Control Engineering
Experiment - 3

Controller design on MATLAB platform using Analog Root Loci

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Aim of the Experiment :

This project requires analysis of a cascade feedback controller for a given digital transfer function, according to desired specifications.
A sensitivity analysis for variation of key parameters is further required.

Theory and Transfer function:

The basic characteristic of the transient response of a closed loop system is closely related to the position of the closed loop poles whose values change with the change in gain if the system has variable gain.
The root locus method for continuous time systems can be extended to discrete time systems without much modifications since the characteristic equation of a discrete control system is of the same form as that of a continuous time control system.
The OLTF of a unity, negative feedback digital system has the form:

$$G_{OL}(z) = K*\left(\frac{0.632*z}{(z-1)*(z-a)} \right)$$

The respective CLTF will be

$$G_{CL}(z) = \frac{0.632*K*z}{((z-1)(z-a)+0.632Kz)}$$

And the characteristic equation is : z^2 - (1+ a -0.632*K)* z + a = 0

<u>In discrete-time, all the poles in the complex z-plane must lie inside the unit circle. The system is marginally stable if it has one or more poles lying on the unit circle.</u>

Matlab Code :

```
z=tf('z',1)
G=@(K,a) K*0.632*z/((z-1)*(z-a))
K=1
a=0.1
for ik=1:length(K)
        for jk=1:length(a)
                H=feedback(G(ik,jk),1);
                rlocus(H)
                figure()
                stepinfo(H)
                zgrid(H)
        end
end
```

Design Requirements and Constraints Design Requirements

The performance requirements of the CLTF are twofold:

- Oscillatory dynamics are to be totally avoided.
- The CLTF dynamics must always be stable.

Given Constraints :

1. The parameter a may assume values in the range 0.1 to 0.9, both limits inclusive.
2. The gain K is restricted to be strictly greater than unity due to some practical limitations of the electronics being employed.

Analysis

So we had a set of values for a=[0.1,0.9] and the value of K had to be greater than 1 so we had fixed the values of a and taken various values for K to see what is the behaviour of the system and then computed the value of K for which the system is stable and for what range of values of K is the oscillation dynamics avoided and later we took the intersection of these two to get the final result.

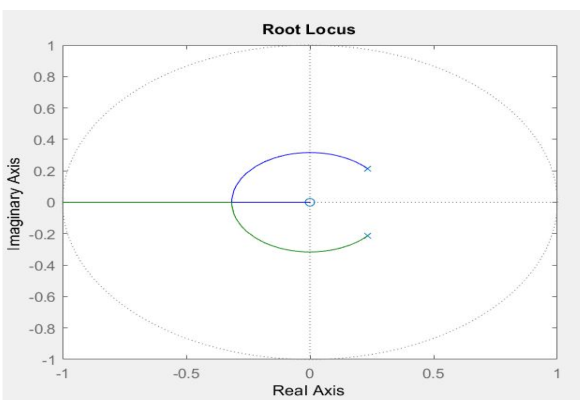
Here is the a sample set for a=0.1 and the values of K we had tried on
Sampling Period was 0.01s

Value of k	Settling Time	Peak Time	Overshoot	Stability	Damping
1	0.03	0.04	0.8626	Stable	negligible
1.1	0.031	0.0175	0.863	stable	negligible
1.2	0.0347	0.0158	2.12	stable	negligible
1.3	0.0342	0.0141	3.01	stable	negligible
1.4	0.0326	0.0122	5.03	stable	negligible
1.5	0.0299	0.00967	7.52	stable	negligible
1.6	0.0287	0.00844	9.21	stable	negligible
1.7	0.0277	0.00791	10.1	stable	negligible
1.8	0.0267	0.00745	10.2	stable	negligible
1.9	0.0335	0.00703	13.8	stable	underdamped
2	0.0344	0.00666	20.1	stable	underdamped
2.5	0.0333	0.00633	26.4	stable	underdamped
3	0.0917	0.00506	89.6	stable	underdamped

