Justifying Geometric optics

The Maxwell Way

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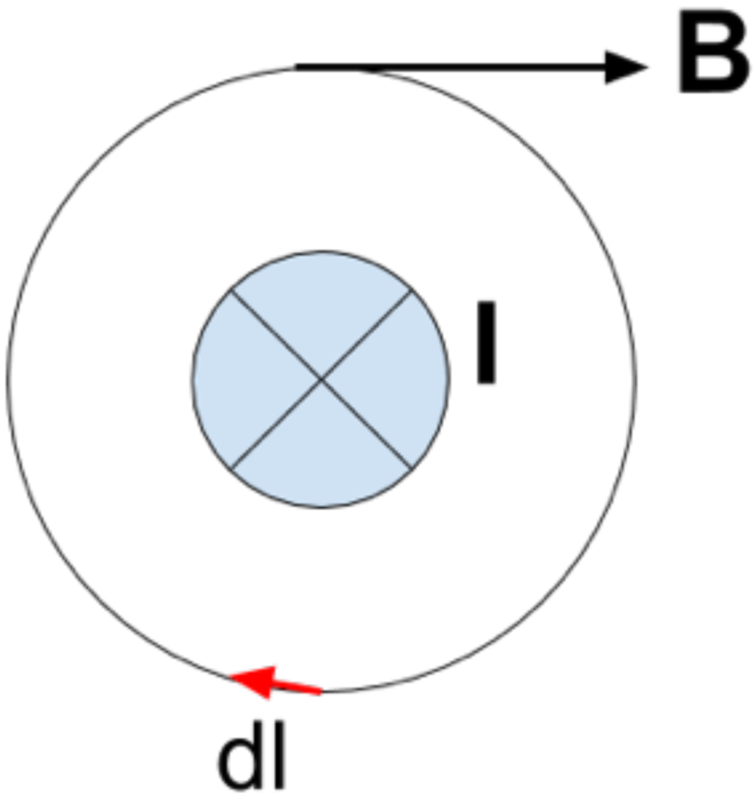
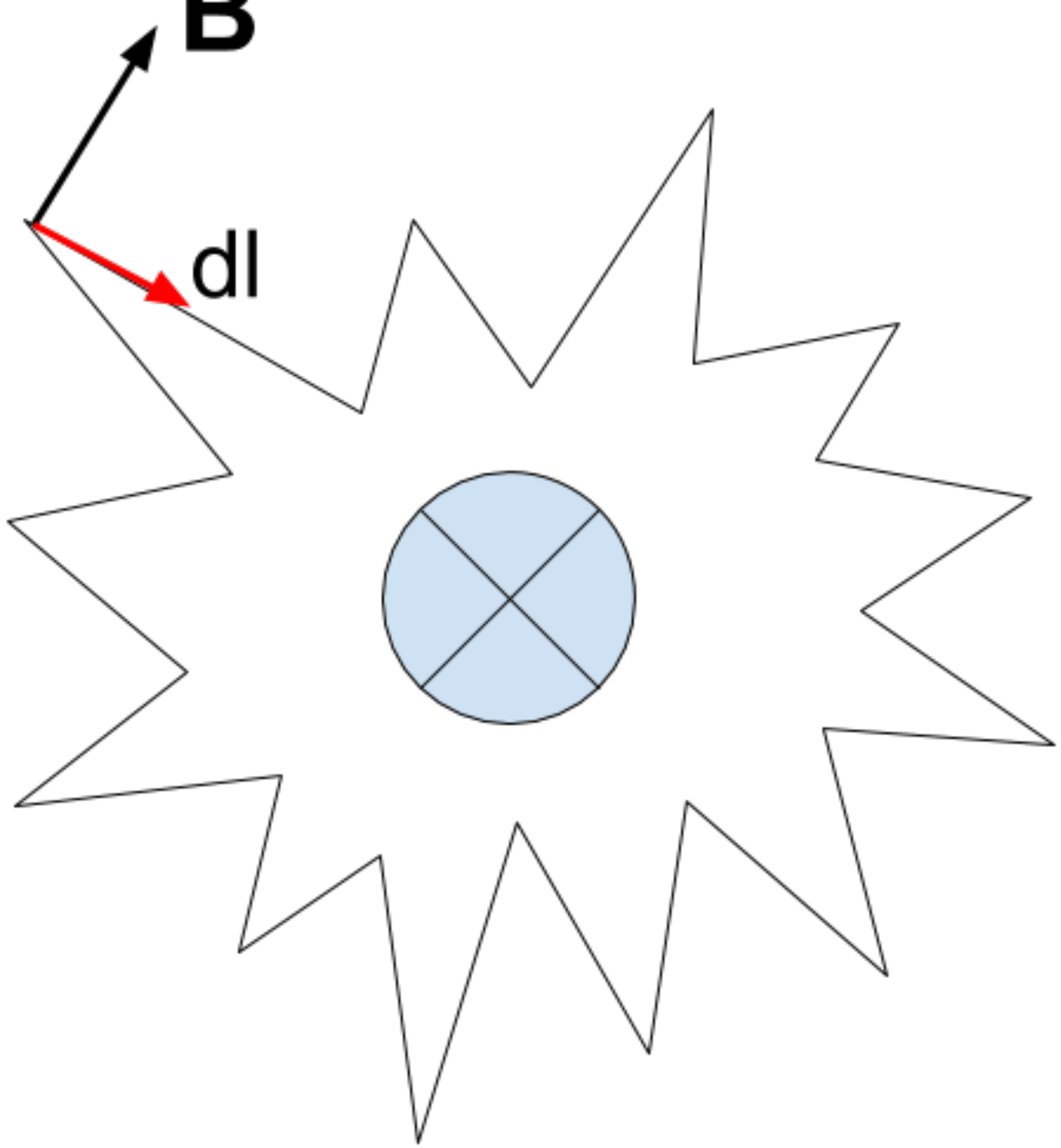


