## Public transport optimization

Public transport is crucial for providing accessible mobility in cities and communities. It goes beyond trams, buses, and trolleybuses, often including minibuses. Public transport enables people to reach work, housing, education, and leisure activities efficiently and economically, particularly in densely populated areas.

As a socially important service, public transport is typically provided or regulated by the government. However, capital investments in its development are rarely fully recovered, and operating costs are often not covered by fares.

The value of public transport lies in time and resource savings for people's mobility, which contributes to increased GDP. Access to high-speed public transport can also enhance the value of real estate through transit-oriented development (TOD).

Ensuring transport accessibility is a pressing issue for cities, regional governments, and large enterprises. Optimizing public transport routes and achieving wide coverage with cost efficiency is crucial. Our experts have extensive experience in optimizing routes, from individual

In our work, we focus on transport demand rather than existing passenger flows. Passenger traffic depends on the interaction between transport demand (people's needs for mobility) and transport supply (specific characteristics of public transport routes). Passenger traffic alone does not necessarily reflect the actual transport demand and can be influenced by factors such as service quality, price, or alternative options.

To optimize public transport routes, we use specialized transport models that reflect current or projected transport demand. Existing passenger flows are primarily used for model calibration, not as the main data source for optimization.

To determine where people need to travel, both public transport trips and car trips are considered, as car trips represent potential demand for public transport. Hypotheses are then created for trunk and access routes based on specified criteria.

With the help of specialized software, potential passengers are redistributed across a hypothetical public transport network, and indicators of network efficiency

are established based on per-passenger or per-kilometer metrics.

The optimization process involves iterative adjustments and the creation of subsequent hypotheses until an optimal scheme and operating parameters for the public transport network are achieved.

Engaging a wider range of stakeholders in the optimization process improves the final result. Our transparent approach allows us to conduct workshops involving citizens, making them co-authors of the new public transport network scheme. This increases the chances of faster implementation.

Our experts have experience in optimizing public transport routes at various planning stages, from strategic concepts to tactical 1-route optimizations for cities or local communities.

To optimize routes and networks effectively, data on available transport, transport demand, and preferably a transport model of the city or region are required, along with passenger flow data for calibration and desired optimization criteria.

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Feel free to reach out to us with any questions. Route and network optimization for public transport is one of our favorite projects as it directly improves people's quality of life, increases their income, and saves their most valuable resource—time.

# Abstract

This research is motivated by the occurrence of inefficiency of the transportation system in the urban area of Sarbagita (Denpasar, Badung, Gianyar, Tabanan) caused by the use of private motorized vehicles. As a limited number of public goods, the efficiency of road space is measured not by the number of vehicles but by the number of people who use the road space. Therefore, the operation of a Mass Public Transportation System (SAUM) which has a large transport capacity optimally is the key to the efficiency of the transportation system in urban areas.This study aims to analyze the variables that affect the optimization of the Mass Public Transport System (SAUM) based on demand and service supply. This research was conducted using quantitative methods. The research location is in the urban area of Sarbagita, an agglomeration area in Bali. The research was conducted in October 2021, during the Covid-19 pandemic. The total population in this study was 14,140 with a sample of 397 passengers on 4 (four) Trans Metro Dewata service routes. Sampling using proportional stratified random sampling. The data used is in the form of a scale of

on the optimization of SAUM, while the supply, certainty and service integration variables have a positive and significant effect on the optimization of SAUM. The request has no significant effect on the integration of SAUM services. On the other hand, service supply and certainty have a positive and significant impact on the integration of SAUM services. This study also finds that the optimization of SAUM services is most influenced by service integration with a coefficient of 0.403, followed by service offerings with a coefficient of 0.374. Indirectly, the supply and demand variables have a positive and significant effect both on the variable between service integration through service certainty and on the endogenous variable of SAUM optimization through service assurance and service integration variables. The novelty of this research is that the service demand variable which has been used as a direct variable in the provision of transportation services does not have a significant effect on the optimization of SAUM. This research answers the phenomenon of the low loading rate of Trans Metro Dewata services, even though the service is free.This study suggests to the government as an institution providing public services that in an effort to realize the efficiency of the transportation system in urban areas, it is necessary to optimize SAUM services. The SAUM optimization efforts are not only based on demand and service supply but also involve variables between service certainty which include indicators of accessibility, service level and service availability and service integration variables which include indicators of physical integration, network integration and tariff integration. The results of this

