Module 8.10 : Ensemble methods

Other forms of regularization

- l_2 regularization
- Dataset augmentation
- Parameter Sharing and tying
- Adding Noise to the inputs
- Adding Noise to the outputs
- Early stopping
- Ensemble methods
- Dropout

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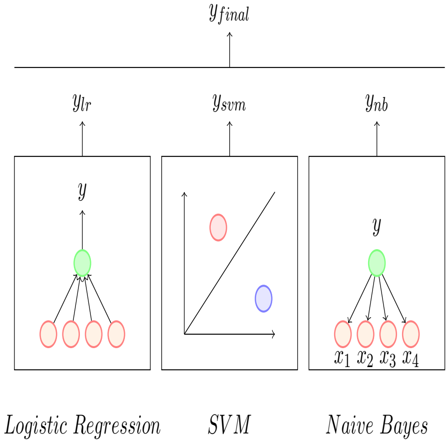
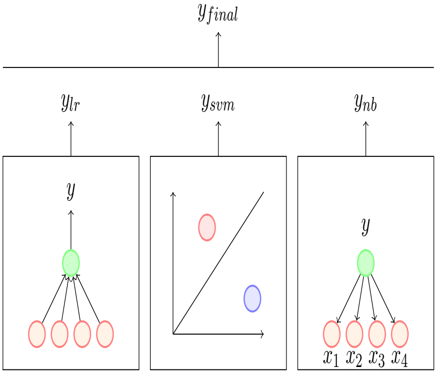


- Combine the output of different models to reduce generalization error



Logistic Regression *SVM* *Naive Bayes*

- Combine the output of different models to reduce generalization error
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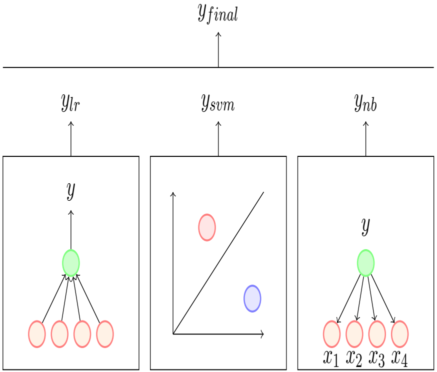

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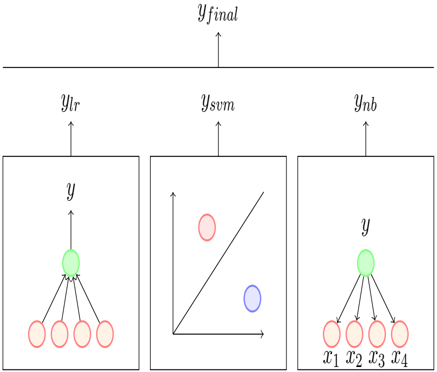

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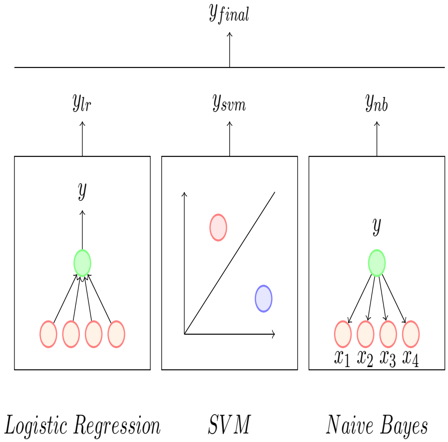

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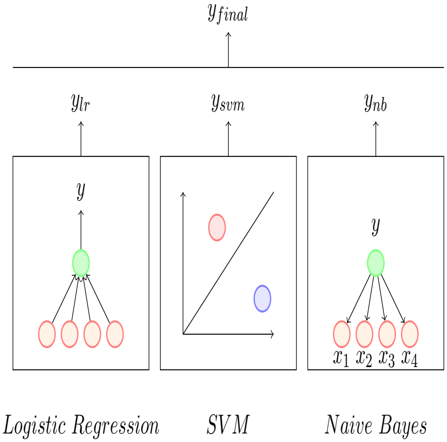