Thing that goes round
and round around the wire
is the magnetic field

Current going into the
plane of the slide
(imagine current carrying wire)

$$\oint \vec{B}.\vec{dl} = B(2\pi r) = \mu_0 I$$

Ampere = Does not have to be a circle,
only has to be closed



$$\oint \vec{B}.\vec{dl} = \mu_0 I_{\text{enclosed}}$$

$$I_{\text{enclosed}} = \oint \vec{J}.d\vec{s}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} \qquad \vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix}$$

Almost 3rd Maxwell equation

Introducing the Maxwell equations

$$\nabla . \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla . \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

Ampere

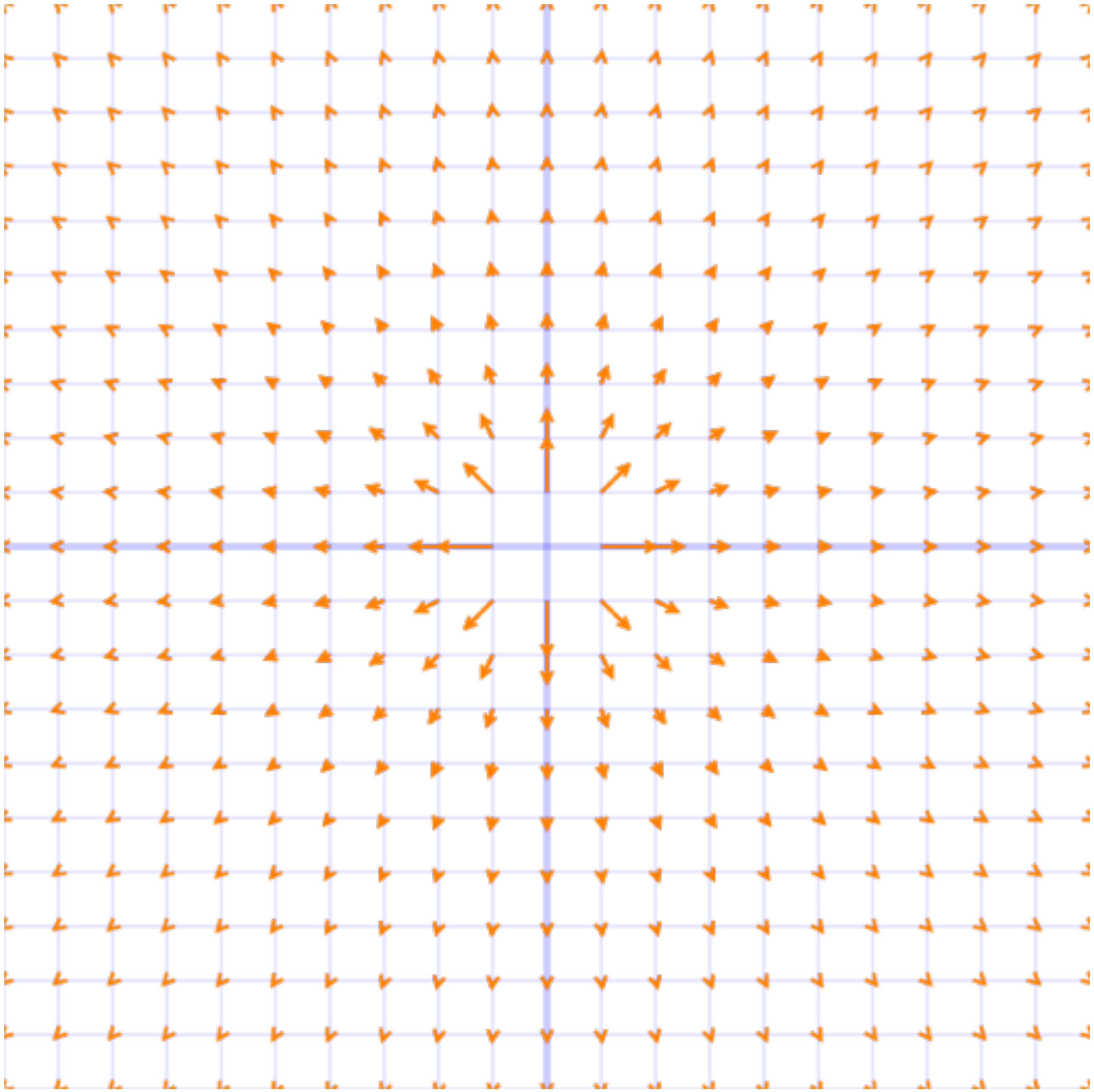
Maxwell

In free space

$$\rho = 0$$

$$\vec{J} = 0$$

Genius idea:
We understood how light travels,
made mobile phones, radio, radar and GPS possible



Explaining divergence

$$\vec{\nabla}.\text{terms}$$

Field with positive divergence

Negative divergence fields will point “inwards” (reverse arrows here)

Zero divergence fields are just flat (think rain falling from sky)

EM waves can exist without any charges or currents

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}(\nabla \times B)$$

