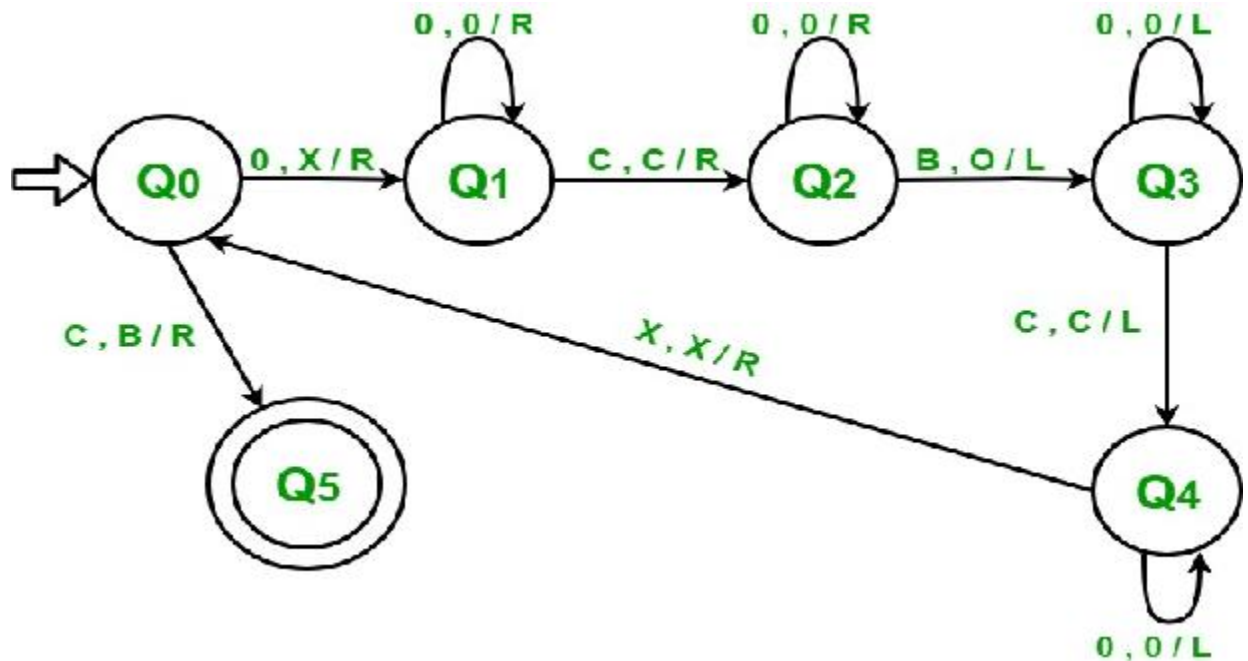
- length.
- a) It accepts language of palindrome over $\{a,b\}^*$ with each string of even length.
 - b) It accepts language of binary addition over $\{a,b\}^*$
 - c) It accepts language of palindrome over $\{a,b\}^*$ with each string of odd length.
 - d) It accepts language of binary subtraction over $\{a,b\}^*$ where $a > b$

6. Which of the accompanying Turing Machine resembles the given transition graph



- (i) Turing Machine for copying data
- (ii) Turing Machine for adding 2 numbers
- (iii) **Turing Machine for subtracting two numbers**
- (iv) Turing Machine for reverse a String

