

Control Engineering

Experiment - 2

Controller design on MATLAB platform using Analog Root Loci

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Aim of the Experiment :

The project requires the design of a cascade feedback controller for a given analog transfer function, according to desired specifications and further performing a sensitivity analysis of key parameters.

Transfer Function :

In this experiment, we were provided with an Open Loop Transfer Function

$$G_{OL}(s) = \frac{s^2 + 0.5s + 5}{s^3 + 5s}$$

Now we needed to design a Controller with a fundamental design objective of stable CLTF operation under all circumstances.

PID controller for the system in unity negative feedback, with the controller the transfer function is given by:

$$C_{OL}(s) = K * \left(\frac{Td(s^2) + s + Ti}{s} \right)$$

So our Closed-Loop Transfer Function is :

$$H(s) = \frac{G(s) * C(s)}{(1 + G(s) * C(s))}$$

A system is said to be stable if its output is under control. The broad notion is the retention of the output at the reference regardless of the dynamics of the state.

For closed-loop stability (the one that matters), all the poles of the transfer function $H(s)$ have to be in the left half-plane. So to ensure stability we must make sure that the roots of $1 + G(s)C(s) = 0$ are in the left half-plane,

$$L(s) = K * \left(\frac{Td(s^2) + s + Ti}{s} \right) * \frac{s^2 + 0.5s + 5}{s^3 + 5s}$$

and $1 + L(s) = 0$ becomes the new characteristic equation.

Controller Design through Manual tuning using Root-loci method:

We were given three variables K , Td , Ti and we need to analyse the system by changing these variables. In the end, we will propose a set for which our PID controller is robust and can handle a 20% variation in these parameters without losing the stability of CLTF.

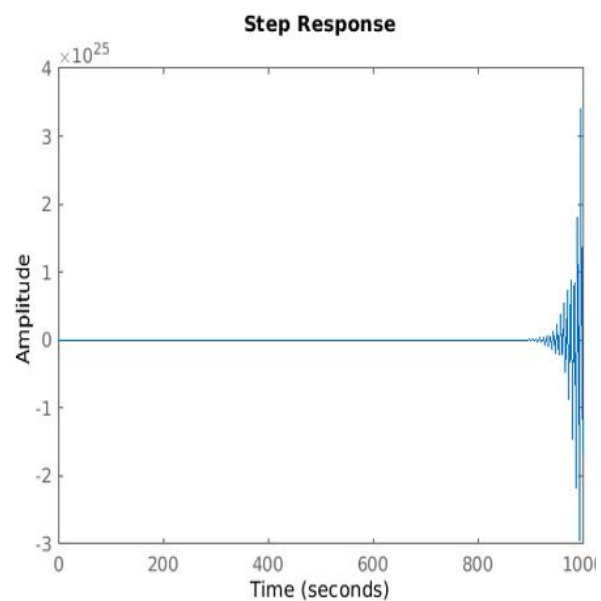
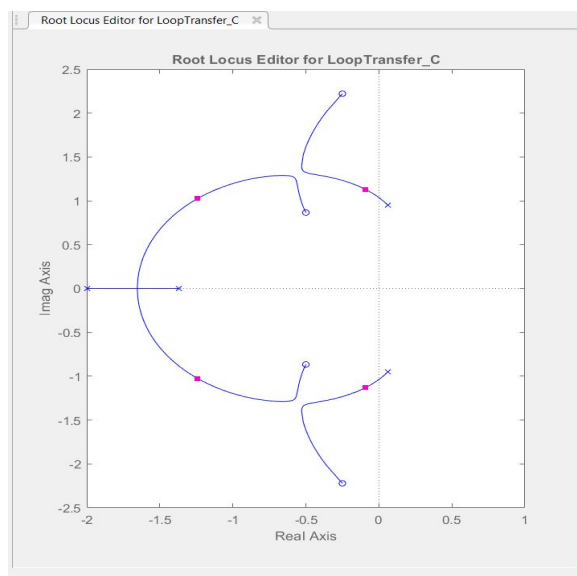
Matlab Code used to get different parameters:

```
G=tf([1,0.5,5],[1,5,0,0])  
s=tf('s');  
C=K*(1+Td/s+Ti*s); %Put values of Td and Ti  
H=feedback(G*C,1);  
controlSystemDesigner('rlocus',H);  
stepinfo(H)  
damp(H)
```