$$\nabla(\nabla . E) - \nabla^2 E = -\epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

$$\epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} = \nabla^2 E$$

wave with velocity = $\frac{1}{\sqrt{\epsilon_0 \mu_0}}$

$$\nabla^2 B = \epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

3D wave equations

Oscillation in
E and B
without any charges
or currents

Propagation with
speed of light

$$\frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

determined by two
static quantities

1D case, A simple solution to Maxwell Equations

$$E_x = E_{0x}\cos(\omega t - kz) \qquad E_y = 0 \qquad E_z = 0 \qquad \frac{\partial^2 E_x}{\partial z^2} = \mu_0\epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

What is B? $\xrightarrow{\text{use}}$ $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \longrightarrow \frac{\partial E_x}{\partial z}\hat{y} = -\frac{\partial \vec{B}}{\partial t}$

$$kE_{0x}\sin(\omega t - kz)\hat{y} = -\frac{\partial \vec{B}}{\partial t}$$

Integration in time

$$\vec{B} = \frac{k}{\omega}E_{0x}\cos(\omega t - kz)\hat{y}$$

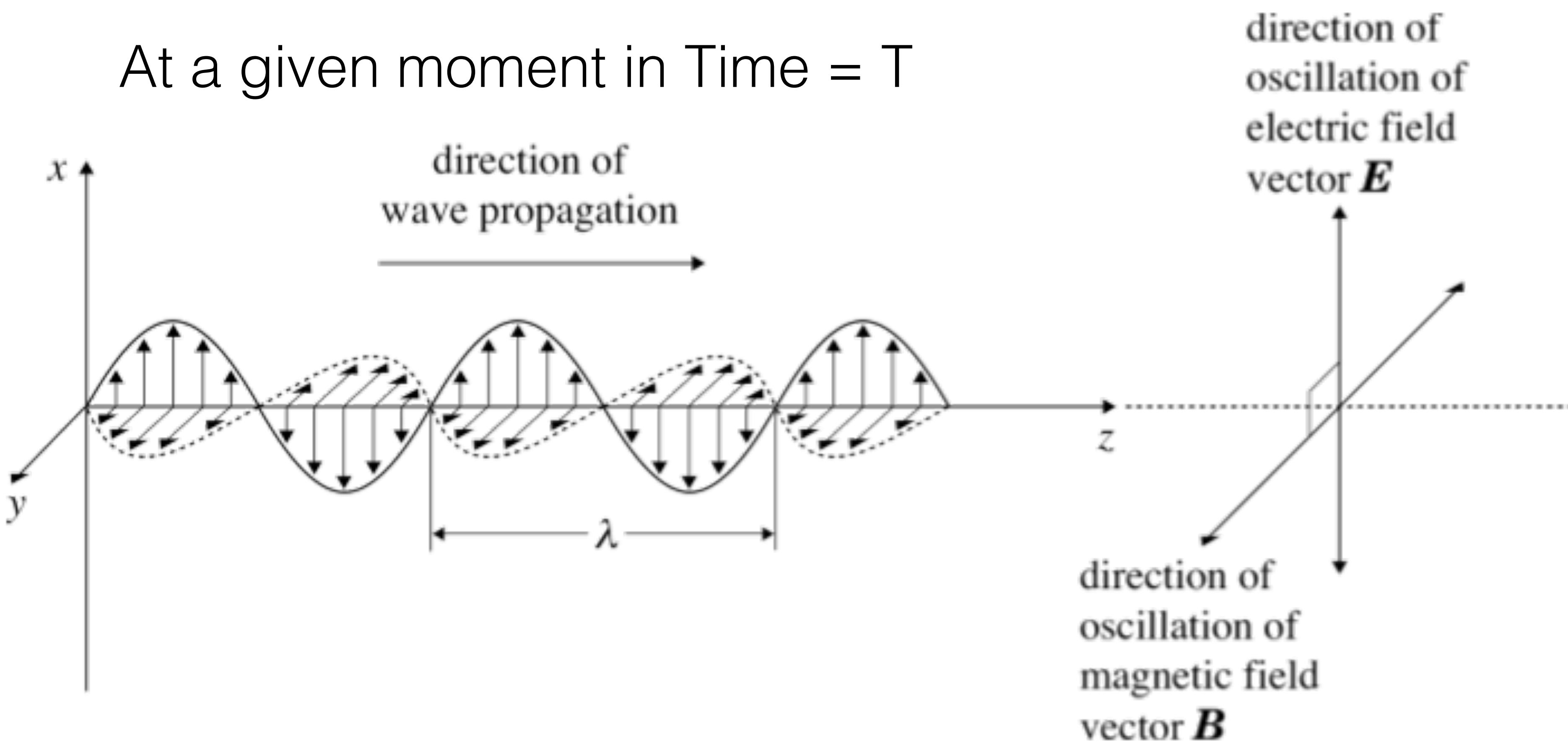
$$= \frac{E_{0x}}{c}\cos(\omega t - kz)\hat{y}$$