7. What does the following transition graph shows



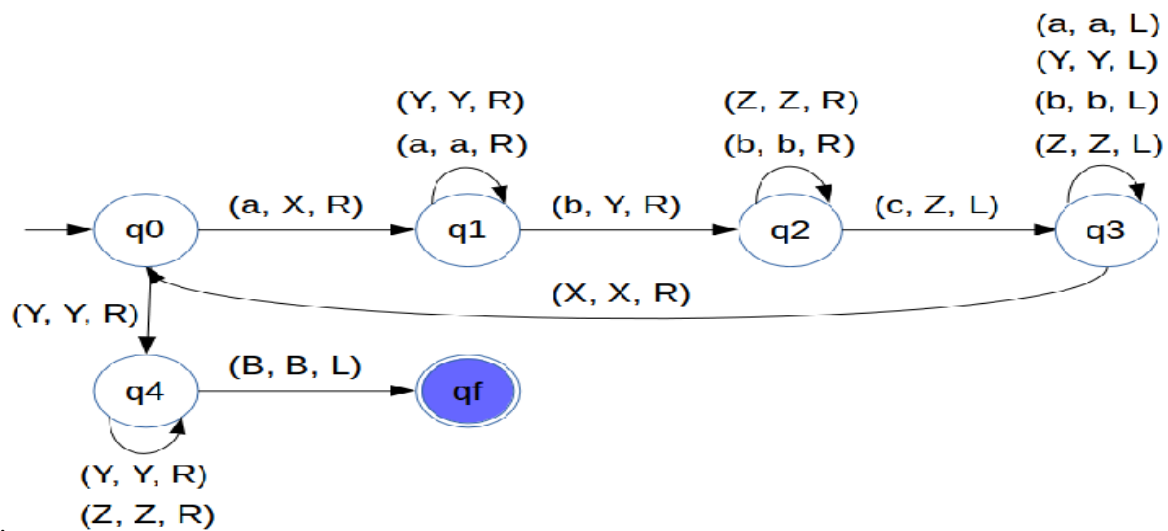
(i) Turing Machine for copying data

(ii) **Turing Machine for adding two unary numbers**

(iii) Turing Machine for subtracting two unary numbers

(iv) Turing Machine for reverse a String

8. What does the following state transition graph shows



(i) **Turing machine for $a^n b^n c^n \mid n \geq 1$**

(ii) **Turing machine for $a^n b^m c^n \mid n, m \geq 1$**

(iii) **Turing machine for $a^n b^n c^n \mid n \geq 2$**

(iv) **Turing machine for $a^n b^m c^n \mid n, m \geq 2$**

9. Here is a description of a TM $M = (\{q_0, q_1\}, \{0, 1\} \{0, 1, B\}, \delta, B, \{q_1\})$

The transition function δ is given below

$\delta(q_0, 1) = (q_0, 1, R)$

$\delta(q_0, B) = (q_1, 1, R)$

$\delta(q_1, B)$ is an accepting state

The following TM acts like

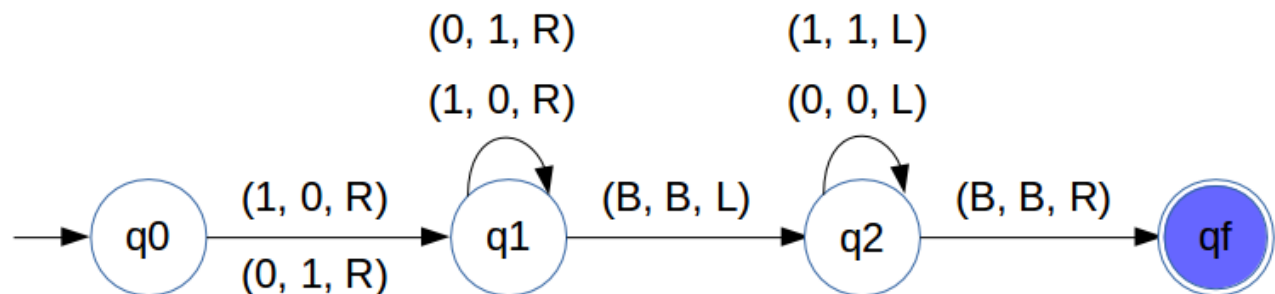
(i) copies a String

(ii) **Successor function of a unary number**

(iii) Reverse a string

(iv) Accepts a Palindrome.

10. Which of the accompanying articulation is right for the state transition diagram



(i) **Turing machine acts for 1's complement.**

- (ii) Turing machine acts comparator
- (iii) Turing machine acts for 2's complement.
- (iv) Turing machine accepts as a Palindrome.

PART C

1. The Turing machine M is defined by the state diagram

δ	B	a	b	c
q_0	q_1, B, R			
q_1	q_2, B, L	q_1, a, R	q_1, c, R	q_1, c, R
q_2		q_2, c, L		q_2, b, L

(1.1) The computational result for the input string "aabca"

- (i) $q_2 B c c b b c B$ (ii) $q_2 B c b c b c B$ (iii) $q_2 B a c c b a B$ (iv) $q_2 B b b a a b B$

(1.2) The computational result for the input string "bcbcb"

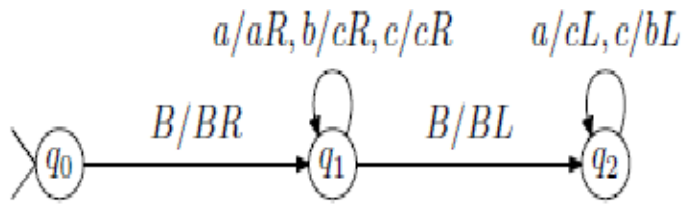
- (i) $q_2 B b b b b B$ (ii) $q_2 B c b c b B$ (iii) $q_2 B c c b b B$ (iv) $q_2 B b b c b B$

(1.3). Describe the result of above computation in M

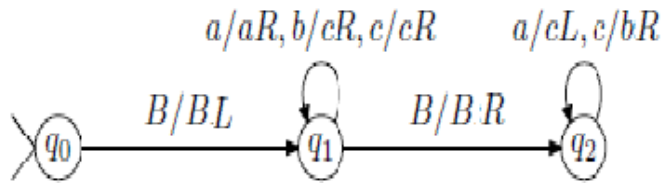
- (i) substitute the a's in the input string with c's and the c's with b's.
- (ii) substitute the a's in the input string with b's and the b's with a's.
- (iii) substitute both b's and c's in the input string with a's
- (iv) substitute the b's in the input string with a's and the c's with b's.

(1.4) The state diagram for the machine M

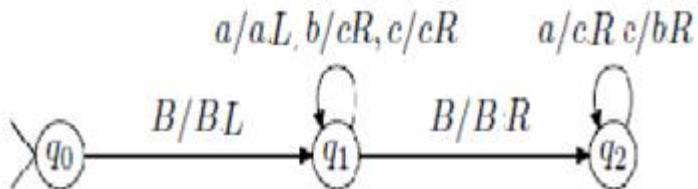
- (i)



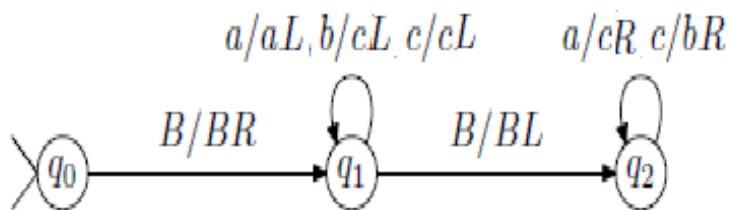
(ii)



(iii)



(iv)

