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History of Graph Theory

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Outline

Introduction to Graph Theory

Graphs are formal way of representing a network or interconnected objects

Introduction to Graph Theory

Graphs are formal way of representing a network or interconnected objects
Example : places in Germany and how they are connected with distances

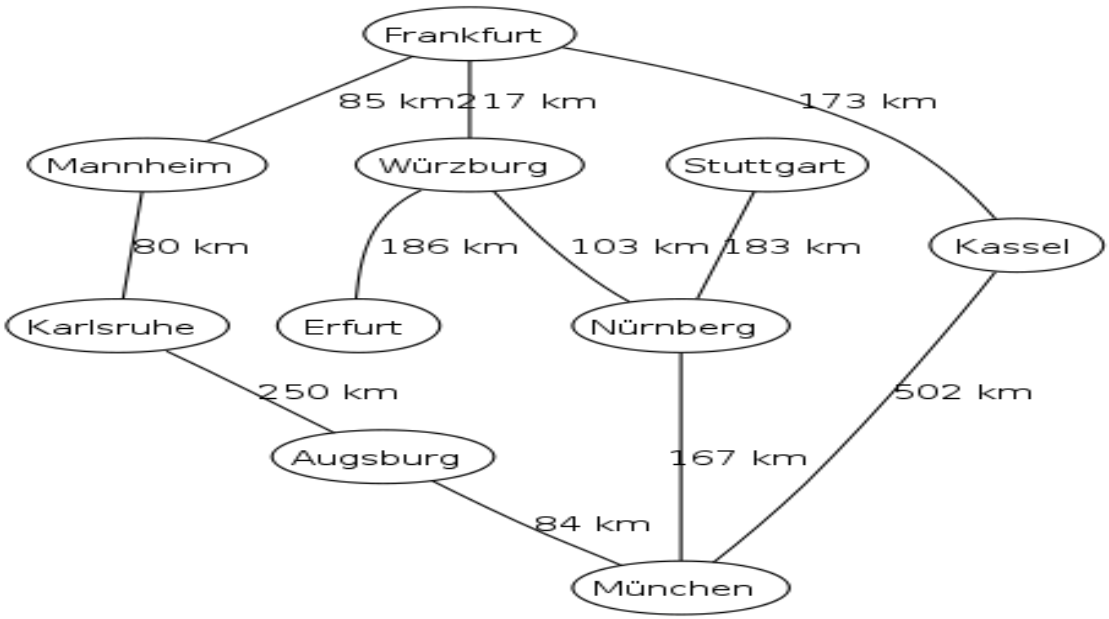

Figure: Graph representation of places in Germany with corresponding distance between them in kilometers

Introduction to Graph Theory

In mathematical terms graph theory is the study of graphs,where graphs are defined as ordered pairs with two parts: Vertices (V) and Edges (E)
Graph $G = (V, E)$

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Graph $G = (V, E)$
Example of graph G with vertices $V(G) = \{0, 1, 2, 3, 4\}$ and edges $E(G) = \{\{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 4\}, \{1, 2\}, \{1, 3\}, \{2, 4\}, \{3, 4\}\}$

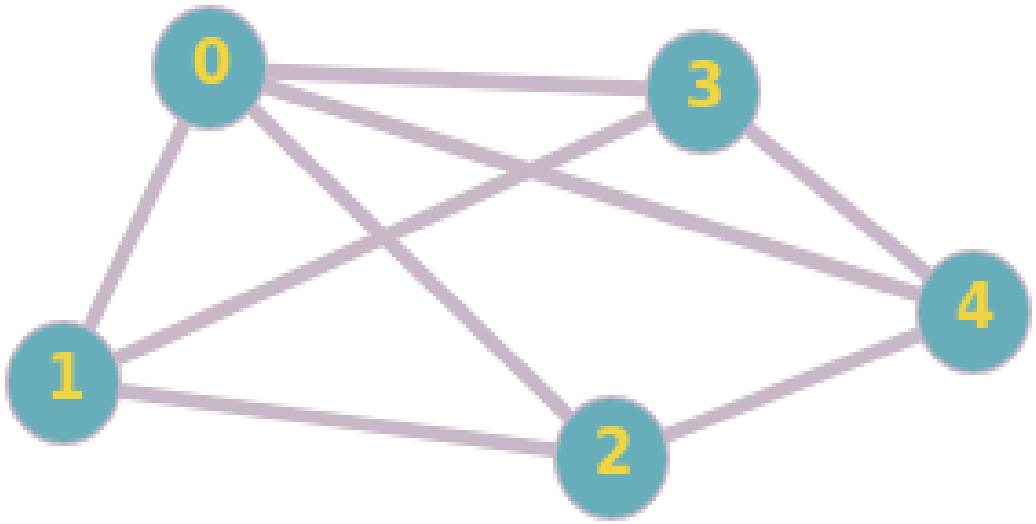

Figure: Example of Graph

History of Graph Theory

Graph Theory began with Leonhard Euler for his study on the Bridges of Königsberg problem

