Dominant dynamic response of Analog systems on MATLAB platform.

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Objective:

The project requires analysis and discussion of the dynamic response of a given linear analog system in terms of different performance measures.

Theory:

- Percentage overshoot - The difference between the magnitude of the highest peak of time response and magnitude of its steady-state expressed in terms of percentage of the steady-state value of the response.
- Settling time - It corresponds to how quickly the system reaches the steady-state value.
- Steady-state error - the difference between actual output and desired output at the infinite range of time. Steady State is a really important parameter while considering the dynamic response. Usually, errors accumulate in the system during the transient phase which is reflected once the steady-state is achieved.

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

  Where E(s) is the Laplace transform of the error signal, e(t)

- Time to first peak - the time required by the response to reach its first peak i.e. the peak of the first cycle of oscillation, or first overshoot.

TRANSFER FUNCTION AND ITS RESPONSE IN CLTF TO A UNIT STEP

The dominant second-order analog OLTF of a system (which has no zeros) is known to be:

$$G_{OL}(s) = \frac{1}{(s+3)(s+5)}$$

for which a given unity negative feedback P-I controller has the form:

$$G_C(s) = \frac{K(s+a)}{s} \quad \text{where, } K = K_p$$

Proportional Integral (PI) Controller

The proportional-integral controller produces an output, which is the combination of outputs of the proportional and integral controllers.

The transfer function of a proportional-integral controller is = $Kp + \frac{Ki}{s}$.

The proportional-integral controller is used to *decrease the steady-state error without affecting the stability of the control system*.

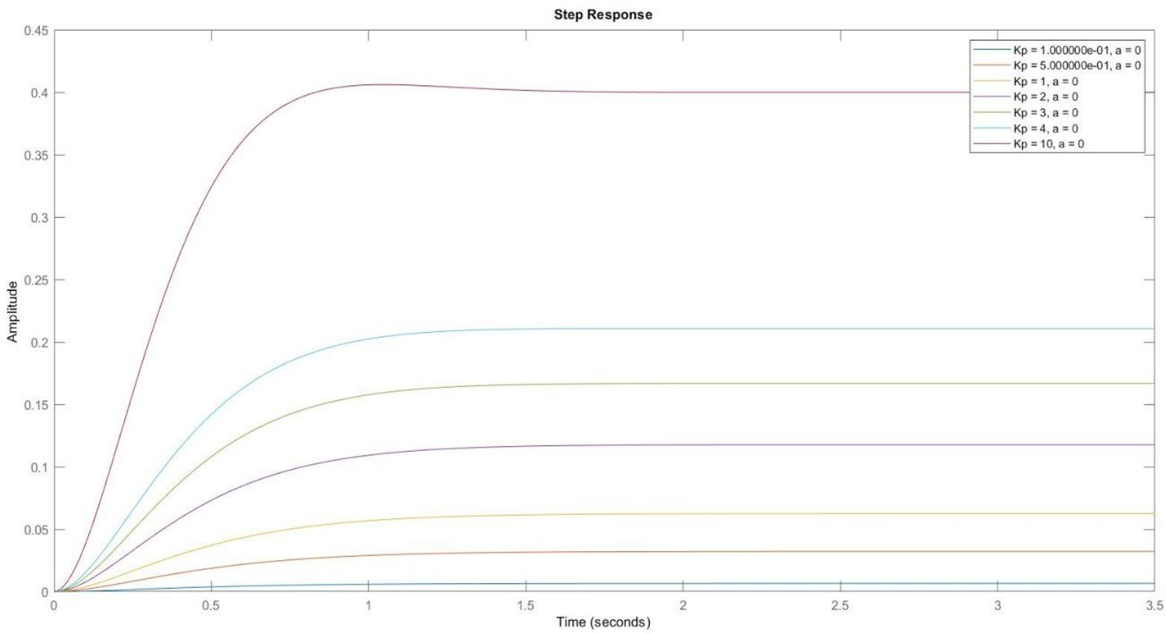
- **Keeping a constant**

For the fixed set of values of Kp (=1,2,3,4,10), 'a' is varied and different parameters of their step response are compared and listed below in the tables.

(a) $a = 0$

For this special case, the controller will not work as a PI controller since the integral constant becomes zero. It has become a *proportional controller*.