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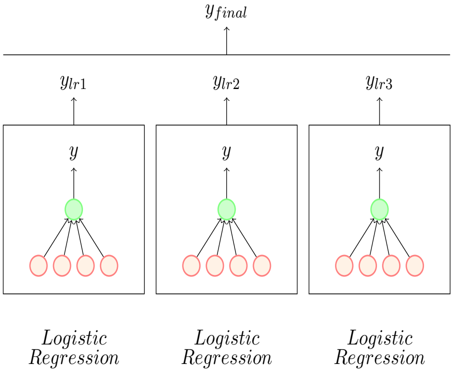
y_{final}
y_{lr1}
y_{lr2}
y_{lr3}
y
y
y
Logistic Regression
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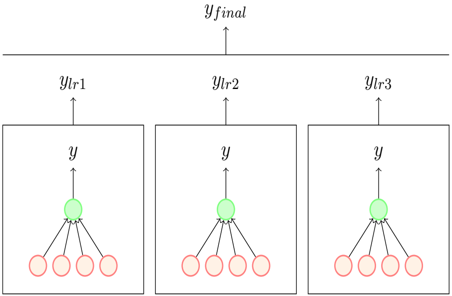

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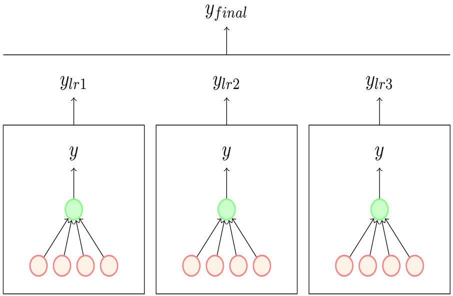

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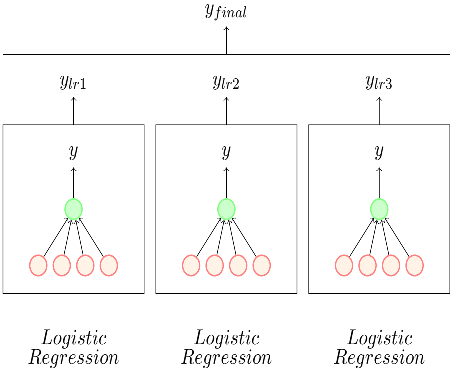

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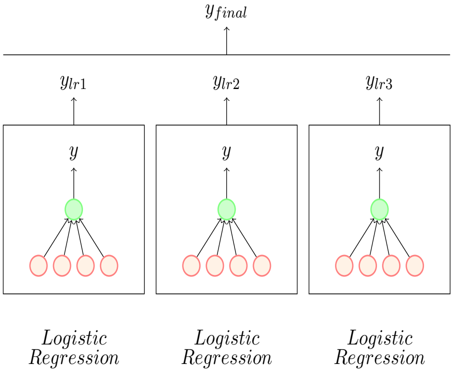

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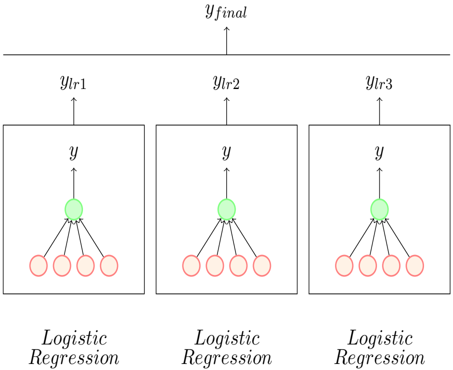

Each model trained with a different sample of the data (sampling with replacement)

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- If the errors of the model are independent or uncorrelated then $C = 0$ and the mse of the ensemble reduces to $\frac{1}{k}V$

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- If the errors of the model are independent or uncorrelated then $C = 0$ and the mse of the ensemble reduces to $\frac{1}{k}V$
- On average, the ensemble will perform at least as well as its individual members