# ENM 5310: Data-driven Modeling and Probabilistic Scientific Computing

## *Lecture #8: Sampling methods*

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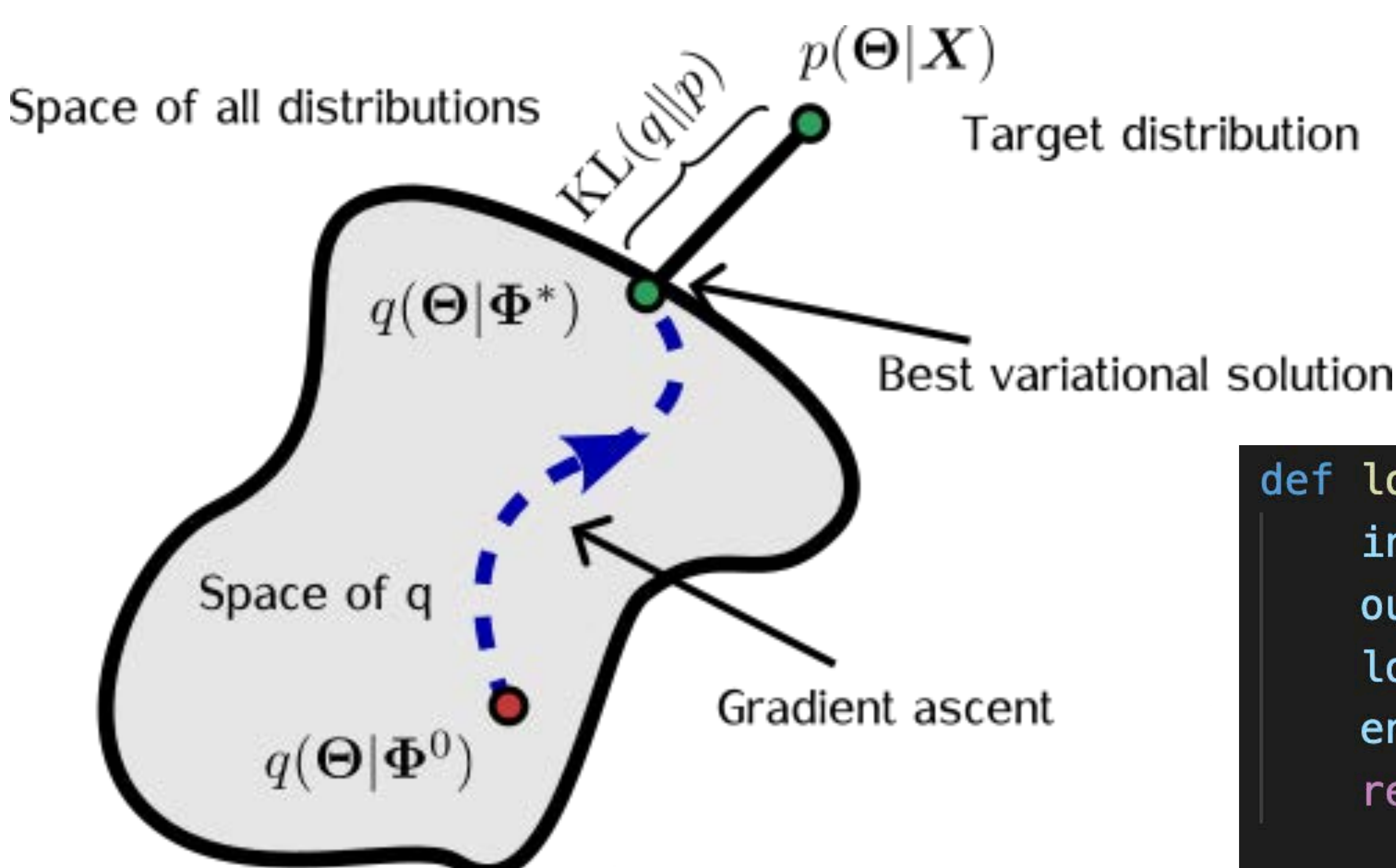
# Variational inference

$$p(\theta|\mathcal{D}) = \frac{p(\mathcal{D}|\theta)p(\theta)}{p(\mathcal{D})} = \frac{p(\mathcal{D}|\theta)p(\theta)}{\int p(\mathcal{D}|\theta)p(\theta)d\theta}$$

Idea: $p(\theta|\mathcal{D}) \approx q_\phi(\theta|\mathcal{D})$




```python
def loss(params, batch):
    inputs, targets = batch
    outputs = model(params, inputs)
    log_likelihood = jnp.mean((targets-outputs)**2)
    entropy = H(params)
    return -entropy-log_likelihood

grads = grad(loss)
```

# Sampling methods

$$p(\theta|\mathcal{D}) = \frac{p(\mathcal{D}|\theta)p(\theta)}{p(\mathcal{D})} = \frac{p(\mathcal{D}|\theta)p(\theta)}{\int p(\mathcal{D}|\theta)p(\theta)d\theta}$$