Leonhard Euler(1707 - 1783)

Leonhard Euler was a Swiss mathematician .Euler discovered his talents in mathematics while attending the University of Basel,Switzerland. By 1726, the 19-year-old Euler had finished his work at Basel and published his first paper in mathematics. In 1727, Euler assumed a post in St. Petersburg, Russia, where he spent fourteen years working on his mathematics. Leaving St. Petersburg in 1741, Euler took up a post at the Berlin Academy of Science. Euler finally returned to St. Petersburg in 1766, where he would spend the rest of his long, productive life[2].

Leonhard Euler

In 1766, because of his worsening eyesight issue he became complete blind and after his blindness by the year 1775 he was publishing one mathematical paper every week with help of his assistants.
Euler wrote over 800 papers covering every branch of mathematics known in his time, and including such applied topics as mechanics, fluid mechanics, naval science, solar and lunar motion, the tides, cartography, astronomical motion and precessions, acoustics, and optics.
He wrote an enormously influential series of calculus textbooks, as well as books on over a dozen other mathematical and scientific fields[3].

Seven Bridges of königsberg

The Königsberg bridge problem originated in the city of Königsberg, formerly in Germany but, now known as Kaliningrad and part of Russia, located on the river Preger.

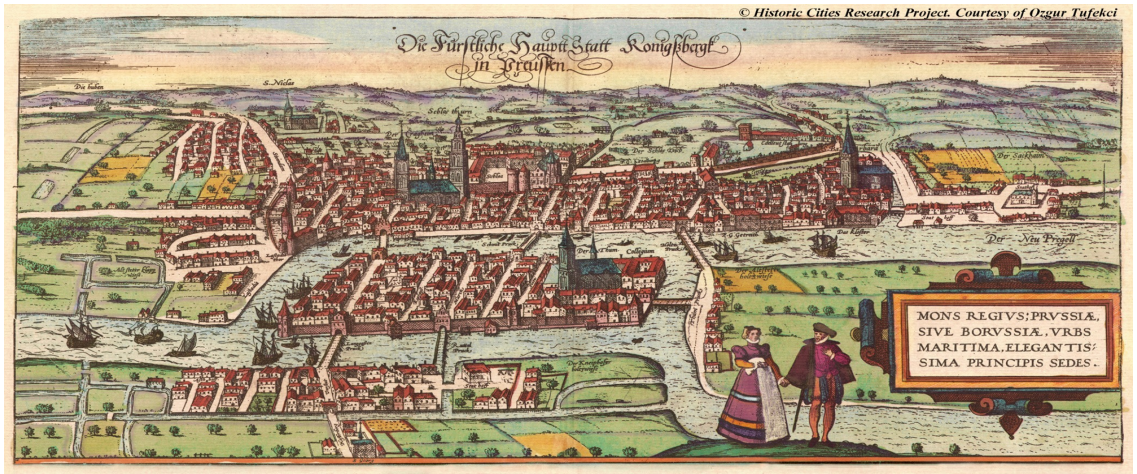

Figure: picture of Königsberg city

Seven Bridges of königsberg

Problem

The city had seven bridges, which connected two islands with the main-land via seven bridges.
People staying there always wondered whether was there any way to walk over all the bridges once and only once ?

