

Control Engineering

Experiment - 8

**Analyse the effectiveness of Ziegler-Nichols rules
in presence of poles with high multiplicity.**

Submitted to: Prof. S. Roy

MADE BY:

Keshav Kishore-2018eeb1158

Mahima Kumawat-2018eeb1162

Preetesh Verma-2018eeb1171

Aim

Analyse the effectiveness of Ziegler-Nichols rules in presence of poles with high multiplicity.

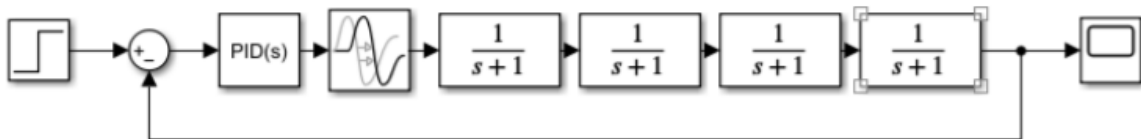
Theory

A chemical reactor is known to have a transfer function of the form

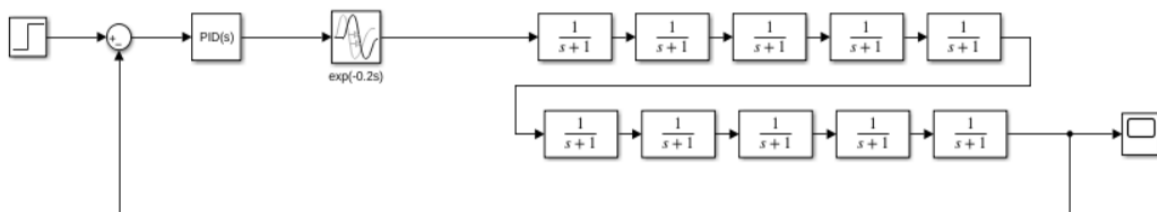
$$G(s) = \frac{e^{-0.2s}}{(s+1)^n}$$

The multiplicity of the pole in the denominator represents multiple chambers in the reactor, each leading to the next one in cascade.

The, $e^{-0.2s}$ term in the numerator is known as transportation lag and in this case, represents a 0.2s lag due to piping leading to the reactor before we reach the first of the n chambers. The value of n may assume integer values from four to ten for different plant configurations.



Simulink model for n=4



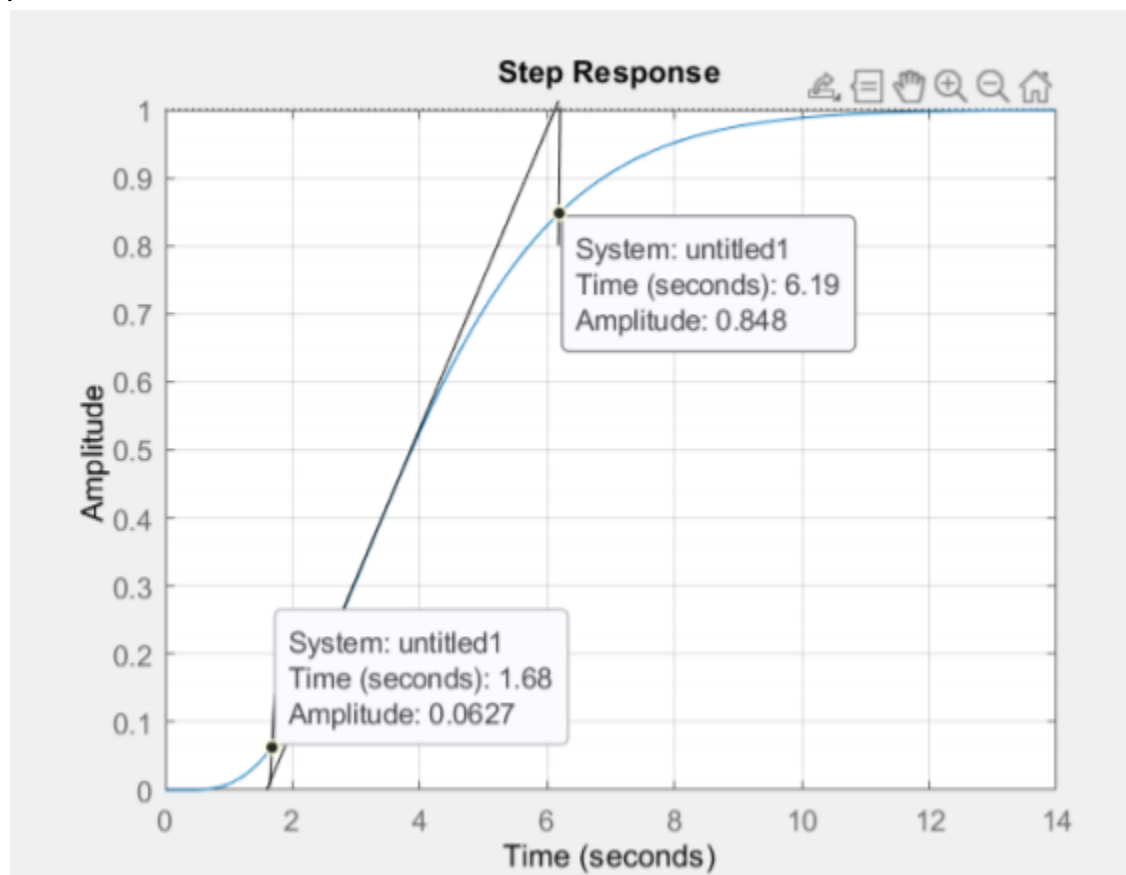
Simulink model for n=10

Plots and Analysis

In the Ziegler-Nichols method of tuning, we have two methods, the open-loop step response method and the closed-loop continuous cycling method. We test both of them for the case of $n=4$ to get PID control gains and compare them. The rest of the cases are solved with the method which provides better results in terms of rise time, peak overshoot and settling time.

