Power Systems-EE309

Assignment 5

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Question1

Page No.

Date : | |

(1) Given: $Z_{12} = 0.12 + j0.16$

$$y_{12} = \frac{1}{Z_{12}} = (3 - 4j) = \cancel{5\angle 0.92}$$
$$= 5\angle -53.13° \text{ pu.}$$

Load power $\Rightarrow P_2^{act} = -100\,MW.$

$Q_2^{act} = -50\,MVAR.$

$$S_2^{act} = -(100 + 50j)\ \ MVA.$$

In pu,

$$S_2^{act} = \frac{-(100+50j)}{100} = (-1 - 0.5j)\ pu$$

Using Newton Raphson method,

$$P_2 = 5|V_2||V_1|\cos(126.87° - \delta_2 + \delta_1) + 5|V_2|^2\cos(-53.13°)$$
$$Q_2 = -5|V_2||V_1|\sin(126.87° - \delta_2 + \delta_1) - 5|V_2|^2\sin(-53.13°)$$

$V_1 = 1\angle 0°$ pu

$|V_2^{(0)}| = 1 \quad \delta_2^{(0)} = 0.0$

$$\therefore\Rightarrow P_2^{(0)} = 5(1)(1)\cos(126.87°) + 5(1)^2\cos(-53.13°)$$
$$= 0.$$
$$Q_2^{(0)} = -5(1)(1)\sin(126.87°) - 5(1)^2\sin(-53.13°)$$
$$= 0.$$
$$\Delta P_2^{(0)} = P_2^{act} - P_2^{(0)} = -1 - 0 = -1\ pu$$
$$\Delta Q_2^{(0)} = Q_2^{act} - Q_2^{(0)} = -0.5 - 0 = -0.5\ pu$$

Page No.

Date : / /

elements of jacobian matrix ⇒

$$J_{11} = \frac{\partial P_2}{\partial \delta_2} = 5\,|V_2|\,|V_1|\,\sin(126.87^\circ - \delta_2 + \delta_1)$$

$$J_{12} = \frac{\partial P_2}{\partial |V_2|} = 5|V_1|\cos(126.87^\circ - \delta_2 + \delta_1) + 10|V_2|\cos(-53.13^\circ)$$

$$J_{21} = \frac{\partial Q_2}{\partial \delta_2} = 5|V_2||V_1|\cos(126.87^\circ - \delta_2 + \delta_1)$$

$$J_{22} = \frac{\partial Q_2}{\partial |V_2|} = -5|V_1|\sin(126.87^\circ - \delta_2 + \delta_1) - 10|V_2|\sin(-53.13^\circ)$$

For 1st iteration,

$$J_{11}^{(0)} = 5(1)(1)\sin(126.87^\circ) = 3.99$$
$$J_{12}^{(0)} = 5(1)\cos(126.87^\circ) + 10(1)\cos(-53.13^\circ) = 3$$
$$J_{21}^{(0)} = 5(1)(1)\cos(126.87^\circ) = -3$$
$$J_{22}^{(0)} = -5(1)\sin(126.87^\circ) - 10(1)\sin(-53.13^\circ) = +3.99$$

For 1st iteration, set of linear eq^n ⇒

$$\begin{bmatrix} -1 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 3.99 & 3 \\ -3 & 3.99 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Solving this, $\Delta\delta_2^{(0)} = -0.1$

$\Delta|V_2^{(0)}| = -0.2$

$$\delta_2^{(1)} = \delta_2^{(0)} + \Delta\delta_2^{(0)}$$
$$|V_2^{(1)}| = |V_2^{(0)}| + \Delta|V_2^{(0)}|$$

Page No.
Date : / /

$\Rightarrow \quad \delta_2^{(1)} = 0 - 0.1 = -0.1 = -5.73^\circ$

$|V_2^{(1)}| = 1 - 0.2 = 0.8$

$P_2^{(1)} = 5(0.8)(1)\cos(126.87^\circ + 5.73^\circ + 0) + 5(0.8)^2\cos(-53.13^\circ)$
$= -0.7875$

