Chapter 7

LOGICAL AGENTS

7.1 Suppose the agent has progressed to the point shown in Figure 7.1(a), having perceived nothing in [1,1], a breeze in [2,1], and a stench in [1,2], and is now concerned with the contents of [1,3], [2,2], and [3,1]. Each of these can contain a pit, and at most one can contain a wumpus. Following the example of Figure 7.2, construct the set of possible worlds. (You should find 32 of them.) Mark the worlds in which the KB is true and those in which each of the following sentences is true:

$$\alpha_2 = \text{“There is no pit in [2,2].”}$$
$$\alpha_3 = \text{“There is a wumpus in [1,3].”}$$

Hence show that $KB \models \alpha_2$ and $KB \models \alpha_3$.

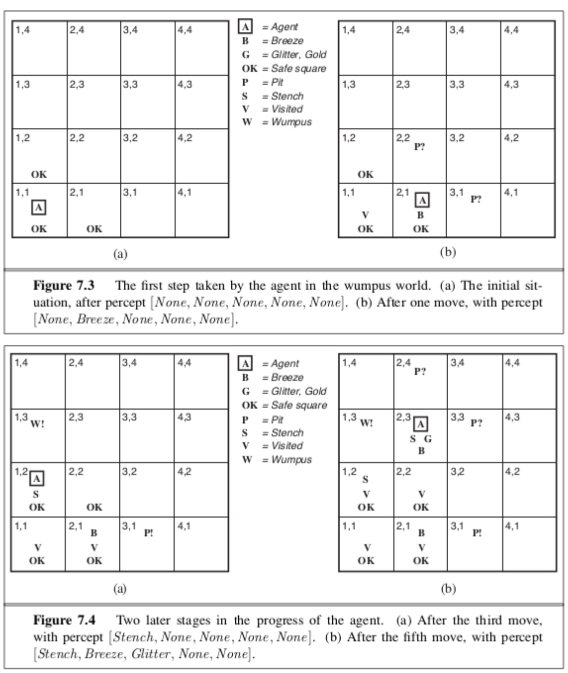

Figure 7.1: Exercise 7.1

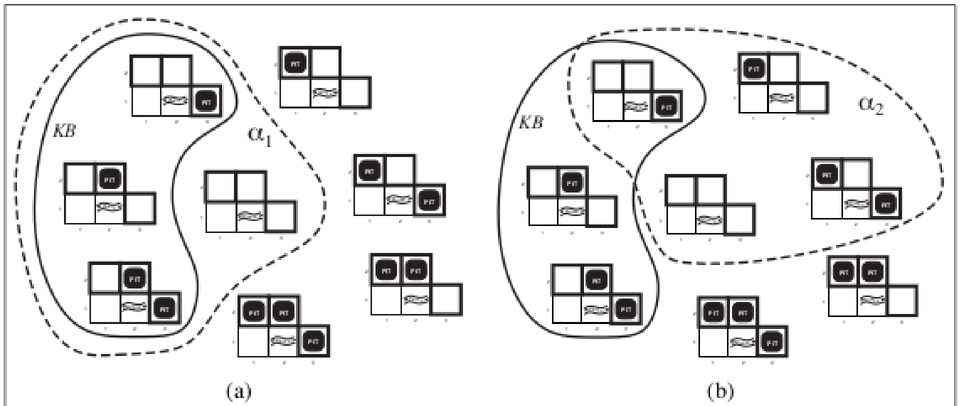

Figure 7.5 Possible models for the presence of pits in squares [1,2], [2,2], and [3,1]. The KB corresponding to the observations of nothing in [1,1] and a breeze in [2,1] is shown by the solid line. (a) Dotted line shows models of α_1 (no pit in [1,2]). (b) Dotted line shows models of α_2 (no pit in [2,2]).

Figure 7.2: Exercise 7.1

7.2 Which of the following are correct?

a. $\mathit{False} \models \mathit{True}$.

b. $\mathit{True} \models \mathit{False}$.

c. $(A \wedge B) \models (A \Leftrightarrow B)$.

d. $A \Leftrightarrow B \models A \vee B$.

e. $A \Leftrightarrow B \models \neg A \vee B$.

f. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$.

g. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$.

h. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$.

i. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.

j. $(A \vee B) \wedge \neg(A \Rightarrow B)$ is satisfiable.

k. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable.

l. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

7.3 Prove, or find a counterexample to, each of the following assertions:

a. If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \wedge \beta) \models \gamma$

b. If $(\alpha \wedge \beta) \models \gamma$ then $\alpha \models \gamma$ or $\beta \models \gamma$ (or both).

c. If $\alpha \models (\beta \vee \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

7.4

a. If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \wedge \beta) \models \gamma$

b. If $\alpha \models (\beta \wedge \gamma)$ then $\alpha \models \beta$ and $\alpha \models \gamma$.

c. If $\alpha \models (\beta \vee \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

7.5 Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

a. $B \vee C$.

b. $\neg A \vee \neg B \vee \neg C \vee \neg D$.

c. $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$.

7.6 Using a method of your choice, verify each of the equivalences in Figure 7.3 (page 3).

$$
\begin{array}{rcll}
(\alpha \wedge \beta) & \equiv & (\beta \wedge \alpha) & \text{commutativity of } \wedge \\
(\alpha \vee \beta) & \equiv & (\beta \vee \alpha) & \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) & \equiv & (\alpha \wedge (\beta \wedge \gamma)) & \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) & \equiv & (\alpha \vee (\beta \vee \gamma)) & \text{associativity of } \vee \\
\neg(\neg \alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg \beta \Rightarrow \neg \alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg \alpha \vee \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) & \equiv & (\neg \alpha \vee \neg \beta) & \text{De Morgan} \\
\neg(\alpha \vee \beta) & \equiv & (\neg \alpha \wedge \neg \beta) & \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) & \equiv & ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) & \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) & \equiv & ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) & \text{distributivity of } \vee \text{ over } \wedge
\end{array}
$$

Figure 7.11 Standard logical equivalences. The symbols α, β, and γ stand for arbitrary sentences of propositional logic.

Figure 7.3: Exercise 7.6

7.7 Use resolution to prove the sentence $\neg A \wedge \neg B$ from the clauses in Exercise 7.8.

7.8 Convert the following set of sentences to clausal form. [S1: $A \Leftrightarrow (B \vee E)$.
S2: $E \Rightarrow D$.
S3: $C \wedge F \Rightarrow \neg B$.
S4: $E \Rightarrow B$.
S5: $B \Rightarrow F$.
S6: $B \Rightarrow C$]

7.9 This exercise looks into the relationship between clauses and implication sentences.

a. Show that the clause $(\neg P_1 \vee \cdots \vee \neg P_m \vee Q)$ is logically equivalent to the implication sentence $(P_1 \wedge \cdots \wedge P_m) \Rightarrow Q$.

b. Show that every clause (regardless of the number of positive literals) can be written in the form $(P_1 \wedge \cdots \wedge P_m) \Rightarrow (Q_1 \vee \cdots \vee Q_n)$, where the Ps and Qs are proposition symbols.

7.10 Consider the following sentence:

$$[(Food \;\Rightarrow\; Party) \vee (Drinks \;\Rightarrow\; Party)] \;\Rightarrow\; [(Food \wedge Drinks) \;\Rightarrow\; Party] \,.$$

a. Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.

b. Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).

c. Prove your answer to (a) using resolution.