Chapter 14

PROBABILISTIC REASONING OVER TIME

14.1 Show that any second-order Markov process can be rewritten as a first-order Markov process with an augmented set of state variables. Can this always be done *parsimoniously*, i.e., without increasing the number of parameters needed to specify the transition model?

14.2 In this exercise, we examine what happens to the probabilities in the umbrella world in the limit of long time sequences.

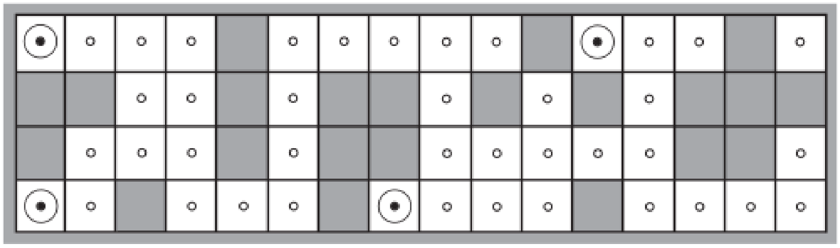
a. Suppose we observe an unending sequence of days on which the umbrella appears. Show that, as the days go by, the probability of rain on the current day increases monotonically toward a fixed point. Calculate this fixed point.

b. Now consider *forecasting* further and further into the future, given just the first two umbrella observations. First, compute the probability $P(r_{2+k}|u_1, u_2)$ for $k = 1 \ldots 20$ and plot the results. You should see that the probability converges towards a fixed point. Prove that the exact value of this fixed point is 0.5.

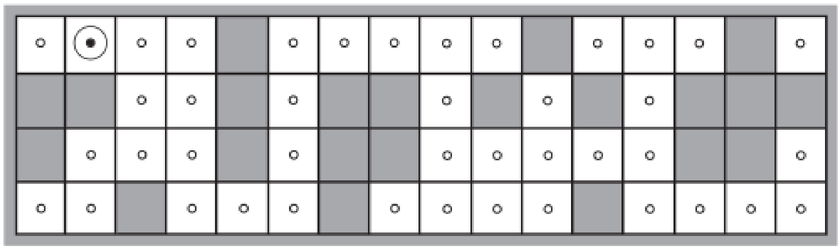
14.3 Consider the vacuum worlds of Figure 14.1 (perfect sensing) and Figure 14.2 (noisy sensing). Suppose that the robot receives an observation sequence such that, with perfect sensing, there is exactly one possible location it could be in. Is this location necessarily the most probable location under noisy sensing for sufficiently small noise probability ϵ? Prove your claim or find a counterexample.

14.4 In Section 14.3.2, the prior distribution over locations is uniform and the transition model assumes an equal probability of moving to any neighboring square. What if those assumptions are wrong? Suppose that the initial location is actually chosen uniformly from the northwest quadrant of the room and the *Move* action actually tends to move southeast. Keeping the HMM model fixed, explore the effect on localization and path accuracy as the southeasterly tendency increases, for different values of ϵ.

14.5 Consider a version of the vacuum robot (page 3) that has the policy of going straight for as long as it can; only when it encounters an obstacle does it change to a new (randomly selected) heading. To model this robot, each state in the model consists of a *(location, heading)* pair. Implement this model and see how well the Viterbi algorithm can track a robot with this model (optional). The robot's policy is more constrained than the random-walk robot; does that mean that predictions of the most likely path are more accurate?



(a) Possible locations of robot after $E_1 = NSW$



(b) Possible locations of robot After $E_1 = NSW, E_2 = NS$

Figure 4.18 Possible positions of the robot, $\odot$, (a) after one observation $E_1 = NSW$ and (b) after a second observation $E_2 = NS$. When sensors are noiseless and the transition model is accurate, there are no other possible locations for the robot consistent with this sequence of two observations.

Figure 14.1: Exercise 14.3

14.6 A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class, and whether they have red eyes. The professor has the following domain theory:

- The prior probability of getting enough sleep, with no observations, is 0.7.
- The probability of getting enough sleep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.
- The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.
- The probability of sleeping in class is 0.1 if the student got enough sleep, and 0.3 if not.