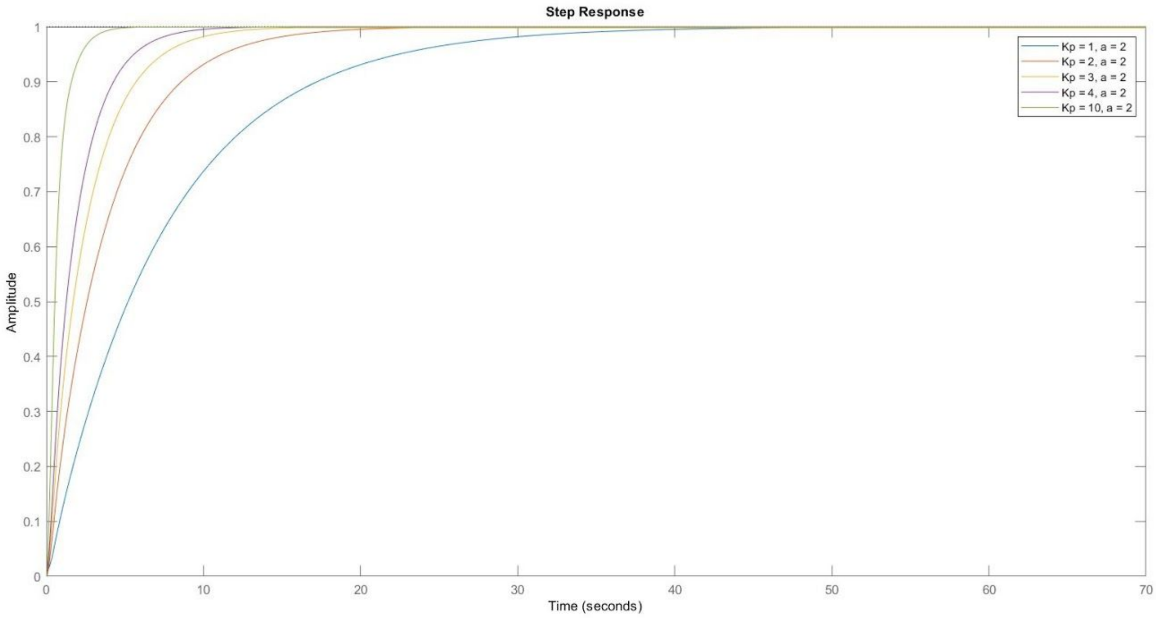
Value of K	**Value of a**	**Peak Time**	**Percentage overshoot**	**Settling time**	**Steady State Error**
0.5	0	2.8732	0	1.5277	0.9679
1	0	2.7900	0	1.4585	0.9375
2	0	2.1759	0	1.3322	0.8826
3	0	2.0608	0.0089	1.2207	0.8335
4	0	1.8190	0.0706	1.2950	0.7895
10	0	1.0477	1.5165	0.7512	0.5997



(b) $a = 2$

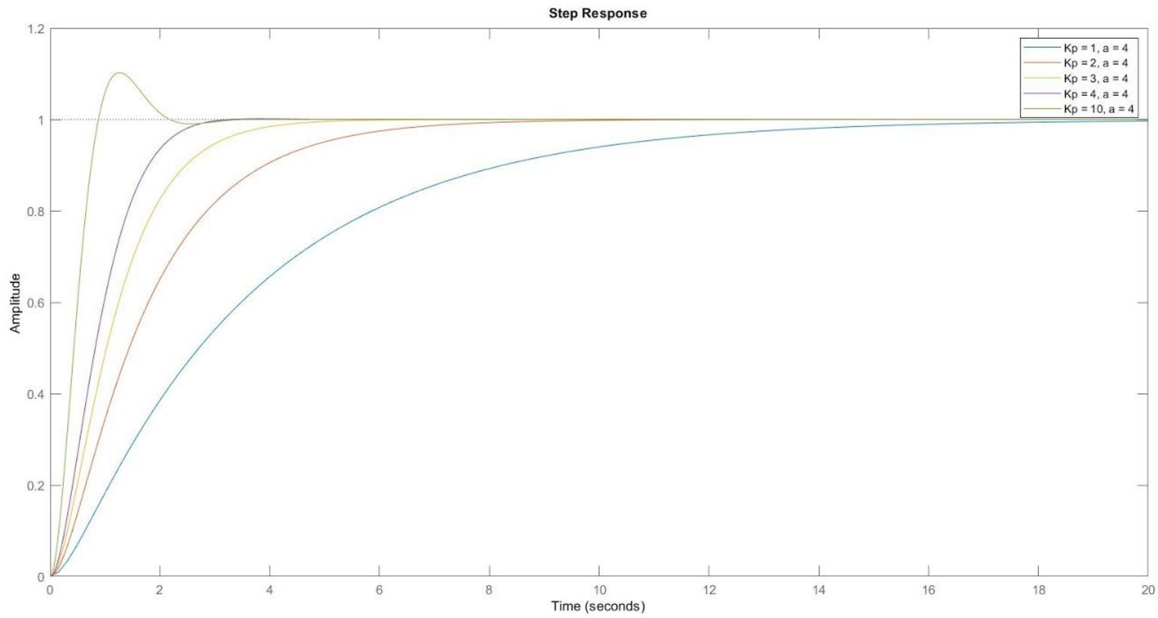
Value of K	**Value of a**	**Peak Time**	**Percentage overshoot**	**Settling time**	**Steady State Error**
1	2	78.8169	0	29.2560	1.2621e-04

2	2	23.4633	0	14.5999	0.0046
3	2	15.9755	0	9.7246	0.0041
4	2	16.7494	0	7.2950	4.4170e-04
10	2	5.9148	0	2.9846	0.0019