## 1. PLANNING OF URBAN PUBLIC TRANSPORTATION

Urban transportation, particularly the phenomena of congestion and pollution, are among the major problems of society [1]. In this sense, empirical evidence has shown, repeatedly, that the construction of new infrastructure or expansion of existing roads is not the best alternative solution. There must be an adequately planning of the system - providing incentives to use public transportation - to mitigate the adverse effects associated with the operation of the system. Transit should also be planned to optimize its performance for the benefit of users and the city. With the growth of cities, the need for people for transport increases, and not all of these journeys may be in private transport due to congestion that is generated. Therefore, the government must provide a transit system well distributed and organized to meet the demand efficiently. All this must be planned with a holistic view of the problem in order to get the best use of economic resources, the best functionality for commuters, preserve the environment and make energy savings [2]. To justify the need for a transit route is necessary to determine the current and future demand of the system and its coverage, i.e., analyze both the current and necessary supply to provide an efficient, comfortable, safe, and economical service. Thus, the demand study constitutes basic information for proper planning of transportation [3]. Predicting future flows of the system to different situations is important and requires, in some degree, to know human behavior, characteristics of land use, economy and others, being necessary to do an analysis of socio-demographic information, of the plans of land use and development, as well as action plans in the region, and all that which affects or is affected by the transportation system being analyzed. When planning transport it is also intended to infer changes in transport demand, due to modifications to the current system. To determine the current demand for transport there are several methods that can corroborate each other, some of the most commonly used are origin-destination studies that can be done with surveys, traffic counts, and traffic studies [3].

The planning of transit systems can be for short-term (operational planning) or for medium to long term (strategic planning). Usually the design of bus routes, frequencies and scheduling of vehicles are short-term problems. This operational planning consists of several sequential steps [2]: 1. Study of the demand that travels from the different origins to different destinations in the city, 2. Modal split, 3. Design of the lines or routes, 4. Frequency determination of the number of passengers for each line, 5. Determining schedules, 6. Vehicle scheduling, and 7. Scheduling of drivers. The first three steps are usually performed by the regulators, i.e. the municipality. The last four steps are generally executed by the service operators. To solve and optimize these stages, it has been used operations research. It integrates a large number of mathematical tools, including optimization and simulation, who allow addressing rationally some of the complex problems of transit planning to support decision making. The global solution to the problem depends on the solution of each of the stages of the process [4].

## 2. STAGES IN THE PLANNING OF THE OPERATION OF COLLECTIVE URBAN TRANSPORTATION

It is essential to study urban transportation as a component of the city and as a function of it [2]. This should ensure that accessibility for people to different activities effectively meets through adequate mobility (in terms of time and monetary cost), so there must be adequate physical-spatial connectivity that makes it possible, neutralizing the negative effect of the distance [5]. To materialize these functions, transportation must be supported by at least two main physical instances: first, a road system that ensures connectivity between different areas or zones of the city, and secondly, a transportation system that using road infrastructure, allows the movement of people between their origins and destinations. Due to the high dimensional complexity, planning of collective public transportation can be broken down into 2 major phases, and each is subdivided into several stages. The two phases are:

*Strategic planning phase of the transportation system:* it is based on the socioeconomic classification of people, the regional distribution of households and firms, and road infrastructure. At this stage we estimate the number of trips that are generated from different origins to different destinations, which is synthesized in a matrix commonly called "origin-destination (OD) matrix." This phase could be considered descriptive: the analyst attempts to describe the possible behavior of people, hence the most useful tools are simulation and forecasting [2]. For more information about transportation planning refer to Ortuzar and Willumsen [6]

*Planning phase of the system operation of the collective public transportation.* This step might be considered normative, because it attempts to establish the best way to operate the system, so that the most useful tool may be optimization [2]. In general, this phase could be divided into the following stages: Study of the demand that travels from the different origins to different destinations in the city: it depends on the configuration of the city, on the projection of the population and their socioeconomic characteristics, etc. The modal split is the estimated number of trips to be made by the mode of transport in evaluation, distributed in time: it depends on the projection of the population, on its socioeconomic characteristics, on the characteristics of the modes, etc. For the design of the lines or routes, the number of lines of transport (bus, subway or other mass transit vehicles) is set as well as the layout of its routes. The frequency determination of the number of passes for each line is based on demand studies and distance or travel time defined by the routes. In determining schedules, for each line, the departure times from the station of origin and arrival at destination stations are determined as well as the synchronization of shipments between routes that share or transfer crossing points. In the vehicle scheduling, the fleet assignment is given based on vehicles available for traveling. Finally, in the scheduling of drivers, available staff and resources are assigned for scheduled trips per line. Readers interested in more detail of the operational and strategic transport planning stage, can refer to Sheffi [7], McNally [8], Ortuzar and Willumsen [6], and Jaramillo and

