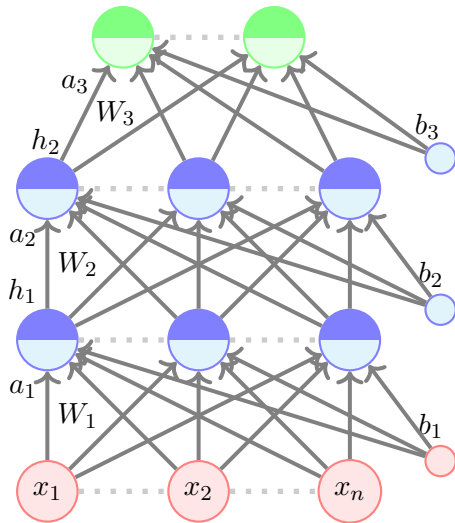# Module 4.2: Learning Parameters of Feedforward Neural Networks (Intuition)

The story so far...

- We have introduced feedforward neural networks
- We are now interested in finding an algorithm for learning the parameters of this model

$h_L = \hat{y} = f(x)$

a_3
W_3
b_3
h_2
a_2
W_2
b_2
h_1
a_1
W_1
b_1
x_1 x_2 x_n

- Recall our gradient descent algorithm

$h_L = \hat{y} = f(x)$

a_3
W_3
b_3
h_2
a_2
W_2
b_2
h_1
a_1
W_1
b_1
$x_1 \quad x_2 \quad x_n$

- Recall our gradient descent algorithm

Algorithm: gradient_descent()

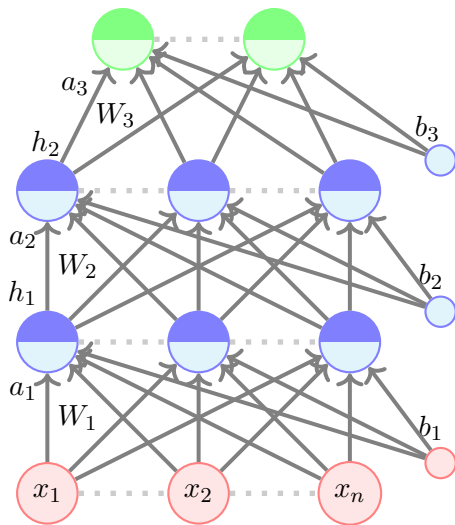$t \leftarrow 0$;
$max\_iterations \leftarrow 1000$;
$Initialize \quad w_0, b_0$;
while $t\text{++} < max\_iterations$ **do**
  $\quad w_{t+1} \leftarrow w_t - \eta \nabla w_t$;
  $\quad b_{t+1} \leftarrow b_t - \eta \nabla b_t$;
end

$h_L = \hat{y} = f(x)$

a_3

W_3

h_2

b_3

a_2

W_2

h_1

b_2

a_1

W_1

b_1

$x_1 \quad x_2 \quad x_n$

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- We can write it more concisely as

Algorithm: gradient_descent()

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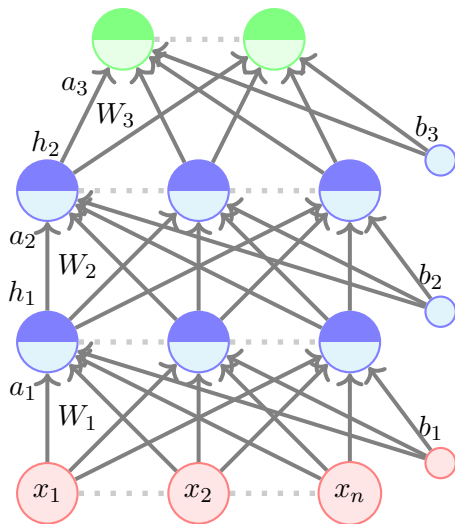**while** $t\text{++} < max\_iterations$ **do**
$\quad w_{t+1} \leftarrow w_t - \eta \nabla w_t$;
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$h_L = \hat{y} = f(x)$

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h_2

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x_1 x_2 x_n

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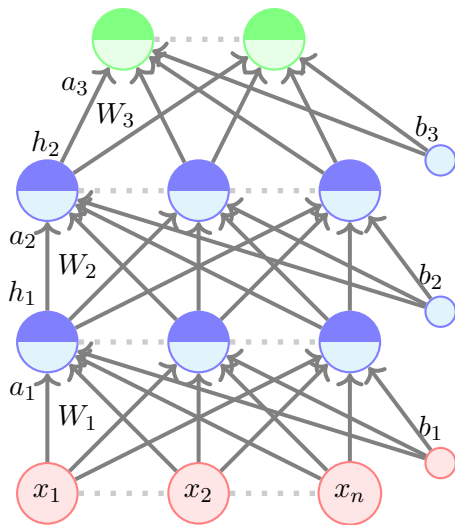$t \leftarrow 0$;
$max\_iterations \leftarrow 1000$;
$Initialize \quad \theta_0 = [w_0, b_0]$;
while $t\text{++} < max\_iterations$ **do**
$\quad \mid \quad \theta_{t+1} \leftarrow \theta_t - \eta \nabla \theta_t$;
end

