Control Engineering
Experiment - 5

CONTROLLER DESIGN ON MATLAB PLATFORM BY DISCRETE FREQUENCY RESPONSE

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Aim of the experiment

To examine the consequences of negative feedback of different gains on Gain and Phase margins in a digital OLTF. As well as looking at the sensitivity of choice of sampling time on the same.

Theory

All the time domain methods are convenient to work in situations where the existing output of an OLTF and desired output by the required CLTF are known in terms of response measures through experiments but there are several systems where we don't have the freedom to apply any inputs we want(i.e. experiments) and are susceptible to a large range of disturbances.

To tackle such issues frequency-domain methods are used through designs by **Bode Diagram** or by **Nyquist Diagram.**

Bandwidth is defined as the frequency between the limits (within which the system is expected to perform) at which the power of the output signals drops to half its maximum value. The main advantage of frequency domain designs by magnitude and phase variations has to do with the additive and subtractive characteristics which they provide.

Consider a system with open-loop gain A_{OL} closed with negative feedback β:

$$\frac{A_{OL}}{1+\beta A_{OL}}$$

We can easily observe that this relation can approach ∞ when the product $\beta A_{OL} = -1$. Bode plots are used to determine just how close a system is to thereby become unstable.

One measure of proximity to instability is the **gain margin.** It is defined as the distance separation of $|\beta A_{OL}|$ in dB from 0 dB at the frequency where the phase of βA_{OL} is -180° (**phase crossover frequency**).

Another equivalent measure of proximity to instability is the **phase margin**. It is defined as the distance of the phase in degrees above $-180°$ at the frequency where its magnitude reaches unity (**gain crossover frequency**).

For discrete-time systems, the bode evaluates the frequency response on the unit circle

$$z = e^{jwT_s} \text{ and } 0 \leq \omega \leq \omega_N = \frac{\pi}{T_s}$$

where Ts is the sampling time and ω_N is the Nyquist frequency

Transfer Function

With A/D and D/A converters included appropriately, the digital OLTF of a system is given by:

$$G_{OL}(z) = 10^{-5} * \frac{4.711*z+4.644}{z^3-2.875*z^2+2.753*z-0.8781}$$

The given digital OLTF is closed through negative feedback of gain K. So, the effective open loop transfer function is given by:

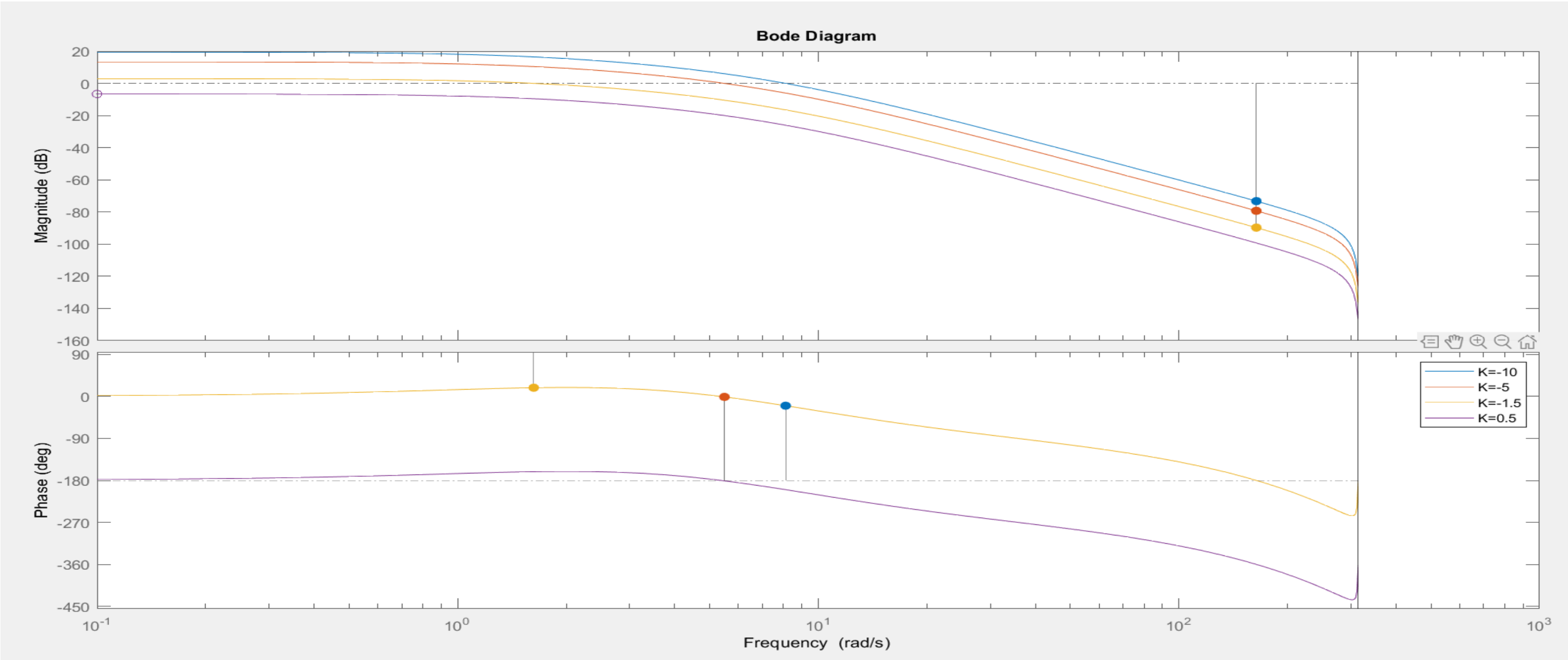
$$G_{Eff}(z) = K * 10^{-5} * \frac{4.711*z+4.644}{z^3-2.875*z^2+2.753*z-0.8781}$$

