Advanced Statistics

Instructor: Shashi Prabh

Problem Set 3. Random variables, Bayes's theorem

References

- FPP: Statistics (4/e), Freedman, Pisani, Purves
- Wasserman: All of Statistics, Springer, 2003
- JWHT: An Introduction to Statistical Learning with Applications in R, James, Witten, Hastie and Tibshirani, Springer, 2017
- VS: An Introduction to R, Venables and Smith

1. Wasserman, pages 13-16, problems 1, 3, 4, 5, 7, 8, 10, 11, 12, 13, 14, 15, 19, 20

2. In a factory producing compact discs, the total quantity of defective items found in a given week is 14%. It is suspected that the majority of these come from two machines, X and Y. An inspection shows that 8% of the output from X and 4% of the output from Y is defective. Furthermore, 11% of the overall output came from X and 23% from Y. A CD is chosen at random and found to be defective. What is the probability that it came from either X or Y?

3. A firm buys 1000 hard disks from two vendors. It buys 900 disks from V_1 and 100 from V_2. The probabilities of shipping a defective disk are 0.01 and 0.005 for V_1 and V_2, respectively. One disk is taken randomly from the lot and is found working. What is the probability that the second disk taken randomly will also be found working? Explain the result.

4. An office secretary puts n letters, all addressed to different individuals, in n labeled envelopes randomly. What is the probability that at-least one of the letters is in correctly labeled envelope when $n = 4$? What happens when n gets large?

5. Show that if three events A, B and C are independent, then $A \cup B$ is independent of C.

6. *Gambler's ruin.* A gambler needs to raise N Rupees and has k Rupees in hand. He bets 1 Rupee on a fair coin toss where he wins 1 Rupee if head shows up and looses the same amount if tail shows up. What is the probability that he looses all his money?

7. Programming exercise: Wasserman, page 16, problem 21