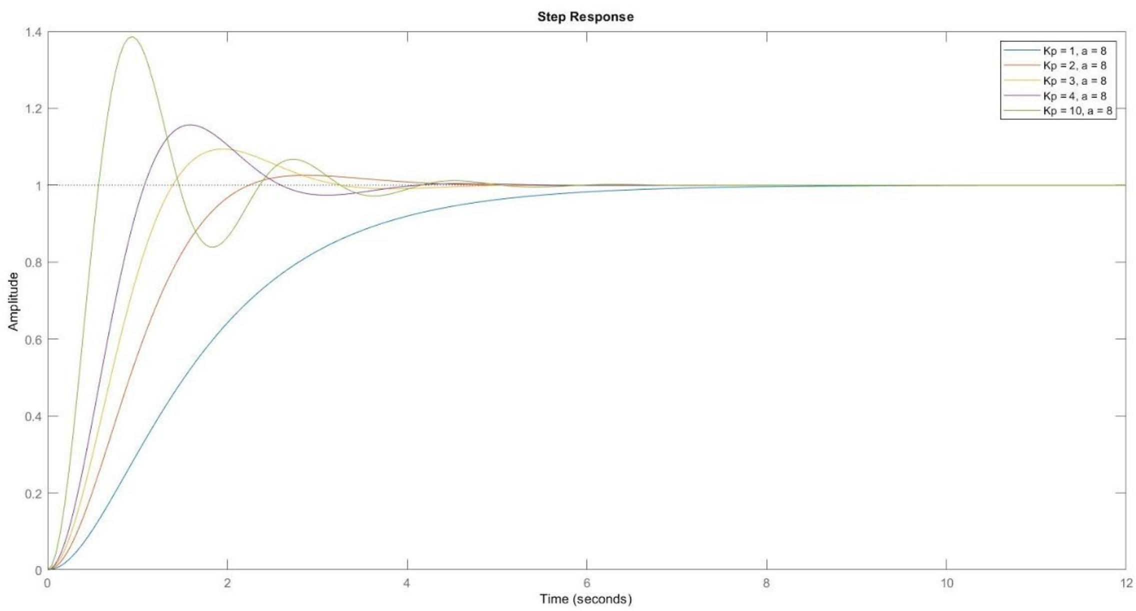
(c) $a = 4$

Value of K	Value of a	Peak Time	Percentage overshoot	Settling time	Steady State Error
1	4	25.1861	0	13.7831	0.0033
2	4	14.5293	0	6.3618	3.3996e-04
3	4	9.7700	0	3.79	6.6957e-05
4	4	3.8319	0.1545	2.4832	0.0014
10	4	1.2707	10.2465	1.9459	0.096



(d) $a = 8$

Value of K	Value of a	Peak Time	Percentage overshoot	Settling time	Steady State Error
1	8	11.9274	0	5.7831	8.0010e-04
2	8	2.8704	2.5752	3.3548	5.5846e-04
3	8	1.9388	9.4003	2.9096	0.0093
4	8	1.5976	15.6733	3.4360	0.0039
10	8	0.9462	38.6182	3.8592	1.4427e-04

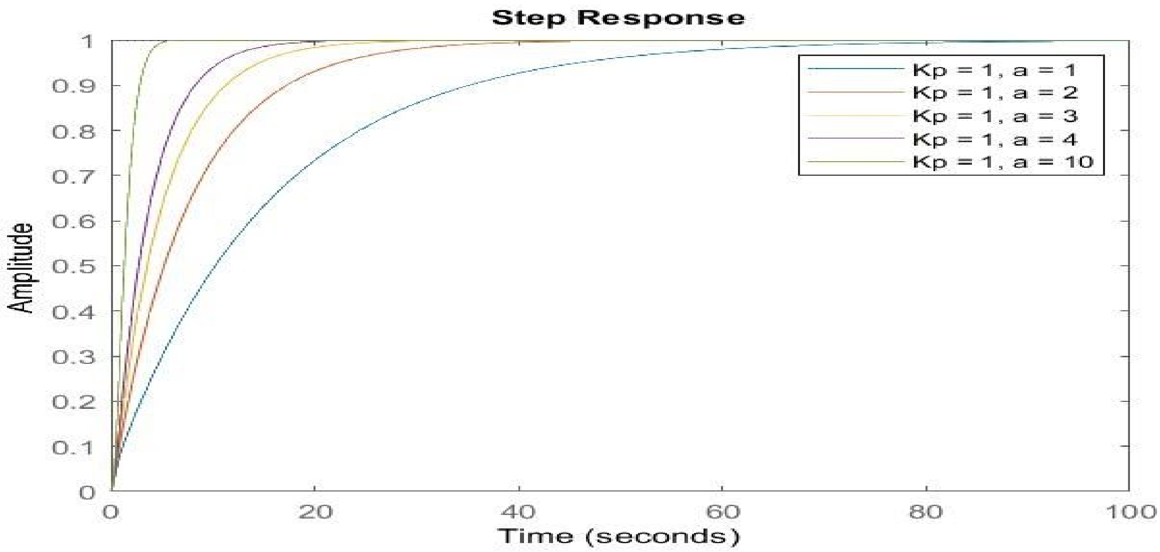


- **Keeping Kp constant**

For the fixed set values of a (=1,2,3,4,10), 'Kp' is varied and different parameters of their step response are compared and listed below in the tables.