Method1: Open-loop step response method taking $n=4$ for PID control.

Open Loop test is used for lag type systems and it is already provided that the exponential term produces a transportation lag. We compute the required parameters as follows:



$$L = 1.68$$

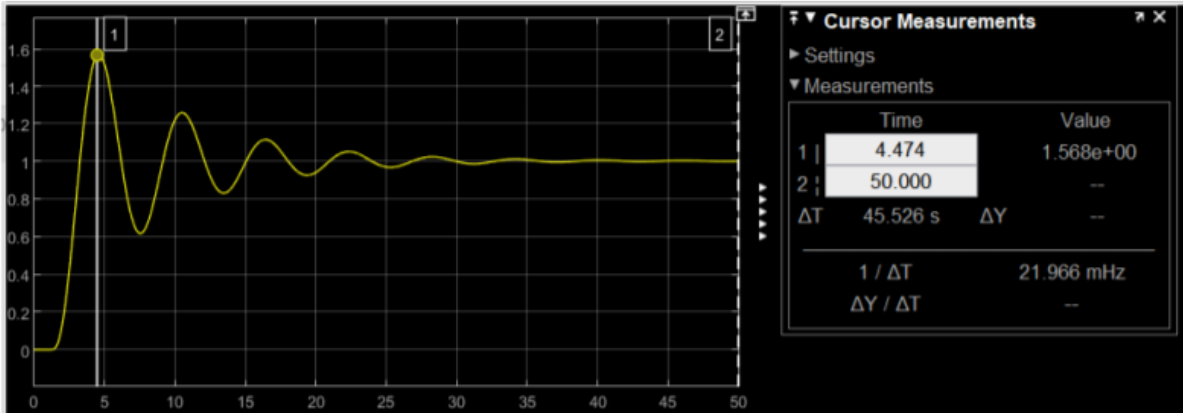
$$T = (6.19 - 1.68) = 4.51$$

$$K_p = 1.2 \cdot (T/L) = 3.2214$$

$$K_i = K_p / (2 \cdot L) = 0.9588$$

$$K_d = K_p \cdot 0.5 \cdot L = 2.7060$$

Rise Time = 1.25 sec
Peak Overshoot = 56.8%
Settling Time = 25.1 sec

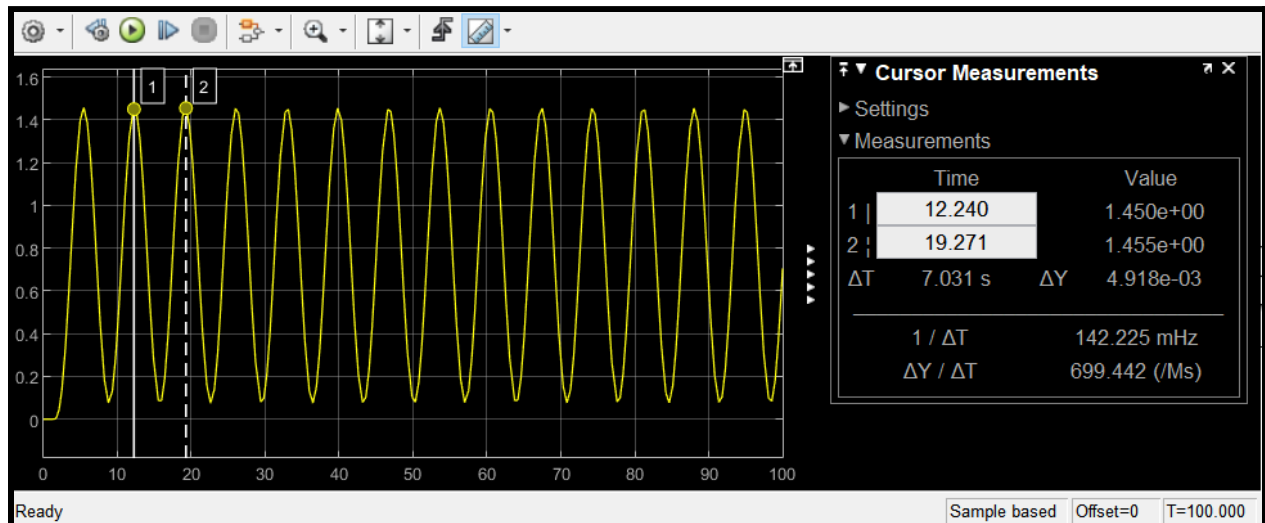


Dynamic response for PID control using OL-ZN tuning

Method2:

Closed loop continuous cycling method taking $n=4$ for PID control.

Here as we increase the gain in the closed loop response and we observe that the system becomes oscillatory and unstable i.e. at some value of k ($k_{critical}$) the system will have sustained oscillations (pure oscillations) synonyms to a sine wave. At this k we calculate the period ($P_{critical}$) of oscillations to compute the gains respectively.