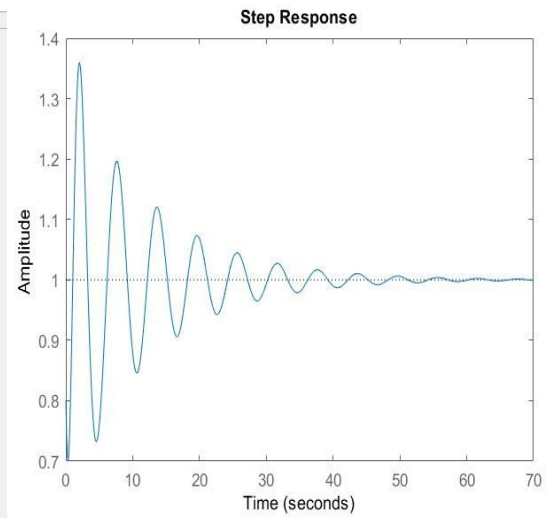
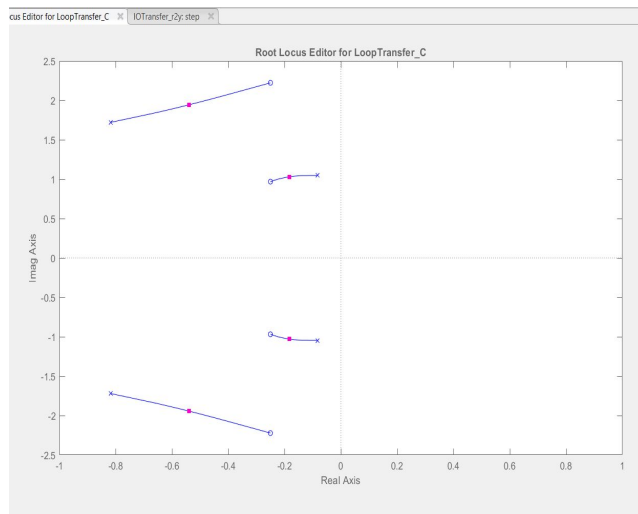
To begin, let's take

1) $K, T_i, T_d \equiv (1, 1, 1)$

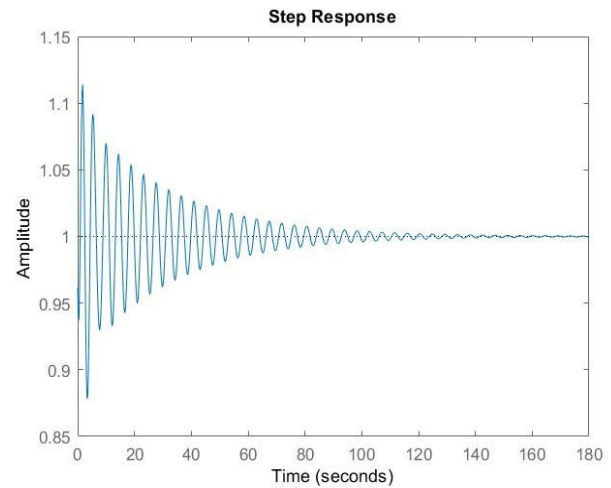
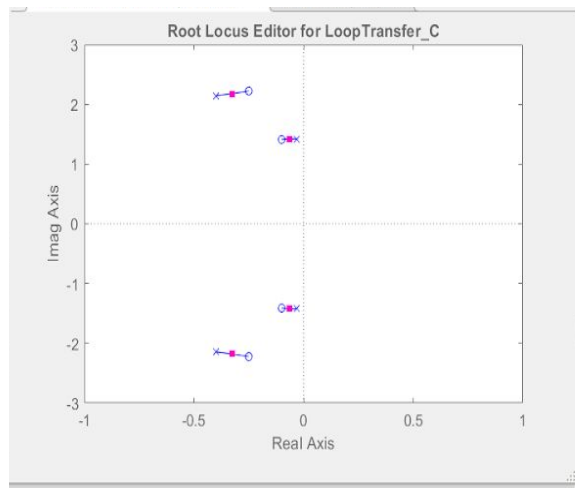
From the root loci plot, we can see that a part of the plot is in RHS, hence our system is unstable. This can be further verified by the step response of the system.



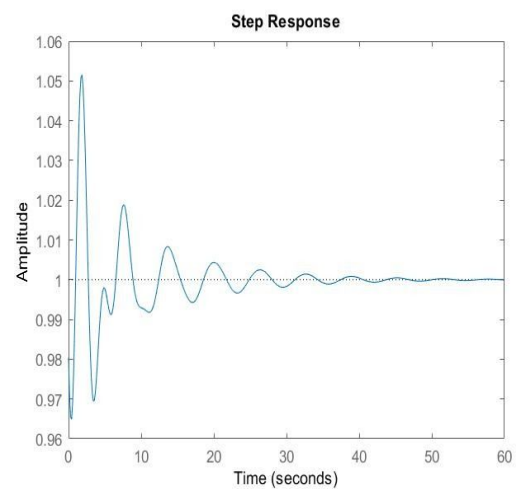
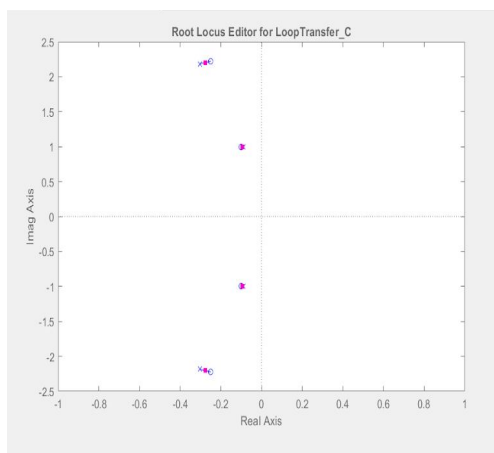
2) $K, T_i, T_d \equiv (2, 2, 2)$