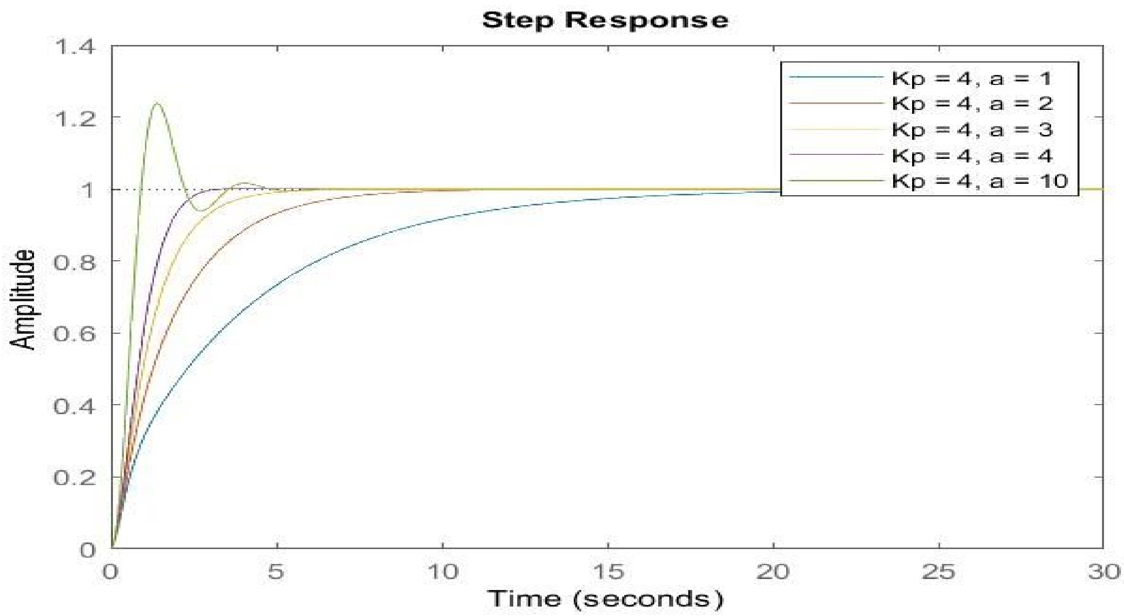
(a) $K = 1$

Value of K	Value of a	Peak Time	Percentage overshoot	Settling time	Steady State Error
1	1	108.2919	0	60.0709	0.0023
1	2	78.8169	0	29.2560	1.2621e-04
1	3	35.0829	0	18.9574	0.0032
1	4	25.1861	0	13.7831	0.0033
1	10	6.6069	0	4.0848	0.0021



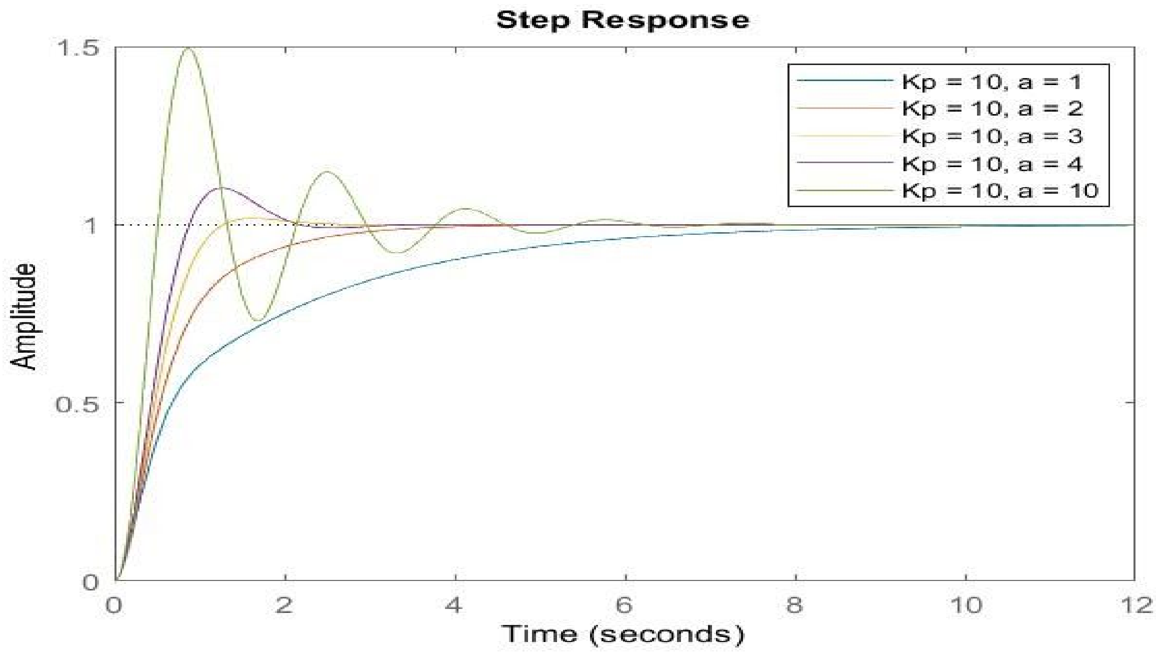
(b) $K = 4$

Value of K	Value of a	Peak Time	Percentage overshoot	Settling time	Steady State Error
4	1	30.3322	0	16.1388	0.0019
4	2	16.7494	0	7.2950	4.4170e-04
4	3	7.1150	0	4.1998	0.0031
4	4	3.8319	0	2.4832	0.0014
4	10	1.3821	23.7673	3.3048	0.0033



(c) $K = 10$

Value of K	Value of a	Peak Time	Percentage overshoot	Settling time	Steady State Error
10	1	12.4438	0	7.4203	0.0044
10	2	5.9148	0	2.9846	0.0019
10	3	1.6210	1.7322	1.1648	0.0018
10	4	1.2707	10.2465	1.9459	0.0096
10	10	0.8619	49.4777	5.1071	0.0036