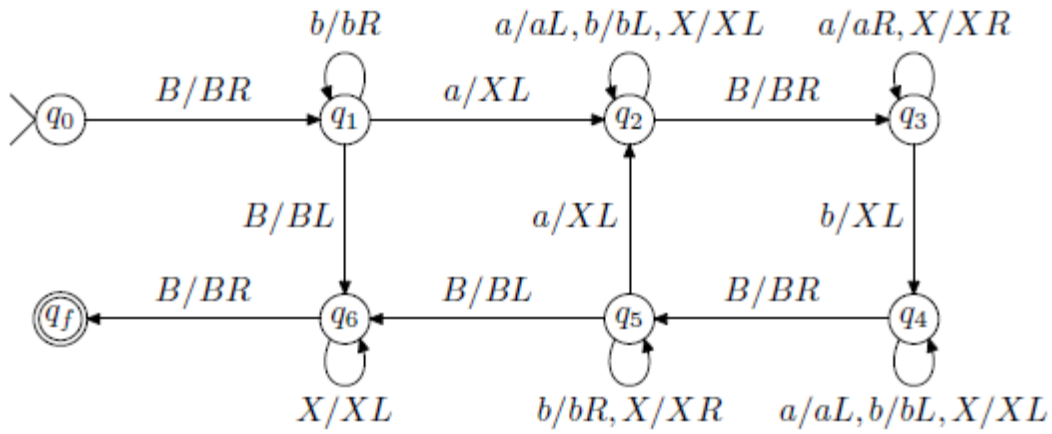


Answer:option(i)

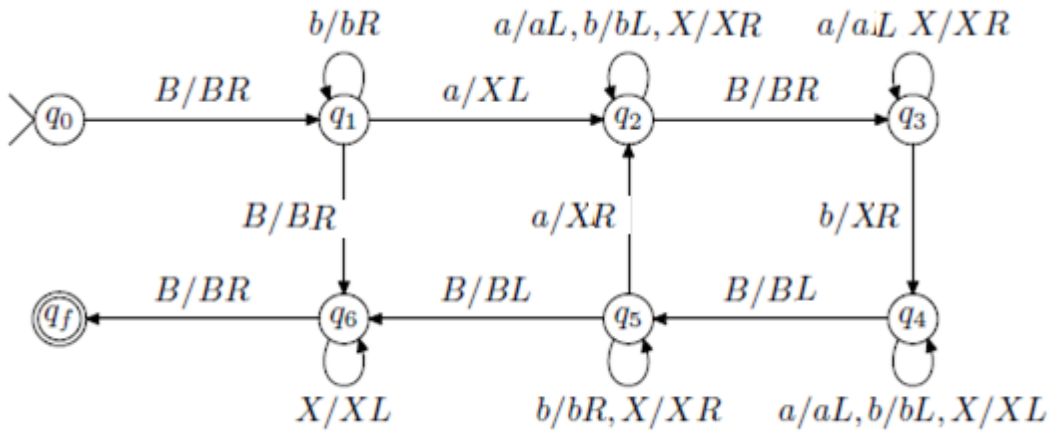
2. Identify the correct Turing machine with input alphabet $\{a, b\}$ to accept for the **Strings**

with the equivalent number of a's and b's

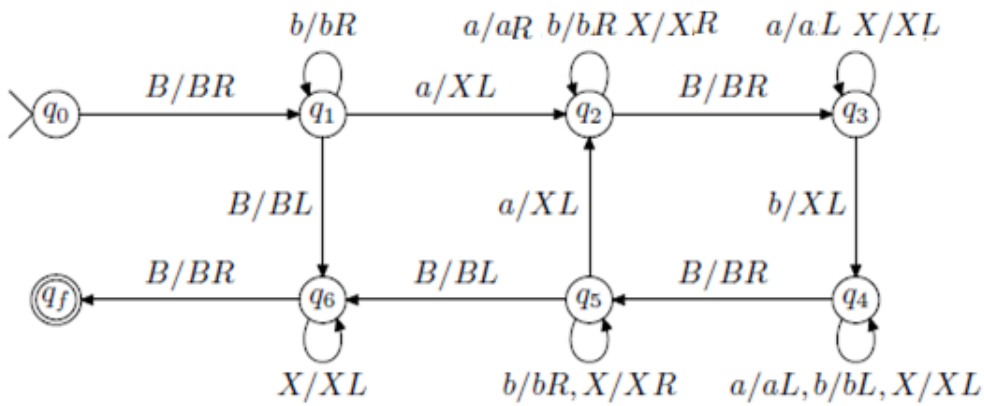
(i)



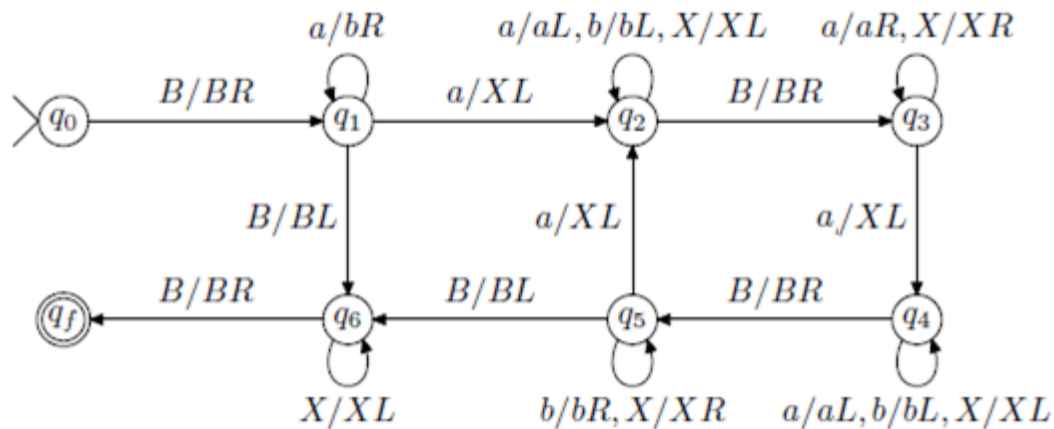
(ii)



