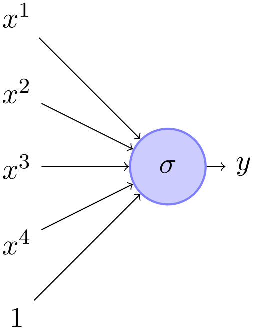
Module 5.9 : Gradient Descent with Adaptive Learning Rate



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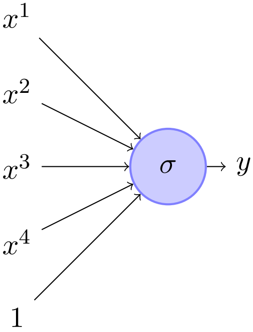

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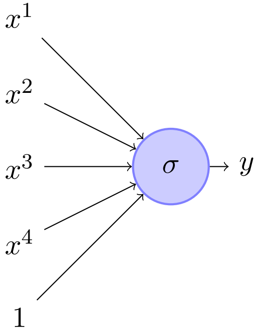

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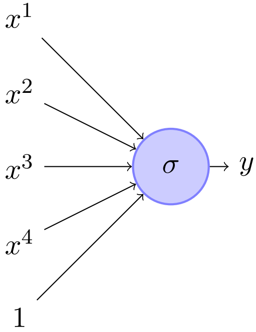

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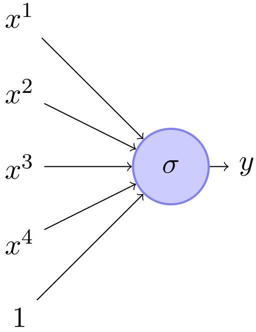

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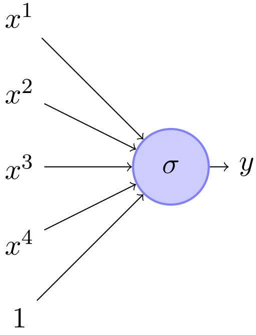

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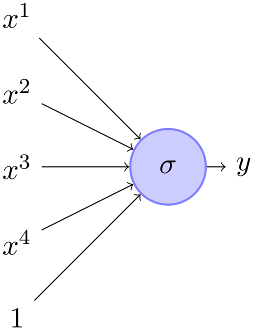

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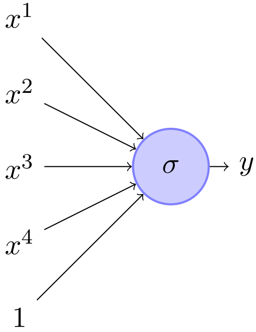

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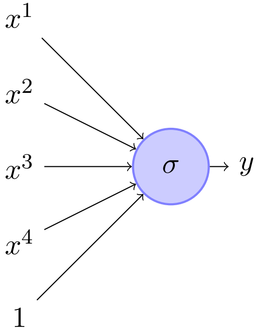

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- Can we have a different learning rate for each parameter which takes care of the frequency of features ?