$Q_2^{(1)} = -5(0.8)(1)\sin(126.87^\circ + 5.73^\circ + 0) - 5(0.8)^2\sin(-53.13^\circ)$
$= -0.3844$

$\Delta P_2^{(1)} = P_2^{act} - P_2^{(1)} = -1 - (-0.7875) = -0.2125$ pu

$\Delta Q_2^{(1)} = Q_2^{act} - Q_2^{(1)} = -0.5 - (-0.3844) = -0.1156$ pu

Jacobian for iteration 2,

$J_{11}^{(1)} = 5(0.8)(1)\sin(126.87^\circ + 5.73^\circ + 0) = 2.94.$

$J_{12}^{(1)} = 5(1)\cos(126.87^\circ + 5.73^\circ + 0) + 10(0.8)\cos(-53.13^\circ)$
$= 1.42.$

$J_{21}^{(1)} = 5(0.8)(1)\cos(126.87^\circ + 5.73 + 0) = -2.71$

$J_{22}^{(1)} = -5(1)\sin(126.87 + 5.73 + 0) - 10(0.8)\sin(-53.13)$
$= 2.72$

$$\begin{bmatrix} -0.2125 \\ -0.1156 \end{bmatrix} = \begin{bmatrix} 2.94 & 1.42 \\ -2.71 & 2.72 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}$$

Solving this, $\Delta\delta_2^{(1)} = -0.035$

$\Delta|V_2^{(1)}| = -0.0773$

$|V_2| = 0.8 + (-0.0773) = 0.7227$

$\delta_2 = -0.1 + (-0.035) = -0.135 = -7.735°.$

$\Rightarrow$ After 2 iterations, we get,

$$\boxed{V_2 = 0.7227 \angle -7.735°}$$

Question 2

Final results:

Y =

```
   0.8824 - 3.4994i  -0.2941 + 1.1765i  -0.5882 + 2.3529i   0.0000 + 0.0000i
  -0.2941 + 1.1765i   0.8627 - 3.0276i  -0.3333 + 1.0000i  -0.2353 + 0.9412i
  -0.5882 + 2.3529i  -0.3333 + 1.0000i   1.2157 - 4.4694i  -0.2941 + 1.1765i
   0.0000 + 0.0000i  -0.2353 + 0.9412i  -0.2941 + 1.1765i   0.5294 - 2.0576i
```

M =

```
    3.6089    1.2127    2.4254         0
    1.2127    3.1482    1.0541    0.9701
    2.4254    1.0541    4.6318    1.2127
         0    0.9701    1.2127    2.1247
```

Ph =

```
   -1.3238    1.8158    1.8158         0
    1.8158   -1.2932    1.8925    1.8158
    1.8158    1.8925   -1.3052    1.8158
         0    1.8158    1.8158   -1.3190
```

Theoretical Calculations

Q.2

1 - 2	$0.2 + j0.8$	$j0.02$
2 - 3	$0.3 + j0.9$	$j0.03$
2 - 4	$0.25 + j$	$j0.04$
3 - 4	$0.2 + j0.8$	$j0.02$
1 - 3	$0.1 + j0.4$	$j0.01$

Inspection method :-

$$y_{12} = y_{21} = \frac{1}{z_{12}} = \frac{1}{0.2+j0.8} = 0.29 - j1.17$$

$$y_{23} = y_{32} = \frac{1}{z_{23}} = \frac{1}{0.3+j0.9} = 0.33 - j$$

$$y_{24} = y_{42} = \frac{1}{z_{24}} = \frac{1}{0.25+j} = 0.23 - j0.94$$

$$y_{34} = y_{43} = \frac{1}{z_{34}} = \frac{1}{0.2+j0.8} = 0.29 - j1.17$$

$$y_{13} = y_{31} = \frac{1}{z_{13}} = \frac{1}{0.1+j0.4} = 0.58 - j2.31$$

Diagonal :