(iii)



(iv)

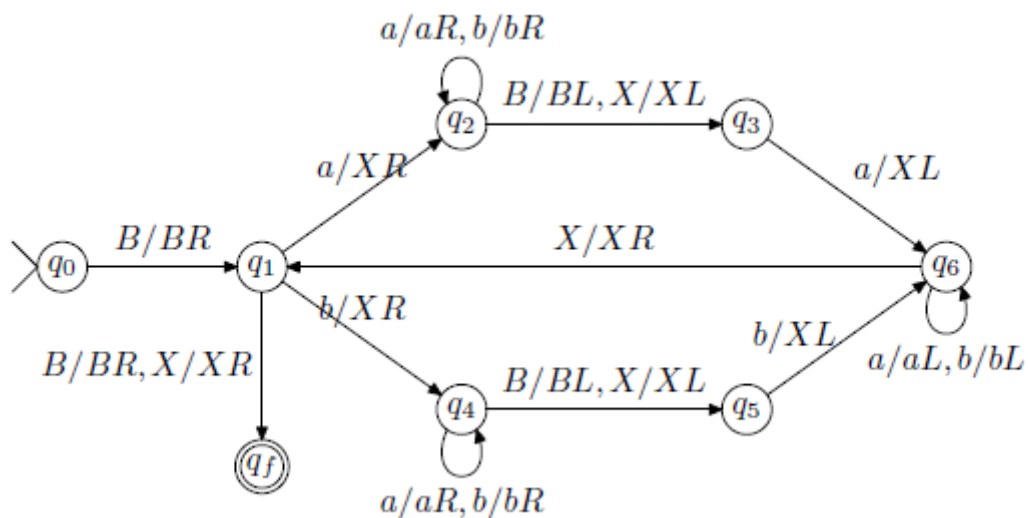


Answer: Option (iv)

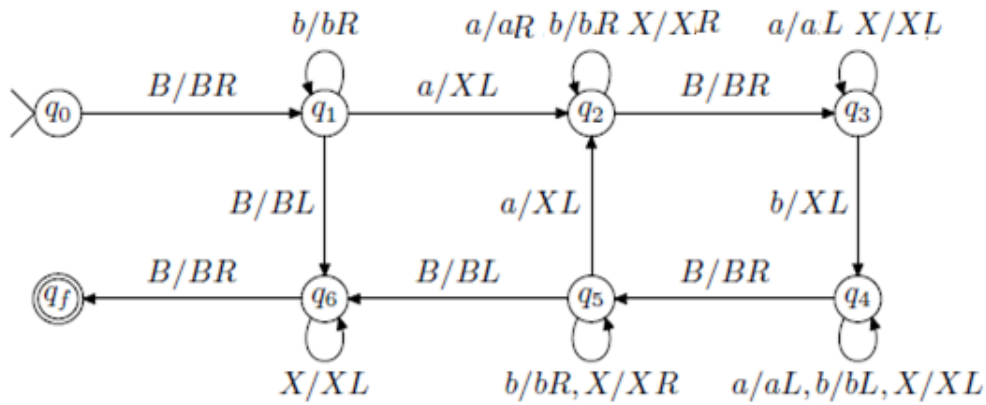
3. Identify the correct Turing machine with input alphabet $\{a, b\}$ to accept for the following languages by final state

$\{uu^R \mid u \in \{a, b\}^*\}$

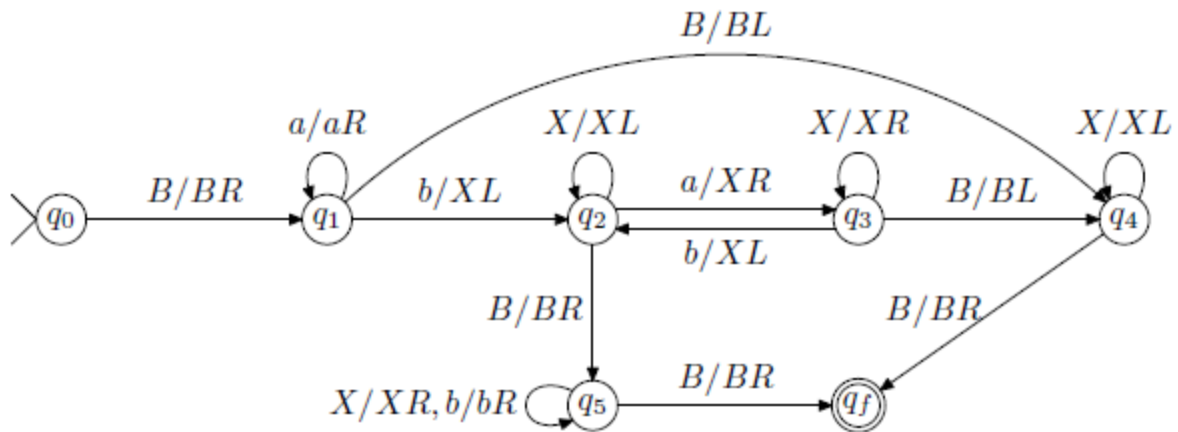
(i)