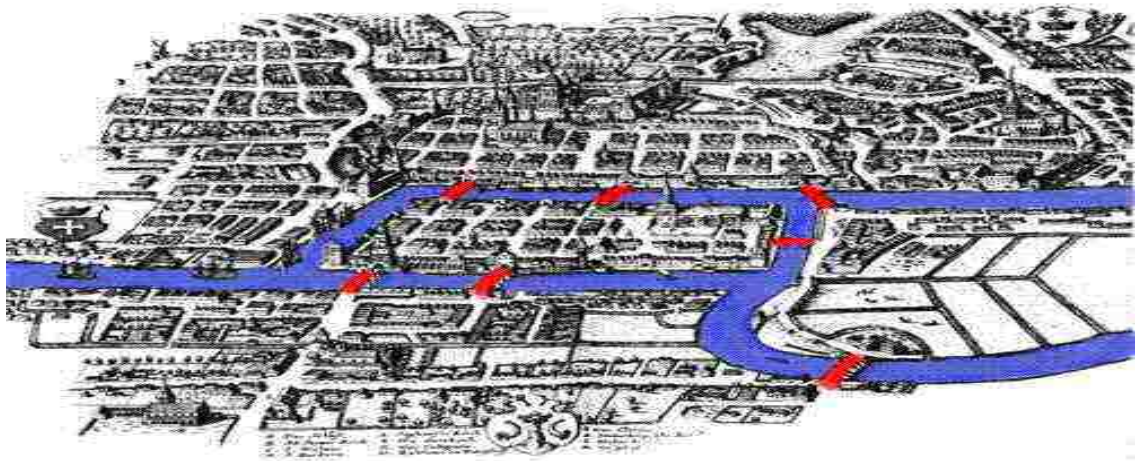

Figure: picture is the map of königsberg highlighting the river Preger and the bridges[6]

Seven Bridges of königsberg

Euler's Intuition

The physical map is not much important,mathematically just need the list of which regions are connected by bridges.

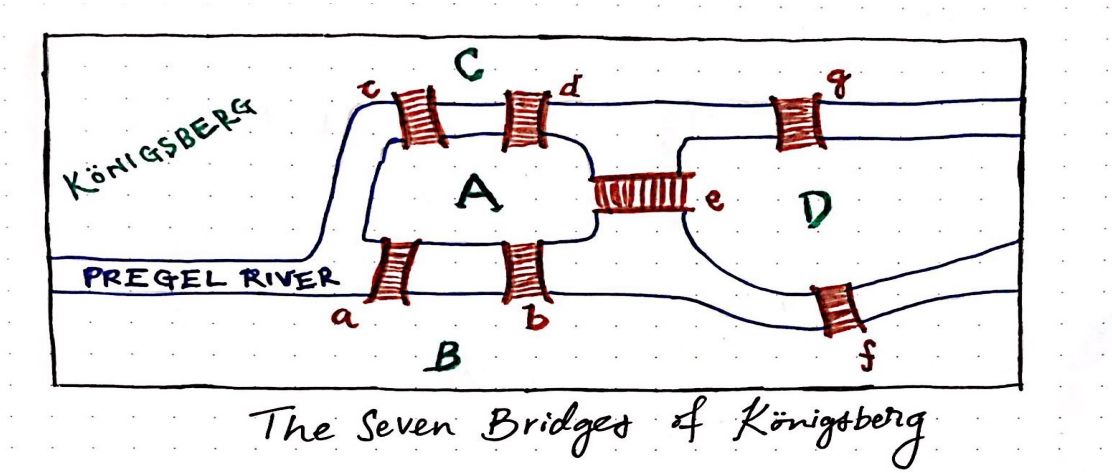

Figure: picture of königsberg bridge problem highlighting the river Preger and the bridges[6]

Seven Bridges of königsberg

Euler's Intuition

considering land as points (Vertices) and bridges as line (Edges) to move from one to another point
königsberg bridge problem physical map is converted to Graph

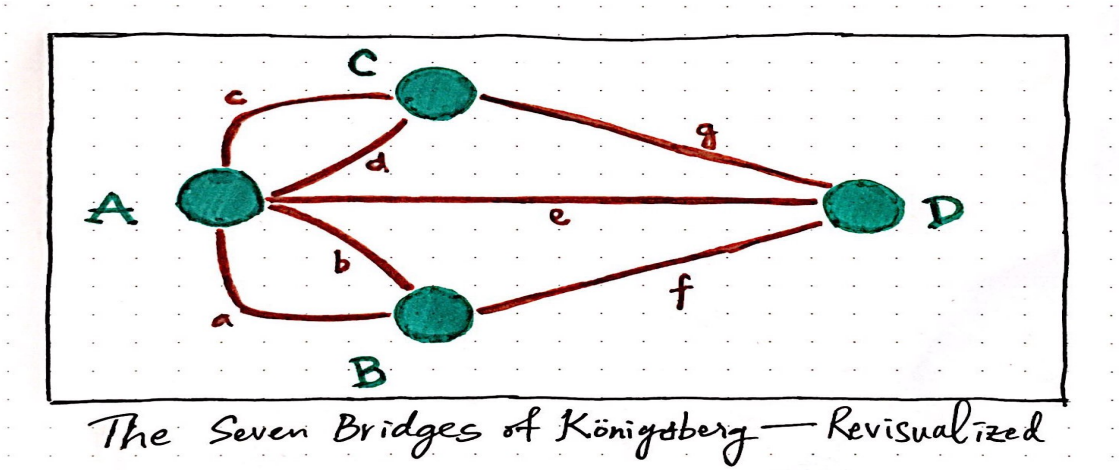

Figure: Graph representation of königsberg bridge problem[6]

