

ENM 360: Introduction to Data-driven Modeling

Lecture #2: Primer on Linear Algebra and Scientific Computing

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Lecture outline



Scientific computing



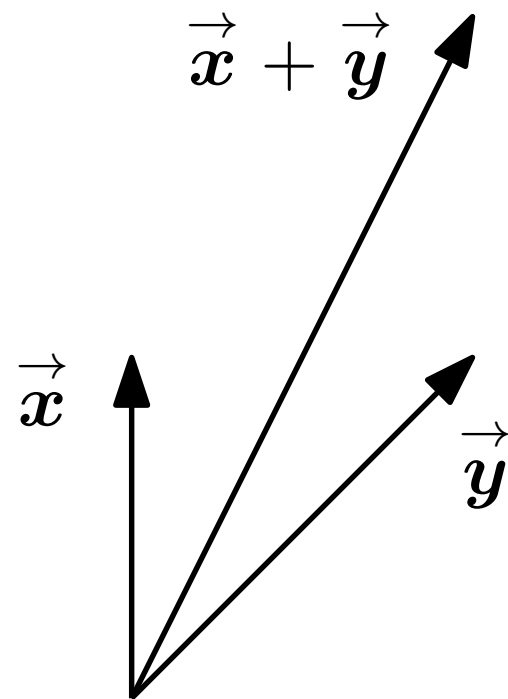
Linear algebra



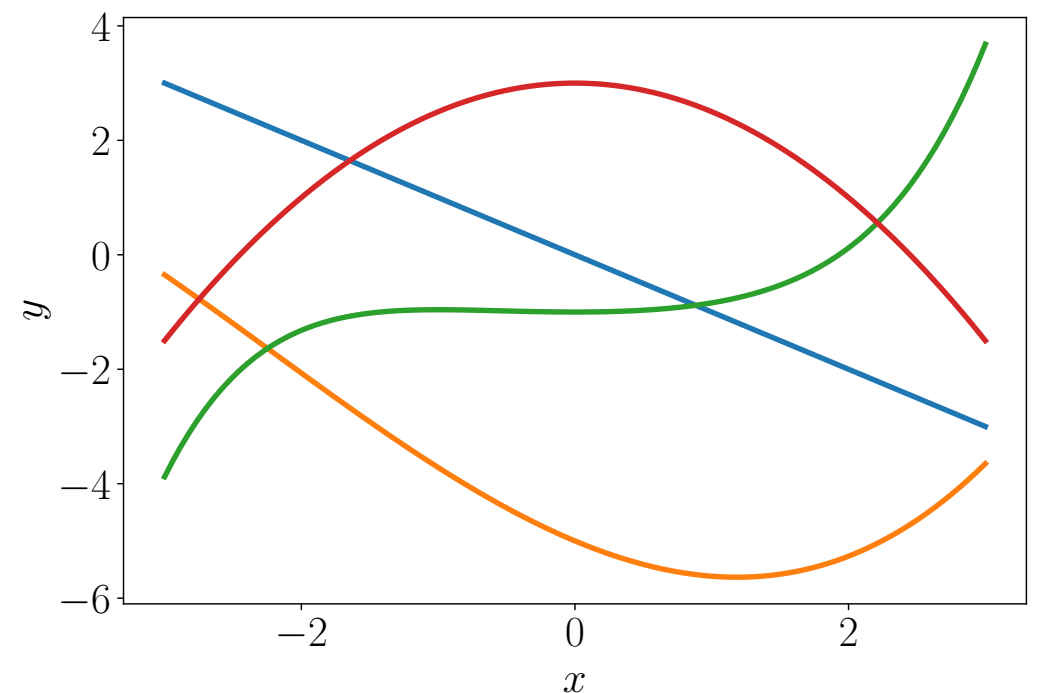
Matrix/vector calculus/operations

Vectors

- Basic definitions
- Vector operations (e.g. addition, subtraction, multiplication, etc.)
- Linear combinations
- Dot products
- Norms
- Vector/Linear spaces
- Linear (in)dependence
- Bases



(a) Geometric vectors.



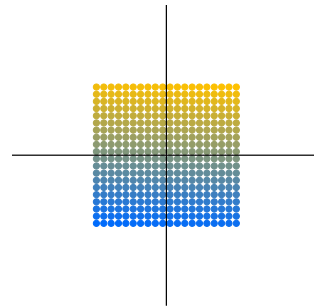
(b) Polynomials.