$\vec{B}$ and $\vec{E}$ in phase

$\vec{B} \perp \vec{E} \perp$ direction of propagation $= \vec{E} \times \vec{B}$

$$B = \frac{E}{c}$$

At a given moment in Time = T



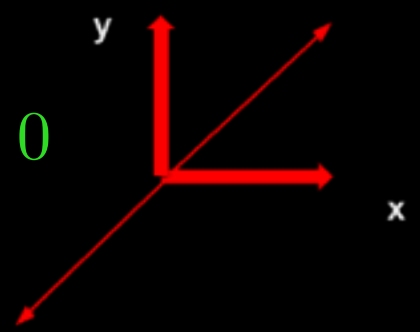
Linearly polarised E field in x direction

If you are standing at a point in z you will see something going up and down in X, that is E field of light

A more complicated solution than before

$$E_x = E_{0x} \cos(\omega t - kz) \qquad E_y = E_{0y} \cos\left(\omega t - kz + \delta\right) \qquad E_z = 0$$

$$\delta = 0$$



Linearly polarised E field in this direction

General Plane wave solutions

$$\vec{E}(\vec{r}, t) = \vec{E}_0 \cos\left(\omega t - \vec{k}.\vec{r}\right)$$

What does this term mean?

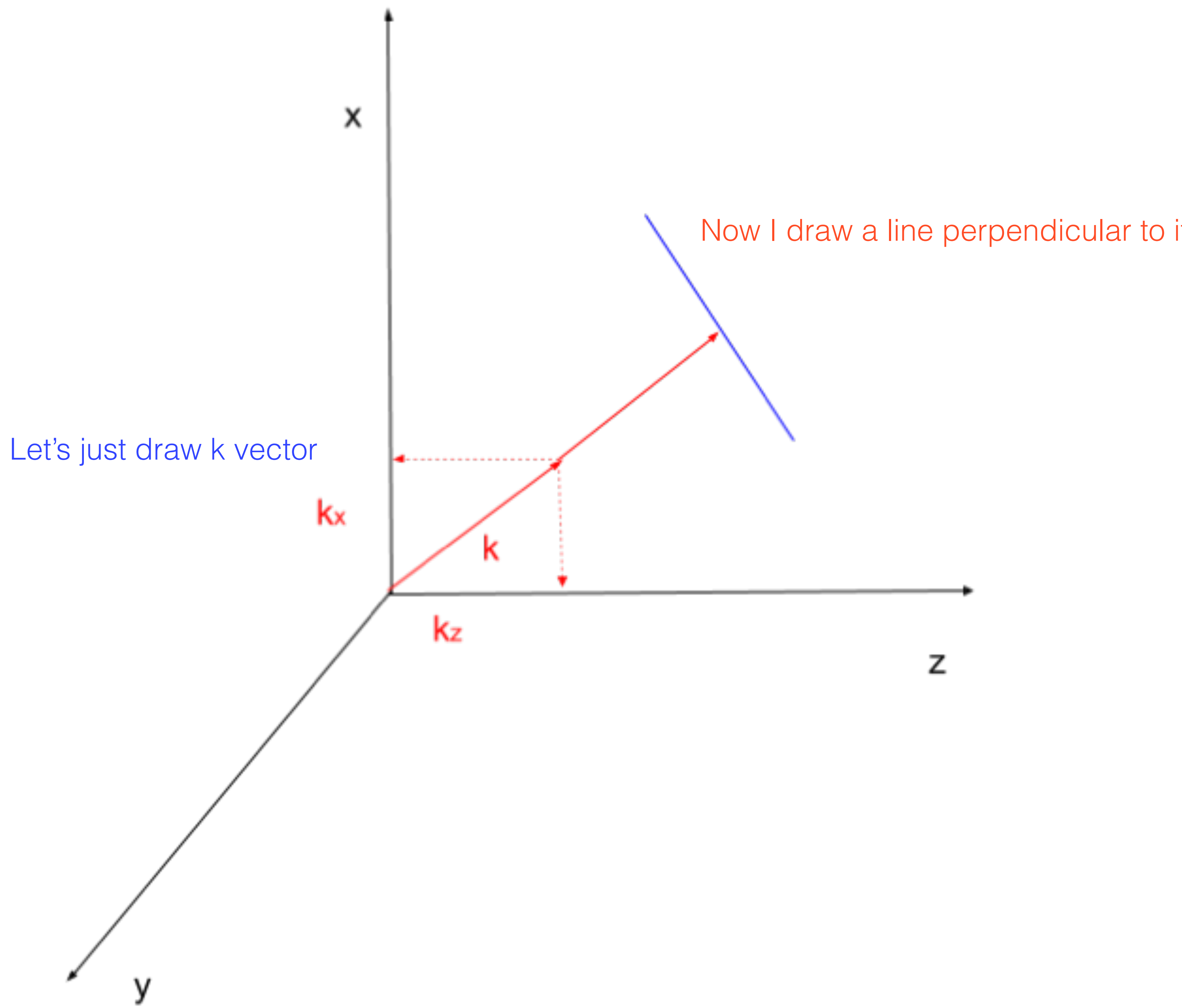
A geometric insight here is
going to justify geometrical optics

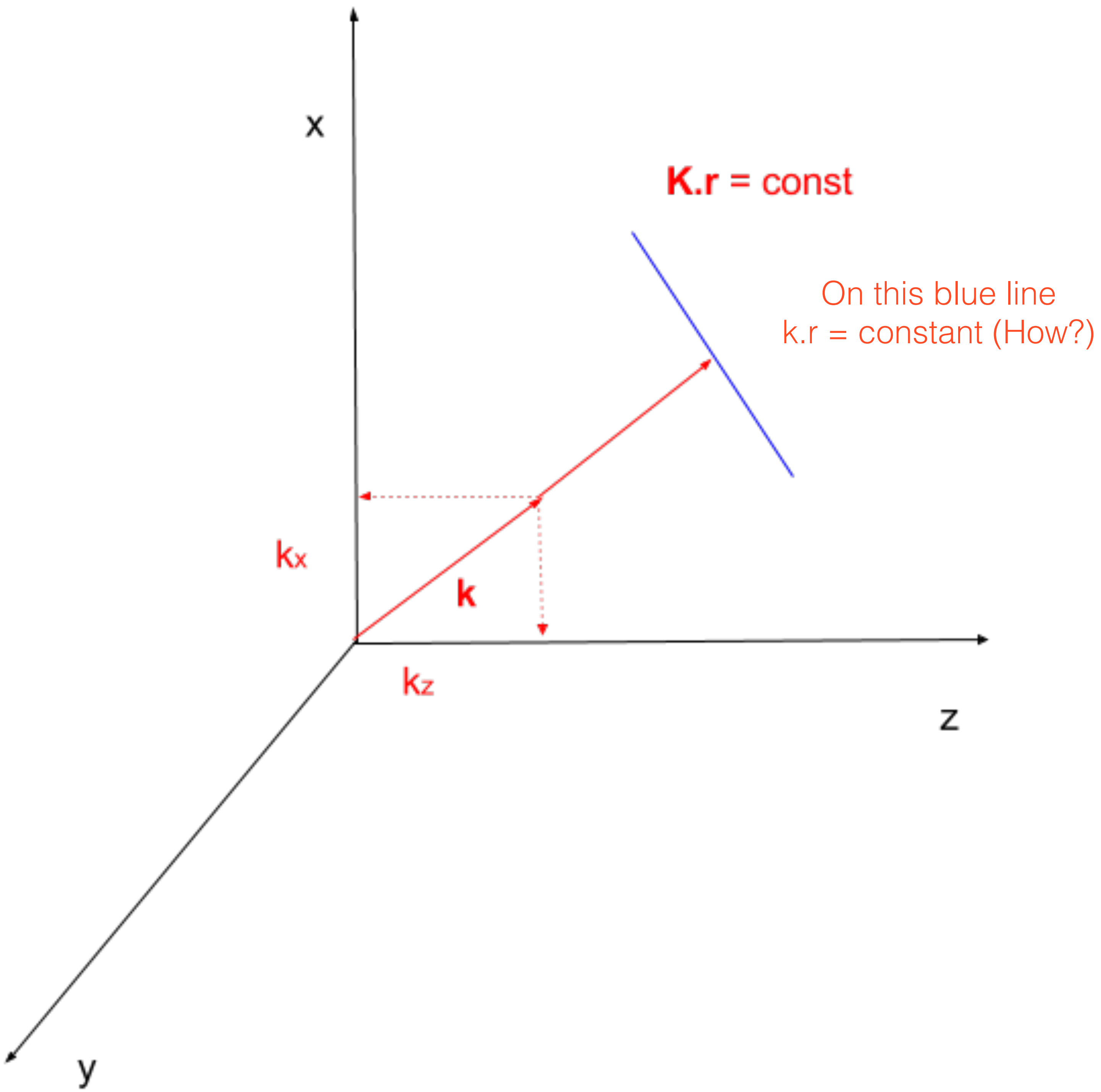
$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