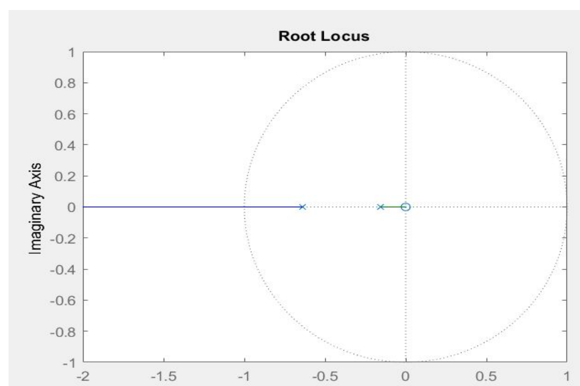
4	NaN	NaN	Inf	Unstable	underdamped
5	NaN	NaN	Inf	Unstable	underdamped
6	NaN	NaN	Inf	Unstable	underdamped
7	NaN	NaN	Inf	Unstable	underdamped
8	NaN	NaN	Inf	Unstable	underdamped
9	NaN	NaN	Inf	Unstable	underdamped
10	NaN	NaN	Inf	Unstable	underdamped
20	NaN	NaN	Inf	Unstable	underdamped
50	NaN	NaN	Inf	Unstable	underdamped
100	NaN	NaN	Inf	Unstable	underdamped
1000	NaN	NaN	Inf	Unstable	underdamped

The rlocus graphs for few cases are:

K=1

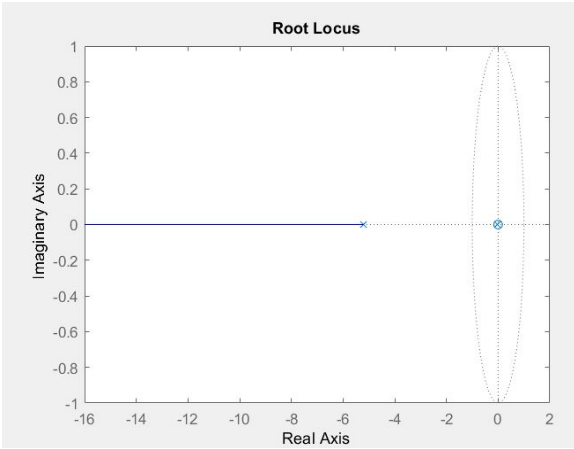


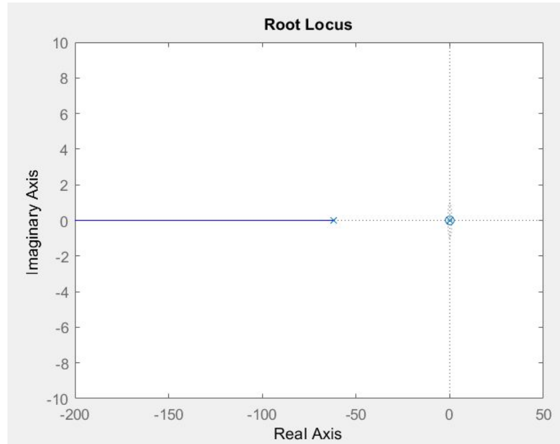
K=3



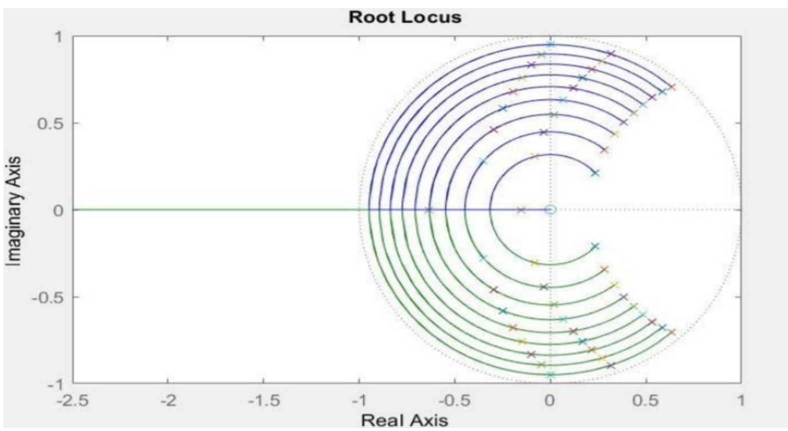
K=10

K=100



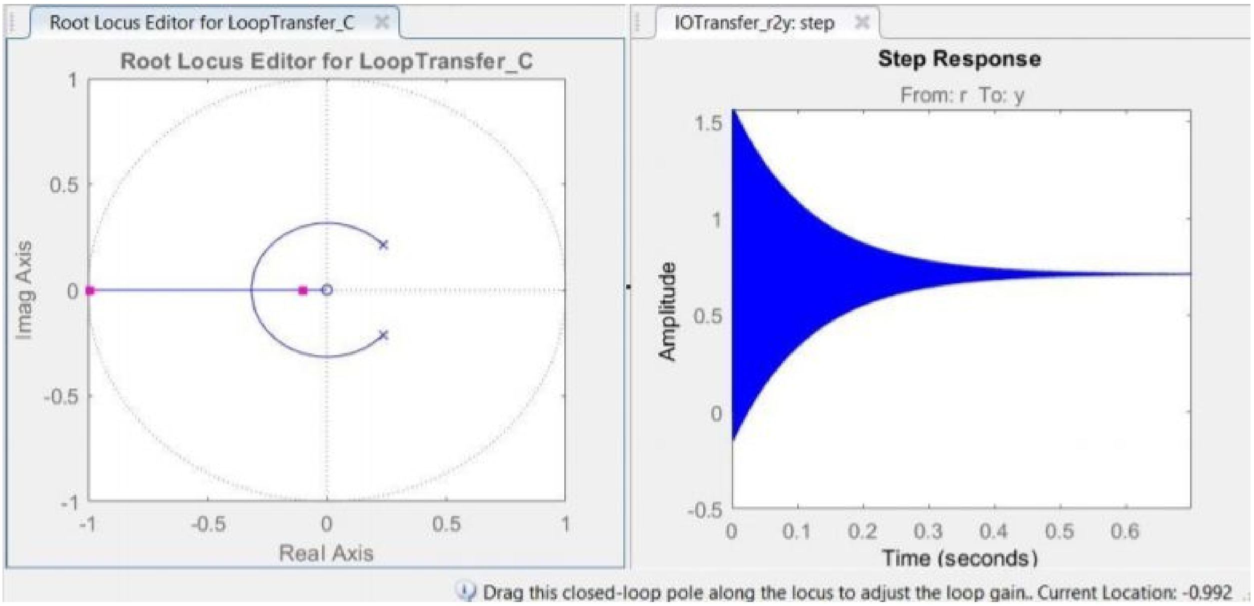


With the increase in k the poles shift towards left on their respective root loci. All poles lie inside the unit circle ensuring stability for the system.

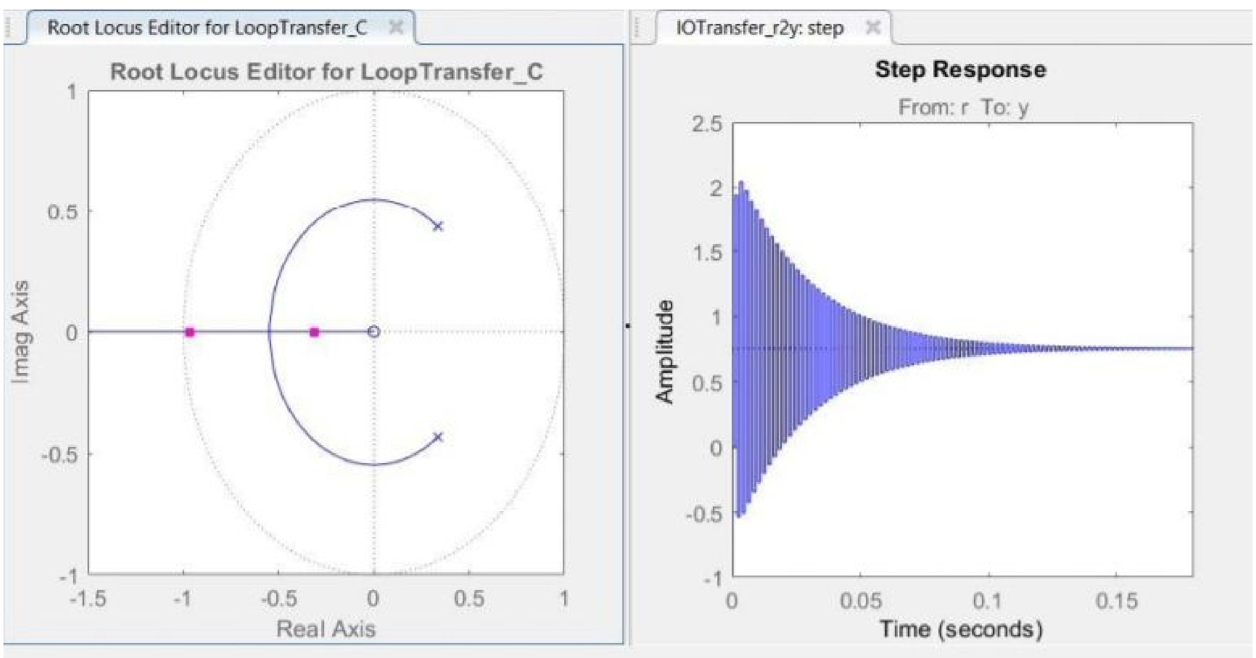


We continue increasing k until one pole reaches z = -1 on the real axis.