Seven Bridges of königsberg

Degree of vertex:number of edges connected to a vertex
example:Degree(A)=5
Degree(B)=3

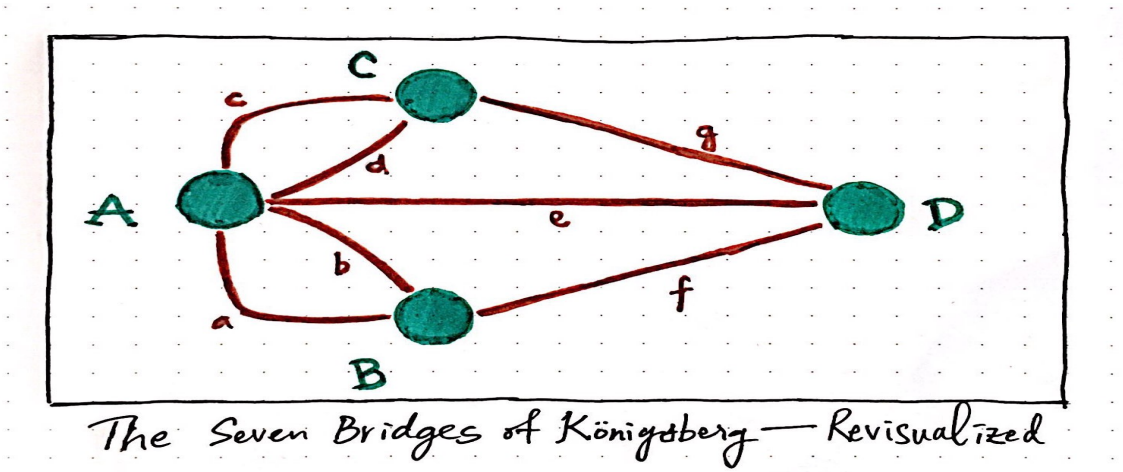

Figure: Graph representation of königsberg bridge problem[6]

Euler stated that the crossing the all bridges(edges) only once possible in only two scenario
first scenario:
when all the vertices of graph are of even degree
the first scenario is called Euler circuit

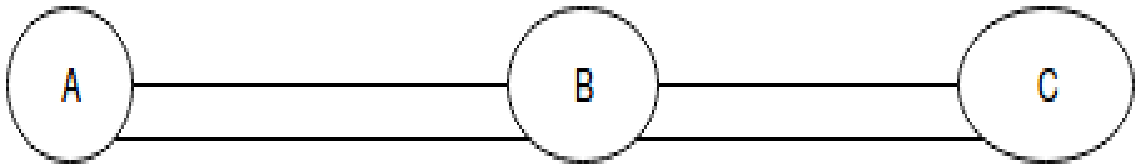

Figure: Graph representation of Euler circuit

Euler circuit for the above graph :
ABCBA or CBABC or BABCB

second scenario :
in a graph degree of two vertices are odd and remaining are even
the second scenario is called Euler path

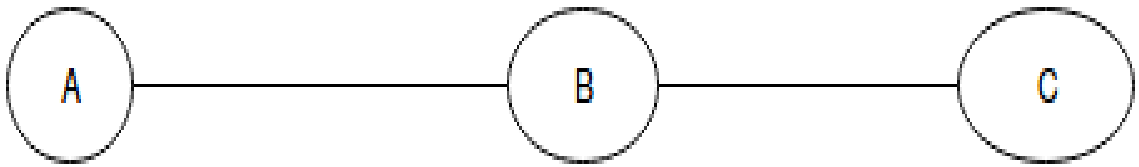

Figure: Graph representation of Euler Path

Euler path for the above graph :
ABC or CBA

Seven Bridges of königsberg

explanation

However,königsberg bridge problem has all odd degree vertices its not possible to cross all the bridges only once