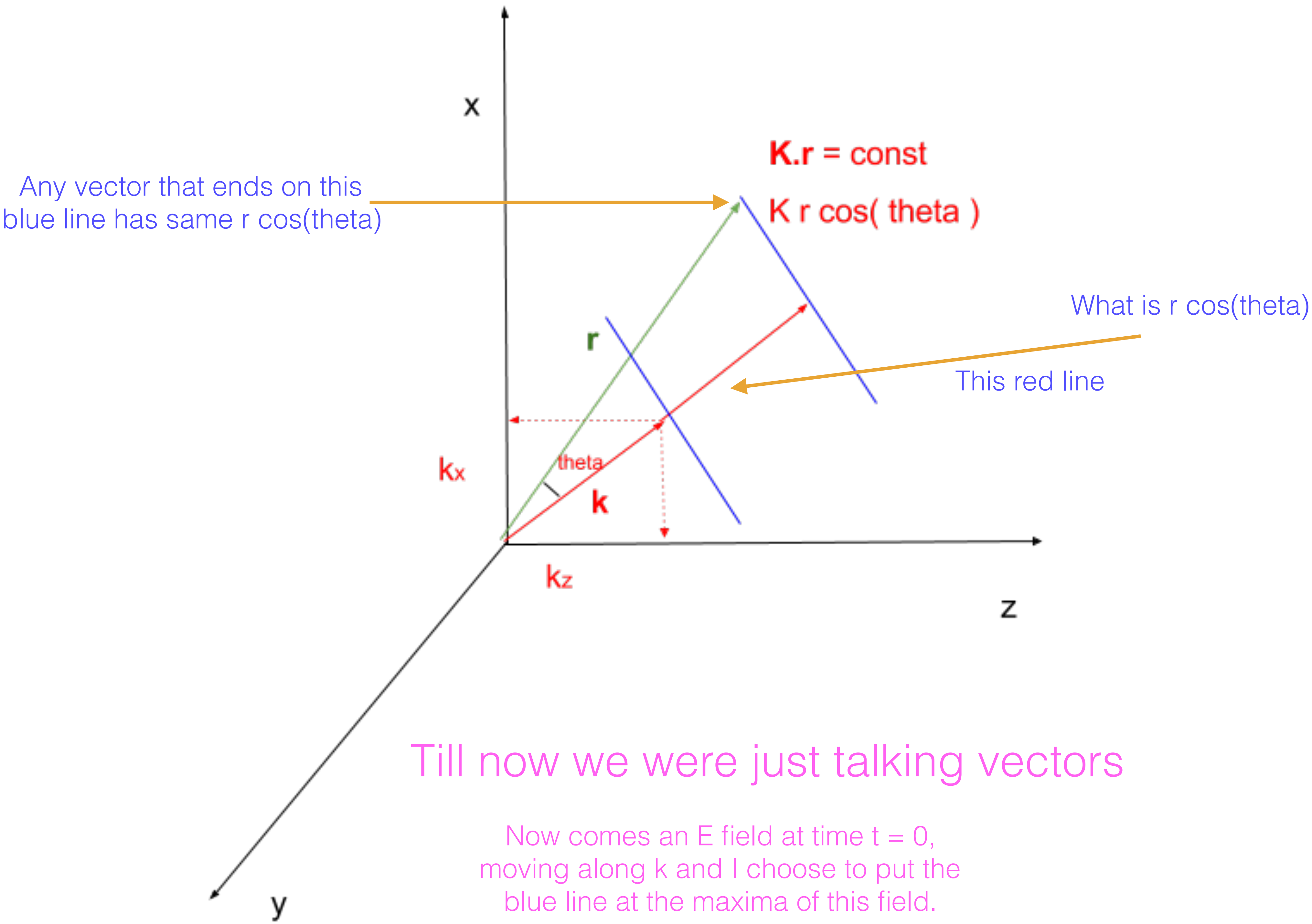
$$\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$$