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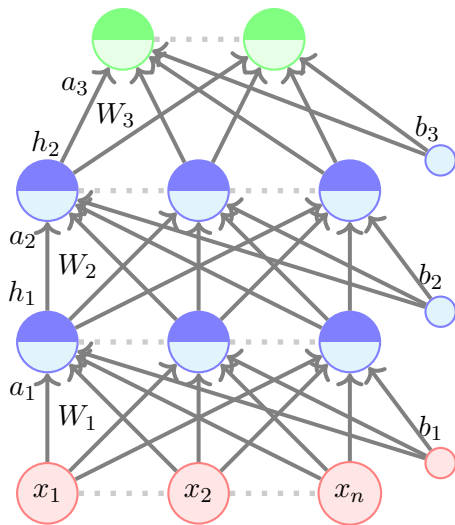
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- where $\nabla \theta_t = \left[\frac{\partial \mathscr{L}(\theta)}{\partial w_t}, \frac{\partial \mathscr{L}(\theta)}{\partial b_t} \right]^T$

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b_3

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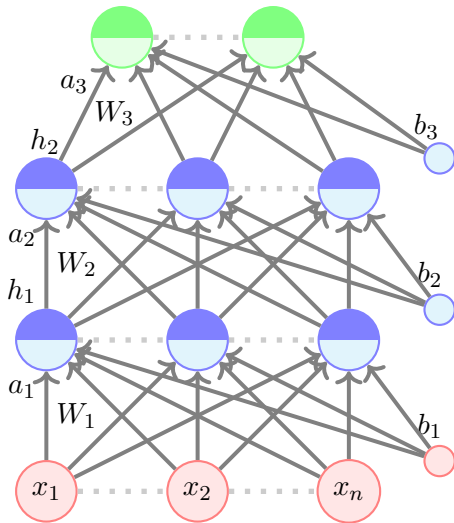$Initialize \quad \theta_0 = [w_0, b_0]$;
while $t{+}{+} < max\_iterations$ **do**
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- where $\nabla \theta_t = \left[\frac{\partial \mathscr{L}(\theta)}{\partial w_t}, \frac{\partial \mathscr{L}(\theta)}{\partial b_t} \right]^T$
- Now, in this feedforward neural network, instead of $\theta = [w, b]$ we have $\theta = [W_1, W_2, .., W_L, b_1, b_2, .., b_L]$

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W_3

h_2

b_3

a_2

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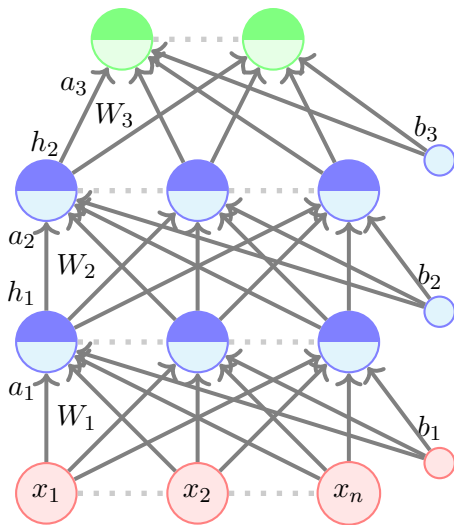$max\_iterations \leftarrow 1000$;
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$\quad \mid \quad \theta_{t+1} \leftarrow \theta_t - \eta \nabla \theta_t$;
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- Now, in this feedforward neural network, instead of $\theta = [w, b]$ we have $\theta = [W_1, W_2, .., W_L, b_1, b_2, .., b_L]$
- We can still use the same algorithm for learning the parameters of our model