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Update rule for Adagrad

$$v_t = v_{t-1} + (\nabla w_t)^2$$

$$w_{t+1} = w_t - \frac{\eta}{\sqrt{v_t + \epsilon}} * \nabla w_t$$

... and a similar set of equations for b_t

- To see this in action we need to first create some data where one of the features is sparse

```python
def do_adagrad():
    w, b, eta = init_w, init_b, 0.1
    v_w, v_b, eps = 0, 0, 1e-8
    for i in range(max_epochs):
        dw, db = 0, 0
        for x,y in zip(X, Y):
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        v_w = v_w + dw**2
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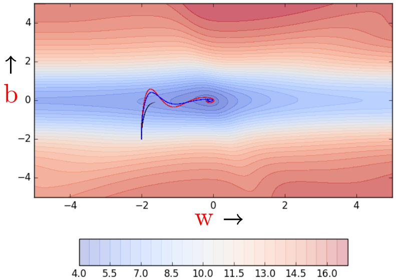

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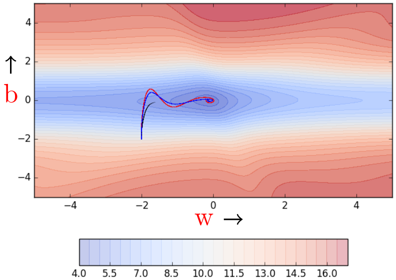

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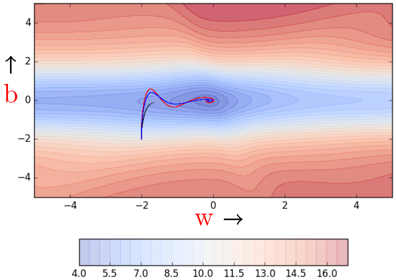

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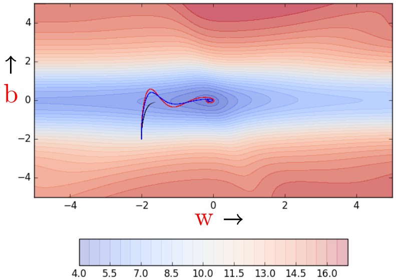

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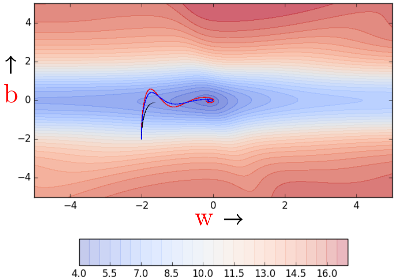

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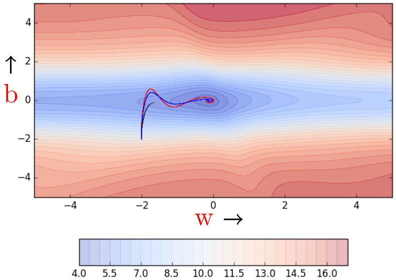

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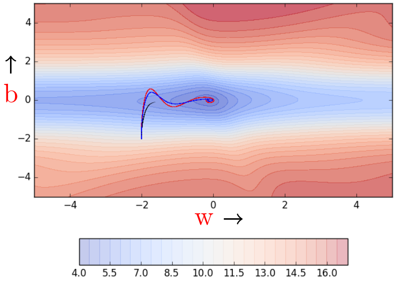

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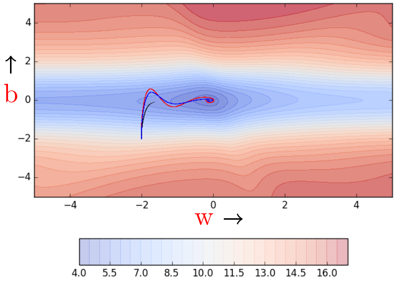

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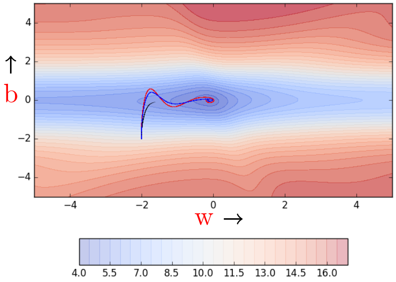
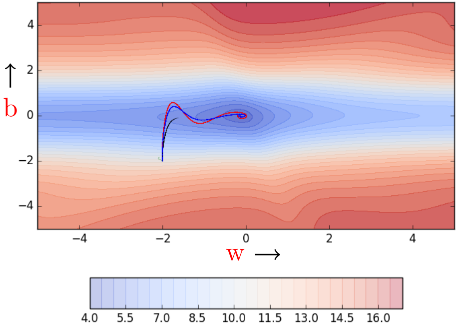


- Let's see what Adagrad does....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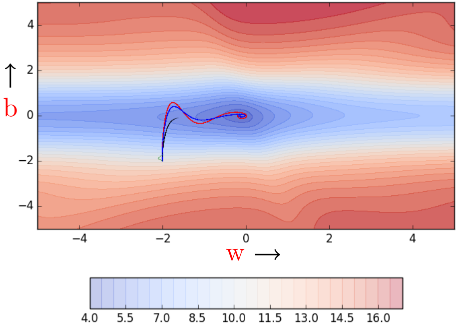

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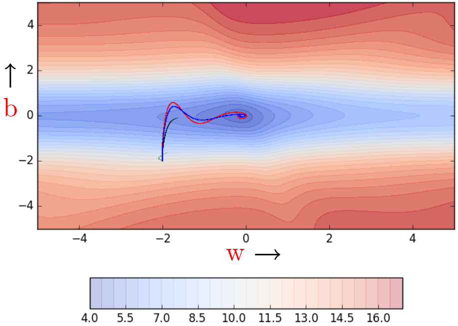
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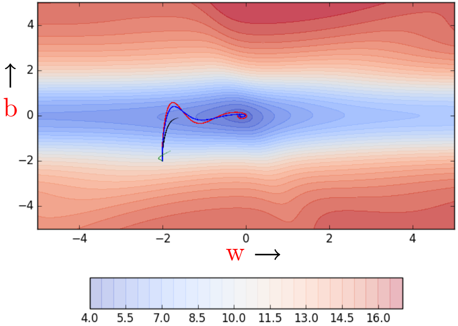

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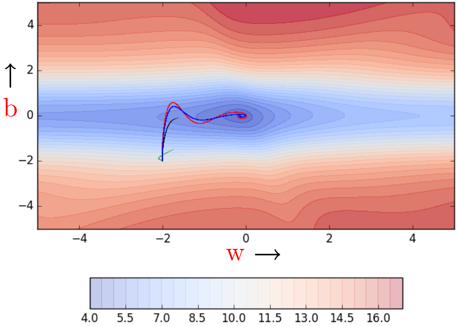

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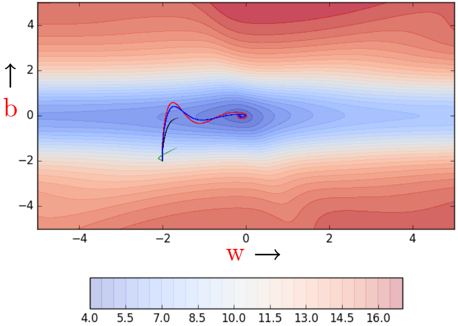
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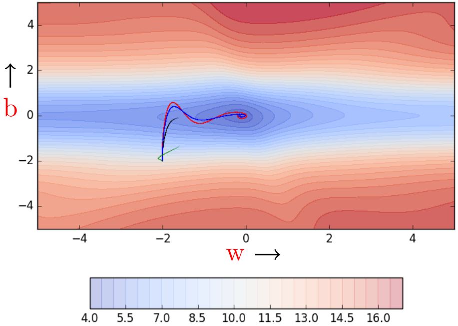
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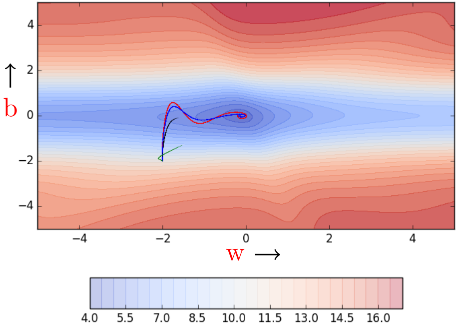
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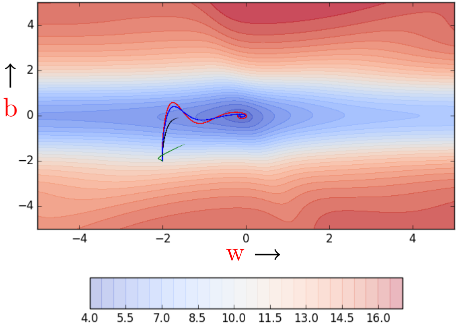
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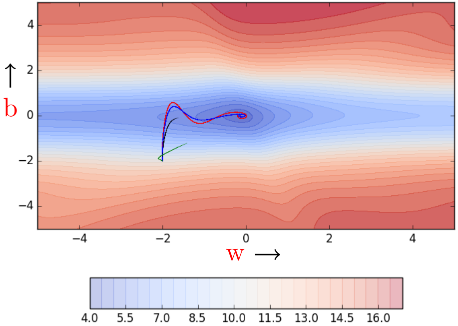

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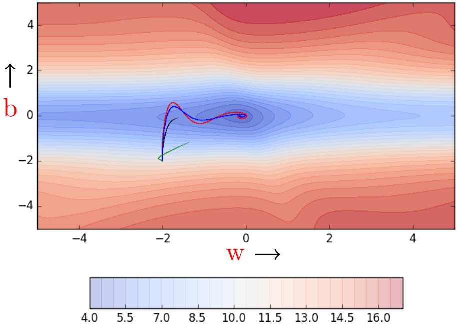

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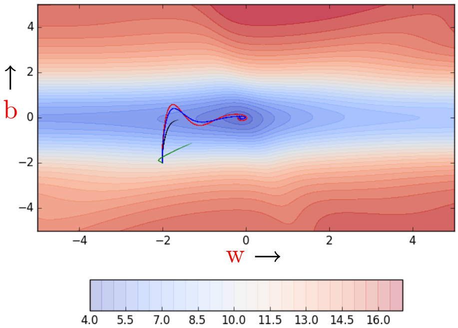
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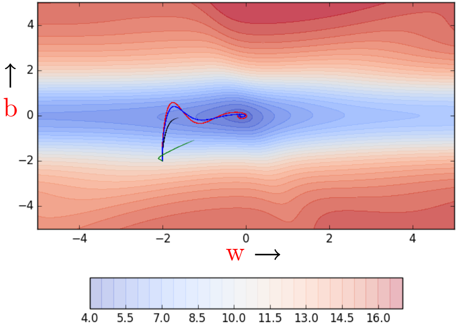

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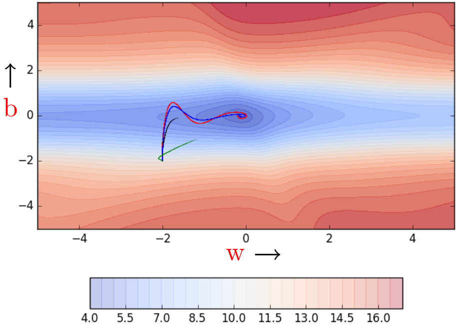
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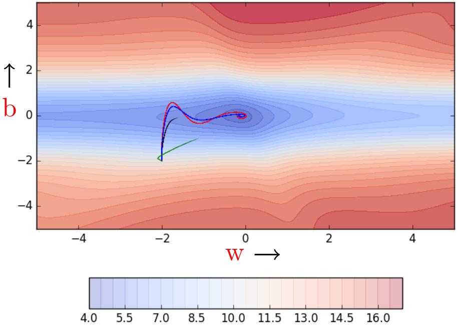
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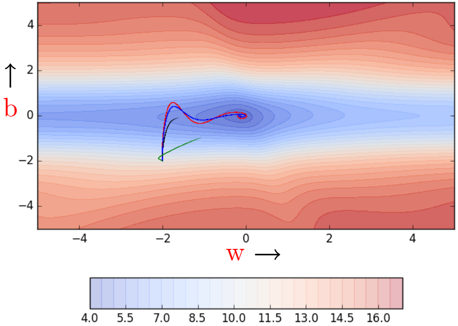
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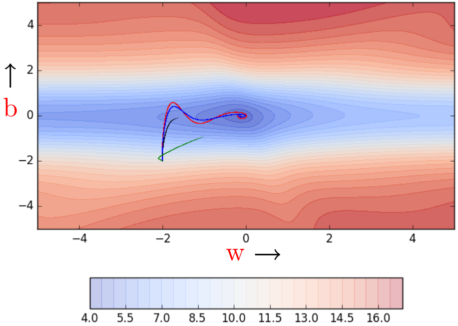

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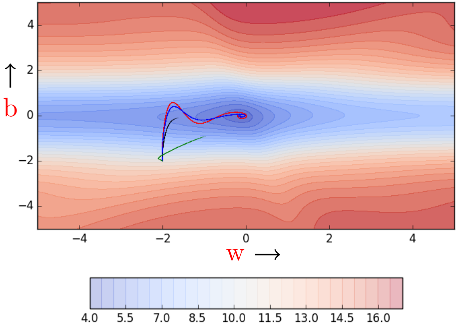