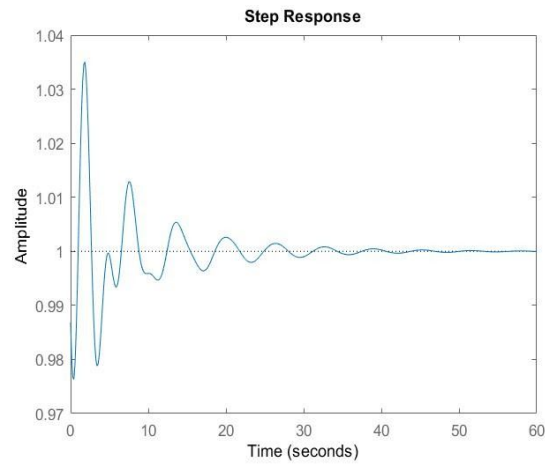
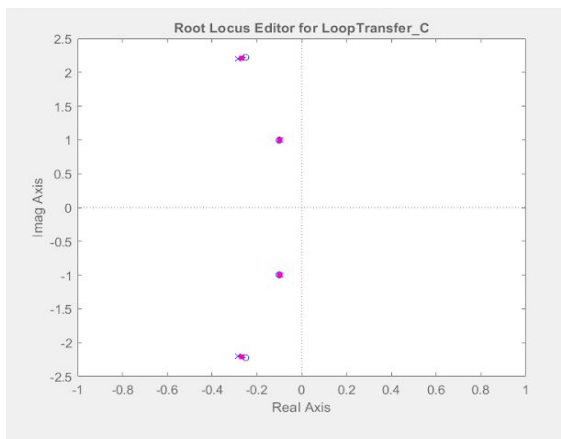
3) $K, T_i, T_d \equiv (5, 10, 5)$



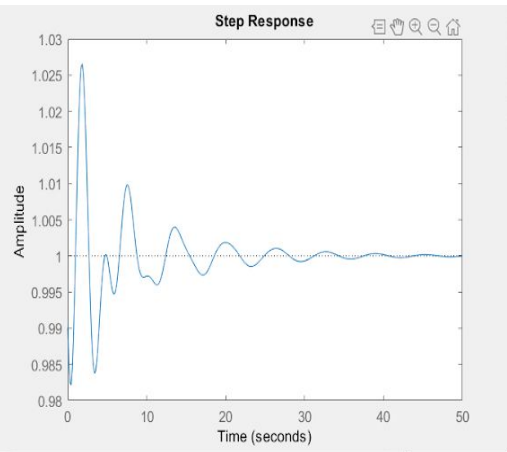
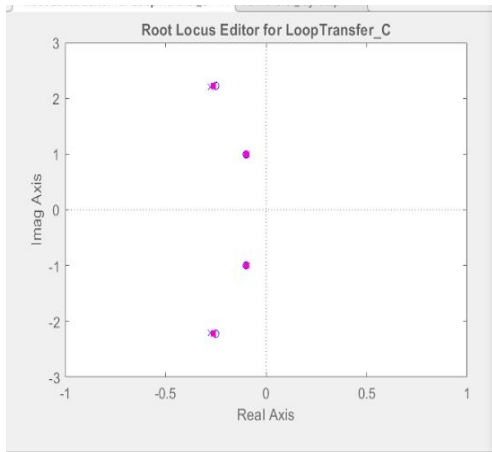
4) $K, T_i, T_d \equiv (10, 5, 5)$