$$k = \frac{2\pi}{\lambda}$$

Direction of propagation, as you will see

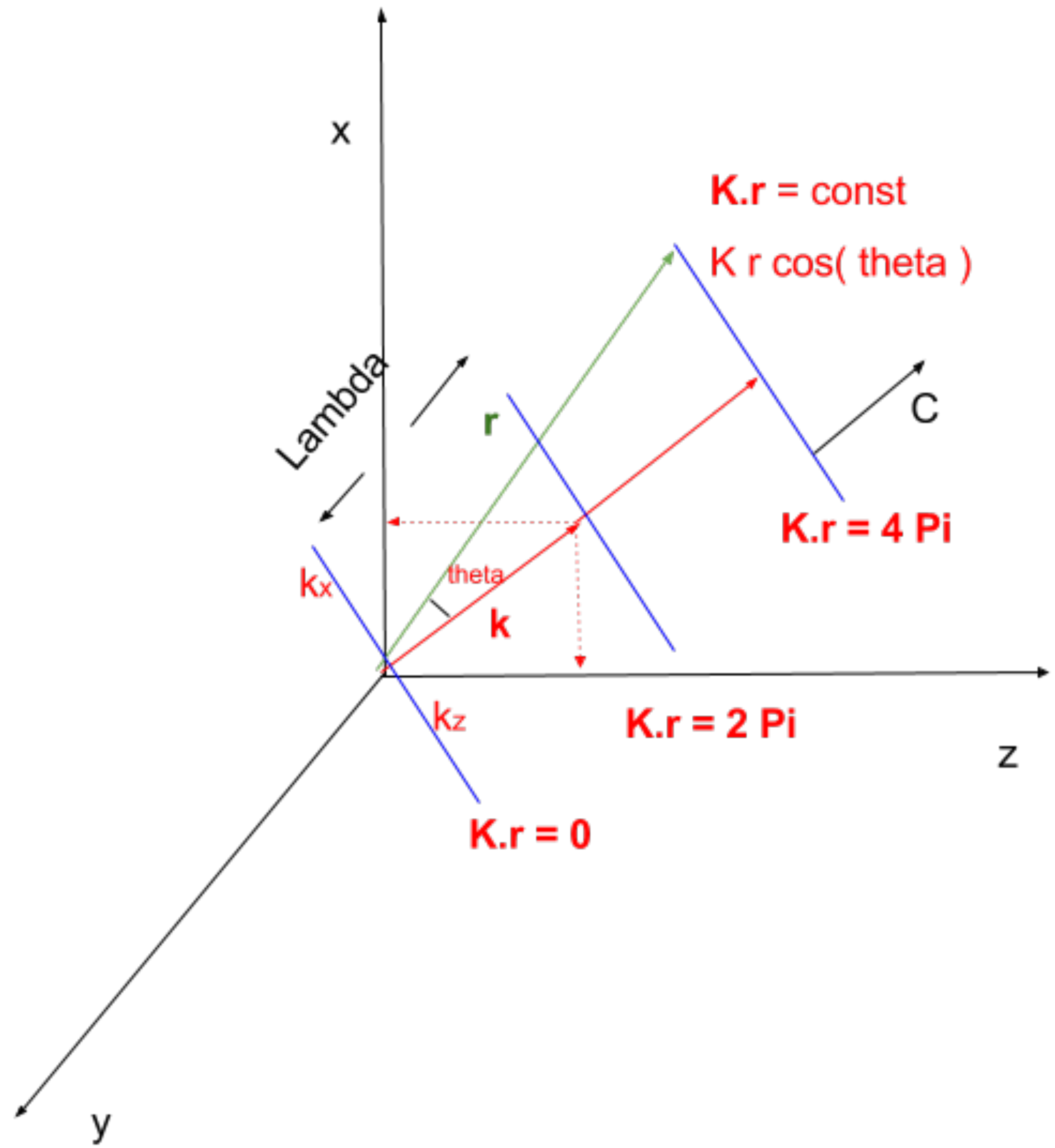

x
Now I draw a line perpendicular to it
Let's just draw k vector
kx
k
kz
z
y


x
K.r = const
On this blue line
k.r = constant (How?)
kx
k
kz
z
y



Till now we were just talking vectors

Now comes an E field at time t = 0, moving along k and I choose to put the blue line at the maxima of this field.