The nominal sampling time for the system is 0.01s, and you are told that the system is suspected to have serious stability concerns close to the band boundaries.

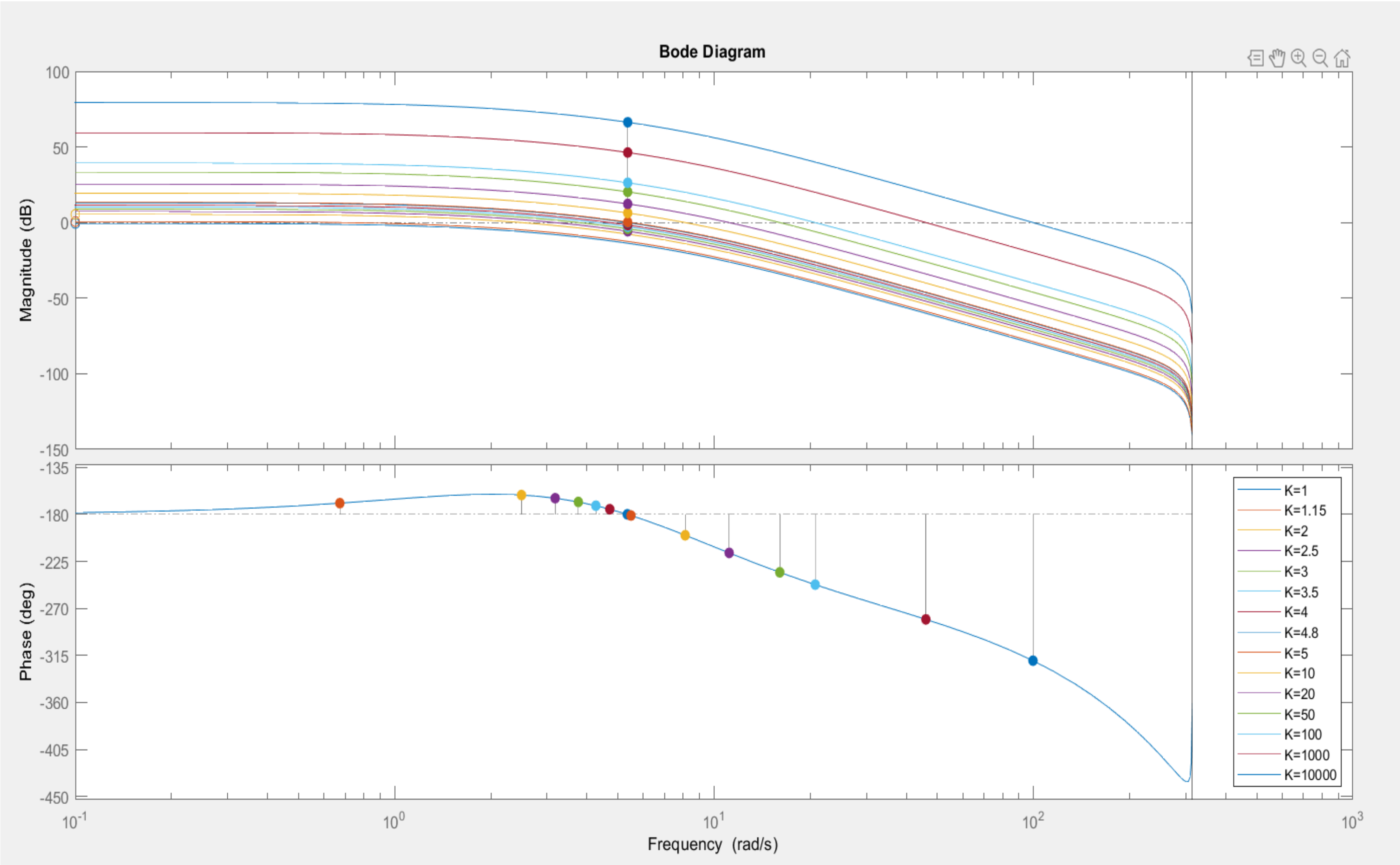
Graphs and Analysis

<u>Effect of Gain 'K' on Gain margin and Phase margin:</u>

In this section, we are trying to see the impact of gain K on the gain margin and phase margin of the system keeping the sampling period constant, Ts=0.01s.



Plot 1: Bode diagram for {K=-10,-5,-1.5,0.5}



Plot 2: Bode diagram for variable K>0