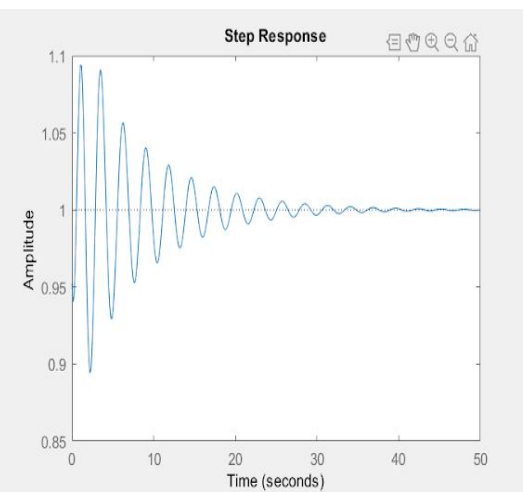
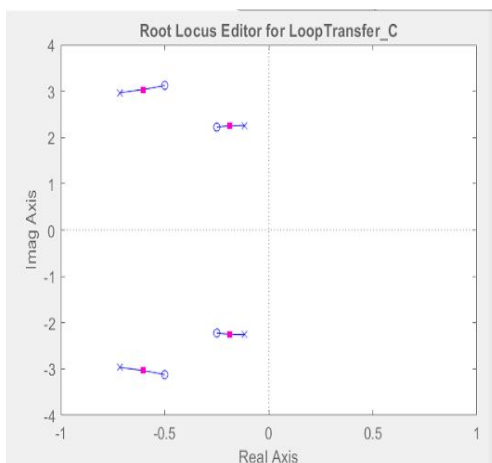
5) $K, T_i, T_d \equiv (15, 5, 5)$



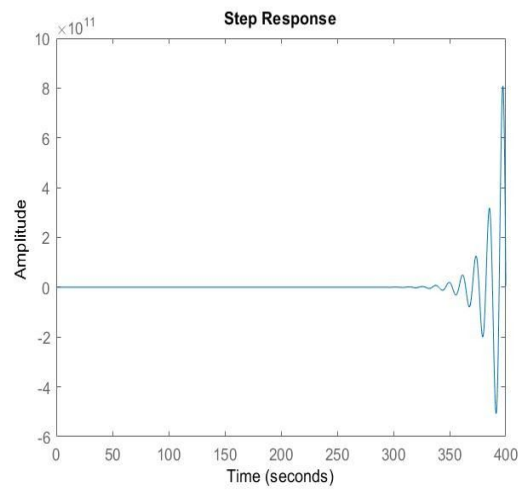
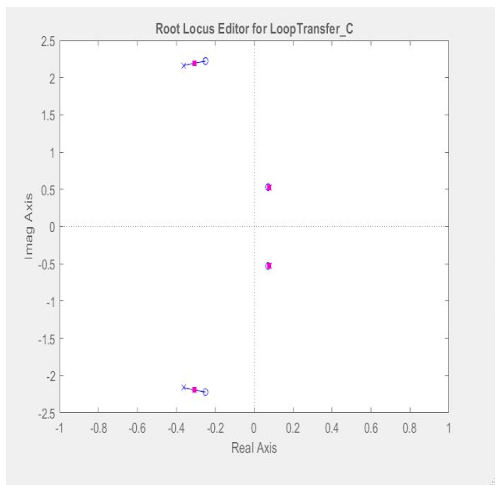
6) $K, T_i, T_d \equiv (20, 5, 5)$



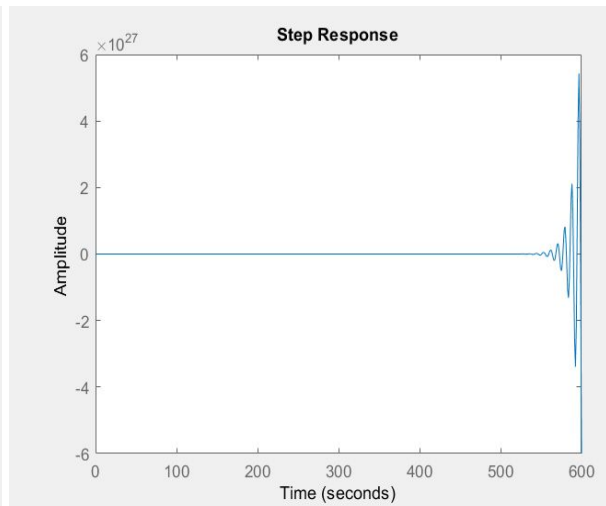
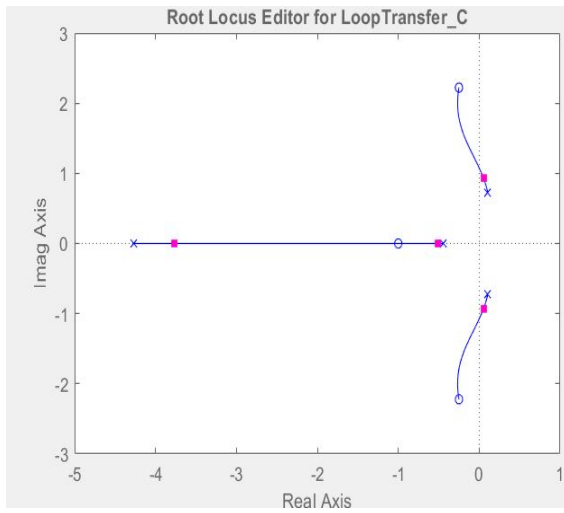
7) $K, T_i, T_d \equiv (20, 10, 1)$



