SUST Final Year Thesis: Homework 2

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Problem 1

Given $V_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, V_2 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$ and $V_3 = \begin{bmatrix} 1 \\ -4 \\ -4 \end{bmatrix}$

Now, when $x = -2, y = 1$ and $z = -1$, $x \cdot v_1 + y \cdot v_2 + z \cdot v_3 = \mathbf{0}$
Therefore, the set $\{v_1, v_2, v_3\}$ is not linearly independent.

Problem 2

Given $V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $V_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Now, $x \cdot v_1 + y \cdot v_2 + z \cdot v_3 = \mathbf{0}$ can happen only when $x = 0, y = 0$ and $z = 0$. That is, there is no nonzero solution for x, y and z .
Therefore, the set $\{v_1, v_2, v_3\}$ is linearly independent and we can say that the set is a basis of $\mathbb{R}^3$.

Problem 3

Given $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Now,

$$\begin{aligned} H^2 &= H \times H \\ &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= I_2 \end{aligned}$$

Hence, H times itself gives the identity matrix.

Problem 4

Given $V_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, V_2 = \begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}$ and $V_3 = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$

Subproblem: i

$$
\begin{aligned}
LHS &= \langle V_1 + V_2, V_3 \rangle \\
&= \langle \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \rangle \\
&= \langle \begin{bmatrix} 8 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \rangle \\
&= \begin{bmatrix} 8 & 3 & 7 \end{bmatrix} \star \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \\
&= 11 \\
RHS &= \langle V_1, V_3 \rangle + \langle V_2, V_3 \rangle \\
&= \langle \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \rangle + \langle \begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \rangle \\
&= \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \star \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \star \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \\
&= 5 + 6 \\
&= 11 \\
\therefore LHS &= RHS
\end{aligned}
$$

Subproblem: ii

$$
\begin{aligned}
LHS &= \langle V_1, V_2 + V_3 \rangle \\
&= \langle \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \rangle \\
&= \langle \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ 1 \\ 6 \end{bmatrix} \rangle \\
&= \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \star \begin{bmatrix} 6 \\ 1 \\ 6 \end{bmatrix} \\
&= 31 \\
RHS &= \langle V_1, V_2 \rangle + \langle V_1, V_3 \rangle \\
&= \langle \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix} \rangle + \langle \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \rangle \\
&= \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \star \begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \star \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \\
&= 26 + 5 \\
&= 31 \\
\therefore LHS &= RHS
\end{aligned}
$$

Problem 5

Let $V = \begin{bmatrix} 4+3i \\ 6-4i \\ 12-7i \\ 13i \end{bmatrix}$

$$
\begin{aligned}
\therefore |V| &= \sqrt{\langle V, V\rangle} \\
&= \sqrt{\langle \begin{bmatrix} 4+3i \\ 6-4i \\ 12-7i \\ 13i \end{bmatrix}, \begin{bmatrix} 4+3i \\ 6-4i \\ 12-7i \\ 13i \end{bmatrix} \rangle} \\
&= \sqrt{\begin{bmatrix} 4+3i \\ 6-4i \\ 12-7i \\ 13i \end{bmatrix}^{\dagger} \star \begin{bmatrix} 4+3i \\ 6-4i \\ 12-7i \\ 13i \end{bmatrix}} \\
&= \sqrt{\begin{bmatrix} 4-3i & 6+4i & 12+7i & -13i \end{bmatrix} \star \begin{bmatrix} 4+3i \\ 6-4i \\ 12-7i \\ 13i \end{bmatrix}} \\
&= \sqrt{25+52+193+169} \\
&= \sqrt{439} \\
&\simeq 21
\end{aligned}
$$