Data collected from the plots are tabulated below.

Gain, K	Gain Margin, Gm	Phase Margin, Pm	Stability of System
-10	73.2	160	Unstable
-5	79.3	179	Unstable
-1.5	89.7	-161	Unstable
0.5	6.6	-	Unstable
1	0.579	-	Unstable
1.15	-0.635	10.8	Stable

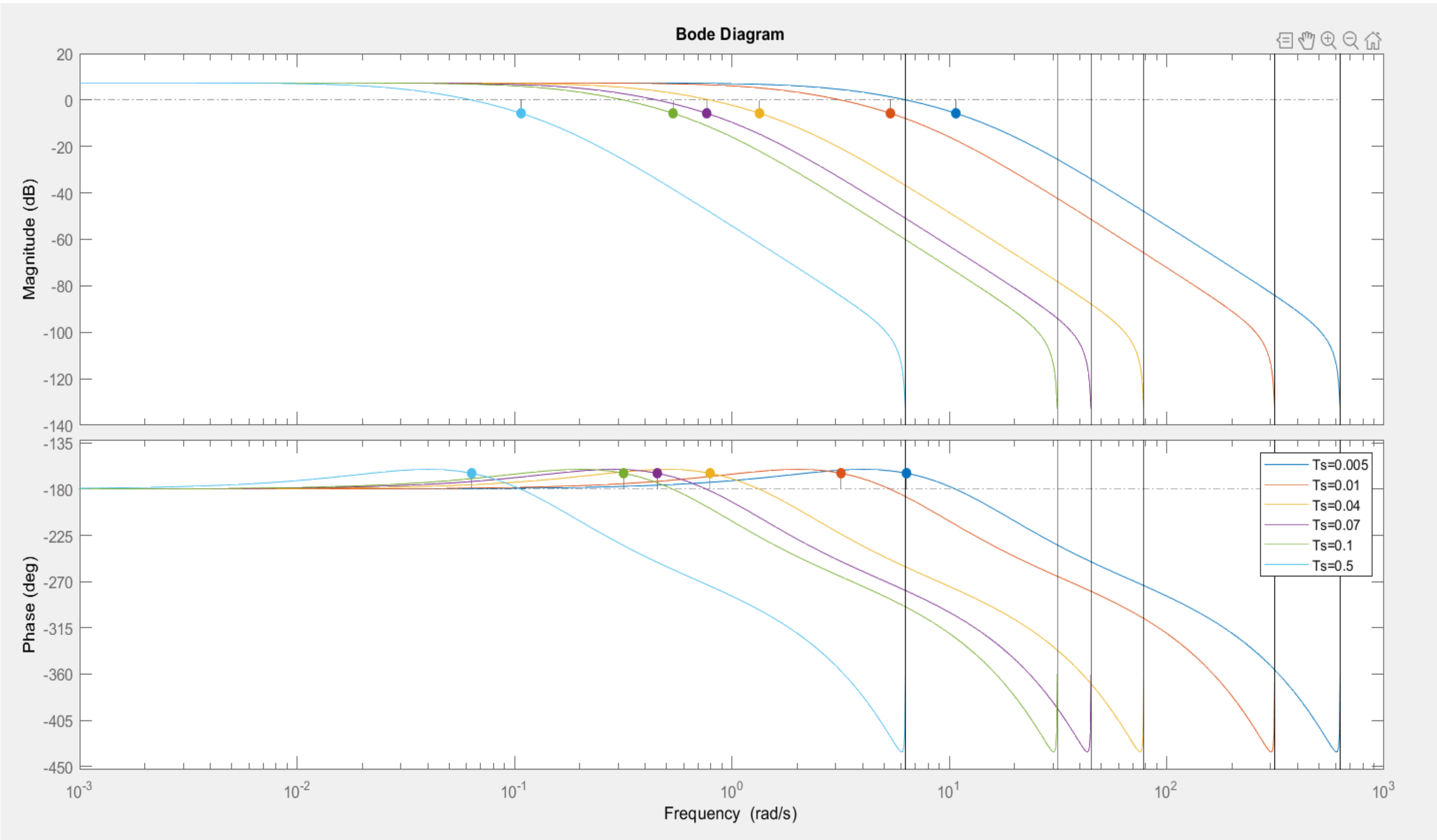
2	-5.44	18.6	Stable
2.5	5.71	15.6	Stable
3	4.12	12	Stable
3.5	2.79	8.4	Stable
4	1.63	5.01	Stable
4.8	0.0422	0.133	Stable
5	-0.312	-0.984	Unstable
10	-6.33	-19.9	Unstable
20	-12.4	-36.8	Unstable
50	-20.3	-55.4	Unstable
100	-26.3	-67.2	Unstable
1000	-46.3	-100	Unstable
10000	-66.3	-140	Unstable