Observation Table

Effect of increasing K and a independently

Variable	Peak Time	Percentage Overshoot	Settling time	Steady-state error
K	decrease	increase	Decrease then increase	Decrease then eliminate
a	decrease	increase	Decrease then increase	Minor change

The CLTF of the given control system is given by

$$H(s) = \frac{Kp(s+a)\cdot}{s(s+3)(s+5) + Kp(s+a)}$$

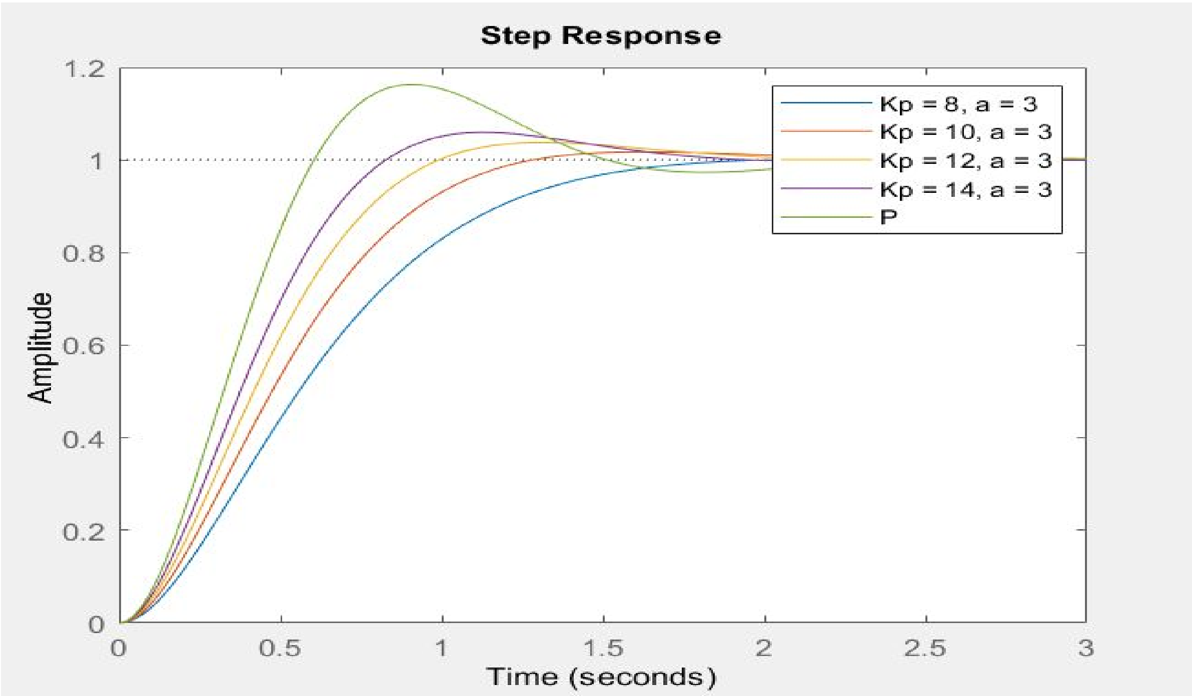
This is a 3rd order system. In the case of a={0,3,5}, the system is reduced to a 2nd order system and is compared to another 2nd order system $P(s)$ with $\zeta = 0.5$.

a) Let us take an Underdamped second-order CLTF P where

$$P(s) = \frac{16}{S^2 + 4s + 16} \quad \text{having} \quad \zeta = 0.5 \ and \ \omega_n = 4$$

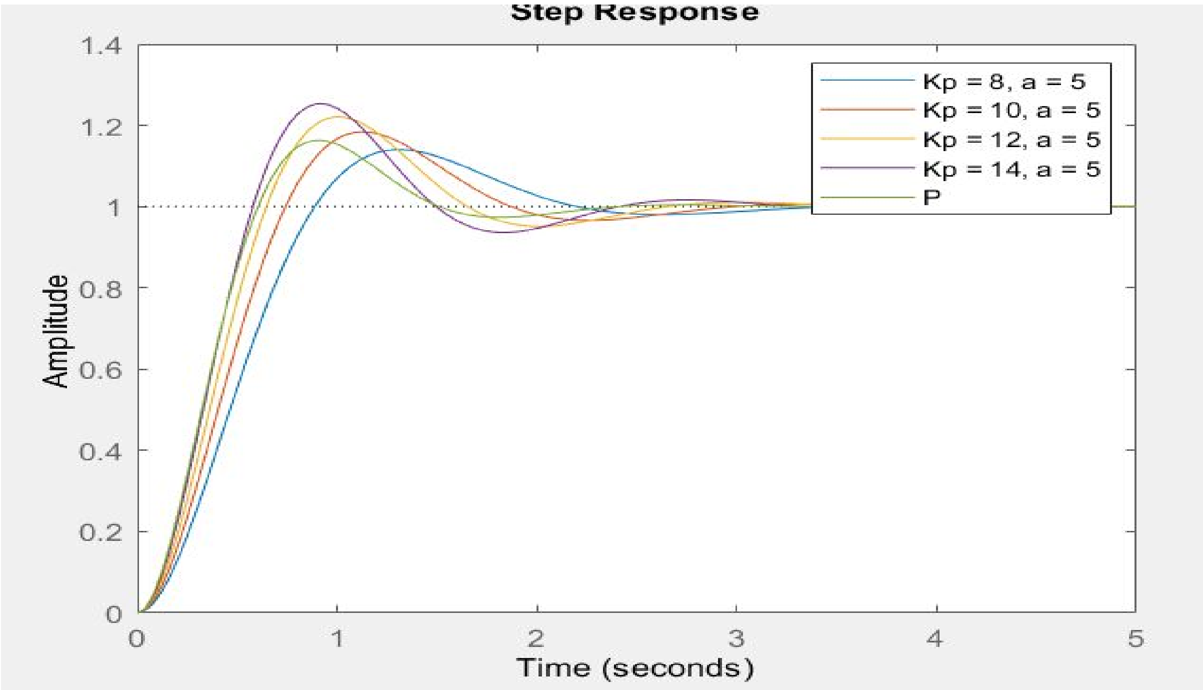
For a = 3
Natural frequency of H(s), $\omega_{n'} = \sqrt{Kp}$