## 3. NETWORK DESIGN OF BUS ROUTES

The design model of public transportation routes identifies the best options for bus routes that travel over a territory, considering that it should serve the maximum possible demand. People must travel from origins to destinations, and have to choose their travel from the bus routes that are offered in competition to other modes of transportation (e.g. car, motorbike) [2]. It also needs to consider the route design to protect the interests of the operators for the bus system to be economically viable. Specifically, the model must choose R bus routes on a network of nodes and potential segments defined a priori. It must consider maximizing coverage, minimizing both transfers and cost of trips made by travelers, but in turn, minimizing operating costs of the bus system. The model characterizes the behavior of the system user on two choices: the first, regarding the choice of route if using the bus system, and second, regarding their choice of transportation mode in which the user will travel eventually. The choice of route is defined as an allocation model, which follows the principles of Wardrop [9] that characterize the equilibrium reached in the system. As for the choice of transportation mode, it is required to have a comparison of the costs that users would assume if they opt for the bus system, and the costs of private options (distance, price, or travel time) which are exogenous parameters to the model.

The model can then be considered as the integration of two sub-models: the first is the design of routes offered by the operator and the second represents the decisions of the end-user of the system (assignment and mode choice). They are modeled simultaneously as a feedback system because the objectives of the operators do not coincide with those of the end-users of the system. Therefore a subsystem must influence the other one and vice versa. The first sub-model represents the decisions of the supply (the operators): bus routes and stations. Their objectives are to minimize operating costs and maximize revenues, which can be assessed indirectly by covering the demand which opts for the bus system. The second submodel represents demand decisions, choice of bus routes considering minimum transfers and costs (distance, time or rate) of travel. If the first sub-model is optimized without considering the second one, the solution would be a local optimum and a global suboptimum, the same for the second sub-model. Therefore they must be integrated into a single model. It would be more complete if it simultaneously makes decisions about the frequency of release of the buses. However, this purpose becomes very complex when solving the model, since it is characterized as NP-hard (nondeterministic polynomial problem) even for models representing territories of small areas, so it is not considered in the current model. Other conditions that influence the choices of the users of the system such as comfort and security are also not considered due to the difficulty in acquiring and treating reliable data, and the computational effort that would be added. Travel demand is highly variable at different times of the day; however, the model represents decisions for a single period, considering a critical time, for example, the peak hours of the morning when demand is usually higher.

### 3.1. Background

Wardrop [9] proposed two principles for the assignment of trips to a transportation network representing the behavior of users when deciding on the route by opting to travel from an origin Oi to a destination Dj. The first principle states that each traveler chooses the path that offers minimum cost (User equilibrium). Wardrop's second principle states that in steady state, the average travel time of the global system is minimal, considering that the travel behavior is cooperative (System optimum). Based on these principles, Beckmann, et al. [10] made the first equilibrium model of transportation networks.

Bruno, et al. [11] incorporated data from OD arrays to this model. Laporte, et al. [12] and Hamacher, et al. [13] address the problem of stations location on a given route. García and Marín [14, 15] studied the problem of network design using bi-level programming considering multimodal assignment in public and private modes. Marín [16] continued the above study without considering previous rail lines. Laporte, et al. [17] proposed the static model of network design of public transportation. Then, Marín and Jaramillo [18] proposed the capacity expansion of these systems using an accelerated Benders decomposition algorithm. Guihaire and Hao [19] present a comprehensive review of design and route scheduling proposing a classification of 69 methodological approaches that have to do with the design of routes, frequency and scheduling. Fan and Machemehl [20] formulated an entire multi-objective mixed model for the design of public transportation routes using Tabu Search. Using this method, Martínez Cuart [21] analyzes from a related point of view the design of public transportation networks and the complementarity of different hierarchy services by introducing the hierarchical structure of traditional modes of urban transportation (bus, tram or LRT, and metro). The author proposes a model and an algorithm to create its own network. Using Tabu Search, Yan, et al. [22] show that the objective functions could converge to the Pareto optimal set and be equally distributed along the Pareto curve with the help of the optimization algorithm proposed by them. Blum and Edsel [23] propose to optimize the design problem of public transportation networks for a reference transit network using a selective solution of Pool Pruning agent. Curtin and Biba [24] use a new method of maximizing the Arc-Node type service to determine optimal routes of public transportation. Finally, Yu, et al. [25] develop a model to design a transit network considering the density of demand in relation to direct demands, transfers, and the length of the routes.