For the negative values of gain K, the system is unstable which is quite obvious. So, we will talk about only the K>0 condition.

- It can be seen from the table that our system will be stable for $1.15 \leq K \leq 4.8$**.**
- In the stable region, when K increases, both gain margin and phase margin are decreasing.
- The phase crossover frequency is the same for all values of K>0.
- Gain crossover frequency will increase with the increase in K value for all K>0 and that's why after certain gain K≃2, we are getting decreasing values of phase margin.
- There are some values of K for which gain crossover frequency does not exist,i.e., the gain is never equal to 0dB and that is observed for $0 < K \leq 1.05$
- We have the values of K, for which the gain margin is negative and the system is stable at the same time and the same is observed for $1.15 \leq K \leq 2.25$
- A negative gain margin indicates that the system will lose its stability if the gain is decreased, while positive gain margins indicate that the system will lose its stability if the gain is increased. This gives us that a lower absolute value of Gain margin implies lower proximity to instability.

Effect of Sampling Time 'Ts' on Gain margin and Phase margin:

You may assume that the change of sampling time changes the Gain and Phase margin but as we mentioned earlier how z and Ts are related, it can be easily concluded that for a given K as we increase or decrease the sampling time, the magnitude and phase plots get squeeze towards left or expand towards right respectively as ωT should remain constant for a phase. It can also be visualized from the plots below.



Plot 3: Bode diagram for K=2.5 and Ts={0.005,0.01,0.04,0.07,0.1,0.5}

- Gain margin and phase margin are not affected by the change in sampling time.
- On increasing the value of sampling time, Gain crossover frequency and Phase crossover frequency will decrease.

Appropriate solutions as compensating blocks for gain and phase, cascaded to the OLTF

Since here our system stability is only concerned with phase and gain margins. So, either we can increase the phase and gain margins for already stable values of K to make the system more stable or we can increase the range of K for which the system is stable.