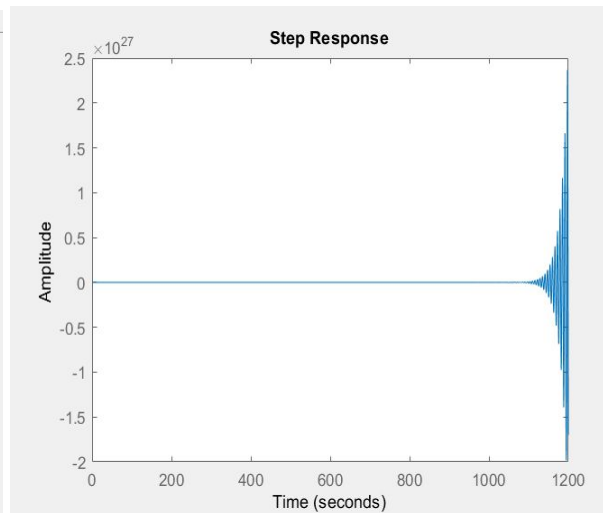
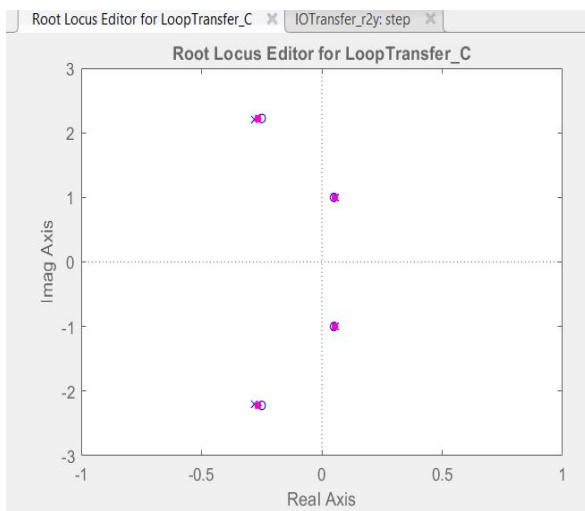
8) $K, T_i, T_d \equiv (-3, -2, -7)$



9) $K, T_i, T_d \equiv (0.5, 0.5, 0.5)$



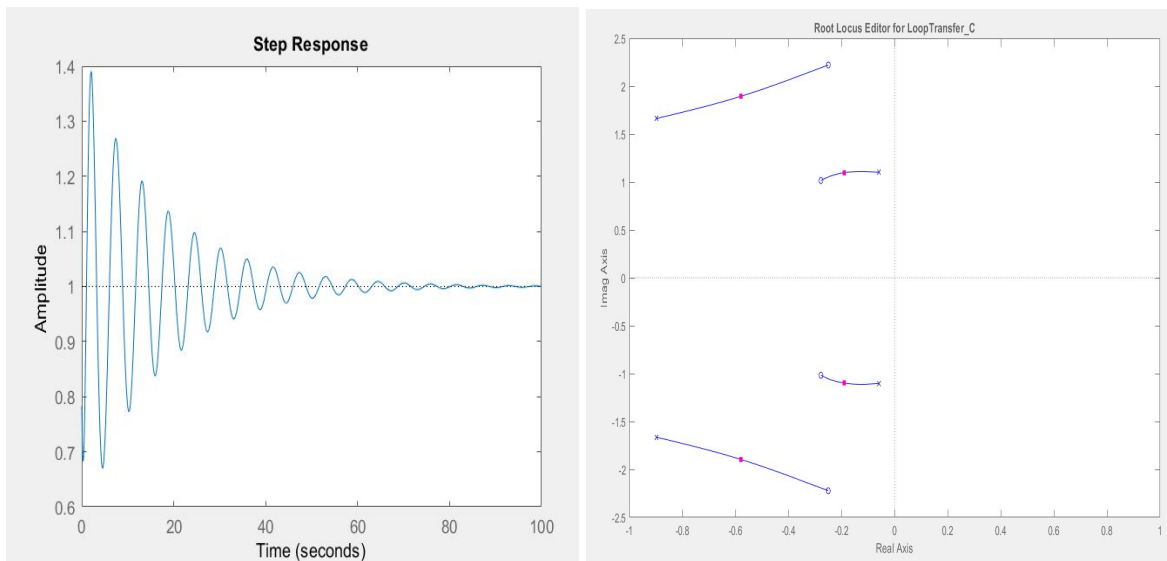
10) $K, T_i, T_d \equiv (-10, -10, -10)$