Sustained oscillations

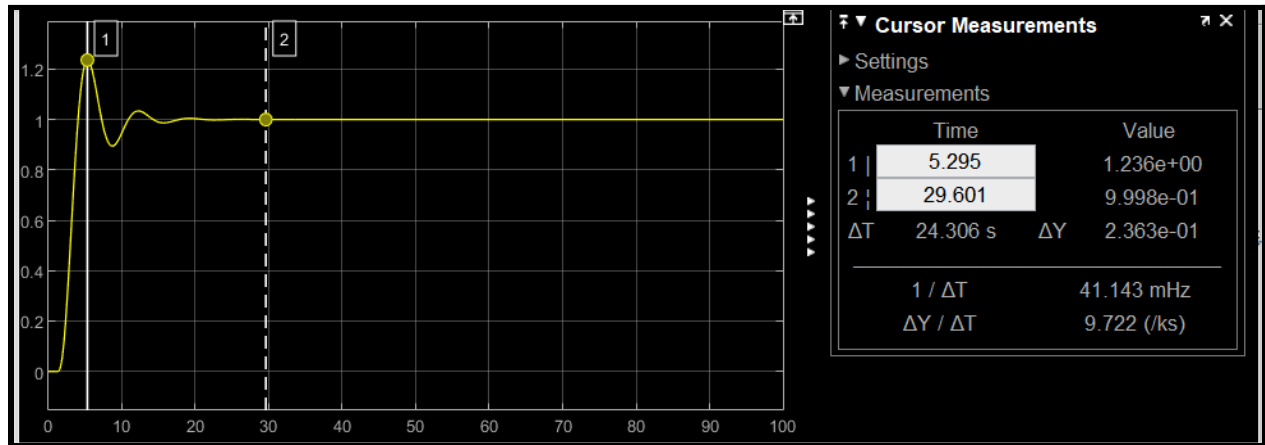
$$K_{cr} = 3.27$$

$$P_{cr} = 7.031$$

$$K_p = 0.6K_{cr} = 1.9620$$

$$K_i = 2K_p/P_{cr} = 0.5581$$

$$K_d = K_p \cdot P_{cr}/8 = 1.7244$$



Dynamic response for PID control using CL-ZN tuning

Rise Time = 1.79 sec
Peak Overshoot = 23%
Settling Time = 13.5 sec

On comparing results from both the methods we observe that there is a huge difference in peak overshoot and settling time. It is easily visible from the plots that CL-ZN tuning has less number of oscillations. Even though CL-ZN tuning has a higher rise time, it is quite marginal. On the other hand CN-ZN gives quite low values of peak overshoot and settling time as compared to OL-ZN tuning method. Therefore for the rest of the cases we shall continue to use the CL-ZN tuning method.

Other parameters calculations is done as follows:

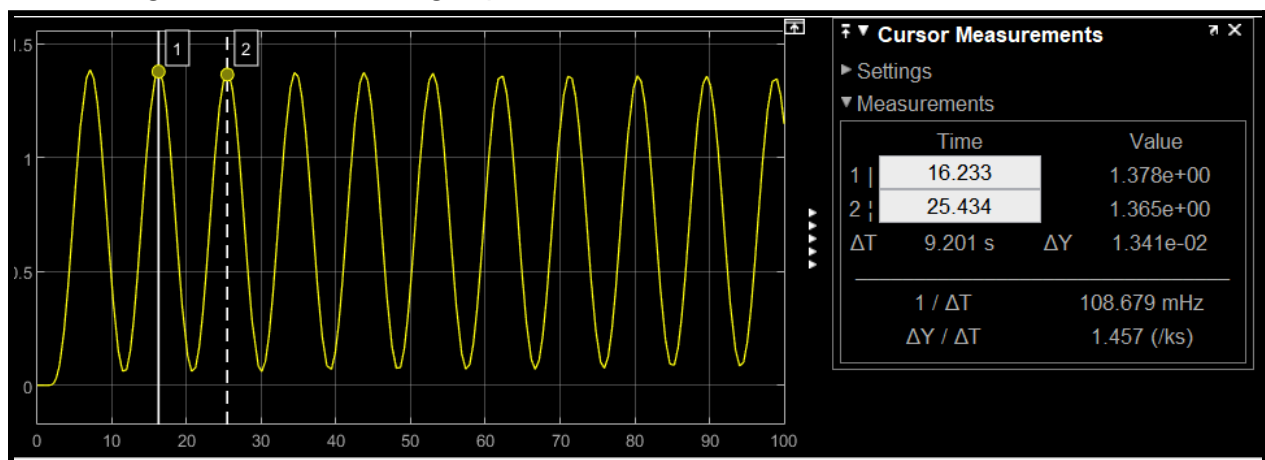
Type of controller	K_p	K_i	K_d
P	$0.5K_{cr}$	infinite	0
PI	$0.45K_{cr}$	$1.2K_p/P_{cr}$	0

PID	$0.6K_{cr}$	$2K_p/P_{cr}$	$K_p P_{cr}/8$
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Taking n=4	K_p	K_i	K_d
P	1.6350	infinite	0
PI	1.4715	0.2511	0
PID	1.9620	0.5581	1.7244