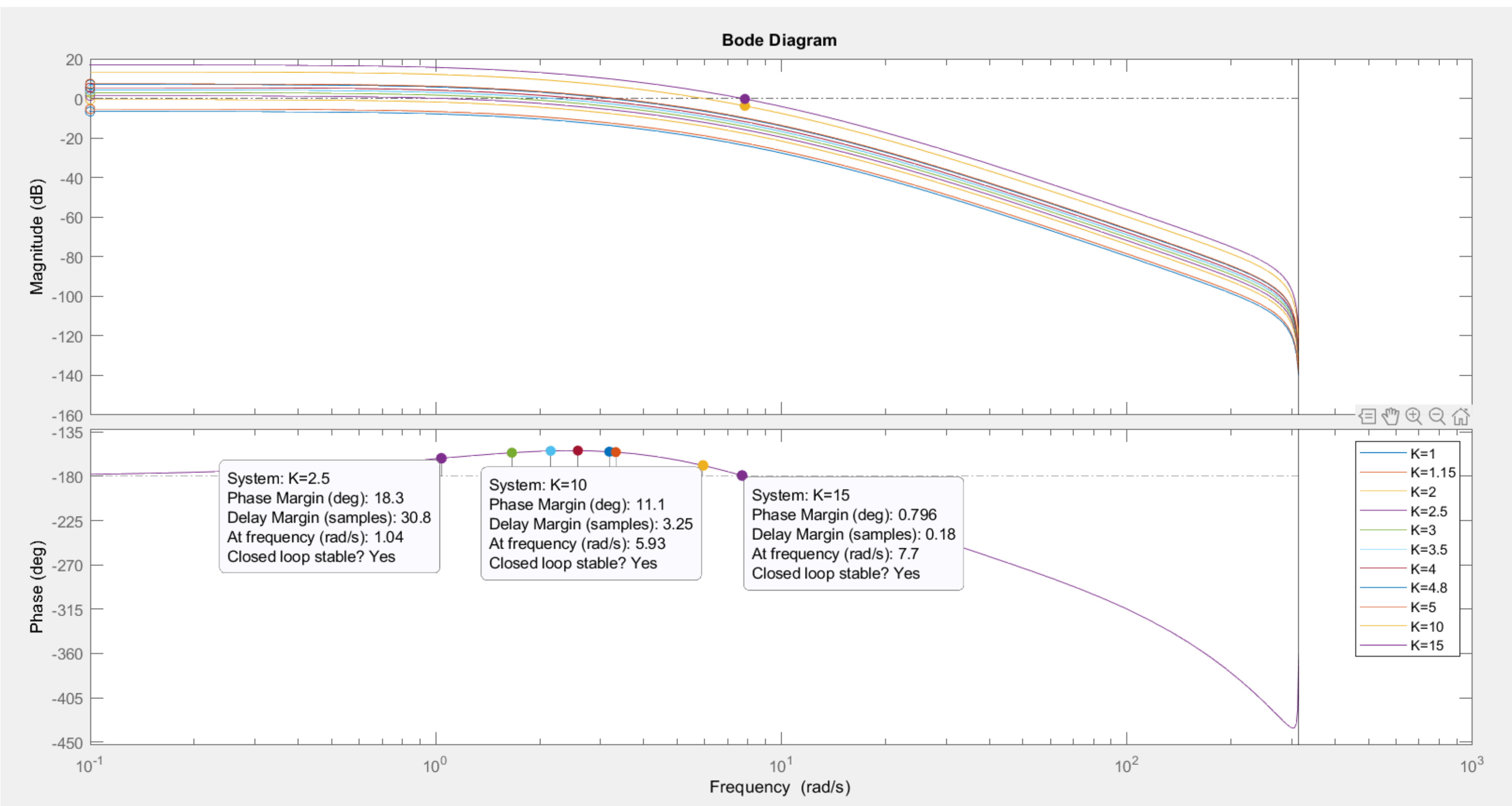
For the first case, we can use a lead lag compensator . A lead-lag compensator combines the phase-lead and phase-lag sections. The phase-lag section improves the phase margin and the dc gain; the phase-lead section improves the bandwidth and the phase margin. Since both lead and lag sections can contribute to the phase margin improvement, the desired PM improvement can be distributed among the two sections.

We are trying to design a lead compensator to increase the range of K.

Lead compensator transfer function is given as

$$G_C(s) = \frac{1+aTs}{a(1+Ts)}$$

Here we plot the bode diagrams for different values of k to find permissible values respectively.



Plot 4: With compensator having 'a=2' and 'T=0.05'

The plot is drawn for various values of K at Ts=0.01. And with this we can say that by using the compensator, the range of K for which the system is stable is increased.

System's stability:
Without compensator : $1.15 \le K \le 4.8$
With compensator (a=2,T=0.05) : $2.5 \le K \le 15$ (our solution for stability)

So, for different combinations of 'a' and 'T', we can increase the range of the gain K for which our system is stable.

Conclusion

- System will be stable for $1.15 \le K \le 4.8$. Further for $1.15 \le K \le 2.25$, gain margin is negative and the system is stable at the same time
- On increasing the value of sampling time, Gain crossover frequency and Phase crossover frequency will decrease.
- There is no impact of Sampling time Ts on the phase and gain margins.
- Using compensators, we can increase the range of the gain K for which our system is stable.