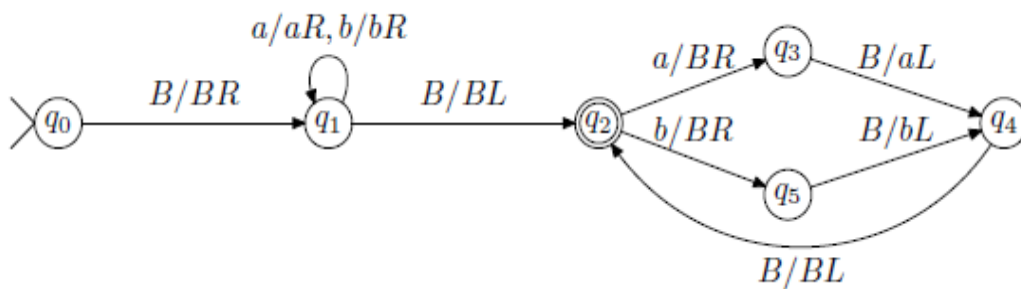
(ii)



(iii)



(iv)



Answer: option(i)

4. The Turing machine described in the below Transition table as

δ	B	a	b	c
q_0	q_1, B, R			
q_1	q_1, B, R	q_1, a, R	q_1, b, R	q_2, c, L
q_2		q_2, b, L	q_2, a, L	

(4.1) The result for the first six state transitions of the input string “abab”

(i) **BababBq1**

(ii) Babbq2aB

(iii) Baaq1abB

(iv) BababBq2

(4.2) The computation for the input string “abcb”.

(i) bbcab

(ii) acbcb

(iii) ccacb

(iv) **bacab**

(4.3) Narrate the result of a computation in M.

(i) Delete the a’s before the first c with b’s and the b’s before the first c with a’s.

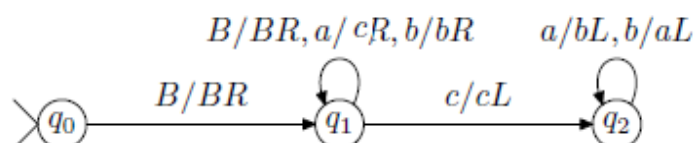
(ii) **Replace the a’s before the first c with b’s and the b’s before the first c with a’s.**

(iii) Erase the a’s before the first c with b’s and the b’s before the first c with a’s.

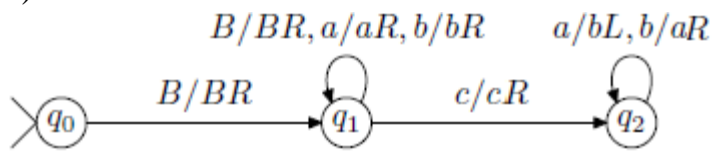
(iv) Insert the a’s before the first c with b’s and the b’s before the first c with a’s.

(4.4) Give the state diagram of the given TM.

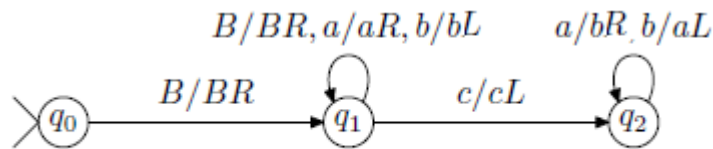
(i)



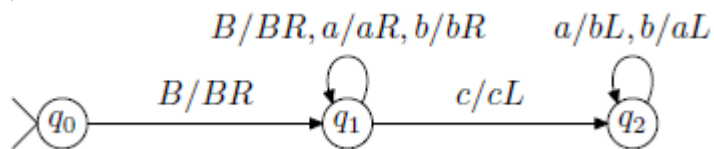
(ii)



(iii)

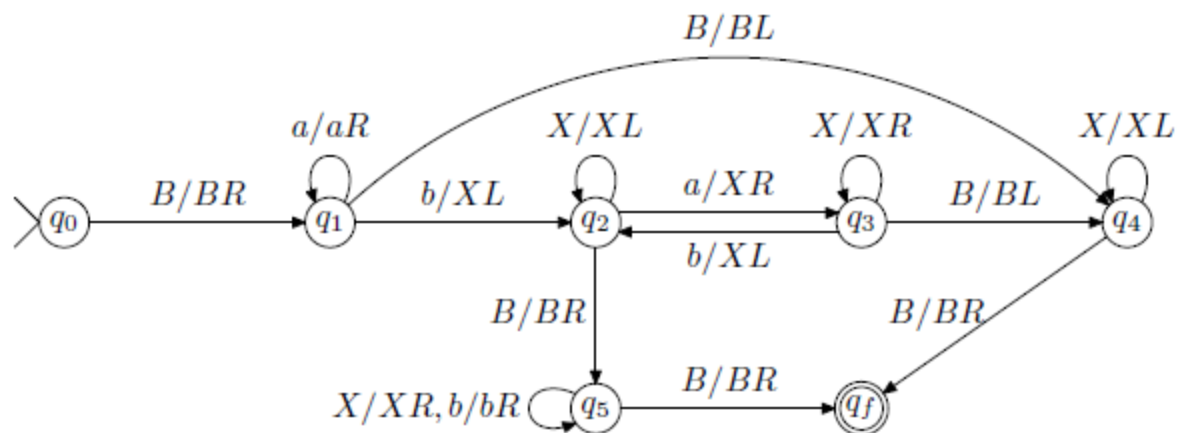


(iv)



Answer:option(iv)

5.What does the following Transition diagram with input alphabet $\{a,b\}$ shows



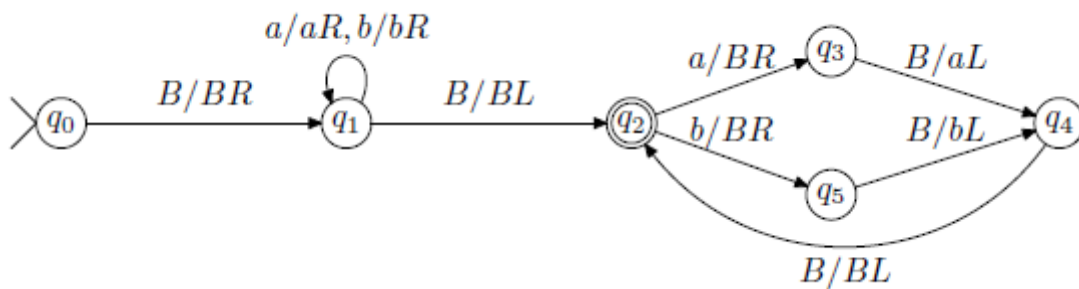
(i) $\{a^i b^j \mid i \geq 0, j \geq i\}$