Useful resources:

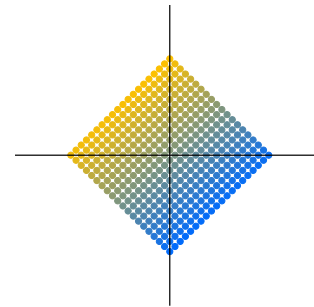
- Gilbert Strang's lectures at MIT OCW: <https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/video-lectures/>
- Pavel Grinfeld's series on linear algebra: <https://www.youtube.com/playlist?list=PLIXfTHzgMRUKXD88ldzSI4F4NxAZudSmv>

Matrices

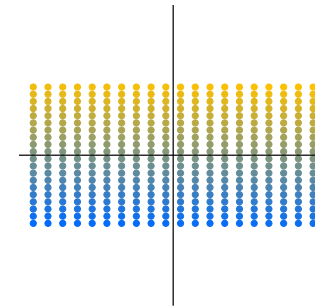
- Basic definitions
- Matrix operations (e.g. addition, subtraction, multiplication, etc.)
- Unit matrices, transposes, inverses
- Basic properties
- Norms
- Linear transformation of vectors
- Eigenvalues and eigenvectors
- Linear systems



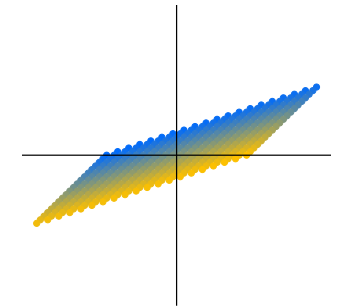
(a) Original data.



(b) Rotation by 45° .



(c) Stretch along the horizontal axis.



(d) General linear mapping.

$$\underbrace{\mathbf{A}}_{n \times k} \underbrace{\mathbf{B}}_{k \times m} = \underbrace{\mathbf{C}}_{n \times m}$$

For $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{2 \times 3}$, $\mathbf{B} = \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3 \times 2}$, we obtain

$$\mathbf{AB} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 5 \end{bmatrix} \in \mathbb{R}^{2 \times 2},$$

$$2x_1 + 3x_2 + 5x_3 = 1$$

$$4x_1 - 2x_2 - 7x_3 = 8$$

$$9x_1 + 5x_2 - 3x_3 = 2$$

$$\begin{bmatrix} 2 & 3 & 5 \\ 4 & -2 & -7 \\ 9 & 5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 2 \end{bmatrix}$$

Useful resources:

- <https://see.stanford.edu/materials/lsoidsee263/Additional1-notes-matrix-primer.pdf>
- https://see.stanford.edu/materials/lsoidsee263/Additional2-matrix_crimes.pdf

Linear systems

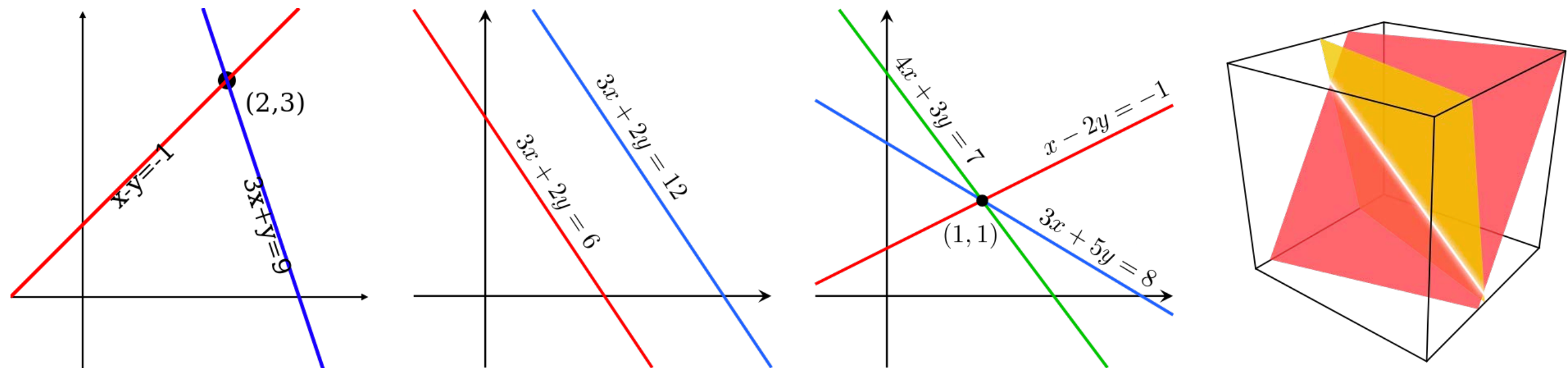
- Direct solvers:
 - Gauss elimination/LU decomposition
 - Cholesky decomposition (SPD matrices)
 - QR decomposition
 - SVD
- Iterative solvers:
 - Jacobi iterations
 - Gauss-Seidel
 - Successive over-relaxation (SOR)
 - Krylov subspace methods (conjugate gradients, etc.)

$$2x_1 + 3x_2 + 5x_3 = 1$$

$$4x_1 - 2x_2 - 7x_3 = 8$$

$$9x_1 + 5x_2 - 3x_3 = 2$$

$$\begin{bmatrix} 2 & 3 & 5 \\ 4 & -2 & -7 \\ 9 & 5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 2 \end{bmatrix}$$



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