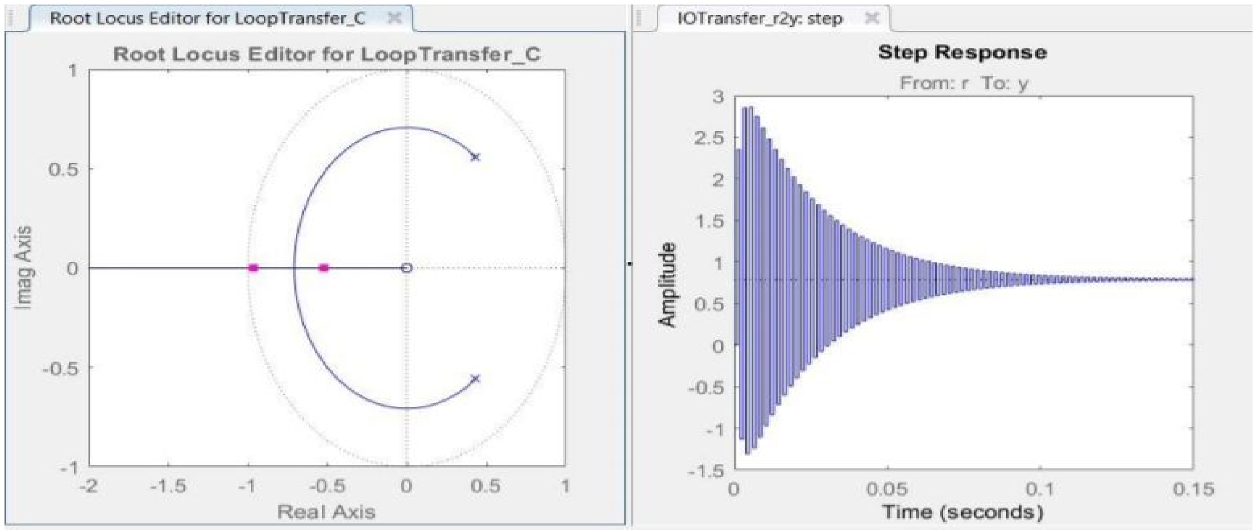
This is the limiting condition of stability. If k is further increased the pole will move out of the unit circle and the system shall become unstable. Here are some of the sample graphs which we obtained and the table thereafter contains the range limit for K.



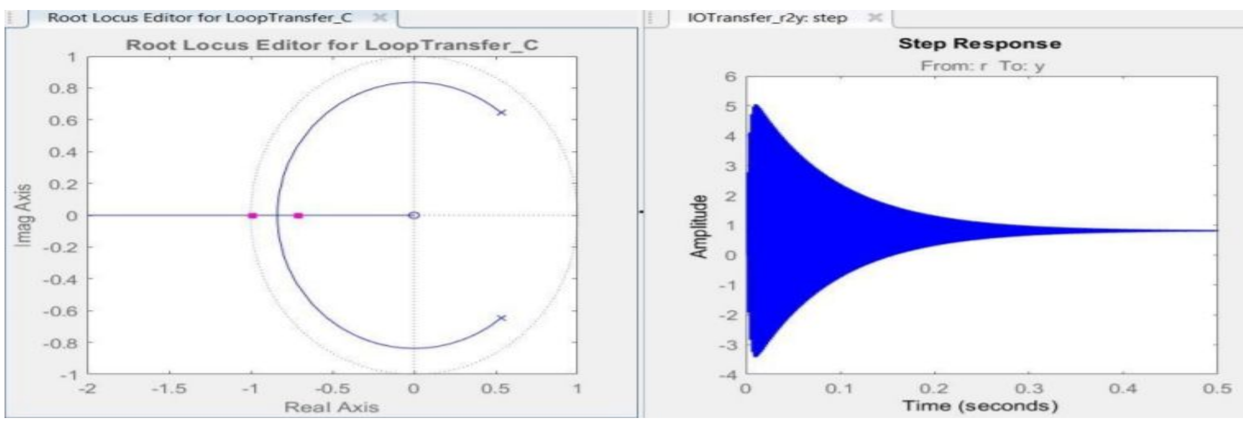
Plot for a = 0.1 and upper limit of k = 3.481