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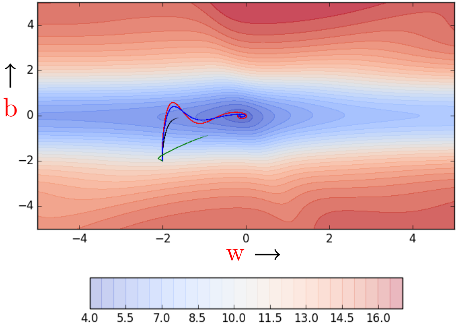
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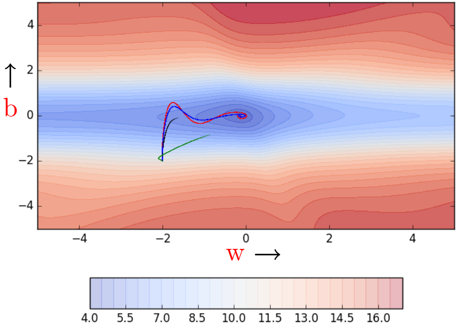
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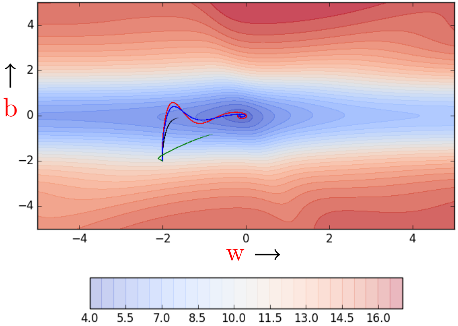
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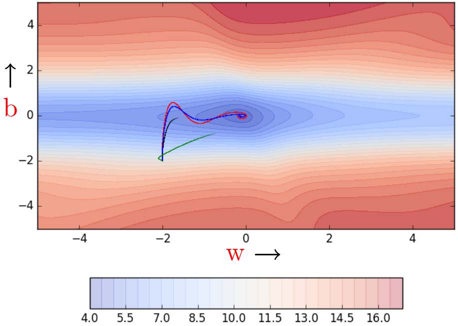

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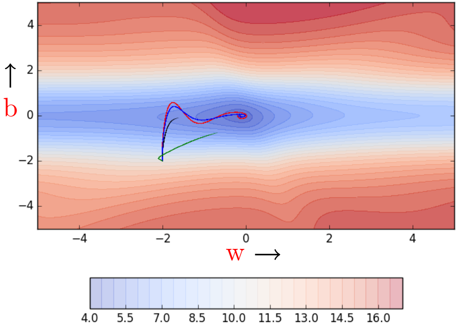
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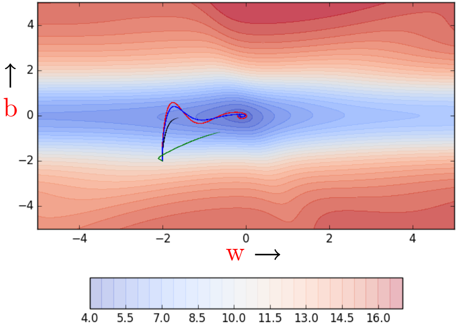
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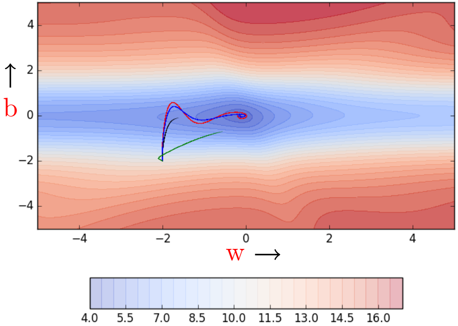

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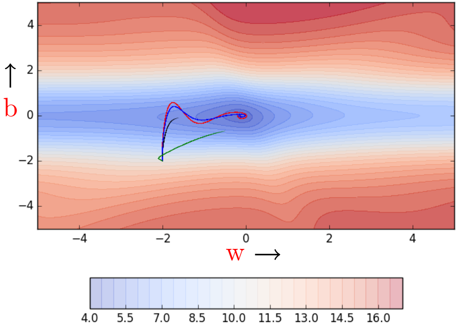
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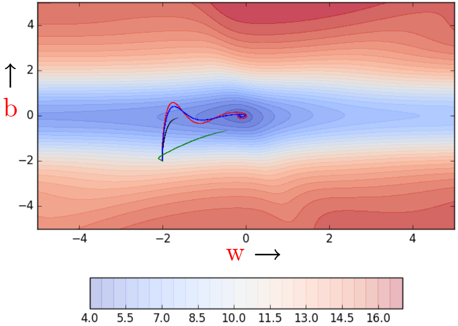

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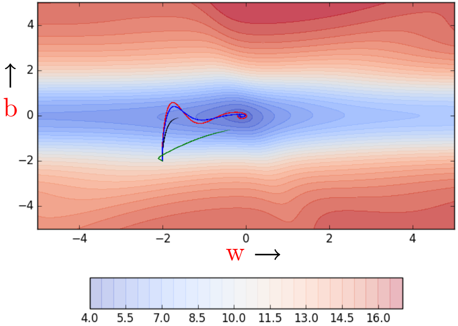
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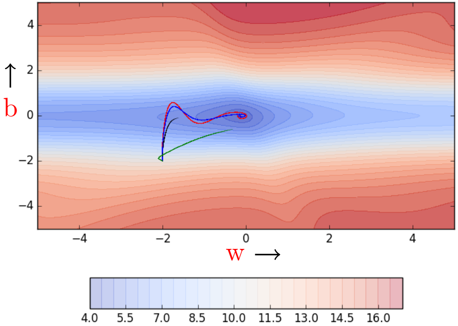

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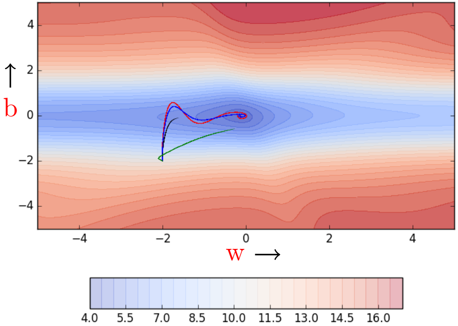

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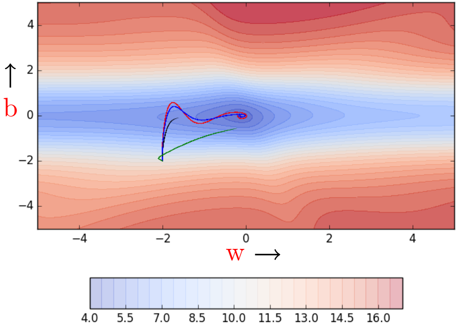

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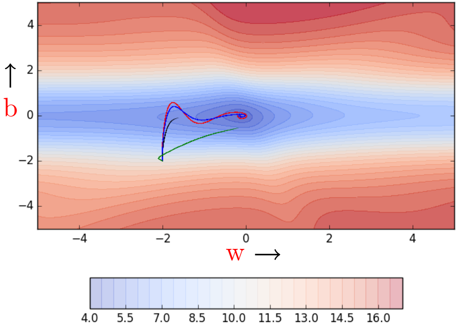
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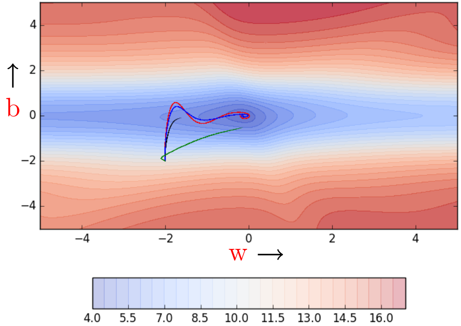
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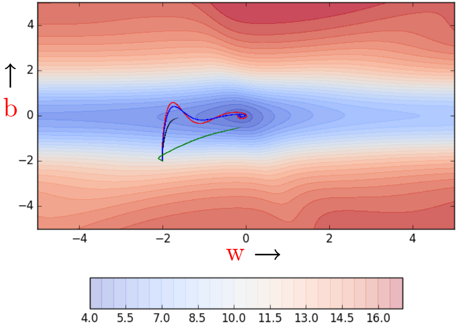
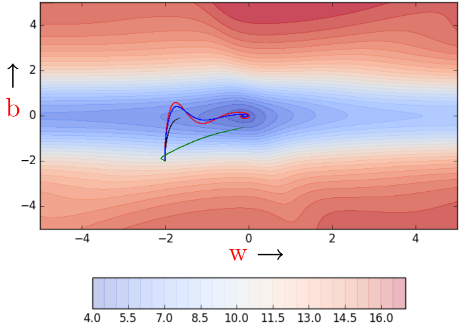
- By using a parameter specific learning rate it ensures that despite sparsity w gets a higher learning rate and hence larger updates



- By using a parameter specific learning rate it ensures that despite sparsity w gets a higher learning rate and hence larger updates
- Further, it also ensures that if b undergoes a lot of updates its effective learning rate decreases because of the growing denominator



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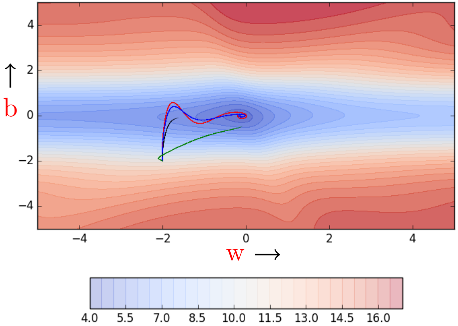

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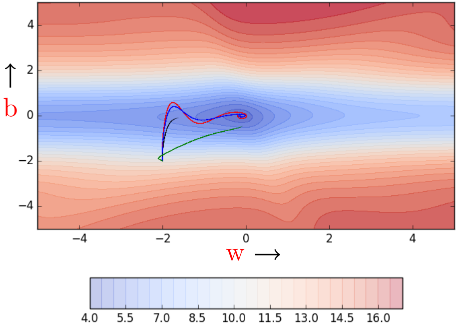

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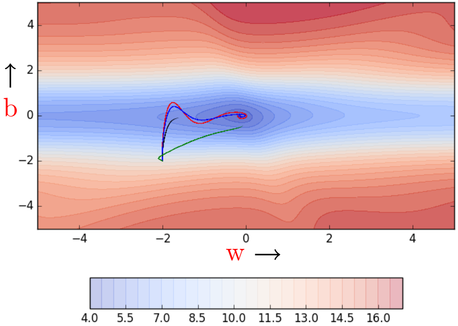
- What's the flipside?



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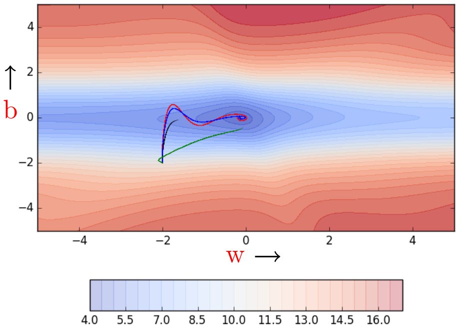
- What's the flipside? over time the effective learning rate for b will decay to an extent that there will be no further updates to b



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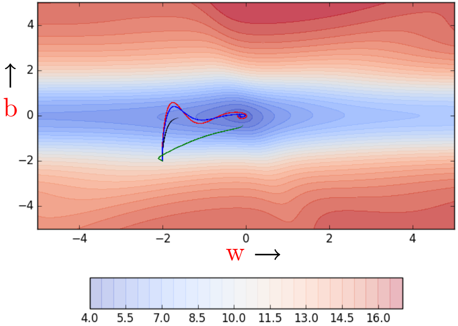
- In practice, this does not work so well if we remove the square root from the denominator (something to ponder about)
- What's the flipside? over time the effective learning rate for b will decay to an extent that there will be no further updates to b
- Can we avoid this?



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Update rule for RMSProp

$$v_t = \beta * v_{t-1} + (1-\beta)(\nabla w_t)^2$$

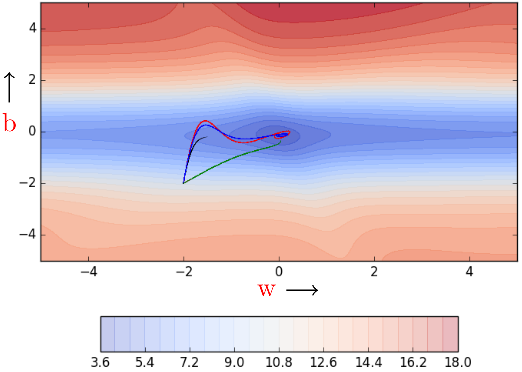
$$w_{t+1} = w_t - \frac{\eta}{\sqrt{v_t + \epsilon}} * \nabla w_t$$

... and a similar set of equations for b_t