(ii) $\{a^i b^j a^i b^j \mid i, j > 0\}$

(iii) $\{uu \mid u \in \{a, b\}^*\}$

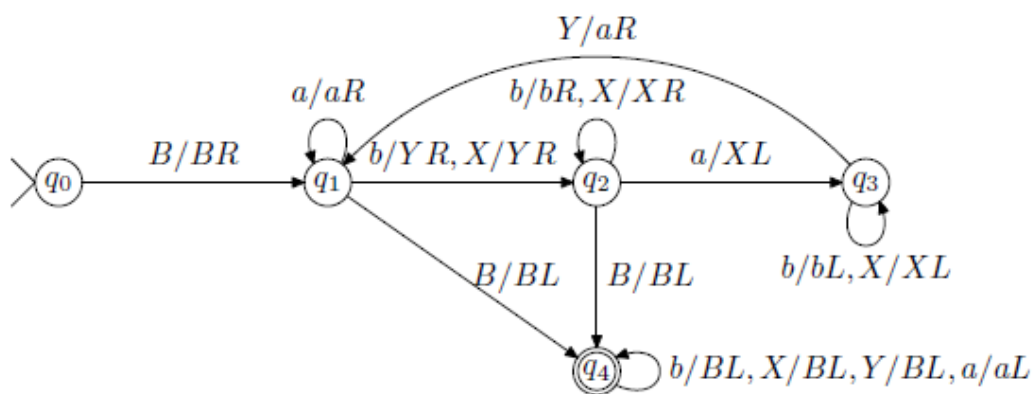
(iv) $\{ a^i b^j \mid i \geq 0, j \geq 0 \}$

(6.1) Given Turing machine M with input alphabet $\{a, b\}$ and input configuration **q0BuB** does perform which of the following operations. .



- (i) Shift the input one space to the left..
- (ii) It reverse the given input
- (iii) **Shift the input one space to the right**
- (iv) The input remains same.

(6.2) Given Turing machine M with input alphabet $\{a, b\}$ and initial configuration **q0BbabaababB** does perform which of the following operations



- (i) Erase the a's from the input
- (ii) Insert .one a's in the input
- (iii) Include .one b's in the input
- Erase the b's from the input**

UNIT-5

***Syllabus:** Undecidability Basic definitions, Decidable problems, Examples of undecidable problems and Problems, Rice's Theorem, Undecidable problems about Turing Machine- Post's Correspondence Problem, Problems related to Post's Correspondence Problem, Properties of Recursive and Recursively enumerable languages, Introduction to Computational Complexity: Definitions, Time and Space complexity of TMs, Complexity classes: Class P, Class NP, Complexity classes: Introduction to NP- Hardness, NP Completeness*

PART-A

1. Diagonalization can be useful in:
 - a) To find a non recursively enumerable language
 - b) To prove undecidability of halting problem
 - c) Both (a) and (b)**
 - d) None of the mentioned

2. If a problem has an algorithm to answer it, we call it _____
a) decidable
b) solved
c) recognizable
d) none of the mentioned
3. Which of the following is true for The Halting problem?
a) It is recursively enumerable
b) It is undecidable
c) Both (a) and (b)
d) None of the mentioned
4. A language L is said to be _____ if there is a turing machine M such that $L(M)=L$ and M halts at every point
a) Turing acceptable
b) decidable
c) undecidable
d) none of the mentioned
5. The language accepted by a turing machine is called _____
a) Recursive Enumerable
b) Recursive
c) Both (a) and (b)
d) None of the mentioned
6. The problems which have no algorithm, regardless of whether or not they are accepted by a turing machine that fails to halts on some input are referred as:
a) Decidable
b) Undecidable
c) Computable
d) None of the mentioned
7. An algorithm is called efficient if it runs in _____ time on a serial computer
a) polynomial
b) non polynomial

- c) logarithmic
 - d) none of the mentioned
8. A problem is called _____ if its has an efficient algorithm for itself
- a) tractable**
 - b) intractable
 - c) computational
 - d) none of the mentioned
9. According to the rice's theorem, If P is a non trivial property, Lp is :
- a) infinite
 - b) decidable
 - c) undecidable**
 - d) none of the mentioned
10. Which of the following statements are undecidable?
- For a given Turing Machine M,
- a) does M halt on an empty input tape
 - b) does M halt for any inputs at all?
 - c) is L(M) regular? Context free? Turing decidable?
 - d) all of the mentioned**
11. Post Correspondence problem is
- a) decidable decision problem
 - b) undecidable decision problem**
 - c) not a decision problem
 - d) none of the mentioned
12. PCP stands for?
- a) Post Correspondence Problem**
 - b) Post Corresponding Problem
 - c) Pre Correspondence problem
 - d) None of the mentioned

13. Can a Modified PCP problem be reduced to PCP?

- a) **yes**
- b) no

14. The complexity class P consist of all the decision problems that can be solved by _____ using polynomial amount of computation time

- a) Push Down automata
- b) DFA
- c) NDFA
- d) **Deterministic Turing machine**

15 What does NP stands for in complexity classes theory?

- a) Non polynomial
- b) **Non-deterministic polynomial**
- c) Both (a) and (b)
- d) None of the mentioned