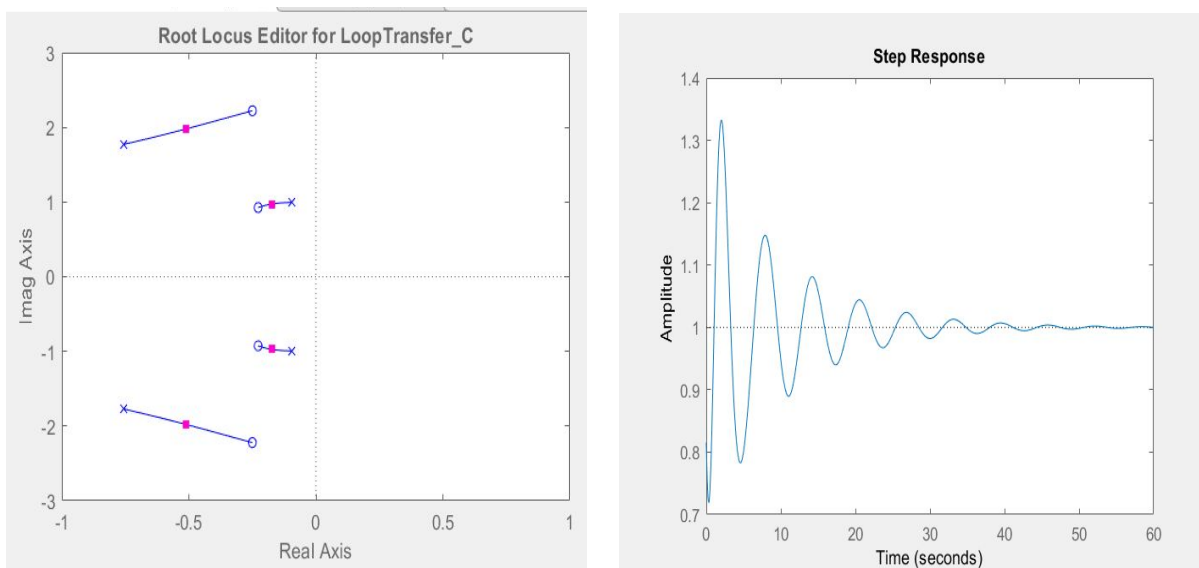
Values	Settling time	Peak Time	Overshoot	Damping Ratio	Nature of System	Stability
-10,-10,-10	NaN	Infinite	NaN	-5.63e-02 1.25e-01	Under damped	Unstable
-3,-2,-7	NaN	Infinite	NaN	-1.47e-01 1.65e-01	Unstable	Unstable
0.5,0.5,0.5	NaN	NaN	NaN	1.00e+00 -1.47e-01	Critically damped	Unstable
1,1,1	NaN	NaN	NaN	-6.23e-02	Unstable	Unstable
2,2,2	47.0649	2.0274	35.9917	7.84e-02 4.29e-01	Under damped	stable
5,10,5	116.1257	1.7671	11.3913	2.22e-021 .84e-01	Under damped	Stable
10,5,5	36.1169	1.8515	5.1535	8.79e-02 1.38e-01	Under damped	stable
15,5,5	33.2543	1.8433	3.5043	9.20e-021 .29e-01	Under damped	Stable
20,5,5	33.1274	1.8391	2.6547	9.40e-021 .25e-01	Under damped	Stable
20,10,1	32.9106	1.0311	9.3860	5.25e-02 2.34e-01	Under damped	Stable

So after using and trying for several values of the parameters, we found that for further variation and analysis we are using the point(2,2,2) as (K, Ti, Td).since it provides stability even on the variation of the 20 percent in the parameters. Now, we will check the robustness of the system by varying the parameters by 20%.

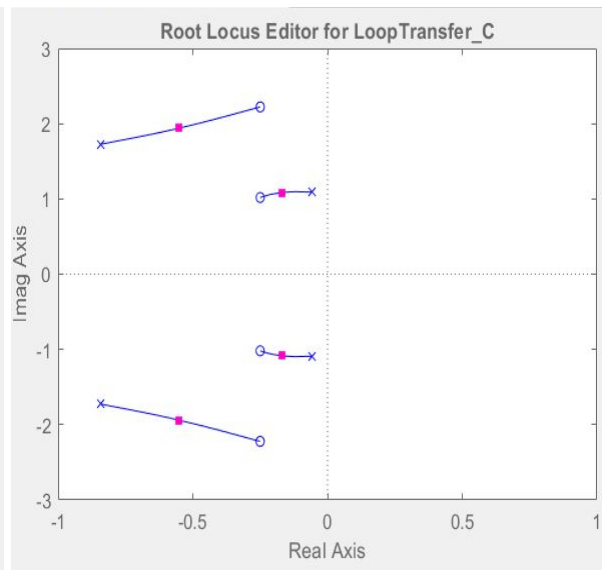
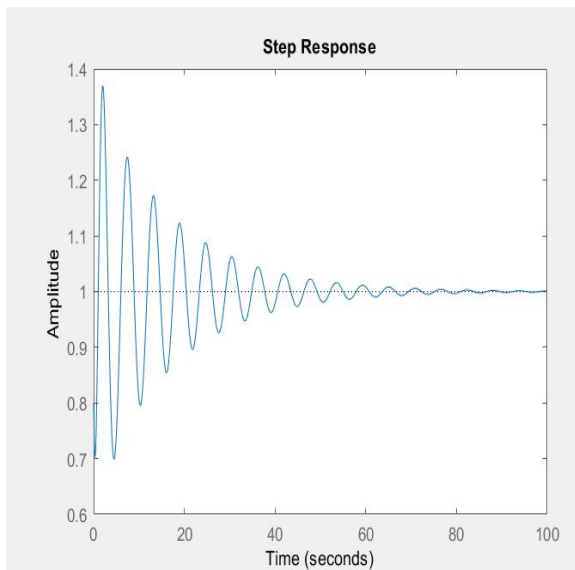
1) $K, T_i, T_d \equiv (2, 2, 1.8)$, decrease in T_i by 20%