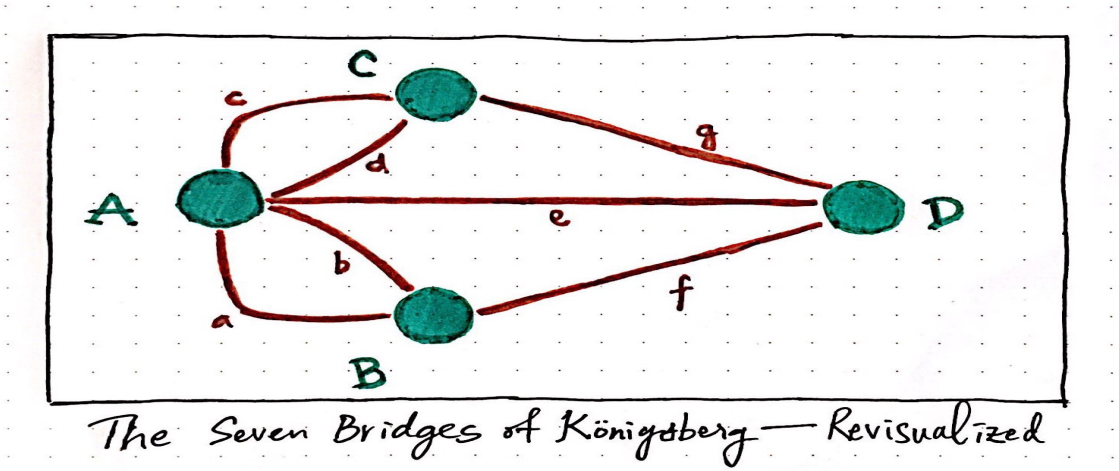

Figure: Graph representation of königsberg bridge problem[6]

Euler circuit

Applications

chinese postman problem:
chinese postman problem is about covering every street one and only once in a village.
if village is converted to graph every street as edge ,when the graph has euler circuit then the circuit is a optimal solution.
the problem was originally studied by the Chinese mathematician Kwan Mei-Ko in 1960.

Hamiltonian Graph

Hamiltonian Graph named after William Rowan Hamilton who invented the icosian game, also known as Hamilton's puzzle

Sir William Rowan Hamilton(1805 - 1865)

Sir William Rowan Hamilton was an Irish mathematician, astronomer, and mathematical physicist, who made important contributions to classical mechanics, optics, and algebra. His studies of mechanical and optical systems led him to discover new mathematical concepts and techniques. His best known contribution to mathematical physics is the reformulation of Newtonian mechanics, now called Hamiltonian mechanics[4].

Hamiltonian Graph

The Hamilton's puzzle involved a dodecahedron on which each of the 20 vertices was labelled by the name of some capital city in the world. The aim of the game was to construct, using the edges of the dodecahedron a closed walk of all the cities which traversed each city exactly once, beginning and ending at the same city.

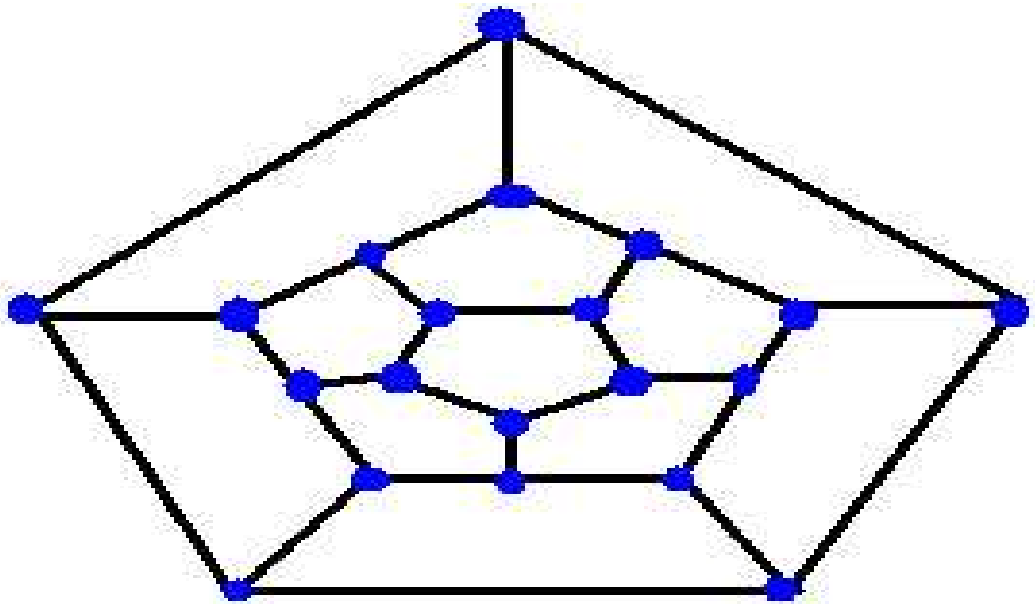

Figure: Dodecahedron

Hamiltonian Graph

Hamiltonian path:
A Hamiltonian path is a path that visits each vertex of the graph exactly once.