```python
def do_rmsprop() :
    w, b, eta = init_w, init_b, 0.1
    v_w, b_updates, eps, beta1 = 0, 0, 1e-8, 0.9
    for i in range(max_epochs) :
        dw, db  = 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)

        v_w = beta1 * v_w + (1 - beta1) dw**2
        v_b = beta1 * v_b + (1 - beta1) db**2

        w = w - (eta / np.sqrt(v_w + eps)) * dw
        b = b - (eta / np.sqrt(v_b + eps)) * db
```



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        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)

        v_w = beta1 * v_w + (1 - beta1) dw**2
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        w = w - (eta / np.sqrt(v_w + eps)) * dw
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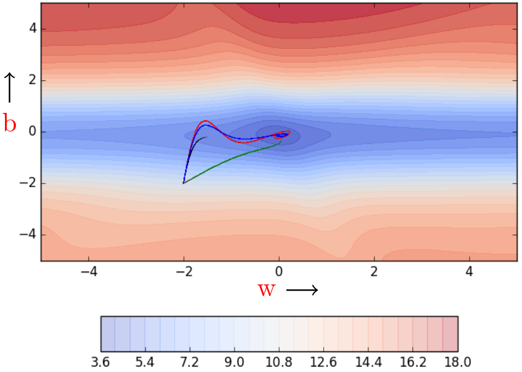
```



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        w = w - (eta / np.sqrt(v_w + eps)) * dw
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```