$$Y_{11} = y_{12} + y_{13} + y_1 = 0.87 - 3.49j$$

$$Y_{22} = y_{21} + y_{23} + y_{24} + y_2 = 0.85 - 3.02j$$

$$Y_{33} = y_{32} + y_{34} + y_{31} + y_3 = 1.58 - 4.46j$$

$$Y_{44} = y_4 + y_{42} + y_{43} = 0.52 - 2.05j$$

Off diagonal elements :-

$$Y_{12} = Y_{21} = -y_{12} = -0.29 + j1.17$$

$$Y_{23} = Y_{32} = -y_{23} = -0.33 + j1$$

$$Y_{24} = Y_{42} = -0.23 + j0.94$$

$$Y_{34} = Y_{43} = -0.29 + j1.17$$

$$Y_{13} = Y_{31} = -0.58 + j2.35$$

admittance matrix :

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

We have verified our results by using intuitive method and theoretical calculation for the same is given.

Question 3

Final result:

Jacobian Matrix =

$$\begin{bmatrix} 36.7888 & -12.0246 & -12.2586 & -4.2842 \\ -11.0250 & 34.6626 & -11.4072 & -6.3504 \\ -12.8325 & -13.0536 & 39.4619 & 12.1328 \\ 1.9886 & -0.2352 & -1.5088 & 15.0581 \end{bmatrix}$$

Theoretical Calculation:

Page No.

Date: / /

(3) No. of buses = 4

no. of PQ buses = 1

$$V_{mag} = \begin{bmatrix} 1.04 \\ 0.9946 \\ 1 \\ 1.077 \end{bmatrix} \qquad delta = \begin{bmatrix} 0 \\ 0.1718 \\ 0 \\ 0.2630 \end{bmatrix}$$

Y - bus matrix is given.

$G = Re[Y]$ 4x4 matrix

$B = Img[Y]$ 4x4 matrix.

$$G_{ij} = \begin{cases} 5.88 & ; \quad i = j \quad \text{(diagonal elements)} \\ -2.94 & ; \quad i \neq j \quad \text{(non - diagonal elements)} \end{cases}$$

$$B_{ij} = \begin{cases} -23.50 & ; \quad i = j \quad \text{(diagonal elements)} \\ 11.76 & ; \quad i \neq j \quad \text{(non diagonal elements)} \end{cases}$$

Real power injected at i^{th} bus,

$$P_i = \sum_{k=1}^{n} |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$P_i = \sum_{k=1}^{4} |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i)$$

Page No.

Date : | |

$$P_i = \sum_{k=1}^{4} |V_i V_k| \left[G_{ik} \cos(\delta_i - \delta_k) + B_{ik} \sin(\delta_i - \delta_k)\right]$$

$$P_1 = |V_1| \Big[|V_1| \left(G_{11} \cos(\delta_1 - \delta_1) + B_{11} \sin(\delta_1 - \delta_1)\right)$$
$$+ |V_2| \left(G_{12} \cos(\delta_1 - \delta_2) + B_{12} \sin(\delta_1 - \delta_2)\right)$$
$$+ |V_3| \left(G_{13} \cos(\delta_1 - \delta_3) + B_{13} \sin(\delta_1 - \delta_3)\right)$$
$$+ |V_4| \left(G_{14} \cos(\delta_1 - \delta_4) + B_{14} \sin(\delta_1 - \delta_4)\right) \Big]$$