Case 2: Taking n=5

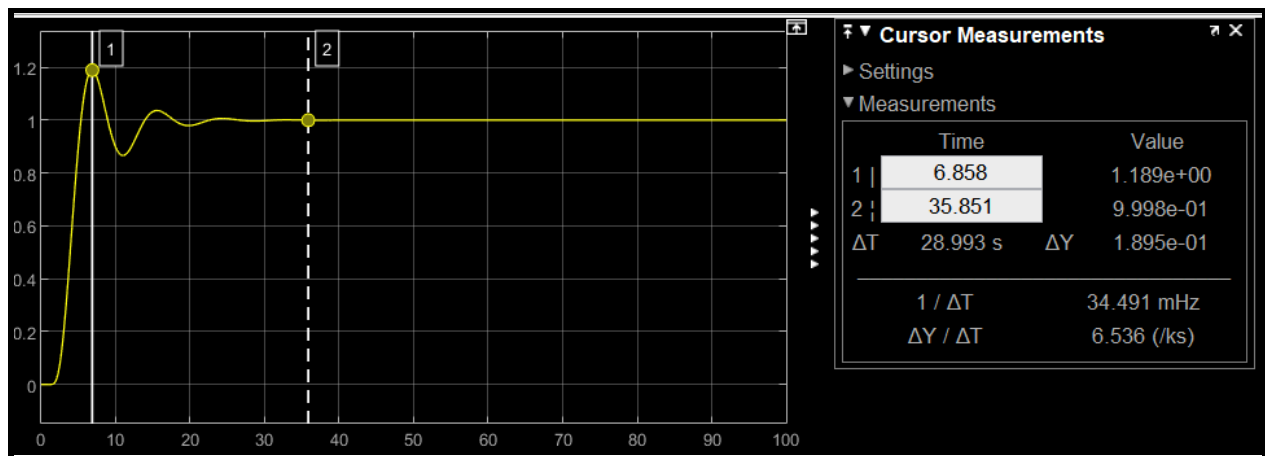
Increasing value of k till we get pure oscillations



$K_{cr} = 2.55$

$P_{cr} = 9.201$

Taking n=5	K_p	K_i	K_d
P	1.2750	infinite	0
PI	1.1475	0.1497	0
PID	1.5300	0.3326	1.7597

