Module 4.9: Derivative of the activation function

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Logistic function

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$$\begin{aligned} g(z) &= \sigma(z) \\ &= \frac{1}{1+e^{-z}} \\ g'(z) &= (-1)\frac{1}{(1+e^{-z})^2}\frac{d}{dz}(1+e^{-z}) \end{aligned}$$

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\begin{aligned}
g(z) &= \sigma(z) \\
&= \frac{1}{1+e^{-z}} \\
g'(z) &= (-1)\frac{1}{(1+e^{-z})^2}\frac{d}{dz}(1+e^{-z}) \\
&= (-1)\frac{1}{(1+e^{-z})^2}(-e^{-z})
\end{aligned}
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&= (-1)\frac{1}{(1+e^{-z})^2}(-e^{-z}) \\
&= \frac{1}{1+e^{-z}}\left(\frac{1+e^{-z}-1}{1+e^{-z}}\right)
\end{aligned}
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Logistic function

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&= \frac{1}{1+e^{-z}}\left(\frac{1+e^{-z}-1}{1+e^{-z}}\right) \\
&= g(z)(1-g(z))
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tanh

$$\begin{aligned} g(z) &= \tanh(z) \\ &= \frac{e^z - e^{-z}}{e^z + e^{-z}} \end{aligned}$$

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&= g(z)(1-g(z))
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$$

tanh

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\begin{aligned}
g(z) &= \tanh(z) \\
&= \frac{e^z - e^{-z}}{e^z + e^{-z}} \\
g'(z) &= \frac{\left(\begin{array}{c}(e^z+e^{-z})\frac{d}{dz}(e^z-e^{-z}) \\ \quad -(e^z-e^{-z})\frac{d}{dz}(e^z+e^{-z})\end{array}\right)}{(e^z+e^{-z})^2}
\end{aligned}
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&= \frac{(e^z+e^{-z})^2-(e^z-e^{-z})^2}{(e^z+e^{-z})^2}
\end{aligned}
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&= \frac{(e^z+e^{-z})^2-(e^z-e^{-z})^2}{(e^z+e^{-z})^2} \\
&= 1-\frac{(e^z-e^{-z})^2}{(e^z+e^{-z})^2}
\end{aligned}
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&= \frac{1}{1+e^{-z}}\left(\frac{1+e^{-z}-1}{1+e^{-z}}\right) \\
&= g(z)(1-g(z))
\end{aligned}
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tanh

$$
\begin{aligned}
g(z) =& \tanh(z) \\
=& \frac{e^z - e^{-z}}{e^z + e^{-z}} \\
g'(z) =& \frac{\begin{pmatrix} (e^z+e^{-z})\frac{d}{dz}(e^z-e^{-z}) \\ \quad -(e^z-e^{-z})\frac{d}{dz}(e^z+e^{-z}) \end{pmatrix}}{(e^z+e^{-z})^2} \\
=& \frac{(e^z+e^{-z})^2-(e^z-e^{-z})^2}{(e^z+e^{-z})^2} \\
=& 1-\frac{(e^z-e^{-z})^2}{(e^z+e^{-z})^2} \\
=& 1-(g(z))^2
\end{aligned}
$$