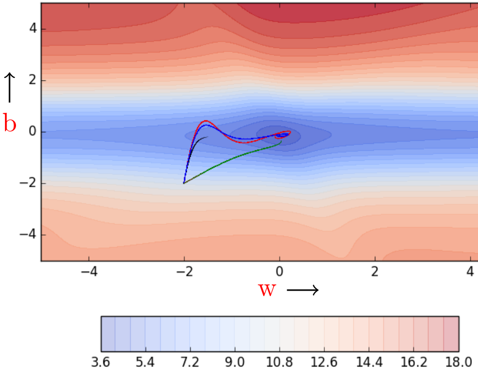

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        w = w - (eta / np.sqrt(v_w + eps)) * dw
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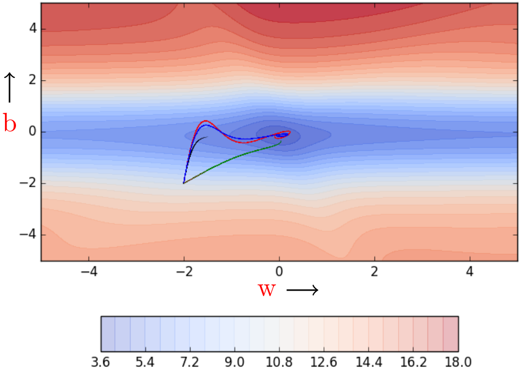
```



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        w = w - (eta / np.sqrt(v_w + eps)) * dw
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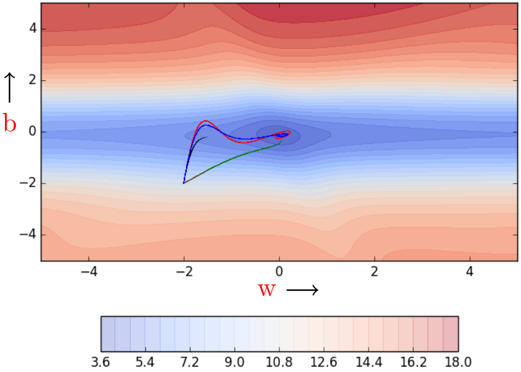

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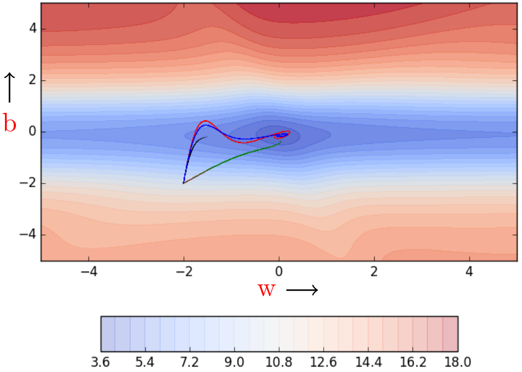

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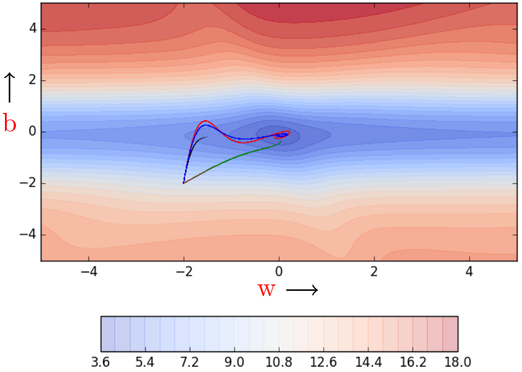

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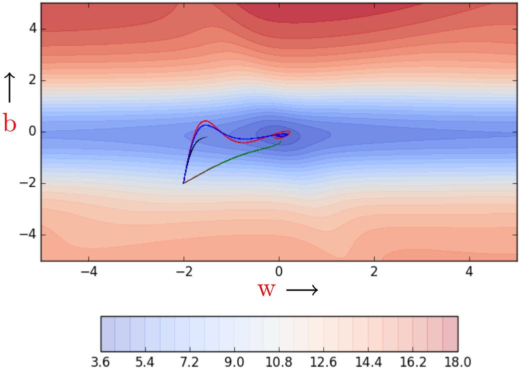

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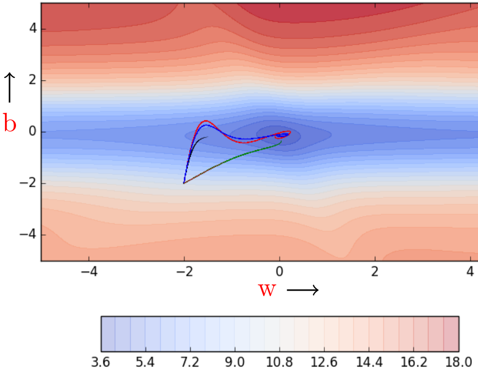

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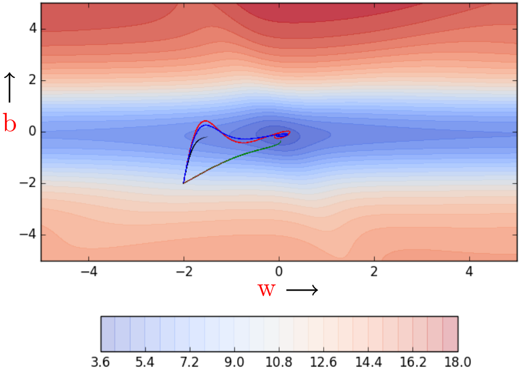

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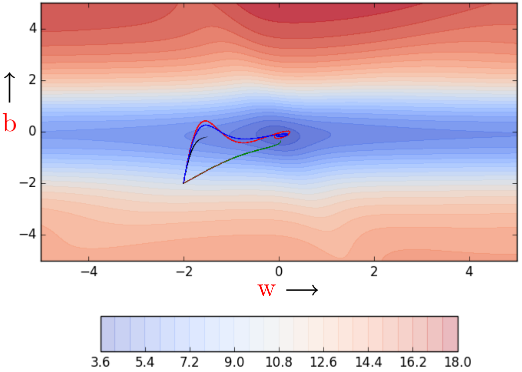

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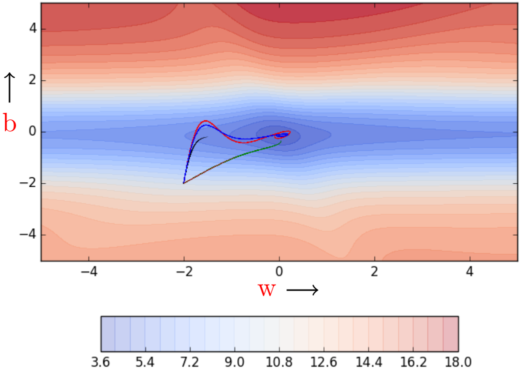

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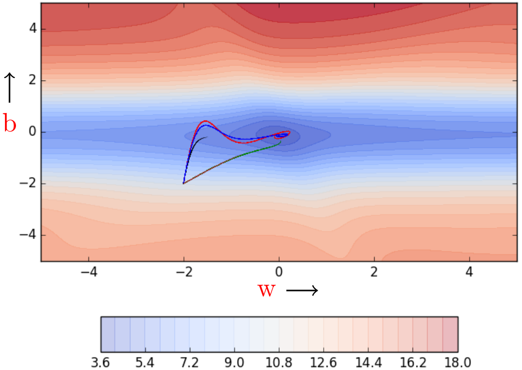

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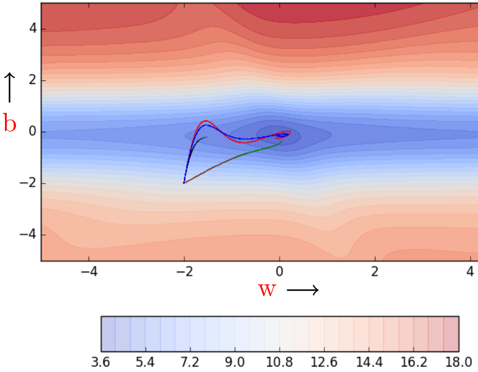

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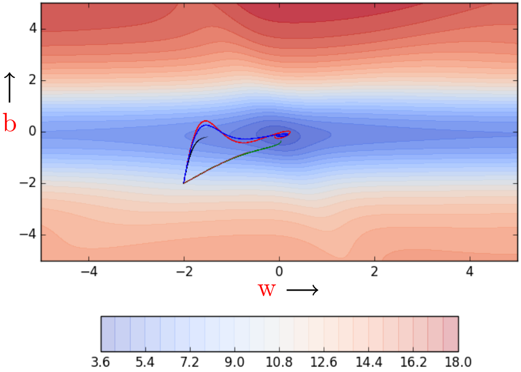

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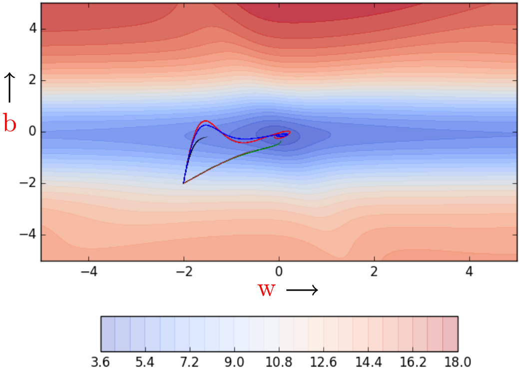

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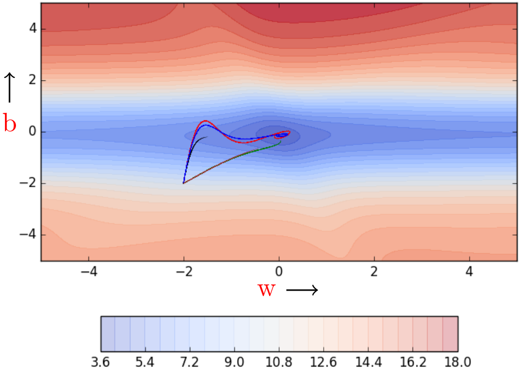

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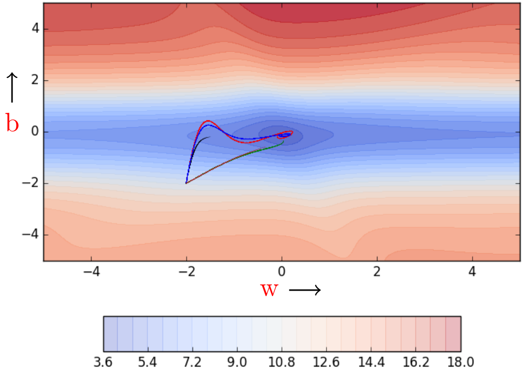

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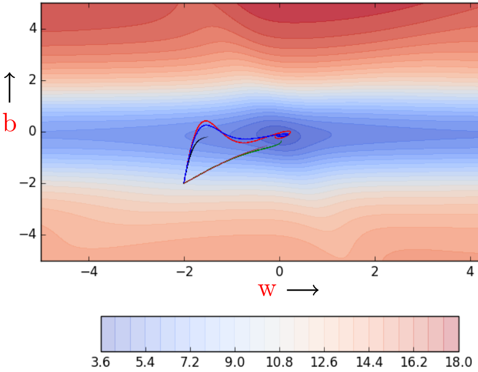

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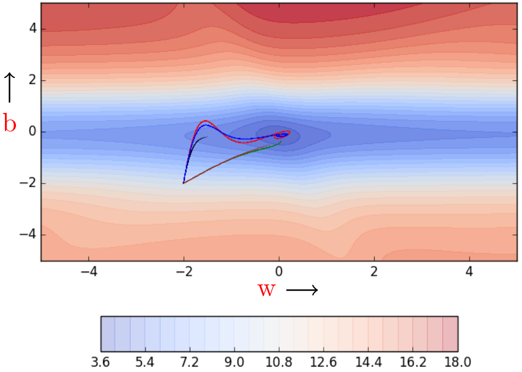

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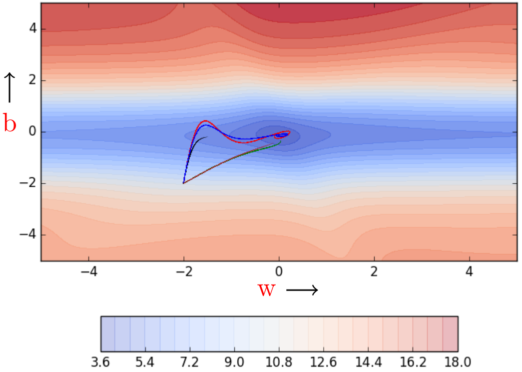

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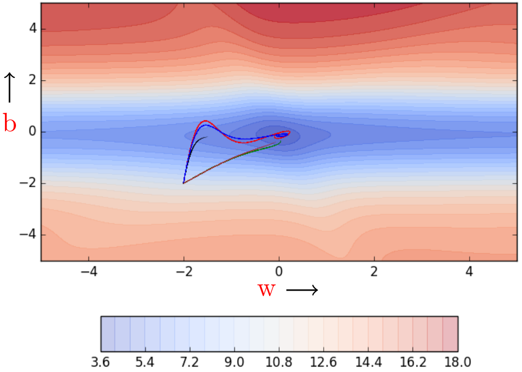

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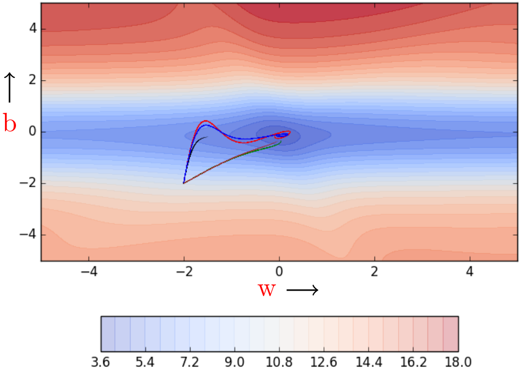

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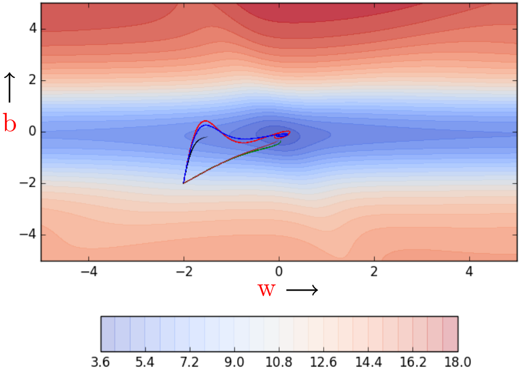

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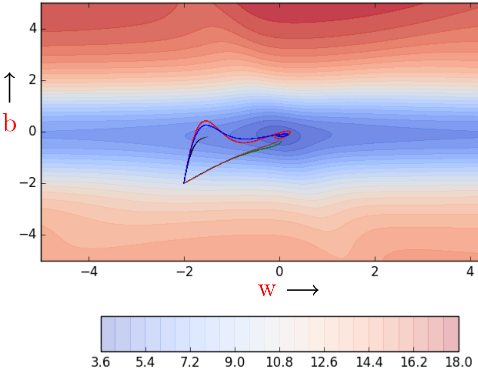

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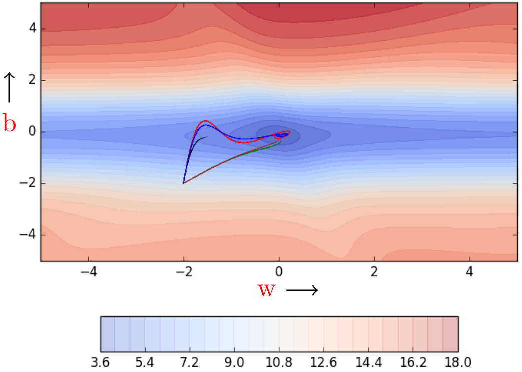

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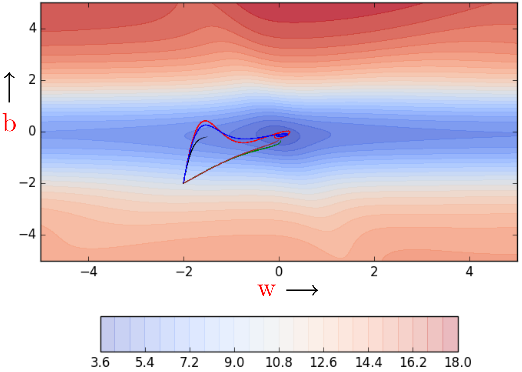

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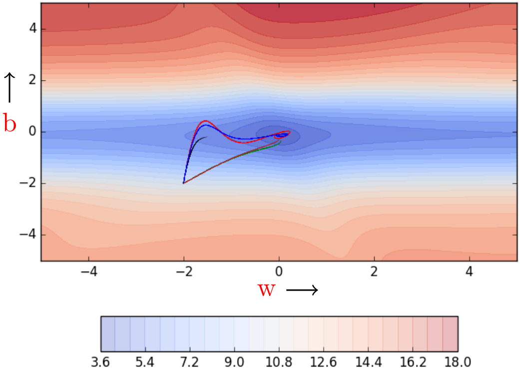

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    for i in range(max_epochs) :
        dw, db  = 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)

        v_w = beta1 * v_w + (1 - beta1) dw**2
        v_b = beta1 * v_b + (1 - beta1) db**2

        w = w - (eta / np.sqrt(v_w + eps)) * dw
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```
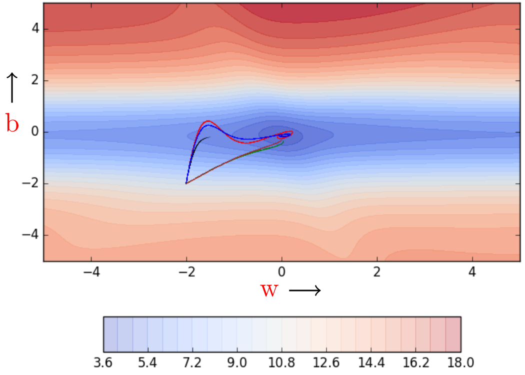
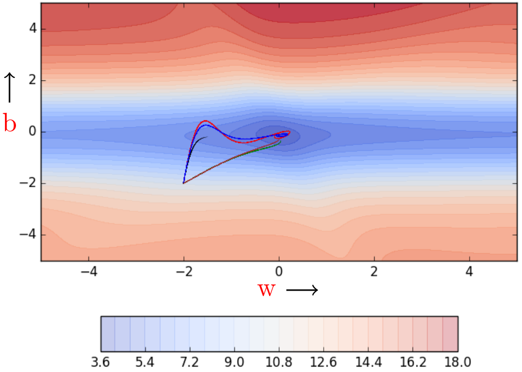



- Adagrad got stuck when it was close to convergence