$$= 1.04 \left[(1.04 \times 5.88) + (-4.8806) + (-2.94) + (-6.35) \right]$$

$$P_1 = -8.3776$$

$$P_2 = |V_2| \Big[|V_1| \left[G_{21} \cos(\delta_2 - \delta_1) + B_{21} \sin(\delta_2 - \delta_1)\right]$$
$$+ |V_2| \left(G_{22} \cos(\delta_2 - \delta_2) + B_{22} \sin(\delta_2 - \delta_2)\right)$$
$$+ |V_3| \left(G_{23} \cos(\delta_2 - \delta_3) + B_{23} \sin(\delta_2 - \delta_3)\right)$$
$$+ |V_4| \left(G_{24} \cos(\delta_2 - \delta_4) + B_{24} \sin(\delta_2 - \delta_4)\right) \Big]$$

$$= |V_2| \left[(-0.9217) + (5.8482) + (-0.88627) (-4.3067) \right]$$

$$P_2 = -0.266.$$

$$P_3 = |V_3| \Big[|V_1| \left(G_{31} \cos(\delta_3 - \delta_1) + B_{31} \sin(\delta_3 - \delta_1) \right)$$
$$+ |V_2| \left(G_{32} \cos(\delta_3 - \delta_2) + B_{32} \sin(\delta_3 - \delta_2) \right)$$
$$+ |V_3| \left(G_{33} \cos(\delta_3 - \delta_3) + B_{33} \sin(\delta_3 - \delta_3) \right)$$
$$+ |V_4| \left(G_{34} \cos(\delta_3 - \delta_4) + B_{34} \sin(\delta_3 - \delta_4) \right) \Big]$$

$$P_3 = |V_3| \left[(-3.0576) + (-4.88066) + (5.88) + (-6.3503) \right]$$

$$\boxed{P_3 = -8.4084}$$

$$P_4 = |V_4| \Big[|V_1| \left(G_{41} \cos(\delta_4 - \delta_1) + B_{41} \sin(\delta_4 - \delta_1) \right) + |V_2| \left(G_{42} \cos(\delta_4 - \delta_2) + B_{42} \sin(\delta_4 - \delta_2) \right) + |V_3| \left(G_{43} \cos(\delta_4 - \delta_3) + B_{43} \sin(\delta_4 - \delta_3) \right) + |V_4| \left(G_{44} \cos(\delta_4 - \delta_4) + B_{44} \sin(\delta_4 - \delta_4) \right) \Big]$$

$$= |V_4| \left[(0.22718) + (-1.84673) + (0.21844) + (6.33276) \right]$$

$$\boxed{P_4 = 5.312}$$

$$Q_i = - \sum_{k=1}^{n} |V_i V_k Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i)$$

$$Q_i = |V_i| \sum_{k=1}^{n} \left[|V_k| \left(G_{ik} \sin(\delta_i - \delta_k) - B_{ik} \cos(\delta_i - \delta_k) \right) \right]$$

$$Q_1 = |V_1| \left[(24.44) + (-11.0244) + (-11.76) + (-11.4068) \right]$$

$$Q_1 = -10.142$$

Similarly,

$Q_2 =$ ~~-13.5043~~ -13.5403.

$Q_3 = -11.1626$

$Q_4 = -12.2019$.

$J_{11} =$ 3×3 matrix.

$$J_{11} = \begin{bmatrix} L_{22} & L_{23} & L_{24} \\ L_{32} & L_{33} & L_{34} \\ L_{42} & L_{43} & L_{44} \end{bmatrix}$$

$$L_{ik} = -|Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \quad ; i \neq k.$$

$$L_{ik} = |V_i V_k| \left[G_{ik} \sin(\delta_i - \delta_k) - B_{ik} \cos(\delta_i - \delta_k) \right] \quad ; i \neq k.$$

$$L_{ii} = -Q_i - |V_i|^2 B_{ii} \quad ; i = k.$$

So,

$$\begin{aligned} L_{22} &= -Q_2 - |V_2|^2 B_{22} \\ &= +(13.5403) - (0.9946)^2(-23.50) \\ &= 36.788. \end{aligned}$$

$$\begin{aligned} L_{33} &= -Q_3 - |V_3|^2 B_{33} \\ &= -(-11.1616) - (1)^2(-23.50) \\ &= 34.662 \end{aligned}$$

