ENM 5310: Data-driven Modeling and Probabilistic Scientific Computing

Lecture #8: Sampling methods

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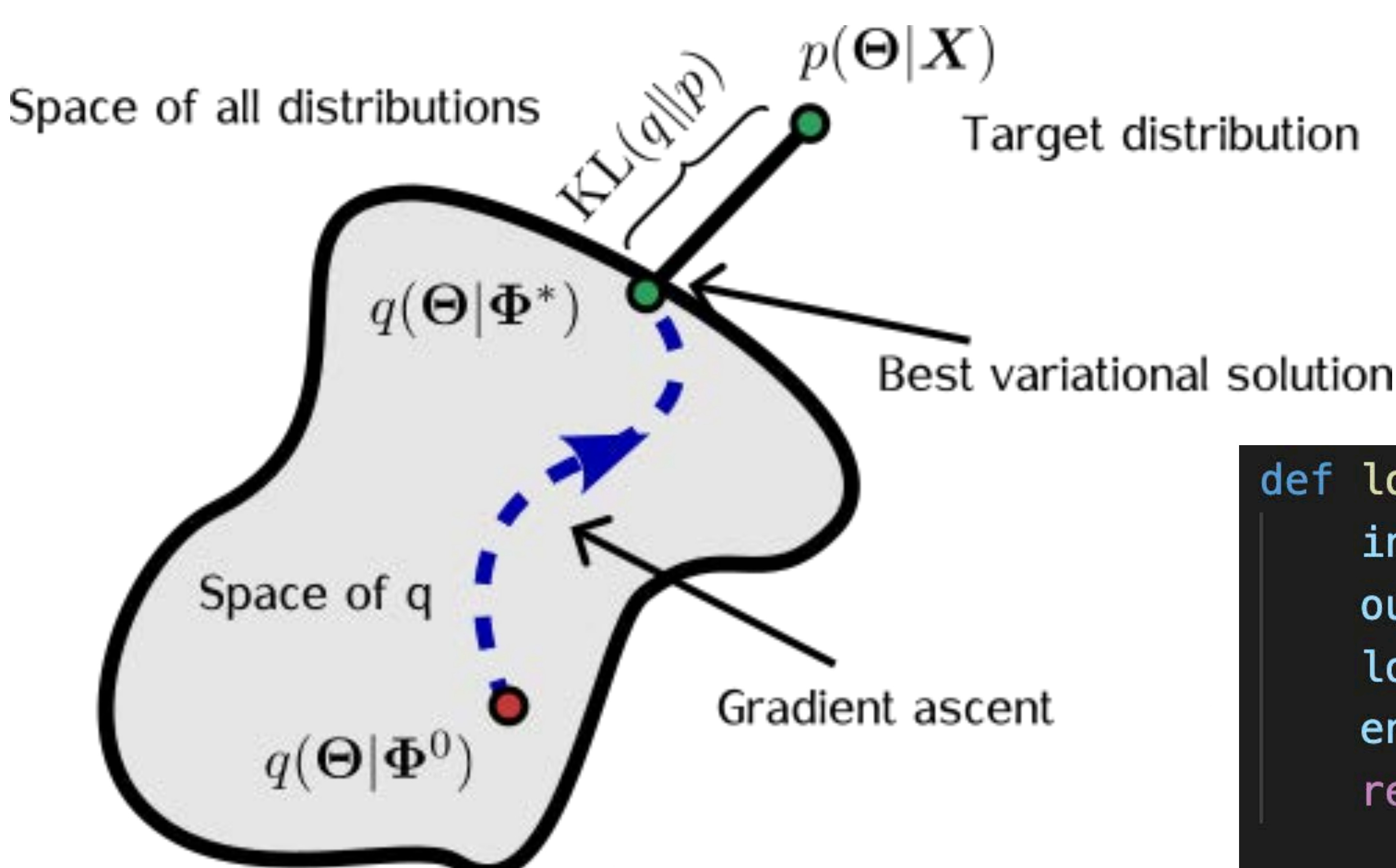
Variational inference

$$p(\theta|\mathcal{D}) = \frac{p(\mathcal{D}|\theta)p(\theta)}{p(\mathcal{D})} = \frac{p(\mathcal{D}|\theta)p(\theta)}{\int p(\mathcal{D}|\theta)p(\theta)d\theta}$$

Idea: $p(\theta|\mathcal{D}) \approx q_\phi(\theta|\mathcal{D})$




```
def loss(params, batch):
    inputs, targets = batch
    outputs = model(params, inputs)
    log_likelihood = jnp.mean((targets-outputs)**2)
    entropy = H(params)
    return -entropy-log_likelihood

grads = grad(loss)
```

Sampling methods

$$p(\theta|\mathcal{D}) = \frac{p(\mathcal{D}|\theta)p(\theta)}{p(\mathcal{D})} = \frac{p(\mathcal{D}|\theta)p(\theta)}{\int p(\mathcal{D}|\theta)p(\theta)d\theta}$$

Idea: $\mathbb{E}_{\theta \sim p(\theta|\mathcal{D})}[\log p(\mathcal{D}|\theta)] \approx \frac{1}{S}\sum_{i=1}^{S} \log p(\mathcal{D}|\theta_i), \qquad \theta_i \overset{\text{iid}}{\sim} p(\theta|\mathcal{D})$

Monte Carlo approximation

$$\mathbb{E}_{x \sim p(x)}[f(x)] = \int f(x)p(x)dx \approx \frac{1}{n}\sum_{i=1}^{n} f(x_i),$$

where x_i are drawn iid from $p(x)$