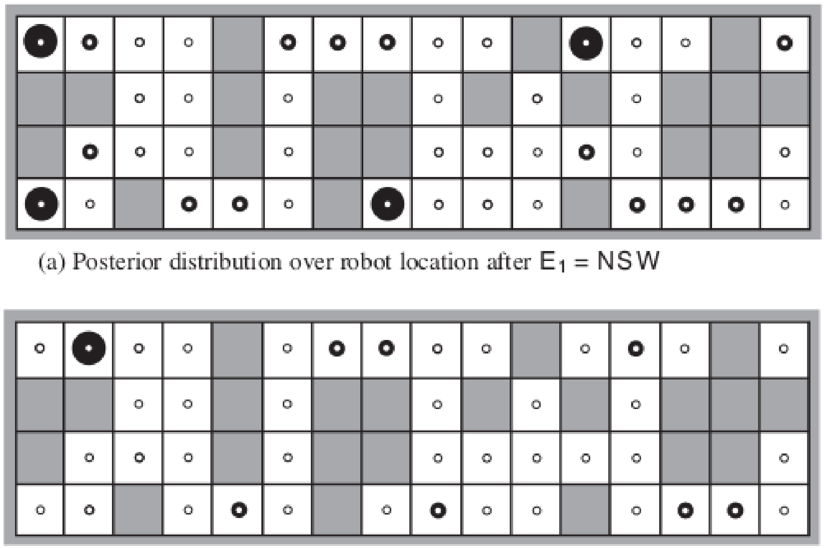
Formulate this information as a dynamic Bayesian network that the professor could use to filter or predict from a sequence of observations. Then reformulate it as a hidden Markov model that has only a single observation variable. Give the complete probability tables for the model.

14.7 For the DBN specified in Exercise 14.7 and for the evidence values

$$\begin{aligned}\mathbf{e}_1 &= \text{not red eyes, not sleeping in class}\\ \mathbf{e}_2 &= \text{red eyes, not sleeping in class}\\ \mathbf{e}_3 &= \text{red eyes, sleeping in class}\end{aligned}$$

perform the following computations:

a. State estimation: Compute $P(EnoughSleep_t|\mathbf{e}_{1:t})$ for each of $t = 1, 2, 3$.



(a) Posterior distribution over robot location after $E_1 = NSW$

(b) Posterior distribution over robot location after $E_1 = NSW, E_2 = NS$

Figure 15.7 Posterior distribution over robot location: (a) one observation $E_1 = NSW$; (b) after a second observation $E_2 = NS$. The size of each disk corresponds to the probability that the robot is at that location. The sensor error rate is $\epsilon = 0.2$.

Figure 14.2: Exercise 14.3

b. Smoothing: Compute $P(EnoughSleep_t|\mathbf{e}_{1:3})$ for each of $t = 1, 2, 3$.

c. Compare the filtered and smoothed probabilities for $t = 1$ and $t = 2$.

14.8 Suppose that a particular student shows up with red eyes and sleeps in class every day. Given the model described in Exercise 14.7, explain why the probability that the student had enough sleep the previous night converges to a fixed point rather than continuing to go down as we gather more days of evidence. What is the fixed point? Answer this both numerically (by computation) and analytically.

14.9 This exercise analyzes in more detail the persistent-failure model for the battery sensor in Figure 14.3(a).

a. Figure 14.3(b) stops at $t = 32$. Describe qualitatively what should happen as $t \to \infty$ if the sensor continues to read 0.

b. Suppose that the external temperature affects the battery sensor in such a way that transient failures become more likely as temperature increases. Show how to augment the DBN structure in Figure 14.3(a), and explain any required changes to the CPTs.

c. Given the new network structure, can battery readings be used by the robot to infer the current temperature?

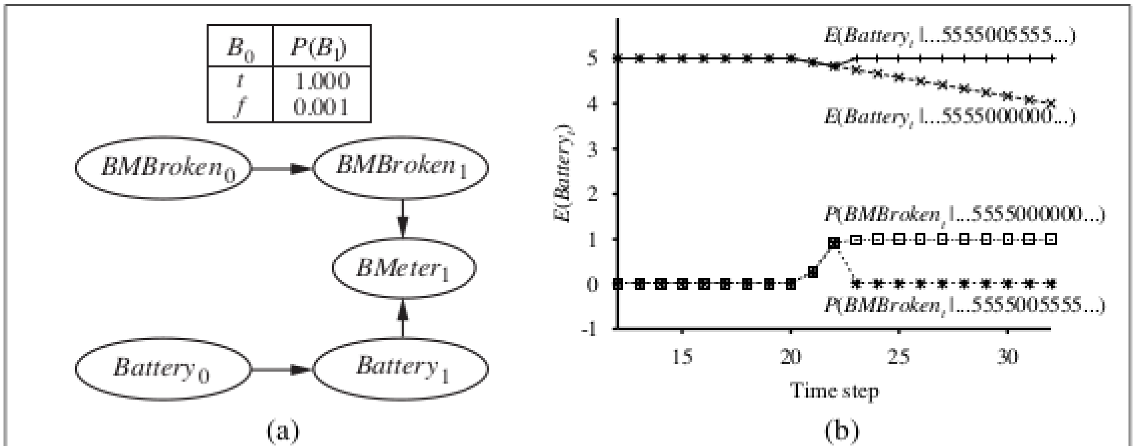


Figure 15.15 (a) A DBN fragment showing the sensor status variable required for modeling persistent failure of the battery sensor. (b) Upper curves: trajectories of the expected value of $Battery_t$ for the "transient failure" and "permanent failure" observations sequences. Lower curves: probability trajectories for $BMBroken$ given the two observation sequences.

Figure 14.3: Exercise 14.9