16. The hardest of NP problems can be:

- a) **NP-complete**
- b) NP-hard
- c) P
- d) None of the mentioned

17. Travelling sales man problem belongs to which of the class?

- a) P
- b) **NP**
- c) Linear
- d) None of the mentioned

18. A problem which is both _____ and _____ is said to be NP complete

- a) **NP, P**
- b) NP, NP hard
- c) P, P complete
- d) None of the mentioned

19. Which of the following does not belong to the closure properties of NP class?

- a) Union

- b) Concatenation
- c) Reversal**
- d) Complement

20. A generalization of P class can be:

- a) PTIME
- b) DTIME
- c) NP**
- d) None of the mentioned

PART-B

1 Let $\langle M \rangle$ be the encoding of a Turing machine as a string over $\Sigma = \{0, 1\}$. Let $L = \{ \langle M \rangle \mid M \text{ is a Turing machine that accepts a string of length } 2014 \}$. Then, L is

- (a) Decidable and recursively enumerable
- (b) undecidable but recursively enumerable**
- (c) undecidable and not recursively enumerable
- (d) decidable but not recursively enumerable

Explanation:

There are finite number of strings of length '2014'. So, a Turing machine will take the input string of length '2014' and test it.

If, input string is present in the language then Turing machine will halt in final state.

But, if Turing machine is unable to accept the input string then it will halt in non-final state or go in an infinite loop and never halt.

Thus, ' L ' is undecidable and recursively enumerable.

2. Consider the following statements:

1. The complement of every Turing decidable language is Turing decidable.
2. There exists some language which is in NP but is not Turing decidable.
3. If L is a language in NP, L is Turing decidable.

Which of the above statements is/are True?

- (a) Only 2
- (b) Only 3
- (c) Only 1 and 2
- (d) Only 1 and 3**

Explanation:

1 is true: Complement of Turing decidable is Turing Decidable

3 is true: All NP problems are Turing decidable

2 is false: The definition of NP itself says solvable in Polynomial time using non-deterministic Turing machine

3. Which of the following statements are TRUE?

- (1) The problem of determining whether there exists a cycle in an undirected graph is in P
- (2) The problem of determining whether there exists a cycle in an undirected graph is in NP
- (3) If a problem A is NP-Complete, there exists a non-deterministic polynomial time algorithm to solve A

- (a) 1, 2 and 3**
- (b) 1 and 3
- (c) 2 and 3
- (d) 1 and 2

Explanation:

1 is true because cycle detection can be done in polynomial time using DFS

2 is true because P is a subset of NP

3 is true because NP complete is also a subset of NP and NP means Non-Deterministic Polynomial time solution exists

4. Let S be an NP-complete problem and Q and R be two other problems not known to be in NP. Q is polynomial time reducible to S and S is polynomial-time reducible to R. Which one of the following statements is true?

- (a) R is NP-complete
- (b) R is NP-hard**
- (c) Q is NP-complete
- (d) Q is NP-hard

Explanation:

- (a) Incorrect because R is not in NP. A NP Complete problem has to be in both NP and NP-hard
- (b) correct because a NP Complete problem S is polynomial time reducible to R
- (c) Incorrect because Q is not in NP
- (d) Incorrect because there is no NP-complete problem that is polynomial time Turing-reducible to Q

5. Assuming $P \neq NP$, which of the following is true?

- (a) NP-complete = NP
- (b) $NP\text{-complete} \cap P = \phi$
- (c) NP-hard = NP
- (d) $P = NP\text{-complete}$

- (a) A
- (b) B**
- (c) C
- (d) D

Explanation:

The answer is B (no NP-Complete problem can be solved in polynomial time). Because, if one NP-Complete problem can be solved in polynomial time, then all NP problems can be solved in polynomial time. If that is the case, then NP and P set become same which contradicts the given condition.

6. A problem X belongs to P complexity class if there exist _____ algorithm to solve that problem, such that the number of steps of the algorithm is bounded by a polynomial in n, where n is the length of the input.

- (a) 1
- (b) 2
- (c) 3
- (d) all of the mentioned**

Explanation:

A problem X belongs to P complexity class if there exist at least 1 algorithm to solve that problem, such that the number of steps of the algorithms bounded by a polynomial in n, where n is the length of the input. Thus, all the options are correct

7. Fill in the blank with reference to Rice's theorem

for any non-trivial property of _____ no general or effective method can decide whether an algorithm computes it with that property

(a) Partial functions

(b) piecewise functions

(c) both (a) and (b)

(d) none of the mentioned

Explanation:

A property of partial functions is called trivial if it holds for all partial computable functions or for none, and an effective decision method is called general if it decides correctly for every algorithm

8. Which of the following set of computable functions are decidable?

(a) The class of computable functions that are constant, and its complement

(b) The class of indices for computable functions that are total

(c) The class of indices for recursively enumerable sets that are co finite

(d) All of the mentioned

Explanation:

According to Rice's theorem, if there exists at least one computable function in a particular class C of computable functions and another computable function not in C then the problem deciding whether a particular program computes a function in C is undecidable

9 Which of the following statements is/are FALSE?

1. For every non-deterministic TM, there exists an equivalent deterministic TM

2. Turing recognizable languages are closed under union and complementation

3. Turing decidable languages are closed under intersection and complementation

4. Turing recognizable languages are closed under union and intersection

- (a) 1 and 4
- (b) 1 and 3
- (c) 2**
- (d) 3

Explanation:

Statement 1 is true as we can convert every non-deterministic TM to deterministic TM