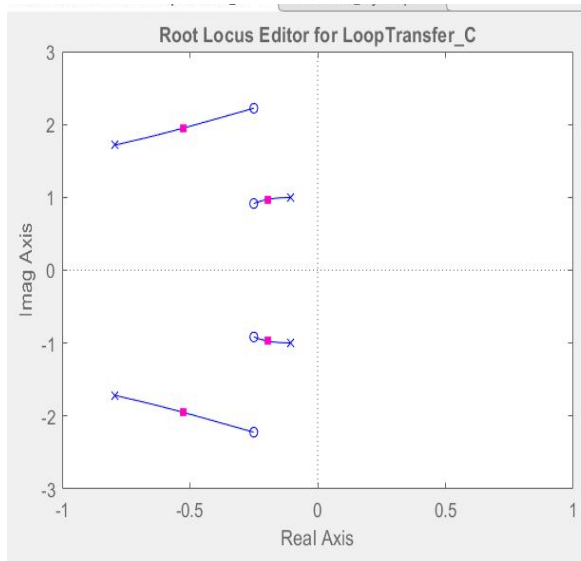
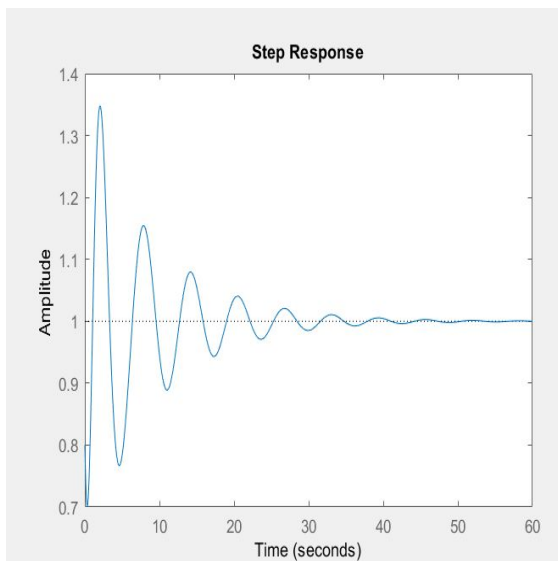
2) $K, T_i, T_d \equiv (2, 2, 2.2)$, increase in T_d by 20%



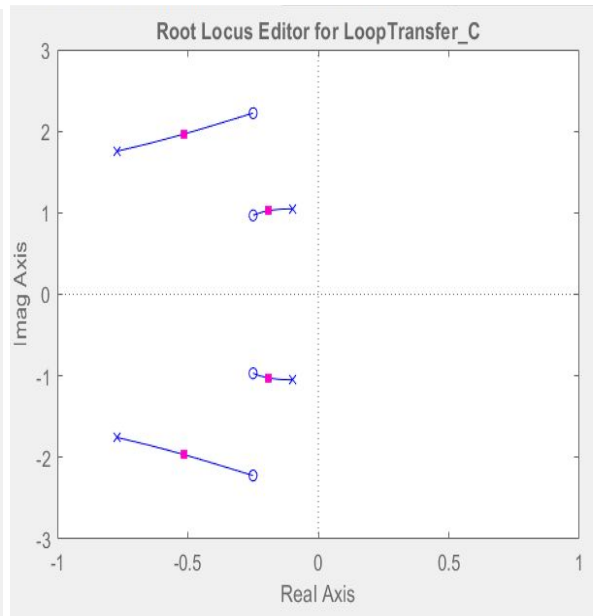
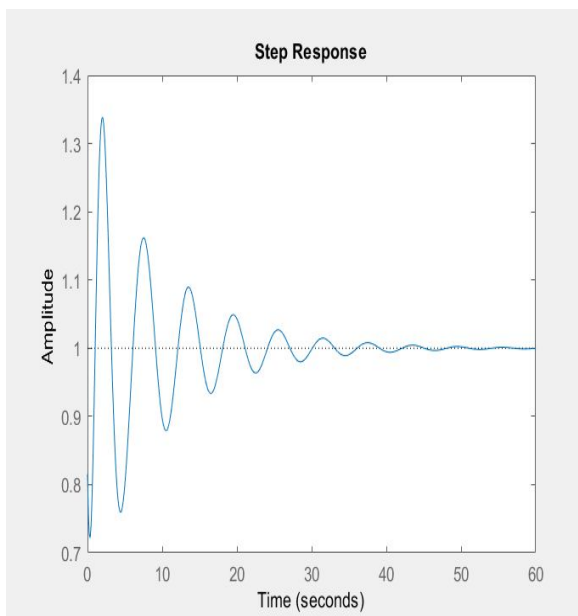
3) $K, T_i, T_d \equiv (2, 2, 2, 2)$, increase in T_d by 20%