Plot for a = 0.4 and upper limit of k = 4.310

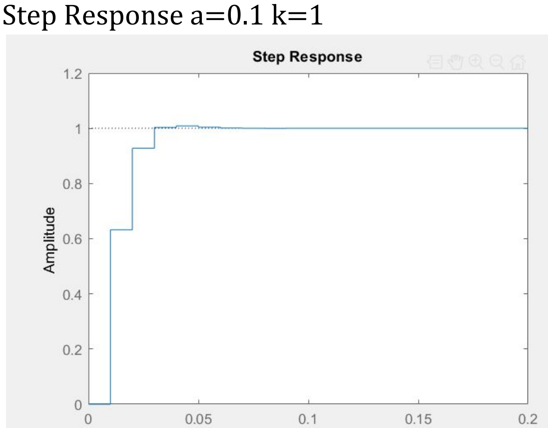


Plot for a = 0.6 and upper limit of k = 5.063

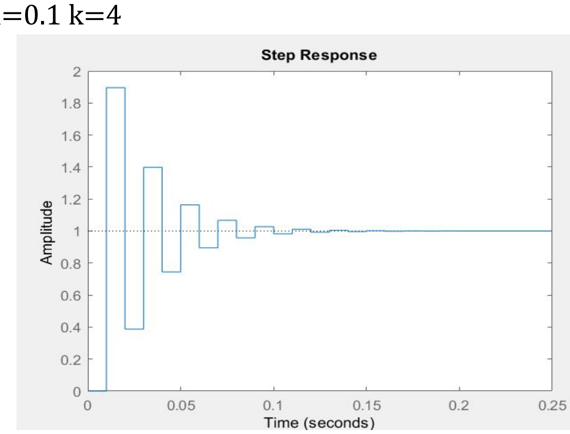


Plot for a = 0.8 and upper limit of k = 5.696

Step Response a=0.1 k=1



a=0.1 k=4



Stability Analysis

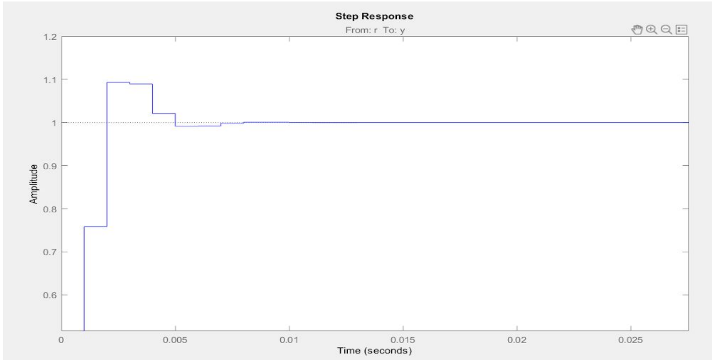
Value of a	Upper limit of K
0.1	3.481
0.2	3.797
0.3	4.113
0.4	4.310
0.5	4.746
0.6	5.063
0.7	5.379
0.8	5.696
0.9	6.012

This table summarizes the range for K for various values of a.

For avoiding oscillatory dynamics we had tried for several values of K for each value of a and found the following results (for a=0.1 the results were already summarized in the table above)

For a=0.2

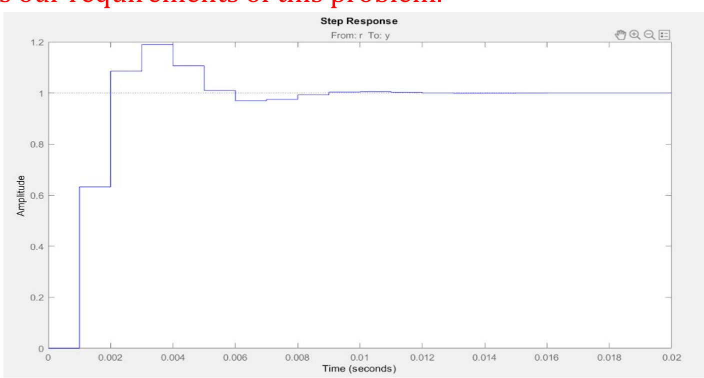
K	**Rise time**	**Overshoot (%)**	**Settling time**	**Stability**	**Oscillatory Dynamics**
1	0.00159	6.85	0.00466	stable	negligible
1.1	0.00144	8.42	0.0044	stable	negligible
1.2	0.00129	9.33	0.00403	stable	negligible
1.3	0.00113	13.2	0.00382	stable	underdamped
1.4	0.000941	16.4	0.00365	stable	underdamped



For a=0.3

K	Rise time	Overshoot (%)	Settling time	Stability	Oscillatory Dynamics
1	0.00148	14.7	0.00487	stable	negligible
1.1	0.00134	16.1	0.00639	stable	underdamped

For any value a $\geq$ 0.3 we are getting oscillatory dynamics in the step response as we can observe it in the below if we want to have no oscillations then we have to take the value of K lesser than 1 which violates our requirements of this problem.