**Idea:** $$\mathbb{E}_{\theta \sim p(\theta|\mathcal{D})}[\log p(\mathcal{D}|\theta)] \approx \frac{1}{S}\sum_{i=1}^{S} \log p(\mathcal{D}|\theta_i), \qquad \theta_i \overset{\text{iid}}{\sim} p(\theta|\mathcal{D})$$

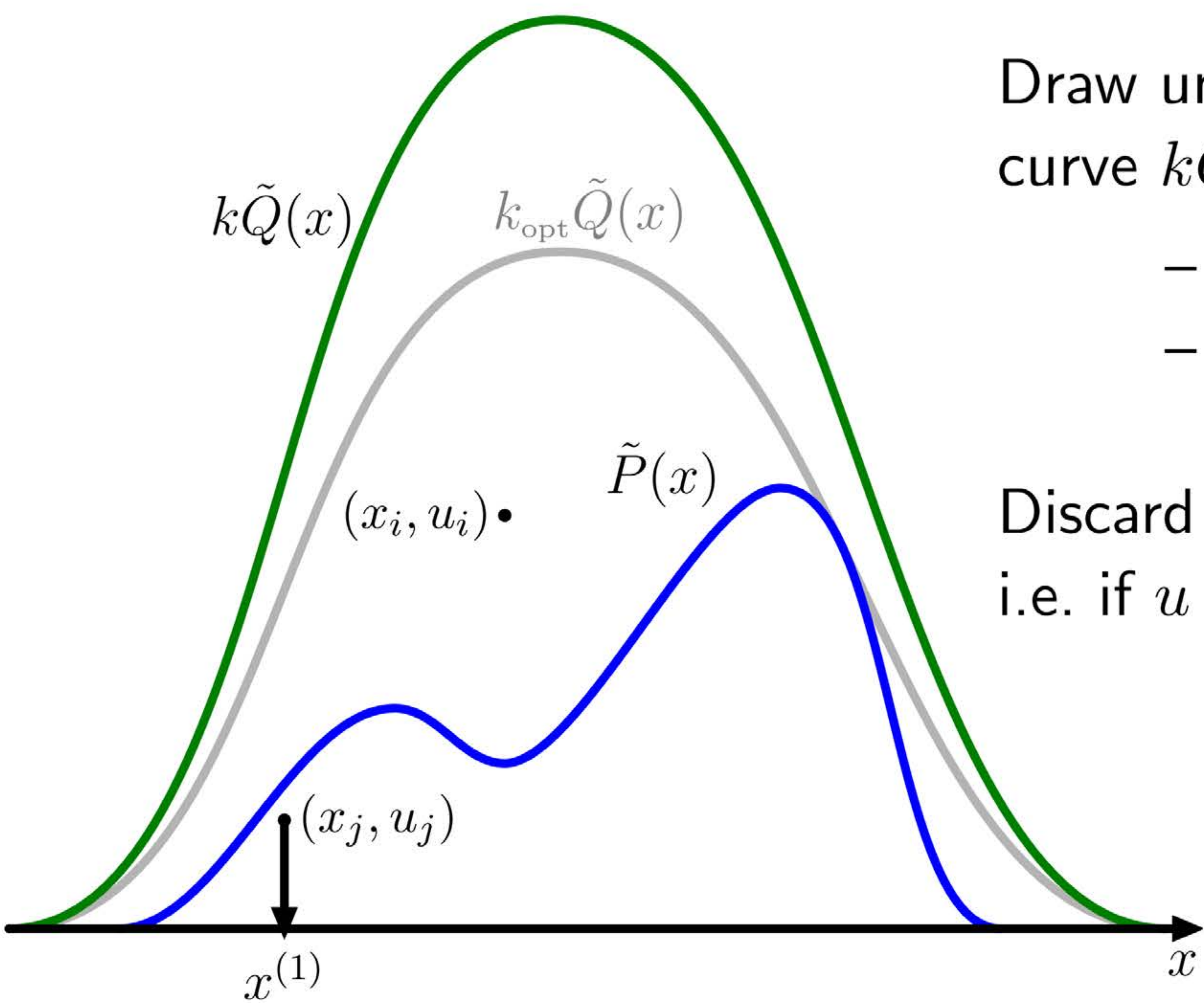
# Monte Carlo approximation

$$\mathbb{E}_{x \sim p(x)}[f(x)] = \int f(x)p(x)dx \approx \frac{1}{n}\sum_{i=1}^{n} f(x_i),$$

where $x_i$ are drawn iid from $p(x)$

# Rejection sampling

Sampling underneath a $\tilde{P}(x) \propto P(x)$ curve is also valid




Draw underneath a simple curve $k\tilde{Q}(x) \geq \tilde{P}(x)$:

- Draw $x \sim Q(x)$
- height $u \sim$ Uniform$[0, k\tilde{Q}(x)]$

Discard the point if above $\tilde{P}$, i.e. if $u > \tilde{P}(x)$