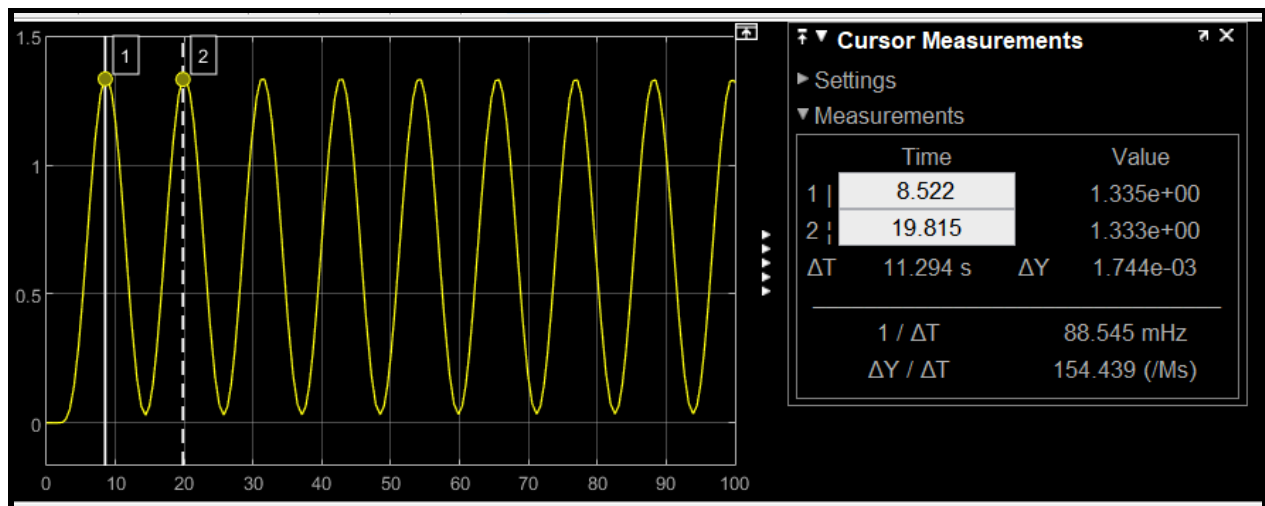


Dynamic response for PID control

Rise Time = 2.46 sec
 Peak Overshoot = 15%
 Settling Time = 18.9 sec

Case 3: Taking $n=6$

Increasing value of k till we get pure oscillations



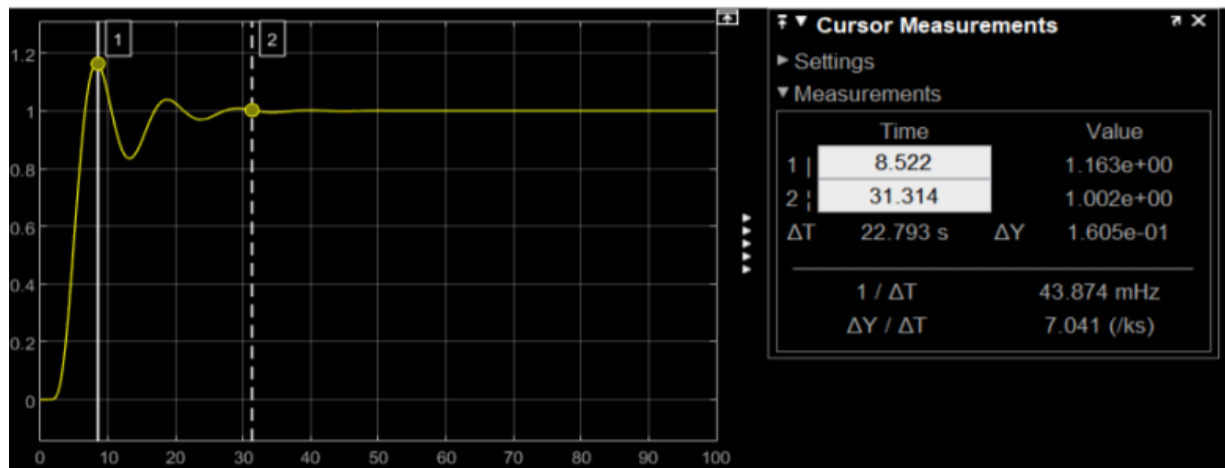
Dynamic response for PID control

$K_{cr} = 2.18$

$P_{cr} = 11.294$

Taking $n=6$	K_p	K_i	K_d
P	1.0900	infinite	0

PI	0.9810	0.1042	0
PID	1.3080	0.2316	1.8466



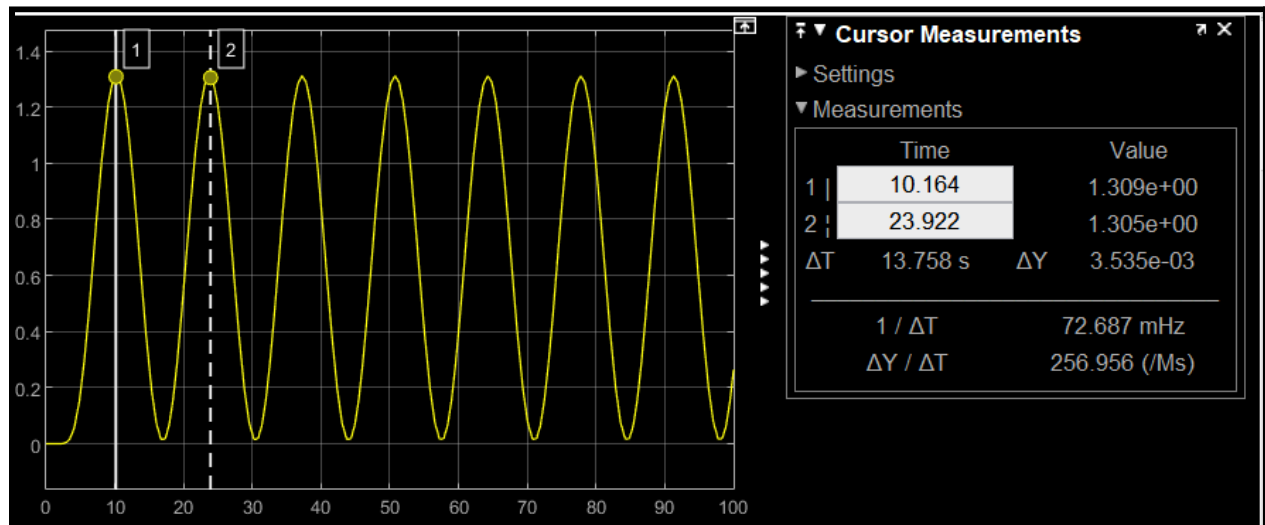
Rise Time = 3.07 sec

Peak Overshoot = 12%

Settling Time = 24.4 sec

Case 4: Taking $n=7$

Increasing value of k till we get pure oscillations

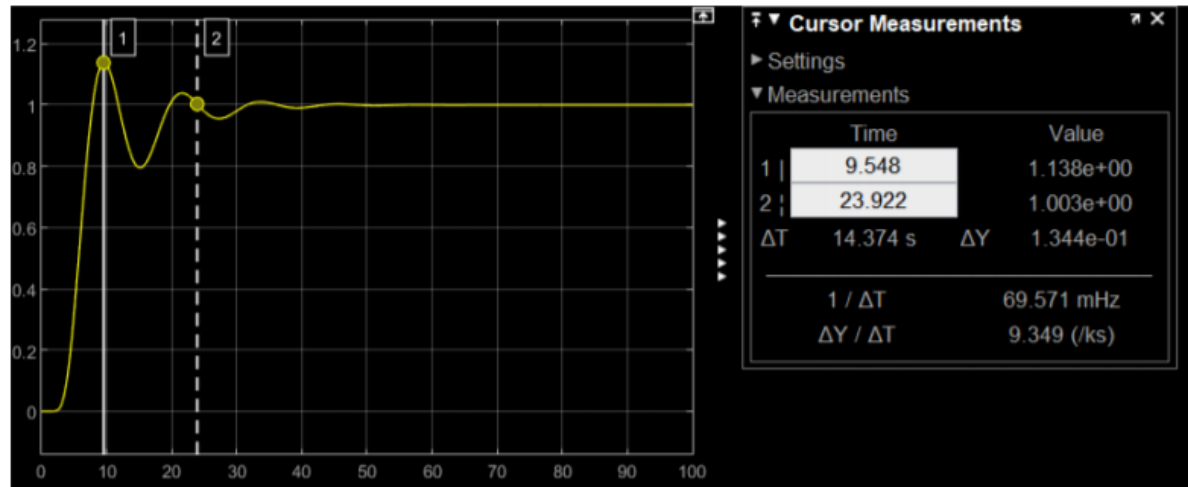


$K_{cr} = 1.95$

$P_{cr} = 13.758$

Taking $n=7$	K_p	K_i	K_d
P	0.9750	infinite	0
PI	0.8775	0.0765	0

PID	1.1700	0.1701	2.0121
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Dynamic response for PID control

