Problem 1

$$\hat{u}_1 = v_1 / |v_1|$$

$$\hat{u}_2 = v_2 - \frac{\langle v_2, u_1 \rangle}{|u_1|^2} u_1$$

$$\hat{u}_3 = v_3 - \frac{\langle v_3, u_1 \rangle}{|u_1|^2} u_1 - \frac{\langle v_3, u_2 \rangle}{|u_2|^2} u_2$$

$$\hat{u}_4 = v_4 - \frac{\langle v_4, u_1 \rangle}{|u_1|^2} u_1 - \frac{\langle v_4, u_2 \rangle}{|u_2|^2} u_2 - \frac{\langle v_4, u_3 \rangle}{|u|^3} u_3$$

$$\langle f, g \rangle = \int_{-1}^{1} f g \, dt$$

orthonormal $= \frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}} t, \sqrt{\frac{5}{8}} (3t^2 - 1), \sqrt{\frac{7}{8}} t(5t^2 - 3)$

$$\langle f, g \rangle = \int_{-1}^{1} f (1 - t^2)^{-0.5} g \, dt$$

orthonormal $= \frac{1}{\sqrt{\pi}}, \frac{2}{\sqrt{\pi}} t, \sqrt{\frac{2}{\pi}} (2t^2 - 1), \sqrt{\frac{2}{\pi}} t(4t^2 - 3)$

```
syms t
clc;
fns = [1,t,t^2,t^3,t^4];
inner_product_version = 1;
GramSchmidt(fns, inner_product_version);

function U = GramSchmidt(V, inner_product_version)
    num_vectors = length(V);
    U = sym(zeros(1,num_vectors));
    for n = 1:num_vectors
        U(n) = V(n);
        for l = 1:n-1
            U(n) = U(n) - polyip(V(n),U(l),inner_product_version)*U(l);
        end
        U(n) = U(n)/sqrt(polyip(U(n),U(n),inner_product_version));
    end
end

function value = polyip(f,g,version)
%inner product for polynomial functions
syms t
    switch version
        case 1
            value = int(f*g,-1,1);
        case 2
            value = int(f*((1-t^2)^-0.5)*g,-1,1);
        otherwise
            value = int(f*g,-1,1);
    end
end
```

Problem 2

Got $\begin{bmatrix} 1 & 0.0001 & 0.0001 \\ 0.0001 & 0.000 & -1.000 \\ 0.0001 & -1.000 & 0.000 \end{bmatrix}$

for both methods I can't tell if I'm supposed to produce different results.

The vectors seem to be rotated to the 0 value which is interesting

```
clc;
e = 0.0001;
v1 = [1, e, e]';
v2 = [1, e, 0]';
v3 = [1, 0, e]';
fns = [v1, v2, v3]
out = GramSchmidt(fns)
out2 = GramSchmidt2(fns)

function U = GramSchmidt(V)
    num_vectors = size(V,2);
    U = zeros(size(V));
    for n = 1:num_vectors
        U(:,n) = V(:,n) - sum((V(:,n).'*U(:,1:n-1)).*U(:,1:n-1),2);
        U(:,n) = U(:,n)/norm(U(:,n));

    end
end

function U = GramSchmidt2(V)
    num_vectors = size(V,2);
    U = zeros(size(V));
    for n = 1:num_vectors
        U(:,n) = V(:,n);
        for l = 1:n-1
            U(:,n) = U(:,n) - dot(V(:,n),U(:,l))*U(:,l);
        end
        U(:,n) = U(:,n)/norm(U(:,n));
    end
end
```

Problem 3

row space = (1, 2)

left null space = (−2, 1)

column space = $\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$

null space = $\begin{pmatrix} -3 & -5 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$

```
clc;

A = [1 2; 3 6; 5 10];

columnspace = A(:,any(rref(A)==1))

nullspace = null(A',"rational")

rowspace = A(any(rref(A')==1),:)

leftnullspace = null(A,"rational")'
```

Problem 4

$$\hat{b} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$$e = b - \hat{b} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$$

$e \cdot a_1 = 0$ $\quad$ $e \cdot a_2 = 0$

```
clc
a1 = [1, 1, 0]';
a2 = [0, 1, 0]';
b = [1, 2, 3]';

A = [a1 a2];

x_hat = (A'*A)\(A'*b);
b_hat = A*x_hat

error = b - b_hat

a1_dot_error = dot(a1,error);
a2_dot_error = dot(a2,error);
```

Problem 5

$$x_{min} = \frac{1}{30}\begin{pmatrix} 17 \\ 8 \\ -1 \end{pmatrix}$$

$$Ax - b = \begin{bmatrix} x_1 + 2x_2 + 3x_3 - 1 \\ 3x_1 + x_2 - x_3 + 2 \end{bmatrix} = 0$$

$$R = \begin{bmatrix} x_1 + 2x_2 + 3x_3 - 1 \\ 3x_1 + x_2 - x_3 + 2 \end{bmatrix}$$

$$R^T R = (x_1 + 2x_2 + 3x_3 - 1)^2 + (3x_1 + x_2 - x_3 + 2)^2$$

Plot graph $x = (0.5667, 0.2667, -0.0333)$

```
clc

A = [1 2  3;
     3 1 -1];
b = [1, 2]';

lambda = -(A*A')\b;
x_min = -A'*lambda;
```