Statement 2 is false as Turing recognizable languages (RE languages) are not closed under complementation

Statement 3 is true as Turing decidable languages (REC languages) are closed under intersection and complementation

Statement 4 is true as Turing recognizable languages (RE languages) are closed under union and intersection

10. State true or false?

Statement: If a problem X is in NP and a polynomial time algorithm for X could also be used to solve problem Y in polynomial time, then Y is also in NP

- (a) True**
- (b) False

Explanation:

This is just a commutative property of NP complexity class where a problem is said to be in NP if it can be solved using an algorithm which was used to solve another NP problem in Polynomial amount of time

PART-C

1. If the number of steps required to solve a problem is $O(n^k)$, then the problem is said to be solved in:

- (a) Non-polynomial time
- (b) polynomial time**
- (c) infinite time
- (d) none of the mentioned

Explanation:

Most of the operations like addition, subtraction, etc as well as computing functions including powers, square roots and logarithms can be performed in polynomial time. In the given question, n is the complexity of the input and k is some non negative integer.

2. Consider three decision problems P_1 , P_2 and P_3 . It is known that P_1 is decidable and P_2 is undecidable. Which one of the following is TRUE?

- (a) P_3 is undecidable if P_2 is reducible to P_3
- (b) P_3 is decidable if P_3 is reducible to P_2 's complement
- (c) P_3 is undecidable if P_3 is reducible to P_2
- (d) P_3 is decidable if P_1 is reducible to P_3

Explanation:

- Option A says $P_2 \leq P_3$. According to theorem 2 discussed, if P_2 is undecidable then P_3 is undecidable. It is given that P_2 is undecidable, so P_3 will also be undecidable. So option (A) is correct.
- Option C says $P_3 \leq P_2$. According to theorem 2 discussed, if P_3 is undecidable then P_2 is undecidable. But it is not given in question about undecidability of P_3 . So option (C) is not correct.
- Option D says $P_1 \leq P_3$. According to theorem 1 discussed, if P_3 is decidable then P_1 is also decidable. But it is not given in question about decidability of P_3 . So option (D) is not correct.
- Option (B) says $P_3 \leq P_2'$. According to theorem 2 discussed, if P_3 is undecidable then P_2' is undecidable. But it is not given in question about undecidability of P_3 . So option (B) is not correct.

3. Consider two languages L_1 and L_2 each on the alphabet Σ . Let $f: \Sigma \rightarrow \Sigma$ be a polynomial time computable bijection such that $(\forall x) [x \in L_1 \text{ iff } f(x) \in L_2]$. Further, let f^{-1} be also polynomial time computable. Which of the following CANNOT be true?

- (a) $L_1 \in P$ and L_2 is finite
- (b) $L_1 \in NP$ and $L_2 \in P$

- (c) **L1 is undecidable and L2 is decidable**
(d) L1 is recursively enumerable and L2 is recursive

Explanation:

We have one to one mapping for all instances of L1 to L2

L1 is given to be undecidable . Further L1 is polynomial time reducible to L2 (By given mapping)
Now if L2 is decidable then there is algorithm to solve L2 in polytime. But then we can solve every instance of L1 in polytime, making L1 also decidable Contradiction

4. Let L1 be a recursive language, and let L2 be a recursively enumerable but not a recursive language which one of the following is TRUE?

- (a) **L1' is recursive and L2' is recursively enumerable**
(b) L1' is recursive and L2' is not recursively enumerable
(c) L1' and L2' are recursively enumerable
(d) L1' is recursively enumerable and L2' is recursive

Explanation:

Option A is False as L2' can't be recursive enumerable (L2 is RE and RE are not closed under complementation)

Option B is correct as L1' is REC (REC languages are closed under complementation) and L2' is not recursive enumerable (RE languages are not closed under complementation)

Option C is False as L2' can't be recursive enumerable (L2 is RE and RE are not closed under complementation)

Option D is False as L2' can't be recursive enumerable (L2 is RE and RE languages are not closed under complementation) As REC languages are subset of RE, L2' can't be REC as well

5. The problem 3-SAT and 2-SAT are

- (a) both in P
(b) both NP complete
(c) **NP-complete and in P respectively**
(d) undecidable and NP-complete respectively

Explanation:

The Boolean satisfiability problem (SAT) is a decision problem, whose instance is a Boolean expression written using only AND, OR, NOT, variables, and parentheses The problem is: given the expression, is there some assignment of TRUE and FALSE values to the variables that will

make the entire expression true? A formula of propositional logic is said to be satisfiable if logical values can be assigned to its variables in a way that makes the formula true

3-SAT and 2-SAT are special cases of k-satisfiability (k-SAT) or simply satisfiability (SAT), when each clause contains exactly $k = 3$ and $k = 2$ literals respectively 2-SAT is P while 3-SAT is NP Complete

6. Which of the following is/are undecidable?

1. G is a CFG. Is $L(G) = \phi$?
 2. G is a CFG. Is $L(G) = \epsilon^*$?
 3. M is a Turing Machine. Is $L(M)$ is regular?
 4. A is a DFA and N is an NFA. Is $L(A) = L(N)$?
-
- a) 3 only
 - b) 3 and 4 only
 - c) 1,2, and 3 only
 - d) 2 and 3 only**

Explanation:

First is Emptiness for CFG; whether a CFG is empty or not, this problem is decidable

Second is everything for CFC; whether a CFG will generate all possible strings(completeness of CFG), this problem is undecidable

Third is Regularity for REC; whether language generated by TM is regular is undecidable

Fourth is equivalence for regular; whether language generated by DFA and NFA are same is decidable. Second and third will be undecidable. Hence, option(D) is correct.