SUST Final Year Thesis: Homework 1

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Problem 1

Given $r_1 = 2, r_2 = 3$, and $V = \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$

$$\begin{aligned}
LHS = & \quad r_1 \cdot (r_2 \cdot V) \\
= & \quad 2 \cdot (3 \cdot \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}) \\
= & \quad 2 \cdot \begin{bmatrix} 6 \\ -12 \\ 3 \end{bmatrix} \\
= & \quad \begin{bmatrix} 12 \\ -24 \\ 6 \end{bmatrix} \\
RHS = & \quad (r_! \times r_2) \cdot V \\
= & \quad (2 \times 3) \cdot \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix} \\
= & \quad 6 \cdot \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix} \\
= & \quad \begin{bmatrix} 12 \\ -24 \\ 6 \end{bmatrix} \\
\therefore LHS = & \quad RHS
\end{aligned}$$

Problem 2

Given $c_1 = 2i, c2 = 1 + 2i$, and $A = \begin{bmatrix} 1 - i & 3 \\ 2 + 2i & 4 + i \end{bmatrix}$

Subproblem: i

$$\begin{aligned}
LHS =& \quad c_1 \cdot (c_2 \cdot A) \\
=& \quad 2i \cdot ((1+2i) \cdot \begin{bmatrix} 1-i & 3 \\ 2+2i & 4+i \end{bmatrix}) \\
=& \quad 2i \cdot \begin{bmatrix} 3+i & 3+6i \\ -2+6i & 2+9i \end{bmatrix} \\
=& \quad \begin{bmatrix} -2+6i & -12+6i \\ -12-4i & -18+4i \end{bmatrix} \\
RHS =& \quad (c_! \times c_2) \cdot A \\
=& \quad (2i \times (1+2i)) \cdot \begin{bmatrix} 1-i & 3 \\ 2+2i & 4+i \end{bmatrix} \\
=& \quad (-4+2i) \cdot \begin{bmatrix} 1-i & 3 \\ 2+2i & 4+i \end{bmatrix} \\
=& \quad \begin{bmatrix} -2+6i & -12+6i \\ -12-4i & -18+4i \end{bmatrix} \\
\therefore LHS =& \quad RHS
\end{aligned}$$

Subproblem: ii

$$\begin{aligned}
LHS =& \quad (c_! + c_2) \cdot A \\
=& \quad (2i + (1+2i)) \cdot \begin{bmatrix} 1-i & 3 \\ 2+2i & 4+i \end{bmatrix} \\
=& \quad (1+4i) \cdot \begin{bmatrix} 1-i & 3 \\ 2+2i & 4+i \end{bmatrix} \\
=& \quad \begin{bmatrix} 5+3i & 3+12i \\ -6+10i & 17i \end{bmatrix} \\
RHS =& \quad c_1 \cdot A + c_2 \cdot A \\
=& \quad 2i \cdot \begin{bmatrix} 1-i & 3 \\ 2+2i & 4+i \end{bmatrix} + (1+2i) \cdot \begin{bmatrix} 1-i & 3 \\ 2+2i & 4+i \end{bmatrix} \\
=& \quad \begin{bmatrix} 2+2i & 6i \\ -4+4i & -2+8i \end{bmatrix} + \begin{bmatrix} 3+i & 3+6i \\ -2+6i & 2+9i \end{bmatrix} \\
=& \quad \begin{bmatrix} 5+3i & 3+12i \\ -6+10i & 17i \end{bmatrix} \\
\therefore LHS =& \quad RHS
\end{aligned}$$

Problem 3

Given $X = \begin{bmatrix} 6-3i & 2+12i & -19i \\ 0 & 5+2.1i & 17 \\ 1 & 2+5i & 3-4.5i \end{bmatrix}$

$\therefore X^T = \begin{bmatrix} 6-3i & 0 & 1 \\ 2+12i & 5+2.1i & 2+5i \\ -19i & 17 & 3-4.5i \end{bmatrix}$

$\therefore \bar{X} = \begin{bmatrix} 6+3i & 2-12i & 19i \\ 0 & 5-2.1i & 17 \\ 1 & 2-5i & 3+4.5i \end{bmatrix}$

$\therefore X^\dagger = \begin{bmatrix} 6+3i & 0 & 1 \\ 2-12i & 5-2.1i & 2-5i \\ 19i & 17 & 3+4.5i \end{bmatrix}$

Problem 4

Suppose $c \in \mathbb{C}$ and A is an $m \times n$ matrix. For $1 \leq i \leq m$, $1 \leq j \leq n$:

$$\begin{aligned} [\overline{cA}]_{ij} &= \overline{[cA]_{ij}} \\ &= \overline{c[A]_{ij}} \\ &= \overline{c}\,\overline{[A]_{ij}} \\ &= \overline{c}[\overline{A}]_{ij} \\ &= [\overline{c}\overline{A}]_{ij} \end{aligned}$$

Since the matrices $\overline{c \cdot A}$ and $\overline{c} \cdot \overline{A}$ are equal in each entry, we can say that $\overline{c \cdot A} = \overline{c} \cdot \overline{A}$.

Problem 5

```python
import numpy as np

A = np.array([[6-3j, 2+12j, -19j], [0, 5+2.1j, 17], [1, 2+5j, 3-4.5j]])
transpose_A = np.transpose(A)
conjugate_A = np.conjugate(A)
adjoint_A = np.transpose(np.conjugate(A))

print(A)
print(transpose_A)
print(conjugate_A)
print(adjoint_A)
```