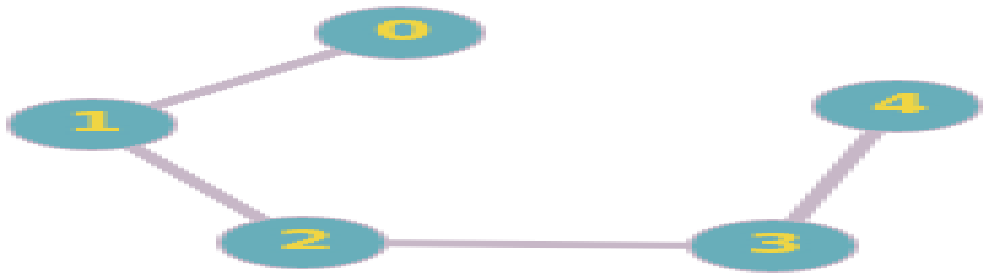

Figure: Example of Hamiltonian path

Hamiltonian path for the above graph:
01234 or 43210

Hamiltonian Graph

Hamiltonian circuit:
A Hamiltonian cycle or Hamiltonian circuit is a cycle that visits each vertex exactly once.

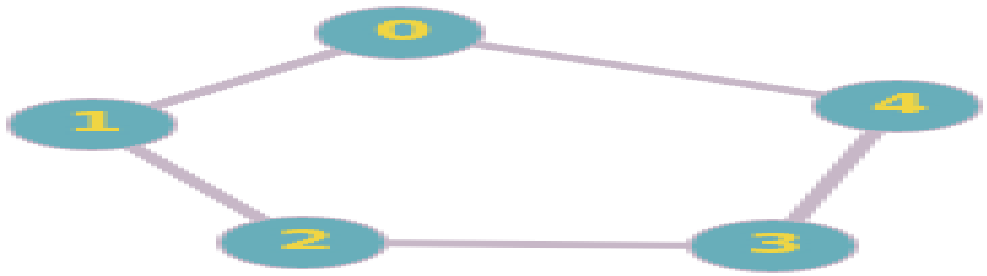

Figure: Example of Hamiltonian circuit

Hamiltonian circuit for the above graph:
012340 or 123401 or 234012 or 340123 or 401234

Hamiltonian Graph

Hamiltonian Graph:
A graph that contains a Hamiltonian cycle is called a Hamiltonian Graph.

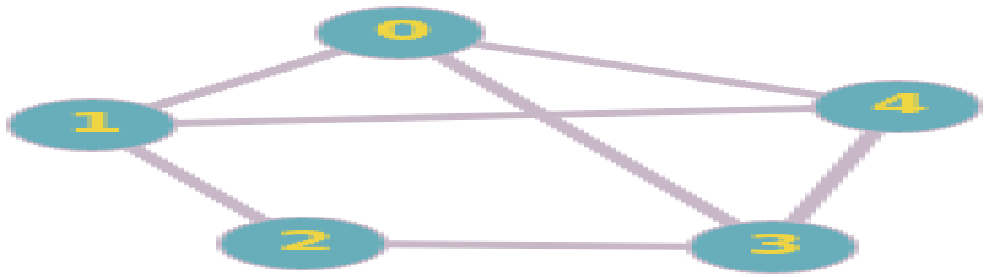

Figure: Example of Hamiltonian Graph

Hamiltonian Graph

Application

Hamiltonian Graph is about finding path traversing through finite vertices in a graph some of the real world applications are:
Mail delivery system.
Pizza delivery.
Travelling salesman.

Euler polyhedral formula

what is polyhedra ?

Polyhedra:
A Polyhedra is a solid in three dimensions with flat polygonal faces, straight edges and sharp corners or vertices.

Figure: Example of polyhedra[7]

Euler polyhedral formula

In 1750 Euler discovered the polyhedral formula $V - E + F = 2$ relating the number of vertices (V), edges (E), and faces (F) of a polyhedron . formula states that for graph with finite vertices and edges there should be finite faces.