```python
def do_rmsprop() :
    w, b, eta = init_w, init_b, 0.1
    v_w, b_updates, eps, beta1 = 0, 0, 1e-8, 0.9
    for i in range(max_epochs) :
        dw, db  = 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)

        v_w = beta1 * v_w + (1 - beta1) dw**2
        v_b = beta1 * v_b + (1 - beta1) db**2

        w = w - (eta / np.sqrt(v_w + eps)) * dw
        b = b - (eta / np.sqrt(v_b + eps)) * db
```



- Adagrad got stuck when it was close to convergence (it was no longer able to move in the vertical (b) direction because of the decayed learning rate)

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    w, b, eta = init_w, init_b, 0.1
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            dw += grad_w(w, b, x, y)
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        v_w = beta1 * v_w + (1 - beta1) dw**2
        v_b = beta1 * v_b + (1 - beta1) db**2

        w = w - (eta / np.sqrt(v_w + eps)) * dw
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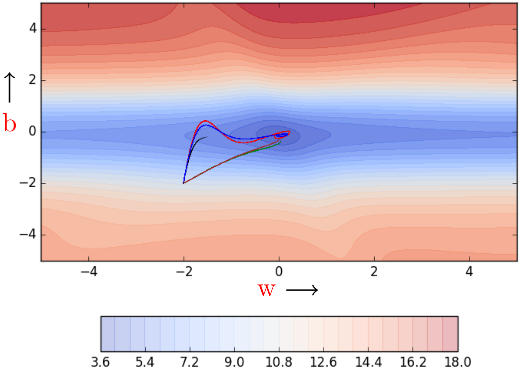
```



- Adagrad got stuck when it was close to convergence (it was no longer able to move in the vertical (b) direction because of the decayed learning rate)

- RMSProp overcomes this problem by being less aggressive on the decay

Intuition

- Do everything that RMSProp does to solve the decay problem of Adagrad

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- Do everything that RMSProp does to solve the decay problem of Adagrad
- Plus use a cumulative history of the gradients

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Update rule for Adam

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Update rule for Adam

$$m_t = \beta_1 * m_{t-1} + (1 - \beta_1) * \nabla w_t$$

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$$\hat{m}_t = \frac{m_t}{1 - \beta_1^t}$$

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- Do everything that RMSProp does to solve the decay problem of Adagrad
- Plus use a cumulative history of the gradients
- In practice, $\beta_1 = 0.9$ and $\beta_2 = 0.999$

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$$w_{t+1} = w_t - \frac{\eta}{\sqrt{\hat{v}_t + \epsilon}} * \hat{m}_t$$

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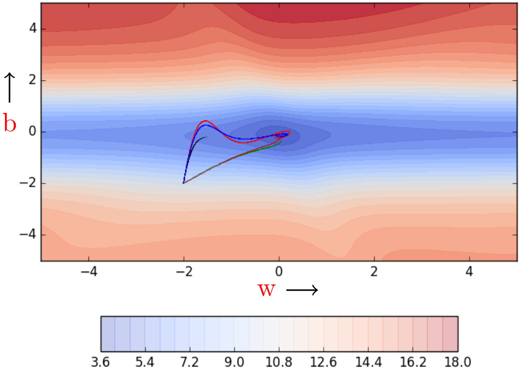
... and a similar set of equations for b_t

```python
def do_adam() :
    w_b_dw_db = [(init_w, init_b,0,0)]
    w_history, b_history, error_history = [], [], [
        ]

    w, b, eta, mini_batch_size, num_points_seen =
        init_w, init_b, 0.1, 10, 0
    m_w, m_b, v_w, v_b, eps, beta1, beta2 = 0, 0, 0
        , 0, 1e-8, 0.9, 0.999
    for i in range(max_epochs) :
        dw, db  = 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)

        m_w = beta1 * m_w + (1-beta1)*dw
        m_b = beta1 * m_b + (1-beta1)*db

        v_w = beta2 * v_w + (1-beta2)*dw**2
        v_b = beta2 * v_b + (1-beta2)*db**2

        m_w = m_w/(1-math.pow(beta1,i+1))
        m_b = m_b/(1-math.pow(beta1,i+1))

        v_w = v_w/(1-math.pow(beta2,i+1))
        v_b = v_b/(1-math.pow(beta2,i+1))

        w = w - (eta / np.sqrt(v_w + eps)) * m_w
        b = b - (eta / np.sqrt(v_b + eps)) * m_b
```