### 3.2. Model

The proposed model is an extension of the models of Marín [16] and Marín and Jaramillo [18] for the design of lines. It is added the objective of minimizing the number of transfers, as a traveler may prefer a longer route with fewer transfers than a shorter path with many transfers. Then we require adding new decision variables and constraints to the original formulation. It increases computational complexity to solve the problem. In contrast, we aim to eliminate variables associated with the stations. For instance, Marín [16] identified stations counting an additional cost of building and defined the optimal locations of the initial and final stations of lines. The model assumes that the scheduler

### 3.2.1 Model considerations

- In the model, the territory is represented by a network of N nodes  Among nodes are defined arcs ij that are potential segments of bus routes.
- The model suggests the design of R routes on the network, which initial and final stations are determined beforehand by the scheduler.
- The model is stationary, i.e., considers only a critical period and the demand is deterministic. This is represented by an OD matrix. Travelers have the option to choose between two modes: the bus system and an alternative. Their choice will depend on the comparison of the costs of traveling in both modes. The costs of the alternative mode are exogenous to the model.
- The nodes o and d belong to the network N. There are dummy arcs od, one for each demand od, which can choose that arc in the event that, given the solution of bus routes, is not feasible to travel through them, or because traveling in the bus system will represent large costs.
- The model does not consider costs of walking to stations or waiting times, because it does not include the frequency design. Future work will incorporate these variables.
- Besides the network definition, we require some additional data: lengths of the potential segments ij, lij; initial stations e(r) and terminal stations f(r) of the r routes; demand od: refers to the number of passengers Dod traveling from node o to destination d; the cost to be assumed by the user in case of choosing the alternative mode Caltod; and the data of traffic conditions through the arcs ij, as average vehicle speed vij and other parameters that will be presented in detail later.

### 3.2.2. Sets

The sets of the model are:

- i: network nodes (represented as j,k,o,d)
- ij: segments that directly connect nodes i and j
- od: segments that directly connect nodes o and d and represent an alternative system.
- r: routes to design
- o: origin nodes of the demand od
- d: destination nodes of the demand od
- od: travelers going from node o to node d

### 3.2.3. Decision Variables

### 3.3. Objective Functions

In this section we present the objective functions for supply, demand and transfers of collective public transportation

### 3.3.1. Supply

Maximize the demand to be served by the bus service

Minimize operating costs. As usually there is no enough information for its evaluation, it is possible to use the length of the designed routes as an indirect evaluator, as shown in Eq. (2):

3.3.2. Demand

Minimize the cost of travel: The costs assumed by users of the global system (all modes) are:

Where Cbus is the function of the total costs of the

approximate and practical approach to the problem because the corresponding function is linear. The objective function is given by Eq. (4):

### 3.3.3. Minimize transfers

### 3.4. Constraints

The constraints are divided into three sections: the first is directly related to the design of routes, the second with the allocation of demand on these routes, and the last one with the link between those two sub-problems. The constraints of the design of routes are:

Initial Station: For each route r, must be assigned a segment originating from the initial station e(r), as shown in Eq. (6):

Final Station: For each route r, must be assigned a segment destined for the terminal station f(r), as shown in Eq. (7):

For each route and the different nodes to the origin and terminal station, there must be balance: if a segment of route r arrives at node i, a segment must also leave from it. The corresponding constraint is defined by:

If desired, in the design of routes that assign segments from e(r) to f(r) and then be returned along the same path, the following constraints can be incorporated. This is defined by:

The former submodel corresponds to the standard problem which is called the problem of minimum-cost path, whose solution often presents the problem of the presence of loops. To ensure that no cycles will occur, we integrate the following constraint given by Eq. (10):

Where E is a subset of the network N, with more than 2 nodes. This constraint adds further complexity to the resolution of the problem so it is usually incorporated only if the optimal solution has a cycle, in which case it is advisable to only incorporate the constraint that points to the solution of that particular cycle. The constraints on the allocation of the demand to routes are given by Eq. (11) to (17). From the trip origin o of each demand od, the entire demand should be allocated as follows:

Toward the final destination trip d of each demand od, the entire demand should be allocated as follows:

In the other nodes i, different from the trip origin or destination of each demand od, balance must be met: if a traveler arrives at that node, he must continue his journey to another node, as shown in Eq. (13).