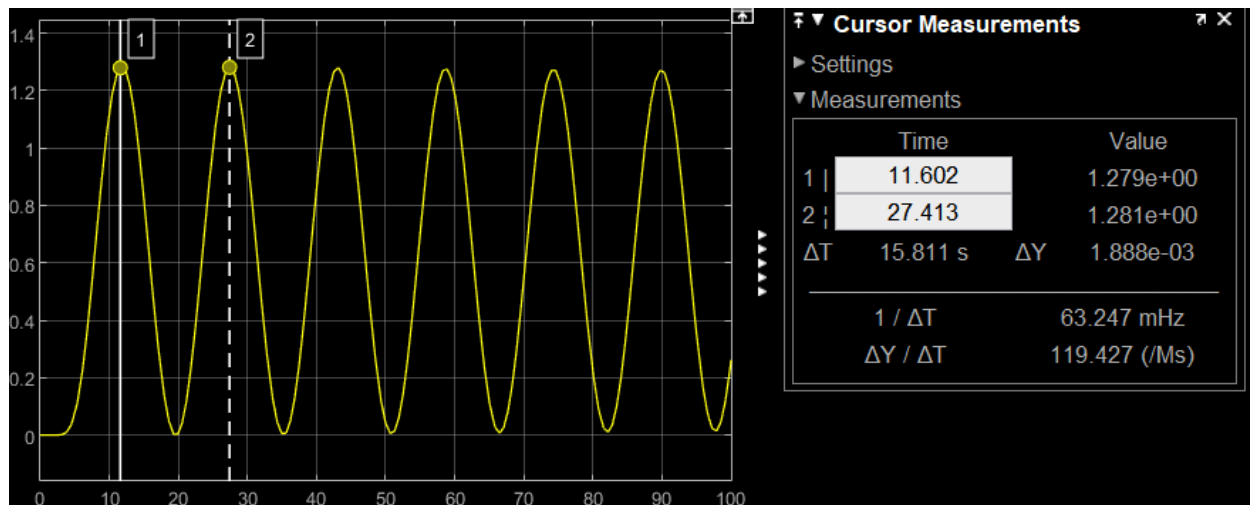
Rise Time = 3.64 sec

Peak Overshoot = 13.8%

Settling Time = 29 sec

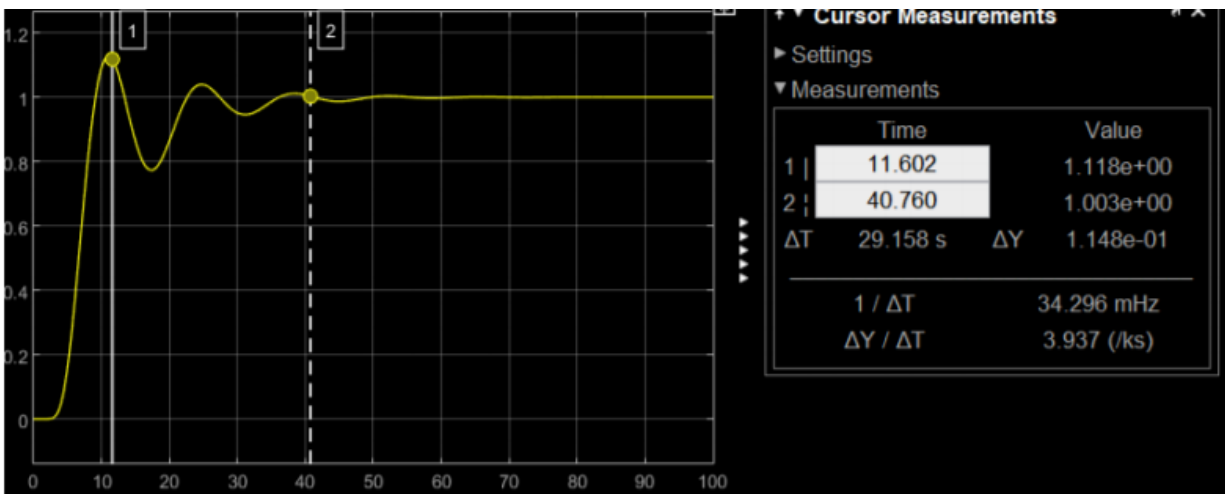
Case 5: Taking n=8

Increasing value of k till we get pure oscillations



$K_{cr} = 1.79$ $P_{cr} = 15.811$

Taking n=8	Kp	Ki	Kd
P	0.8950	infinite	0
PI	0.8055	0.0611	0
PID	1.0740	0.1359	2.1226