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        v_w = beta2 * v_w + (1-beta2)*dw**2
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        m_w = m_w/(1-math.pow(beta1,i+1))
        m_b = m_b/(1-math.pow(beta1,i+1))

        v_w = v_w/(1-math.pow(beta2,i+1))
        v_b = v_b/(1-math.pow(beta2,i+1))

        w = w - (eta / np.sqrt(v_w + eps)) * m_w
        b = b - (eta / np.sqrt(v_b + eps)) * m_b
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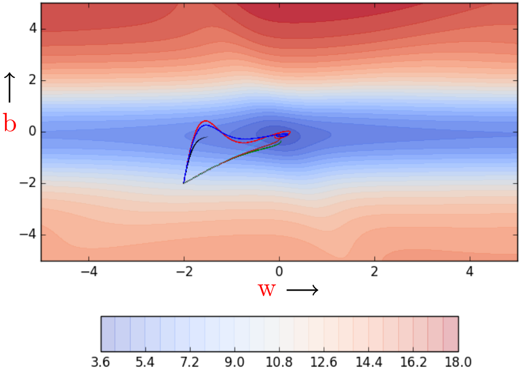

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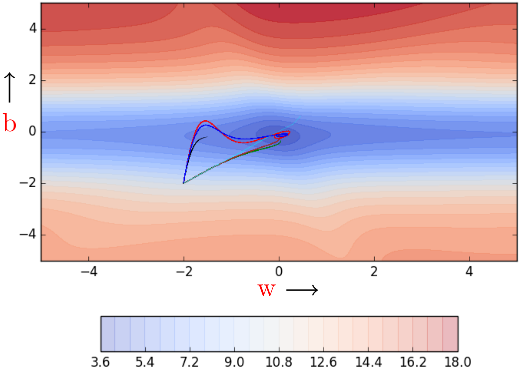

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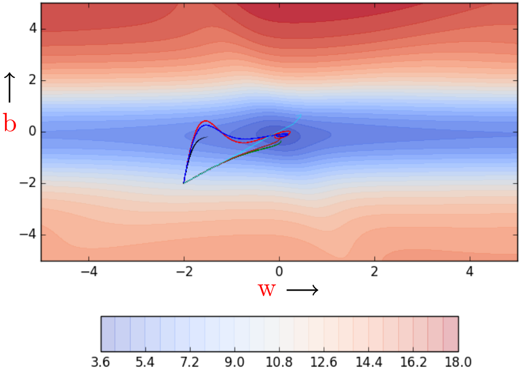

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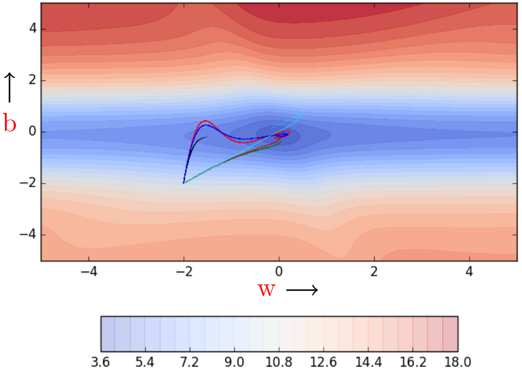

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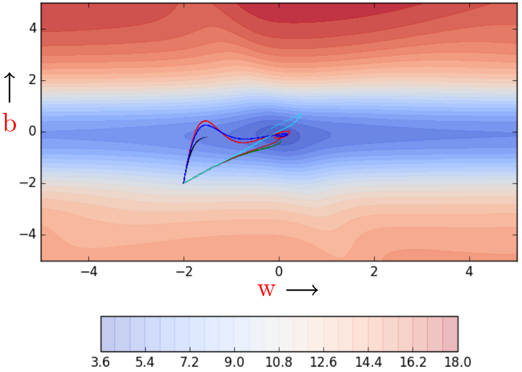

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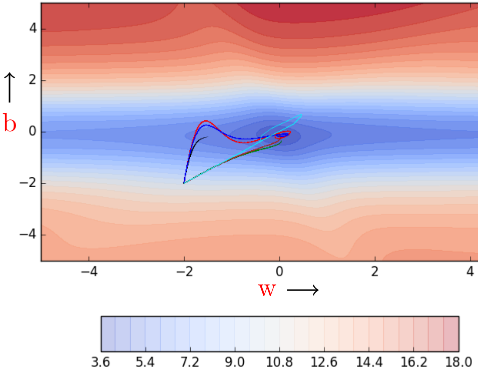

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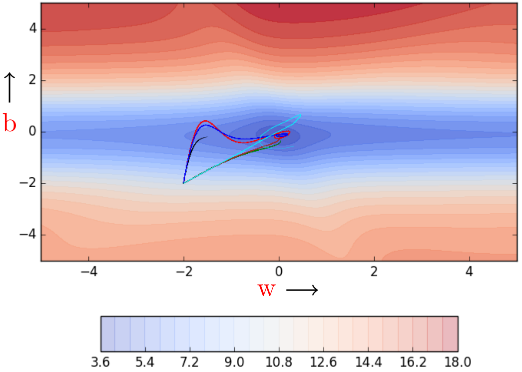

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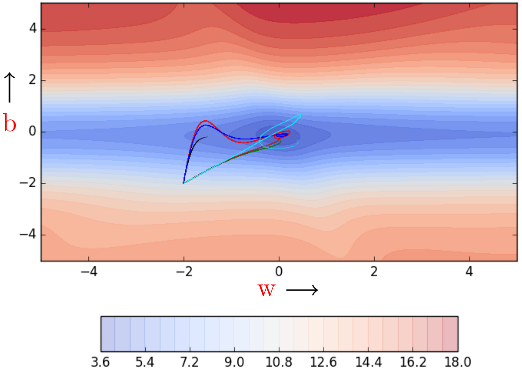

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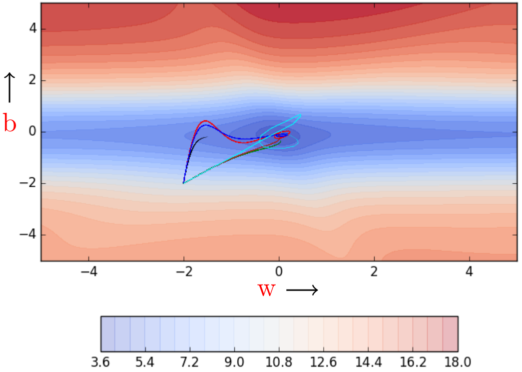

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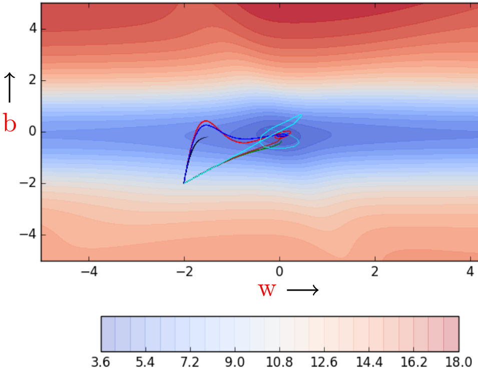
        m_w = m_w/(1-math.pow(beta1,i+1))
        m_b = m_b/(1-math.pow(beta1,i+1))

        v_w = v_w/(1-math.pow(beta2,i+1))
        v_b = v_b/(1-math.pow(beta2,i+1))

        w = w - (eta / np.sqrt(v_w + eps)) * m_w
        b = b - (eta / np.sqrt(v_b + eps)) * m_b
```



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def do_adam() :
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    w_history, b_history, error_history = [], [], [
        ]

    w, b, eta, mini_batch_size, num_points_seen =
        init_w, init_b, 0.1, 10, 0
    m_w, m_b, v_w, v_b, eps, beta1, beta2 = 0, 0, 0
        , 0, 1e-8, 0.9, 0.999
    for i in range(max_epochs) :
        dw, db  = 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)

        m_w = beta1 * m_w + (1-beta1)*dw
        m_b = beta1 * m_b + (1-beta1)*db

        v_w = beta2 * v_w + (1-beta2)*dw**2
        v_b = beta2 * v_b + (1-beta2)*db**2

        m_w = m_w/(1-math.pow(beta1,i+1))
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        w = w - (eta / np.sqrt(v_w + eps)) * m_w
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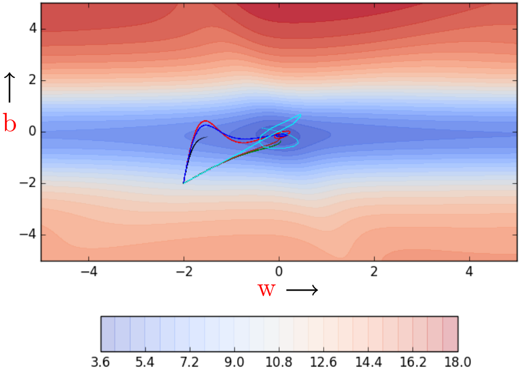

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        w = w - (eta / np.sqrt(v_w + eps)) * m_w
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```

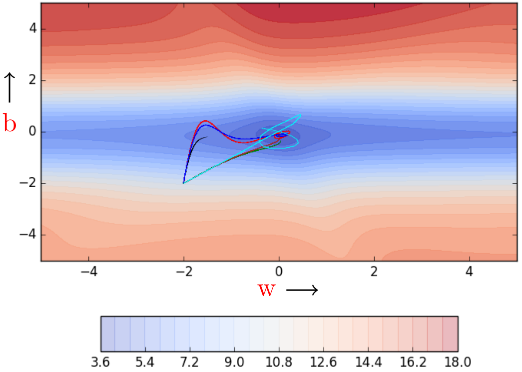

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        m_w = m_w/(1-math.pow(beta1,i+1))
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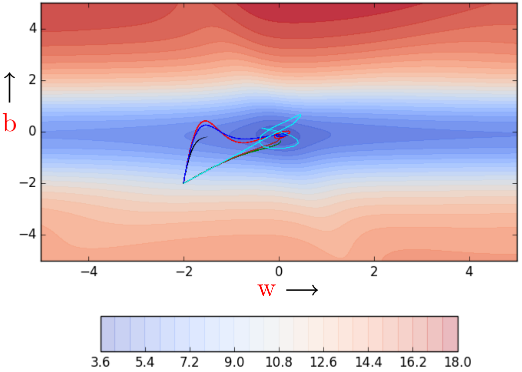

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        v_b = v_b/(1-math.pow(beta2,i+1))

        w = w - (eta / np.sqrt(v_w + eps)) * m_w
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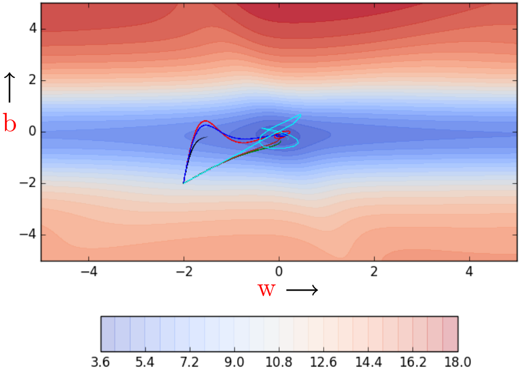
```