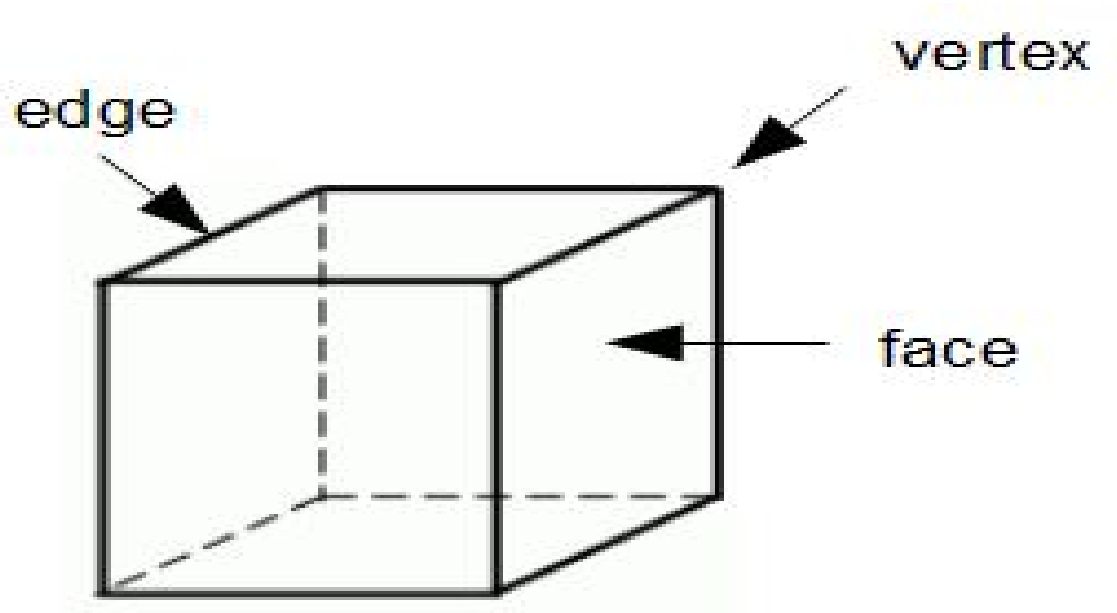

Figure: Example of polyhedra[7]

Polyhedra shape must not have any holes

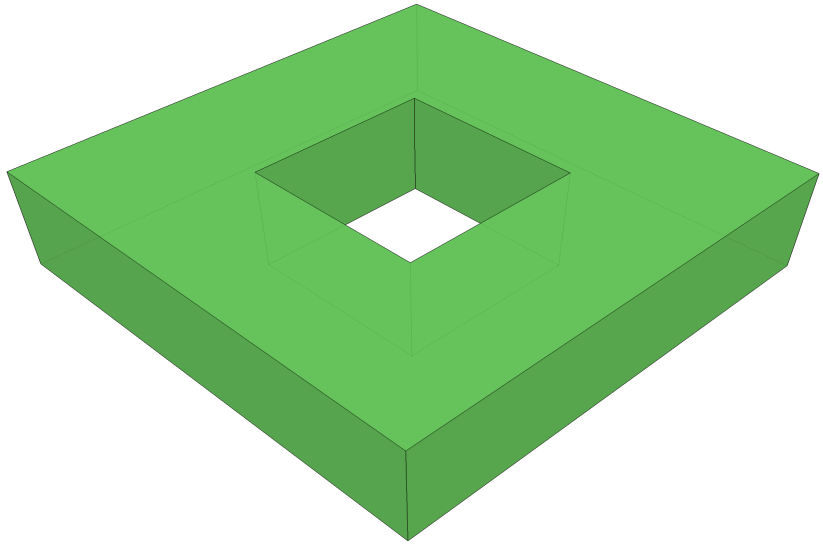

Figure: Example of polyhedral shape with hole

Euler polyhedral formula

Euler's Polyhedra rules

Polyhedra cannot be made up of two pieces stuck together, such as two cubes stuck together by one edge or one vertex