System	Peak Time	Percentage overshoot	Settling time	SS error	Rise time
Wn = 4, P(s)	0.8980	16.2929	2.0190	0.0040	0.4098
Kp = 8, H(s)	2.3763	0.2640	1.6009	0.0014	0.9940
Kp =10, H(s)	1.6210	1.7322	1.1648	0.0018	0.7693
Kp =12, H(s)	1.3079	3.7803	1.7076	2.0475e-04	0.6335



For a = 5
Natural frequency of H(s), $\omega_{n'} = \sqrt{Kp}$

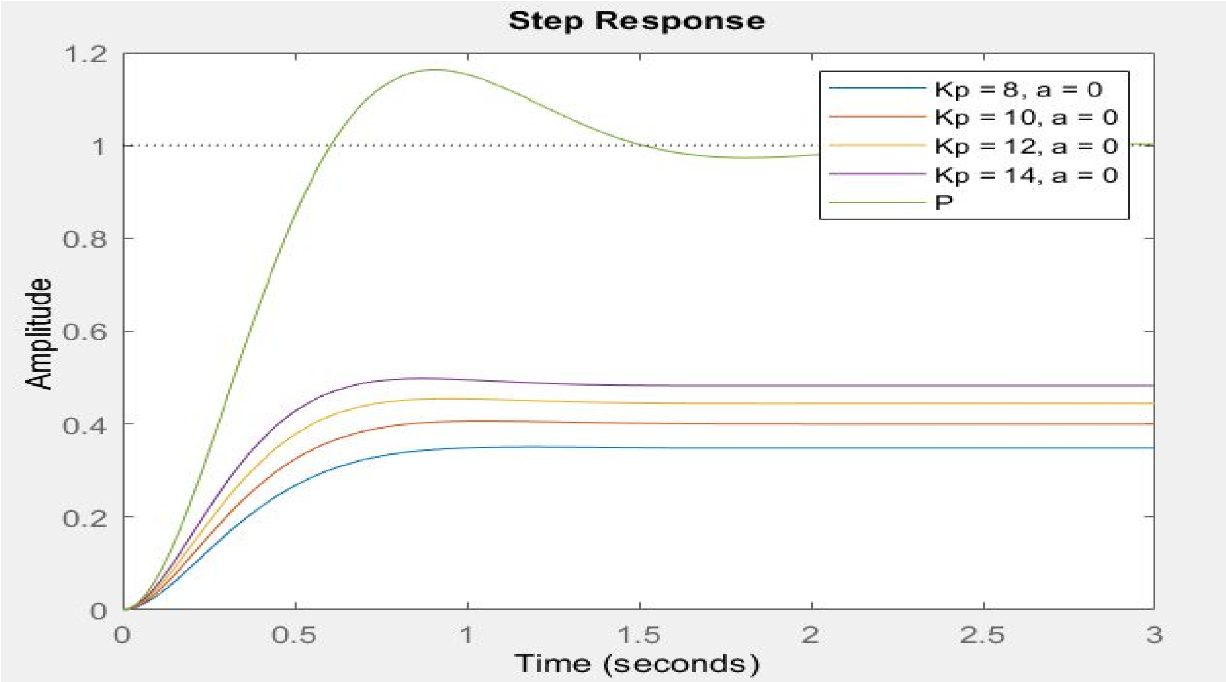
System	Peak time	Percentage overshoot	Settling time	SS error	Rise time
Wn = 4, P(s)	0.8980	16.2929	2.0190	0.0040	0.4098
Kp = 8, H(s)	1.3201	14.0072	2.0435	0.0027	0.6010
Kp = 10, H(s)	1.1359	18.3964	2.6100	0.0059	0.5029
Kp = 12, H(s)	1.0131	22.1028	2.4195	3.4466e-04	0.4387



For a = 0

Natural frequency of H(s), $\omega_{n'} = \sqrt{(15 + Kp)}$

System	Peak time	Percentage overshoot	Settling time	SS error	Rise time
Wn = 4, P(s)	0.8980	16.2929	2.0190	0.0040	0.4098
Kp=8, H(s)	1.1858	0.8655	0.8430	0.6492	0.5425
Kp=10, H(s)	1.0477	1.5165	0.7512	0.5997	0.4936
Kp=12, H(s)	0.9441	2.2617	1.0543	0.5558	0.4536



Pole -Zero map to compare the stability of a given system with assumed 2nd order system P(s) at different sets of values of ‘a’ and ‘K’.