Dynamic response for PID control

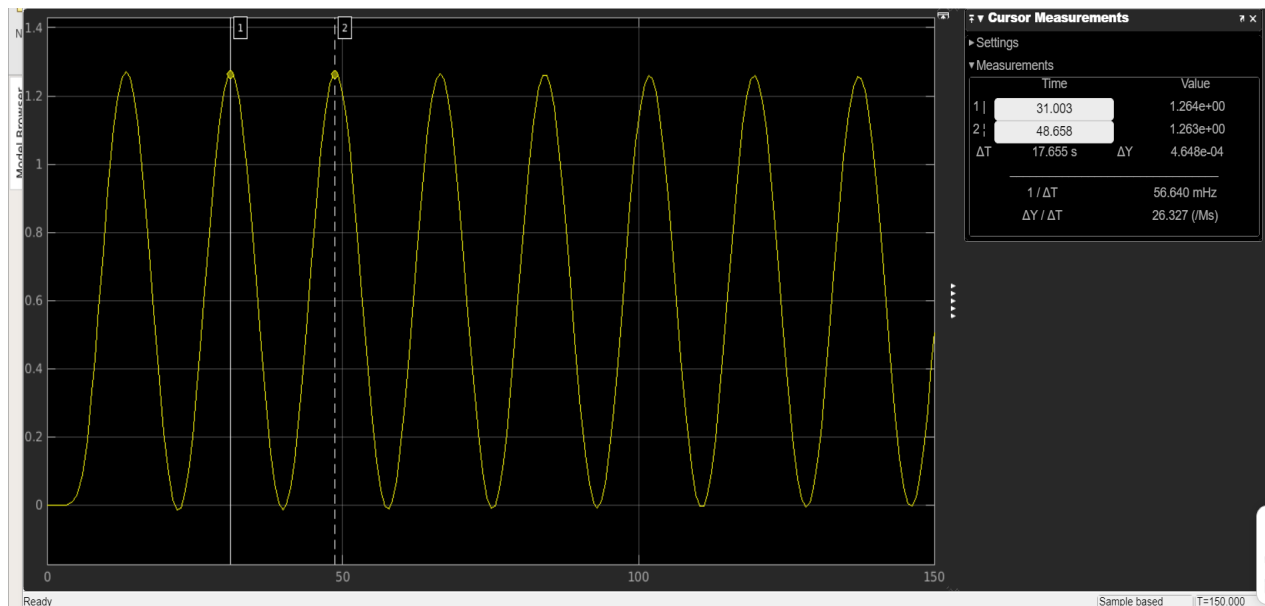
Rise Time = 4.18 sec

Peak Overshoot = 11.8%

Settling Time = 33.7 sec

Case 6: Taking $n=9$

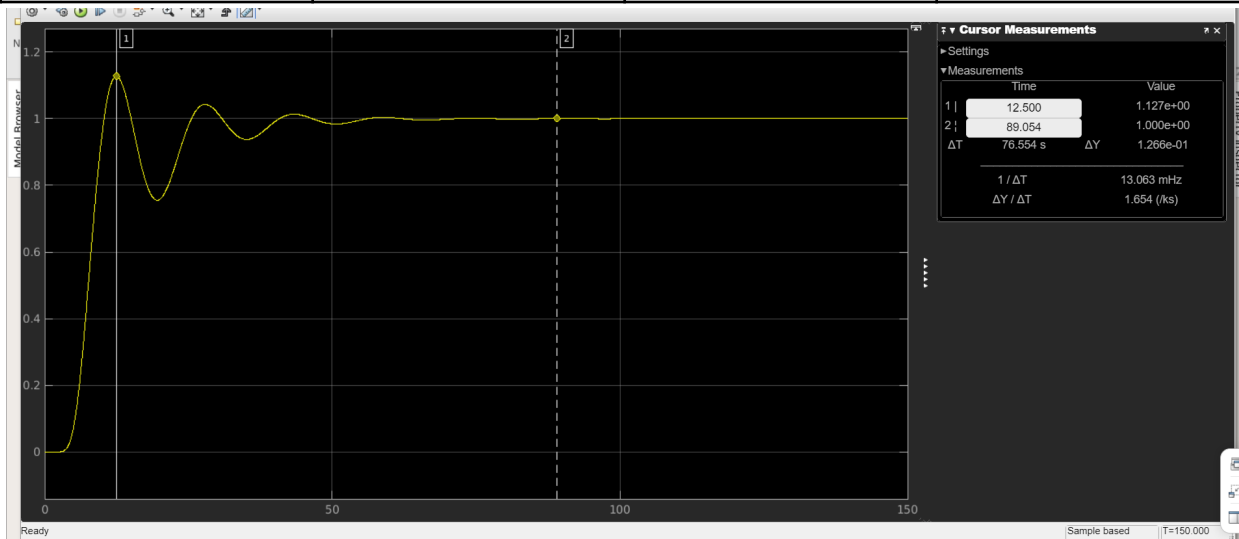
Increasing value of k till we get pure oscillations



