Preetham - 4NM18EC130
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CONTROL SYSTEM - TASK - 2

1) A unity feedback system has given open loop transfer function
2) Sketch root locus plot
3) Determine the value of K for damping ratio $\xi = 0.5$
4) Plot the range of value of K for which the system is stable, marginally stable & unstable
5) Show rootlocus and bode simulation in MATLAB
6) Find step response for closed loop transfer function obtained from (3)

$$G(s)H(s) = \frac{K(s+6)}{s^2+4s+20}$$

Number of poles $P = 2$

$s_1 = -2+4i$ $\quad$ $s_2 = -2-4i$

Number of zeroes $Z = 1$

$z = -6$

$q = 0 \ldots P-Z-1$

$q = 0$

Asymptotes

$$\theta = \frac{(2q+1)180^\circ}{P-Z}$$

$$\theta_0 = \frac{(2(0)+1)\,180^\circ}{1} = \underline{\underline{180^\circ}}$$

Centroid

$$\alpha = \frac{\Sigma \text{ Real part of poles} - \Sigma \text{ real part of zeroes}}{P-Z}$$

$$\alpha = \frac{-2+(-2)-(-6)}{1}$$

$$\alpha = \underline{\underline{2}}$$

Breakaway point

$1 + G(s)H(s) = 0$

$$1 + \frac{K(s+6)}{s^2+4s+20} = 0$$

$$s^2 + 4s + 20 + Ks + 6K = 0$$

$$K = \frac{-s^2-4s-20}{s+6}$$

$$\frac{dK}{ds} = \frac{(s+6)(-2s-4) - (-s^2-4s-20)}{(s+6)^2}$$

$$\frac{dK}{ds} = 0$$

$$\Rightarrow \quad \frac{-s^2-12s-4}{(s+6)^2} = 0$$

$$-s^2-12s-4 = 0$$

$s_1 = -0.343$ invalid $\qquad s_2 = -11.65$ valid

Intersection points

RH Array

$s^2 + s(K+4) + 20 + 6K$

s^2	1	$20+6K$
s^1	$K+4$	
s^0	$20+6K$	

$20+6K > 0$ $\qquad K+4 > 0$

$6K > -20$ $\qquad K > -4$

$K > -\frac{20}{6}$

$K > -3.33$

system is stable $K > -3.33$

marginally stable for $K = -3.33$

system is unstable for $K < -3.33$

Intersection point

$A(s) = s^2 + 20 + 6(-3.33)$

$s^2 = 0$

No. intersection point with imaginary axis

Damping ratio $\xi = 0.5$

$\theta = \cos^{-1}(\xi)$

$\theta = \cos^{-1}(0.5)$

$= \underline{\underline{60^\circ}}$

$1 + G(s)H(s)$

$s^2 + s(K+4) + 20 + 6K = 0$

$s^2 + 2\xi\omega_n s + \omega_n^2$

$\omega_n = K + 4 \qquad \omega_n^2 = 20 + 6K$

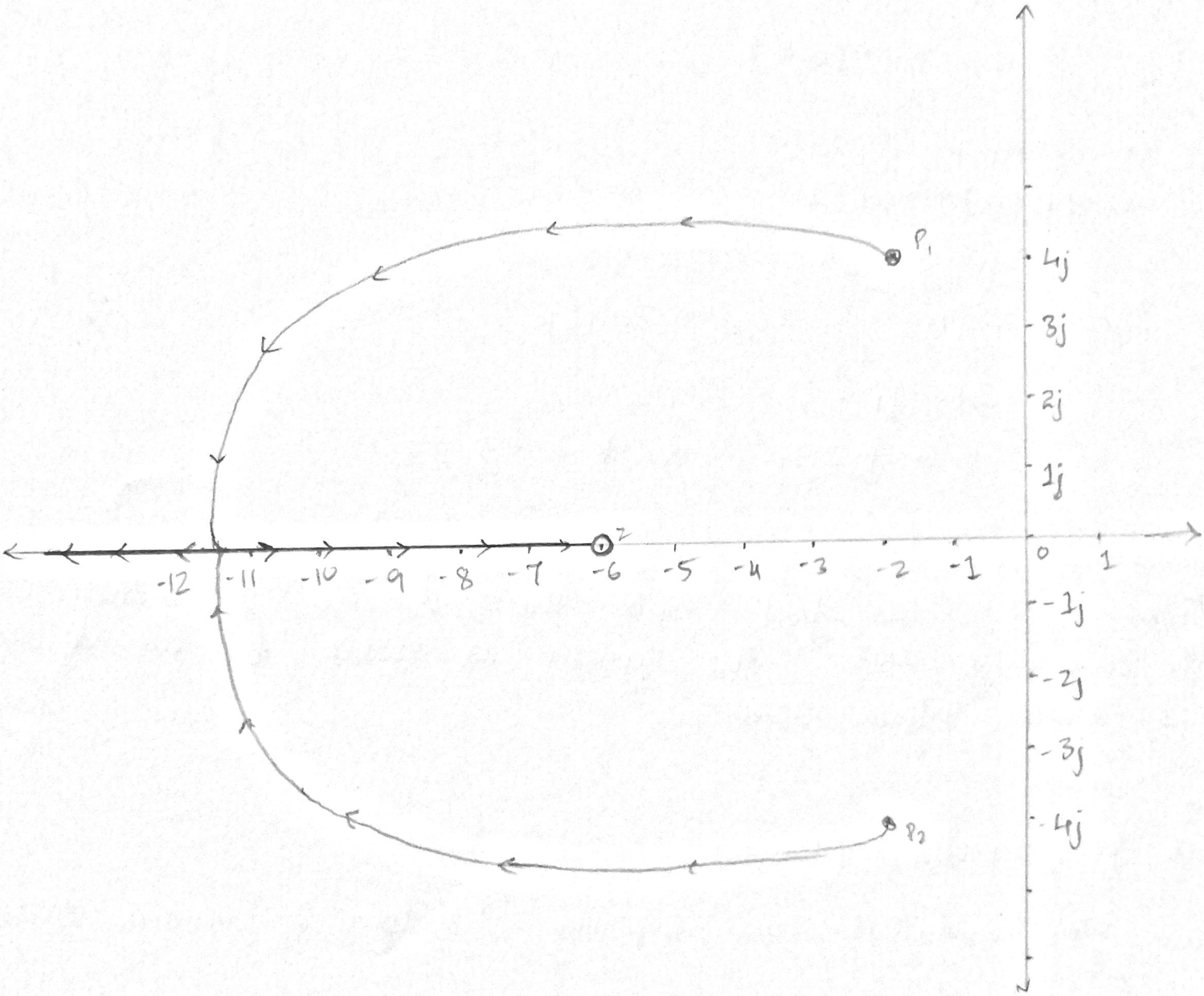
$K^2 + 2K - 4 = 0$

$K = \underline{\underline{1.23}}$ or $K = \underline{\underline{-3.23}}$

From root locus plot since there is no intersection with imaginary axis, the system is stable for all values of K greater than zero.

Bode plot stability ($K = 1$)

Since PM and GM are infinity, system is always stable.



Root locus plot