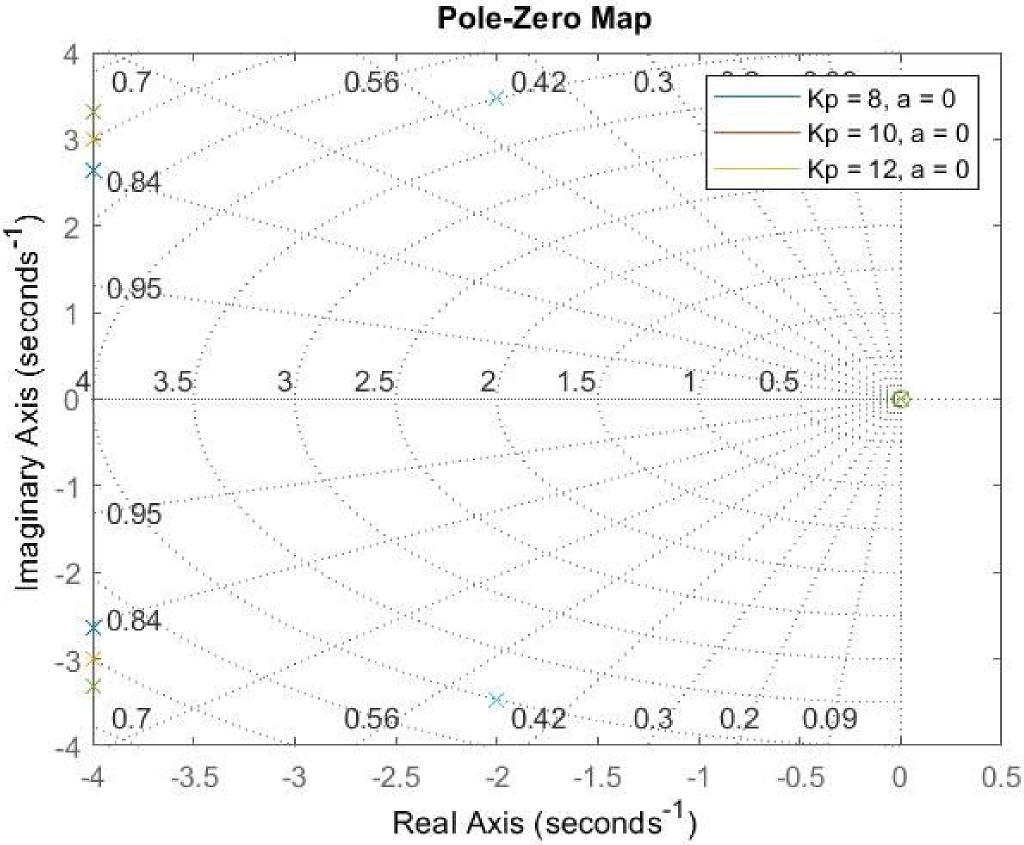
The broad notion of stability is the retention of output at the reference regardless of the dynamics of the state.
Stability of any system is decided by the position of its poles.
If the real part of the poles lie on

- ➢ Negative real axis : the system is Stable
- ➢ Imaginary axis : the system is Marginally Stable
- ➢ Positive real axis : the system is Unstable

(i) $a = 0$

$$H(s) = \frac{Kp}{(s+3)(s+5) + Kp}$$

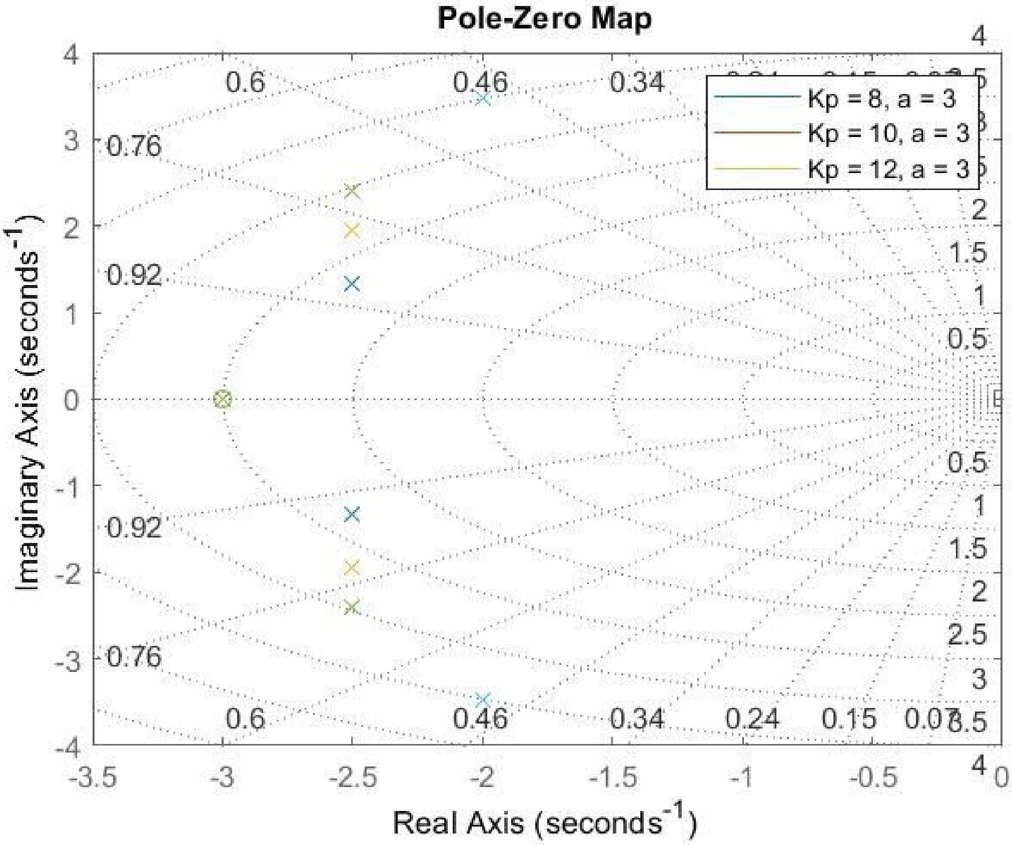
Real part of poles of H(s)= -4
Real part of the pole of P(s)= -2
All the poles of H(s) have their real part more negative compared to the pole of P(s).
So, H(s) system is more stable than the P(s).



