Terms you'll find helpful in completing today's challenge are outlined below.

Negative Binomial Experiment

A negative binomial experiment is a statistical experiment that has the following properties:

- The experiment consists of n repeated trials.
- The trials are independent.
- The outcome of each trial is either success (s) or failure (f).
- $P(s)$ is the same for every trial.
- The experiment continues until x successes are observed.

If X is the number of experiments until the x^{th} success occurs, then X is a discrete random variable called a negative binomial.

Negative Binomial Distribution

Consider the following probability mass function:

$$b^*(x, n, p) = \binom{n-1}{x-1} \cdot p^x \cdot q^{(n-x)}$$

The function above is negative binomial and has the following properties:

- The number of successes to be observed is x.
- The total number of trials is n.
- The probability of success of 1 trial is p.
- The probability of failure of 1 trial q, where $q = 1 - p$.
- $b^*(x, n, p)$ is the negative binomial probability, meaning the probability of having $x - 1$ successes after $n - 1$ trials and having x successes after n trials.

Note: Recall that $\binom{n}{x} = \frac{n!}{x!(n-x)!}$. For further review, see the Combinations and Permutations Tutorial.

Geometric Distribution

The geometric distribution is a special case of the negative binomial distribution that deals with the number of Bernoulli trials required to get a success (i.e., counting the number of failures before the first success). Recall that X is the number of successes in n independent Bernoulli trials, so for each i (where $1 \leq i \leq n$):

$$X_i = \begin{cases} 1 & \text{if the } i^{th} \text{ trial is a success} \\ 0 & \text{otherwise.} \end{cases}$$

The geometric distribution is a negative binomial distribution where the number of successes is 1. We express this with the following formula:

$$g(n, p) = q^{(n-1)} \cdot p$$

Example

Bob is a high school basketball player. He is a 70% free throw shooter, meaning his probability of making a free throw is 0.70. What is the probability that Bob makes his first free throw on his fifth shot?

For this experiment, $n = 5$, $p = 0.7$ and $q = 0.3$, So, $g(n = 5, p = 0.7) = 0.3^4 0.7 = 0.00567$

Solve Problem