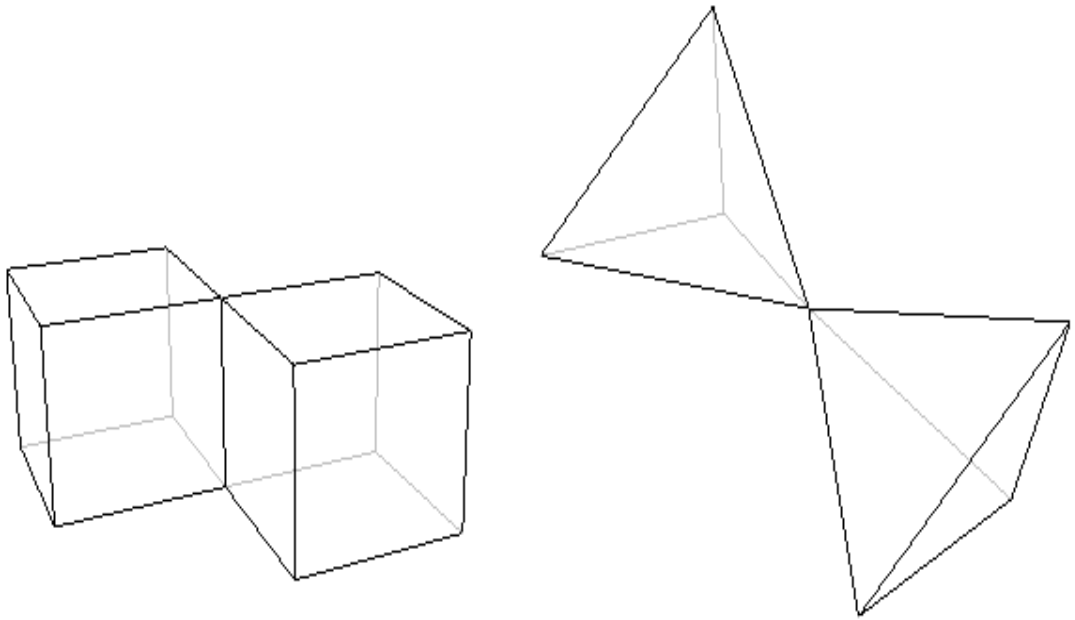

Figure: Example of polyhedra stuck with one edge and one vertex[7]

Four Color Map Problem

In 1852, Frances Guthrie trying to color a map of the counties(areas used for the purposes of administrative, geographical, cultural or political demarcation) of England, he noticed that at least four colors were required so that no two regions sharing a common border were the same color.

Francis Guthrie (1831 - 1899)

Francis Guthrie was a South African mathematician and botanist who first posed the Four Color Problem in 1852. He studied mathematics under Augustus De Morgan, and botany under John Lindley at University College London. Guthrie obtained his B.A. in 1850, and LL.B. in 1852 with first class honors[5].

Four Color Map Problem

Francis asked his brother Frederic Guthrie(was a student at university of London) whether true for all maps ?
Frederic Guthrie asked the same question to his professor Augustus De Morgan (at university of London) ?
De Morgan unable to give answer at that time wrote a letter to sir Hamilton about the question.
that's how the four color map problem spread.

Four Color Map Problem

In 1879, Alfred kempe(mathematician) gave the first proof for the problem.
Every planar graph can be four colorable.
Planar Graph :a graph drawn in a plane without any of its edges crossing or intersecting.

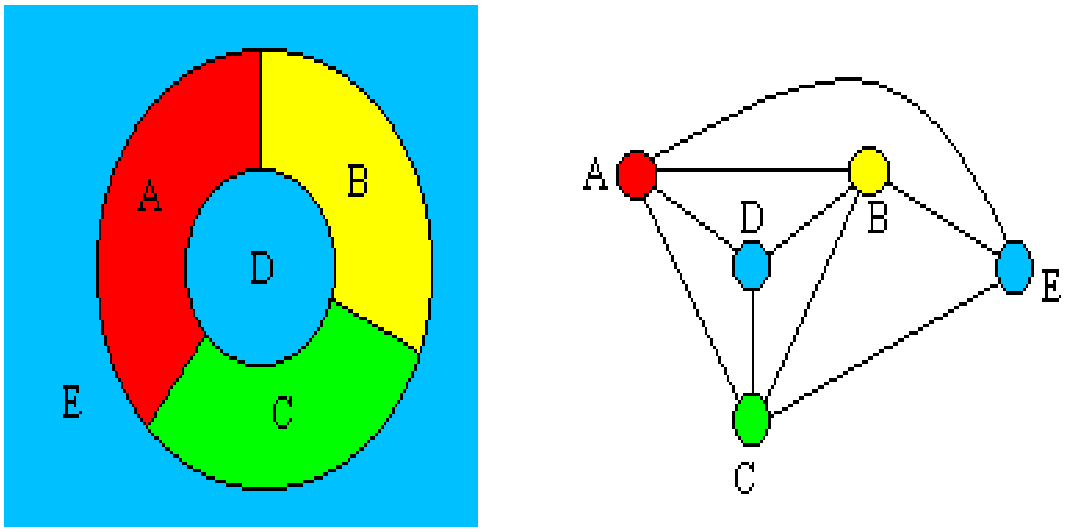

Figure: Example of map to planar Graph using four colors

Four Color Map Problem

After a decade Percy Heawood(mathematician) in 1890 found a flaw in the proof and reformulated it as five color map and provided a proof for it. Many mathematicians have offered various insights, re-formulations and even proofs of the theorem.

Four Color Map Problem

In 1976, Kenneth Appel and Wolfgang Haken (mathematicians,University of Illinois,) gave a corrected proof of Alfred kempe using computer and were the first to rely on computer checking thousands of cases.

Four Color Map Problem

Applications

some of the application where four color map problem is used are:
Making a schedule.
Assigning radio frequency to mobile station .

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