(ii) $a = 3$

$$H(s) = \frac{Kp}{(s)(s+5) + Kp}$$

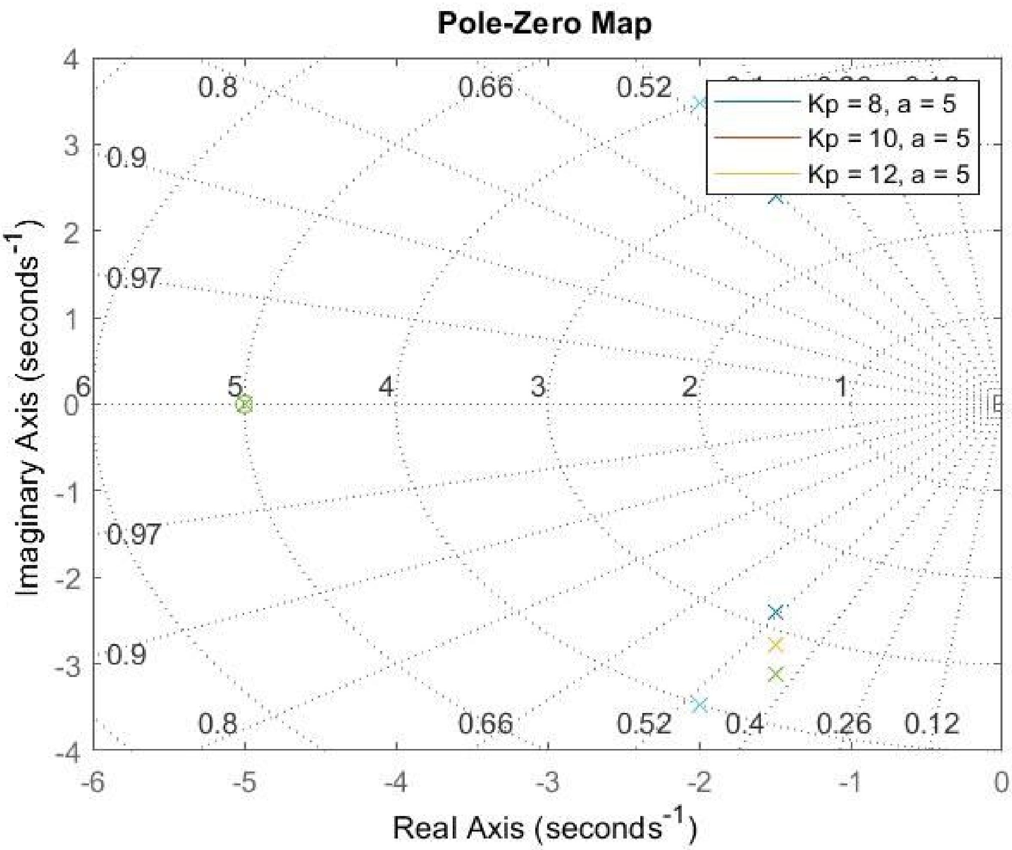
Real part of poles of H(s)= -2.5
Real part of the pole of P(s)= -2
All the poles of H(s) have their real part more negative compared to the pole of P(s).
So, H(s) system is more stable than the P(s).



(iii) $a = 5$

$$H(s) = \frac{Kp}{(s)(s+3) + Kp}$$

Real part of poles of H(s)= -1.5
Real part of pole of P(s)= -2
The real part of the pole of P(s) is more negative than the real part of poles of H(s). So, P(s) is more stable compared to H(s).



Observations

If the given system H(s) is compared to the 2nd order system P(s), there is the tradeoff between steady-state error, settling time and stability of the system. If the value of 'a' is increased to match the steady-state error and settling time of P(s), the stability of the system will become less compared to that of P(s).

Conclusion

The integral term accelerates the movement of the process towards setpoint and eliminates the residual steady-state error that occurs with a pure proportional controller. However, since the integral term responds to accumulated errors from the past, it can cause the present value to overshoot the setpoint value. Also, as we keep increasing the integral coefficient, instability increases due to excess gain.