x
K.r = const
K r cos(theta)
Lambda
r
C
K.r = 4 Pi
kx
theta
k
kz
K.r = 2 Pi
z
K.r = 0
y

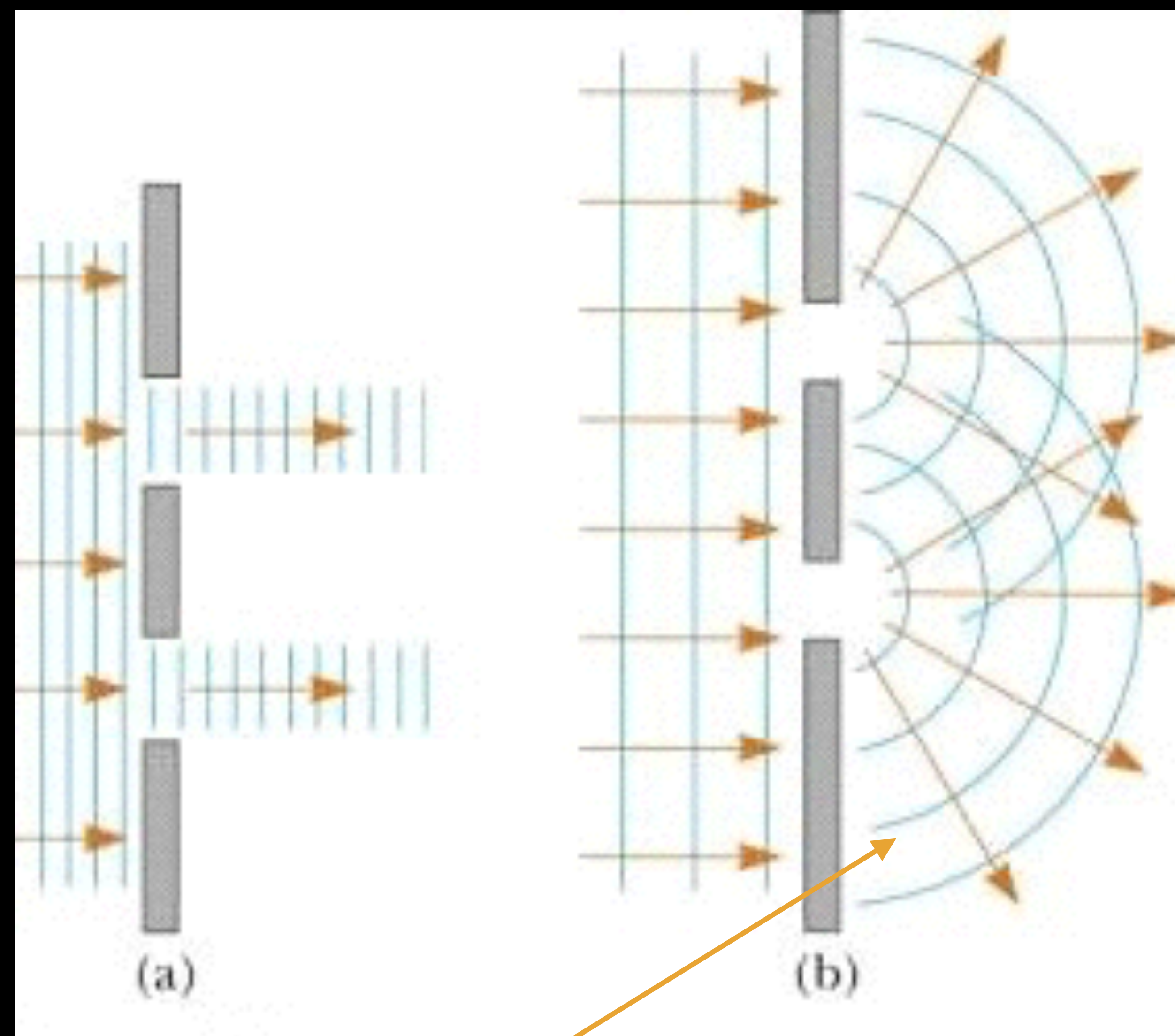
In previous club talks, you have seen:

Travelling/Plane waves

It shows you the lines
where E field is maxima
and where k.r = const

Separation between blue lines
is the wavelength of light

Wavefront moves
with the speed of light



Travelling/Spherical waves

k.r = constant along blue curve

Circularly polarised E field, Ex = max, Ey = 0, vice-versa

$$\delta = \frac{\pi}{2}$$