$$\begin{aligned} L_{44} &= -Q_4 - |V_4|^2 B_{44} \\ &= -(-12.2019) - (1.077)^2(-23.50) \\ &= 39.46. \end{aligned}$$

$$\begin{aligned} L_{23} &= |V_2 V_3| \left[G_{23} \sin(\delta_2 - \delta_3) - B_{23} \cos(\delta_2 - \delta_3) \right] \\ &= -12.024 \end{aligned}$$

$$\begin{aligned} L_{24} &= |V_2 V_4| \left[G_{24} \sin(\delta_2 - \delta_4) - B_{24} \cos(\delta_2 - \delta_4) \right] \\ &= -12.258. \end{aligned}$$

$$\begin{aligned} L_{32} &= |V_3 V_2| \left[G_{32} \sin(\delta_3 - \delta_2) - B_{32} \cos(\delta_3 - \delta_2) \right] \\ &= -11.025 \end{aligned}$$

Page No.
Date: / /

$$L_{34} = |V_3 V_4| \left[G_{34} \sin(\delta_3 - \delta_4) - B_{34} \cos(\delta_3 - \delta_4) \right]$$
$$= -11.407$$

$$L_{42} = |V_4 V_2| \left[G_{42} \sin(\delta_4 - \delta_2) - B_{42} \cos(\delta_4 - \delta_2) \right]$$
$$= -12.832$$

$$L_{43} = |V_4 V_3| \left[G_{43} \sin(\delta_4 - \delta_3) - B_{43} \cos(\delta_4 - \delta_3) \right]$$
$$= -13.053$$

$$\Rightarrow \quad J_{11} = \begin{bmatrix} 36.788 & -12.024 & -12.258 \\ -11.025 & 34.662 & -11.407 \\ -12.832 & -13.053 & 39.46 \end{bmatrix}$$

$J_{21} = 1 \times 3$
$J_{12} = 3 \times 1$
$J_{22} = 1 \times 1$

For $J_{21} \Rightarrow \quad J_{21} = [M_{42} \quad M_{43} \quad M_{44}]$

$$M_{42} = -|Y_{42} V_4 V_2| \cos(\theta_{42} + \delta_2 - \delta_4)$$
$$= -|V_4 V_2| \left[G_{42} \cos(\delta_4 - \delta_2) + B_{42} \sin(\delta_4 - \delta_2) \right]$$
$$= 1.988$$

$$M_{43} = -|V_4 V_3| \left[G_{43} \cos(\delta_4 - \delta_3) + B_{43} \sin(\delta_4 - \delta_3) \right]$$
$$= -0.2352$$

$$M_{44} = P_4 - |V_4|^2 (G_{44})$$
$$= 5.312 - (1.077)^2 (5.88)$$
$$= -1.508$$

Page No.

Date : / /

$$J_{12} = \begin{bmatrix} -4.2842 \\ -6.3504 \\ 12.132 \end{bmatrix}$$

$\Rightarrow J_{22} \Rightarrow [1\times1] = [O_{44}]$.

$$O_{44} = Q_{44} - |V_4|^2 B_{44}$$
$$= (-12.2019) - (1.077)^2 (-23.50)$$
$$= 15.057.$$

$\Rightarrow$ Jacobian matrix $= \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$

Jacobian matrix =

$$\begin{bmatrix} 36.788 & -12.024 & -12.258 & -4.2842 \\ -11.025 & 34.662 & -11.407 & -6.3504 \\ -12.832 & -13.053 & 39.46 & +12.132 \\ 1.988 & -0.2352 & -1.508 & 15.057 \end{bmatrix}$$

➢ No assumptions are made, only given data is used in all the questions.

Conclusion:

Newton-Raphson method is useful to calculate the voltages of all the buses in the power system after a certain number of iterations to converge the del(P) and del(Q) values to 0 or a very small number. Once knowing the bus voltages, we can have an overall idea of the load power flow in the system.