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        v_w = beta2 * v_w + (1-beta2)*dw**2
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        m_w = m_w/(1-math.pow(beta1,i+1))
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        w = w - (eta / np.sqrt(v_w + eps)) * m_w
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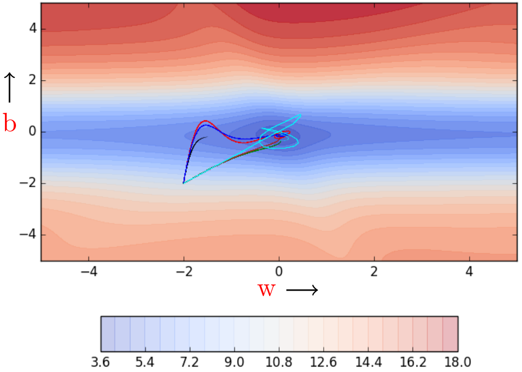

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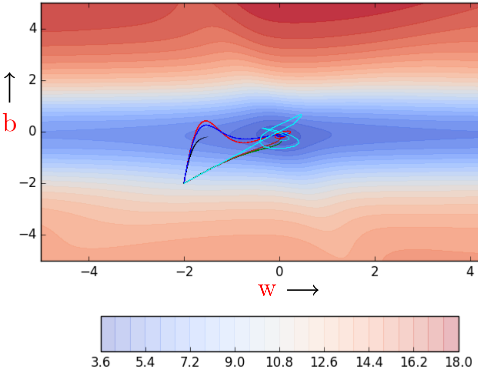

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        m_w = m_w/(1-math.pow(beta1,i+1))
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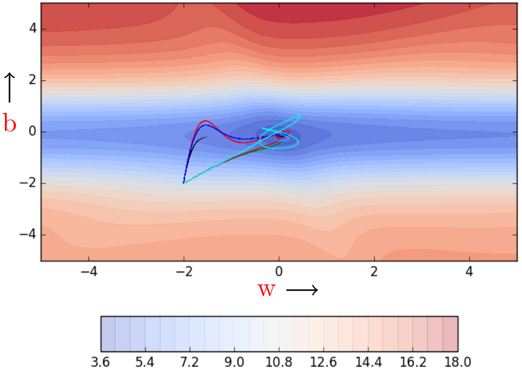

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        m_w = m_w/(1-math.pow(beta1,i+1))
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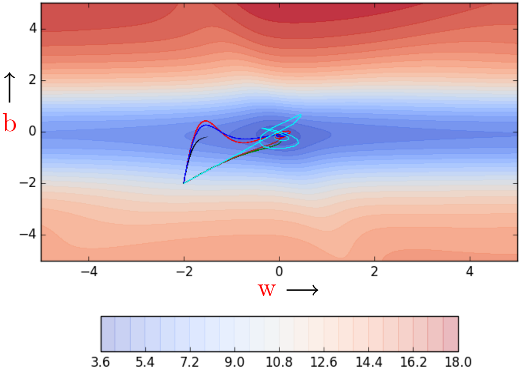

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        m_w = m_w/(1-math.pow(beta1,i+1))
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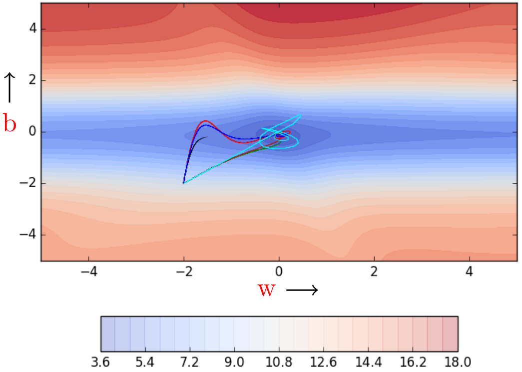

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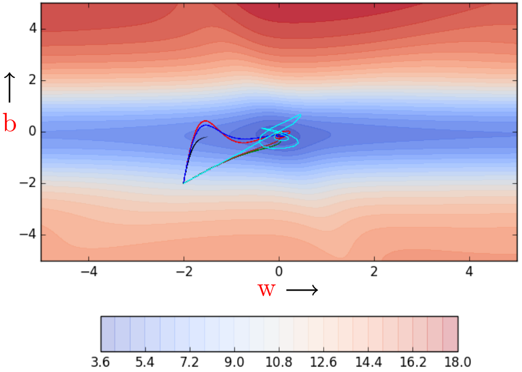

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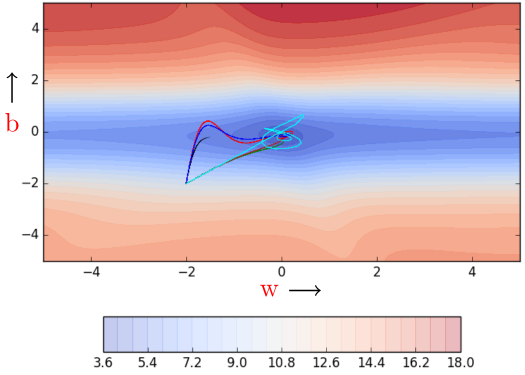

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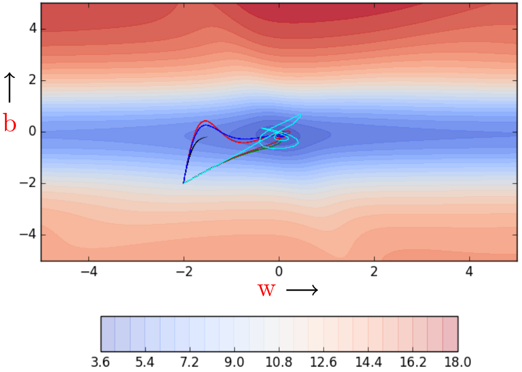

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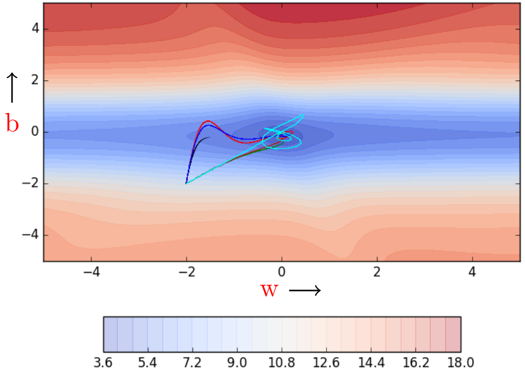

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```

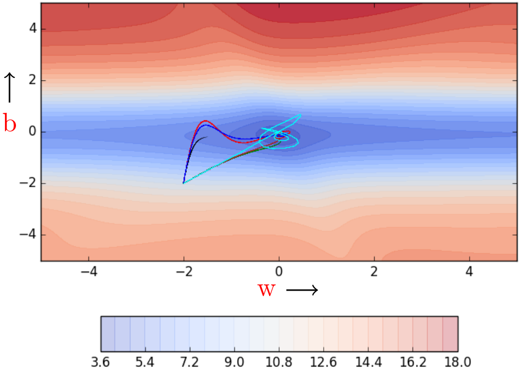

- As expected, taking a cumulative history gives a speed up ...

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- Some recent work suggest that there is a problem with Adam and it will not converge in some cases