Oscillatory Analysis

Value of a	Range of K
0.1	1.74
0.15	1.6
0.2	1.4
0.25	1.3
0.3	-

0.4	-
0.5	-
0.6	-
0.7	-
0.8	-
0.9	-

Observation and Discussion

To fulfill performance requirements we take an intersection of the above two tables and obtain a the range for k for different values of k as follows:

Value of a	Range of K
0.1	1-1.74
0.15	1-1.6
0.2	1-1.4
0.25	1-1.3
0.3	1-1.1
0.4	Not possible for K>1
0.5	Not possible for K>1
0.6	Not possible for K>1
0.7	Not possible for K>1
0.8	Not possible for K>1
0.9	Not possible for K>1

For the value of $K \in (1, 1.74)$ the performance requirements are met regardless of the values assumed by a=0.1. We can clearly see that as we increase the value of a there is no such range of K for which the system is both stable and avoiding oscillation dynamics.Although for these values if we are allowed to use K<1 then we can have sets which would suffice both of the design objectives.

Sensitivity function:

It is the ratio of the change in the system transfer function to a change in the process transfer function(or parameter) for a small increment change.

1. **with parameter α**

$$S_{\alpha}^{T}(z) = \frac{\alpha}{T}\,\frac{\partial T}{\partial \alpha}\Big|_{\alpha=\alpha_o} = \frac{\alpha}{N}\,\frac{\partial N}{\partial \alpha} - \frac{\alpha}{D}\,\frac{\partial D}{\partial \alpha}\Big|_{\alpha=\alpha_o}$$

When we take $\alpha = K$

$$S_{K}^{T}(z) = 1 - \frac{0.632Kz}{(z-1)(z-a)+0.632Kz}$$

When we take $\alpha = a$

$$S_{a}^{T}(z) = \frac{a(z-1)}{(z-1)(z-a)+0.632Kz}$$

Values of a	Values of K	Magnitude of Sensitivity
0.1	(1, 1.8]	[0.134,0.187]
0.15	(1, 1.6]	[0.187,0.233]
0.2	(1, 1.2]	[0.234,0.245]

Observation:

- As the value of K increases the system sensitivity increases for the variations in parameter ‘a’. Also, our control system is very less sensitive towards the variation in parameter ‘a’.
- As the value of parameter ‘a’ is increased the magnitude of $\left|S_{a}^{T}(z)\right|$ is also increasing implying that as the value of ‘a’ increases our system becomes more sensitive towards variations in ‘a’ itself.

2. **with OLTF G**

$$S_{G}^{T}(z) = \frac{1}{1+GH} = \frac{(z-1)(z-a)}{(z-1)(z-a)+0.632Kz}$$

Values of a	Values of K	Magnitude of Sensitivity
0.1	(1, 1.8]	[1.46,2.07]
0.15	(1, 1.6]	[1.43,1.78]
0.2	(1, 1.2]	[1.39,1.46]

Observations

- As the value of K increases the system sensitivity increases for the variations in open loop transfer function which implies the system becomes more sensitive.
- Normally, we want our system to be as insensitive for the variations.

A typical value of magnitude of sensitivity $\left|S_G^T(z)\right|$ should be in the *range of 1.3 to 2 for designing a good control system* . And we can also observe that as the value of 'a' increases the system becomes more and more insensitive to the variations regardless of the values of K.

3. Sensitivity analysis for variation of parameter K:

$$S_K^T(z) = 1 - \frac{0.632Kz}{(z-1)(z-a)+0.632Kz} = \frac{a(z-1)}{(z-1)(z-a)+0.632Kz} = S_G^T(z)$$

Since sensitivity function of CLTF w.r.t. variations in parameter K is same as that of the sensitivity function of CLTF w.r.t. variations in OLTF. So our observations will be the same as described above.

Which implies that:

- As K increases , sensitivity towards variation in K increases.
- As 'a' increases , sensitivity towards variation in K decreases.

Finally, It can be said that as K increases no matter what parameter we are considering , sensitivity for any variations increases.