Modal disaggregation of demand: this constraint represents the behavior of the traveler, which is often complex because it depends on objective and subjective evaluations of the decision maker on the system. For computational simplicity, in this model we consider that the demand od opt for the bus system if the cost (time) of its course is cheaper than an alternative way Codalt, which is considered an exogenous parameter to the model. Thus the constraint can be stated

### 3.5. Multi-objective Solution

In general, the model is mixed integer programming. Because it contains large amounts of binary variables, it is difficult to solve. Then it is necessary to use powerful solvers, decomposition techniques or metaheuristics techniques such as Genetic Algorithms, Tabu Search, Ant Colony, etc. In this paper we used the software GAMS 22.7 and its solver CPLEX.

The problem considers multiple conflicting objectives so it does not have a global optimal solution, but it has a set of Pareto-optimal solutions. They are defined as the set of solutions that meet the condition that no other feasible solution surpasses them in all objectives. Then, it is required to face the problem by a multi-objective optimization method. There are many of these methods and they all try to solve the problem from different points of view. Interested readers in multi-objective optimization methods can refer to Smith, et al. [26]. In our case, we will apply the method of weightings [27] for being the most recognized and applied, due to its low computational effort.

### 3.6. Example

To test the model we used a network that represents a region with 15 nodes corresponding to possible stops and strategic interconnections. We want to design 4 bus routes knowing the demand matrix of potential travels D for a peak period of demand. Each arc has a length lij. In Figure 2 we show the corresponding network.

The matrix of demands of travels D is exogenous to the model and is as follows:

In order to satisfy all possible demand from the bus network we consider $C_{alt}$= M (a very high value) for all demands, such that the demand $D^{od}$ only uses the alternate mode if it is not possible for it to access any of the designed routes.

The multi-objective optimization method of weighting factors integrates into a single objective function, the weighted sum of all objective functions of the problem. Those weights correspond to the relative importance that the decision maker gives to each one and all must sum 1. Each objective function must be normalized by a factor that let the functions to be dimensionless because they are all represented in different units (passengers, costs, etc.). The normalizing factor must be such that the values of the normalized objective function takes values between 0 to 1. In addition, the objective functions to be minimized must be multiplied previously by -1 because they are included in a global objective function to be maximized, as shown in Eqs. (18) to (22):

The relative importance weights used are $w_1$=0.25, $w_2$=0.25, $w_3$=0.25 and $w_4$=0.25, in order to neutralize its influence on the model analysis. For simplicity, costs are considered in units of distance, so that $v_{ij}$=1. The maximum number of transfers is $tr_{max}$=2. We consider all the constraints defined for the model, except for the constraints that prevent loops, which may be included in the solution only if loops are detected, incorporating them to break them. In the example it was not necessary to incorporate any of them.

## Results

## 4. CONCLUSIONS AND FUTURE WORK

The proposed model advances over those found in the literature as it includes additional features such as minimizing transfers. Previous models did not included this aspect, so they gave inconsistent solutions with the usual wishes of the decision-makers, who may prefer more expensive trips in time or length but do less transfers.

Regarding the solution of the model we suggest the application of meta-heuristic techniques as Genetic Algorithms. This would allow better management of non-linear functions, including binary variables and application to large networks. We also propose the incorporation of non-linear functions of congestion and modal choice, such as logit models, as well as other functions which represent the decision maker's behavior, including latent variables and characteristics of the bounded rationality of the decision-maker. It is also advisable to include stochasticity in some variables, demand, travel times, etc. in order to obtain more robust models.