$K_{cr} = 1.68$

$P_{cr} = 17.655$

Taking n=9	Kp	Ki	Kd
P	0.84	infinite	0
PI	0.756	0.0685	0
PID	1.008	0.1141	2.2245



Dynamic response for PID control

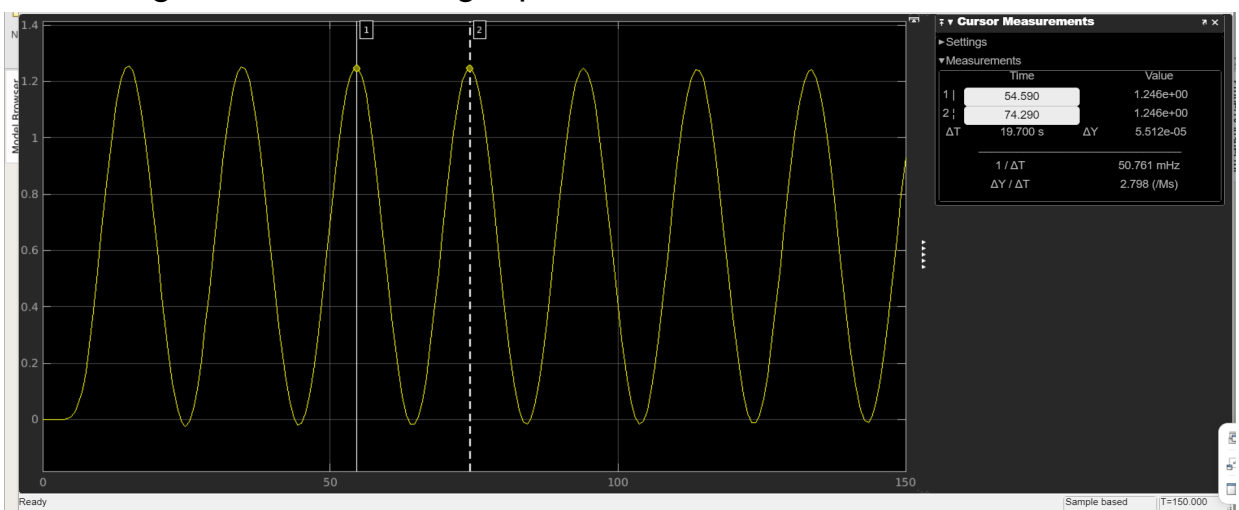
Rise Time = 4.379 sec

Peak Overshoot = 12.66%

Settling Time = 39.336 sec

Case 7: Taking n=10

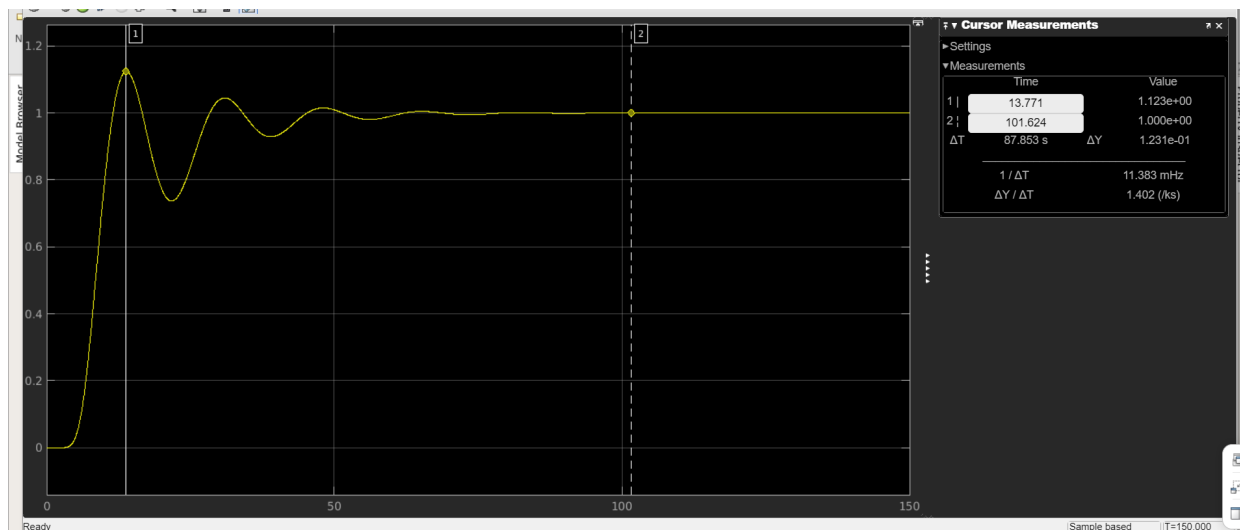
Increasing value of k till we get pure oscillations



$K_{cr} = 1.595$

$P_{cr} = 19.7$

Taking n=10	Kp	Ki	Kd
P	0.7975	infinite	0
PI	0.7177	0.0582	0
PID	0.957	0.0971	2.3566



Dynamic response for PID control

Rise Time = 4.802 sec

Peak Overshoot = 12.31%

Settling Time = 43.573 sec

Observation and Discussion

1. We observe a maximum peak overshoot of 23% for different values of n . Which can be considered a decent enough result i.e. we are able to get good enough results considering peak overshoot, rise time and settling time using closed loop continuous cycling method for multiple pole systems.
2. As n increases the rise time and settling time also increases but the increase is marginal in value.

3. As n increases the plots show that there is an increase in the peak undershoot value, which can be of concern and may be a design requirement to be included in the performance measures.

4. Merits and demerits:

Merits of using ZN method

1. The tuning rules are easy and very simple to use.
2. It often produces a good initial K_p , K_i and K_d .
3. It doesn't require an actual model/simulation of the system.

Demerits of using ZN method

1. Not mathematically rigorous, i.e. it works for a lot of systems but there is no guarantee that it will work for every system.
2. Disadvantage in using step response methods is that controller settings are aggressive, resulting in large overshoot and oscillatory responses as observed in the very first plot.
3. A disadvantage of the continuous cycling method is that the system is driven towards instability, which can lead to dangerous situations in practical applications.