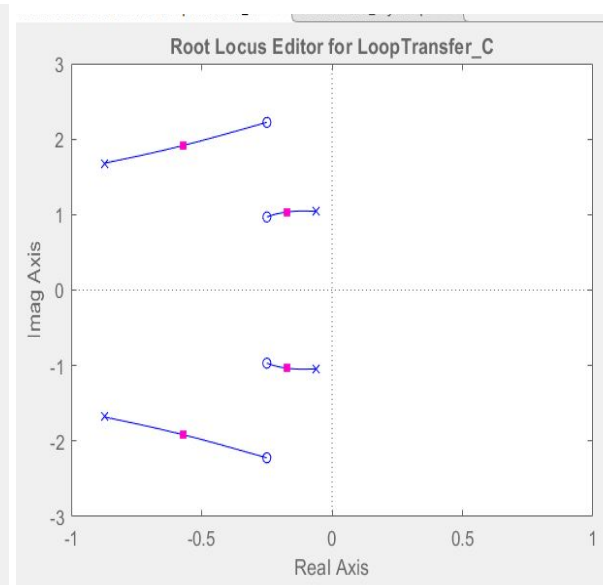
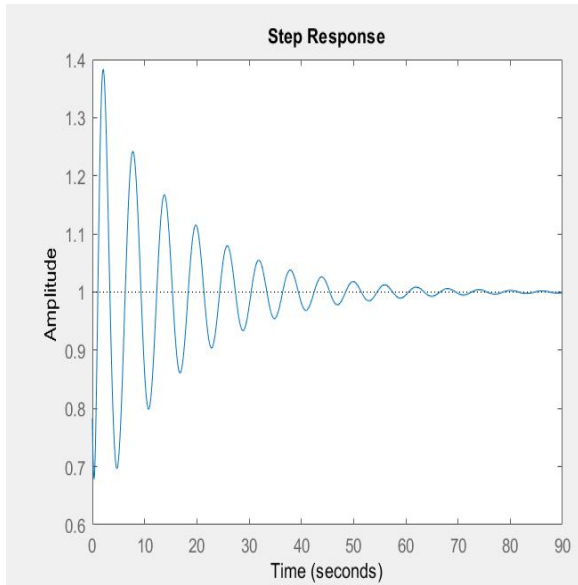
4) $K, T_i, T_d \equiv (2, 1.8, 2)$, decrease in T_d by 20%



5) $K, T_i, T_d \equiv (2.2, 2, 2)$, increase in K by 20%



6) $K, T_i, T_d \equiv (1.8, 2, 2)$, decrease in K by 20 percent



Observation Table for variation in parameters and its effect on various functions

Values	Settling time	Peak Time	Overshoot	Damping Ratio	Nature of System	Stability
2,2,1.8	64.9242	2.0529	39.0374	5.36e-02 4.75e-01	Under damped	stable
2,2,2.2	39.8598	1.3323	33.2332	9.63e-02 3.92e-01	Under damped	stable
2,2,2,2	65.4542	1.9708	36.4916	5.38e-02 4.38e-01	Under damped	stable
2,1.8,2	36.6184	1.3478	34.7790	1.06e-01 4.20e-01	Under damped	stable
2.2,2,2	38.0545	1.3386	33.8583	9.52e-02 4.02e-01	Under damped	stable
1.8,2,2	62.4274	2.1095	38.3014	5.89e-02 4.61e-01	Under damped	stable

Discussion:

- When T_d is increased \Rightarrow Settling time decreases, Peak time decreases, Overshoot decreases
- When T_i is increased \Rightarrow Settling time increases, Peak time increases, Overshoot increases
- When K is increased \Rightarrow Settling time decreases, Peak time decreases, Overshoot
- In our case, the system always remains stable and Underdamped.

The system was stable even for higher values of K , T_i , T_d . For e.g, we tried for (100000,100000,100000) and the system was still stable.