Today's public transportation systems, especially in large cities, face many challenges. Some common challenges of the public transportation systems are a surge in the number of passengers and transportation costs, continuous driver shortage due to an income decrease caused by hours-of-service regulations and limiting the number of miles drivers can drive, and increasing demand for real-time reporting for improving efficiency and service levels. Moreover, the main reasons for many complaints about public transportation in many transportation systems are delays. When a vehicle is delayed, not only its passengers will reach their destination with delay, but also, they may miss their connection bus or train even when small delays occur. Public transportation managers should focus on customers' convenience by analyzing the roots of

Transportation planning is a very complicated problem because planners must take into account multiple competing criteria (e.g., service requirements, asset utilization, cost minimization, workload fairness, etc.), components of the transportation system, and their interconnection. Moreover, transportation planners should consider many factors to create efficient plans, including but not limited to vehicle and driver availability, vehicle size and capacity, traffic details, travel time windows, and passengers' locations, which are too many for one to handle efficiently in his head. On the other hand, plans often are required to be modified later due to unexpected events, such as vehicle breakdown, drivers' sickness, and severe weather conditions, which make transportation planning even more complex. However, many

technologies, which is very hard and time-consuming for even the most experienced planners. Besides, increasing demand for faster planning, increasing pressure to reduce transportation costs through better decision making, and increasing complexity of transportation problems due to a significant increase in the size of transportation networks make manual transportation planning almost impossible.

Public transportation planning mainly includes short-term (operational) planning, medium-term (tactical) planning, and long-term (strategic) planning. Strategic planning consists of decisions that have an impact lasting many years. Decisions about the locations of terminals and stops are an example of strategic planning. Operational planning consists of problems like designing bus routes,

timetable design, fleet assignment, and crew assignment, where decisions taken at a given stage influence decisions taken at subsequent stages. The frequency setting problem determines the time interval between subsequent buses on the lines, based on their itinerary and the demand given by an origin-destination matrix. When designing the itinerary of the lines, i.e., the route network, a preliminary setting of frequencies is needed. Also, during the tactical planning it is necessary to adjust the frequencies to demand variations along different seasons of the year or time of day, or as a response to changes in the route network design. The frequencies impact both the passengers (waiting time, the capacity of the lines) and also the operators.

The good news is that most operational, tactical, and strategic decisions for designing a public transportation system can be optimized and automated by Mathematical Optimization, also known as Mathematical Programming. Mathematical Optimization is a powerful prescriptive analytics methodology that enables decision-makers to instantly identify the best decision out of a large number of alternatives (e.g., millions of possible decisions) that leads to the best possible result according to prespecified criteria such as profitability, service level, resource utilization, etc. Mathematical Optimization includes mathematical modeling methods that capture the key features of a complex business problem including business rules, objectives, and decisions as data-driven mathematical models. Optimization models are solved by

decision, also known as the optimal solution. Mathematical Optimization has many advantages including but not limited to considering interdependencies of complex systems, supporting what-if scenario analysis, avoiding personal bias, and significant flexibility in constantly changing business environments.

In what follows, we will introduce some of the most important use cases of Mathematical Optimization in public transportation systems:

- **Transit Frequency:** Transit frequency optimization models can determine the optimal time interval between subsequent buses for a set of public transportation lines given by their itineraries, i.e., sequences of stops and street sections, that reaches a

- **Stop/Terminal Location:** Decision-makers can use facility location optimization models to determine the optimal locations of stops or terminals from a given set of candidate locations based on a pre-determined objective such as maximization of the population covered by the facilities.

- **Vehicle Routing:** Vehicle routing optimization models identify the best routes for vehicles (e.g., buses) that travel over a region to serve the maximum possible demand, minimize the transportation cost, or minimize passengers' waiting time.

- **Delay Management:** When a delay occurs, two alternatives are available: 1) A connection bus waits and causes a delay for both

- **Maintenance Scheduling:** Maintenance scheduling optimization models determine when and where should roadway or vehicle maintenance be scheduled to minimize the consequences of non-working assets on the efficiency of the public transportation system.

- **Driver Scheduling:** Resource allocation and scheduling optimization models work seamlessly together to assign available drivers to vehicles and shifts in order to maximize driver's job satisfaction, resource utilization, and service level by generating fair and efficient driver schedules.

In conclusion, Mathematical Optimization can help public transportation systems, especially in large cities, overcome their existing challenges and unlock next-level business growth. The transportation planning tools that are equipped with advanced optimization models and algorithms can reduce planning time, make the transportation planning process more efficient, improve drivers' and passengers' convenience, decrease transportation costs, and increase visibility in the transportation system, to name but a few. Moreover, they enable managers to test various what-if scenarios, including suggested changes, to learn how they would affect the performance of their transportation system.