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W_3

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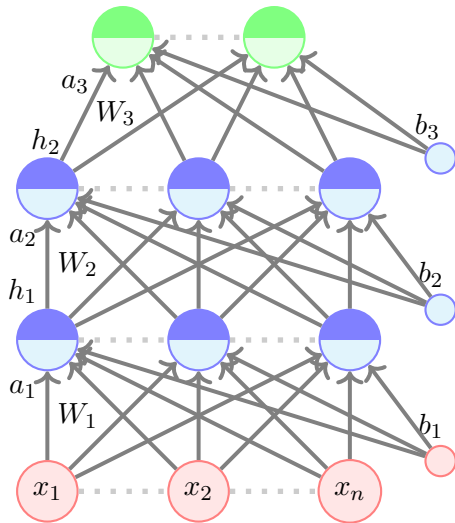$Initialize \quad \theta_0 = [W_1^0, ..., W_L^0, b_1^0, ..., b_L^0]$;
while $t{+}{+} < max\_iterations$ **do**
$\quad \Big| \quad \theta_{t+1} \leftarrow \theta_t - \eta \nabla \theta_t$;
end

- where $\nabla \theta_t = \left[\frac{\partial \mathscr{L}(\theta)}{\partial W_{1,t}}, .., \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,t}}, \frac{\partial \mathscr{L}(\theta)}{\partial b_{1,t}}, .., \frac{\partial \mathscr{L}(\theta)}{\partial b_{L,t}} \right]^T$
- Now, in this feedforward neural network, instead of $\theta = [w, b]$ we have $\theta = [W_1, W_2, .., W_L, b_1, b_2, .., b_L]$
- We can still use the same algorithm for learning the parameters of our model

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$$\begin{bmatrix} \frac{\partial \mathscr{L}(\theta)}{\partial W_{111}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{11n}} & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{bmatrix}$$

- Except that now our $\nabla\theta$ looks much more nasty

$$
\begin{bmatrix}
\frac{\partial \mathscr{L}(\theta)}{\partial W_{111}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{11n}} \\[2ex]
\frac{\partial \mathscr{L}(\theta)}{\partial W_{121}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{12n}} \\
\vdots & \vdots & \vdots \\
\frac{\partial \mathscr{L}(\theta)}{\partial W_{1n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{1nn}}
\end{bmatrix}
$$

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$$\begin{bmatrix} \frac{\partial\mathscr{L}(\theta)}{\partial W_{111}} & \cdots & \frac{\partial\mathscr{L}(\theta)}{\partial W_{11n}} & \frac{\partial\mathscr{L}(\theta)}{\partial W_{211}} & \cdots & \frac{\partial\mathscr{L}(\theta)}{\partial W_{21n}} \\[2ex] \frac{\partial\mathscr{L}(\theta)}{\partial W_{121}} & \cdots & \frac{\partial\mathscr{L}(\theta)}{\partial W_{12n}} & \frac{\partial\mathscr{L}(\theta)}{\partial W_{221}} & \cdots & \frac{\partial\mathscr{L}(\theta)}{\partial W_{22n}} \\[1ex] \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial\mathscr{L}(\theta)}{\partial W_{1n1}} & \cdots & \frac{\partial\mathscr{L}(\theta)}{\partial W_{1nn}} & \frac{\partial\mathscr{L}(\theta)}{\partial W_{2n1}} & \cdots & \frac{\partial\mathscr{L}(\theta)}{\partial W_{2nn}} \end{bmatrix}$$

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$$\begin{bmatrix} \frac{\partial \mathscr{L}(\theta)}{\partial W_{111}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{11n}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{211}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{21n}} & \cdots \\ \\ \frac{\partial \mathscr{L}(\theta)}{\partial W_{121}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{12n}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{221}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{22n}} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \mathscr{L}(\theta)}{\partial W_{1n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{1nn}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{2n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{2nn}} & \cdots \end{bmatrix}$$

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$$\begin{bmatrix} \frac{\partial \mathscr{L}(\theta)}{\partial W_{111}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{11n}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{211}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{21n}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,11}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,1k}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,1k}} & \frac{\partial \mathscr{L}(\theta)}{\partial b_{11}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial b_{L1}} \\[2ex] \frac{\partial \mathscr{L}(\theta)}{\partial W_{121}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{12n}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{221}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{22n}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,21}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,2k}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,2k}} & \frac{\partial \mathscr{L}(\theta)}{\partial b_{12}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial b_{L2}} \\[1ex] \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\[1ex] \frac{\partial \mathscr{L}(\theta)}{\partial W_{1n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{1nn}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{2n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{2nn}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,nk}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,nk}} & \frac{\partial \mathscr{L}(\theta)}{\partial b_{1n}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial b_{Lk}} \end{bmatrix}$$

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$$\begin{bmatrix} \frac{\partial \mathscr{L}(\theta)}{\partial W_{111}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{11n}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{211}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{21n}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,11}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,1k}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,1k}} & \frac{\partial \mathscr{L}(\theta)}{\partial b_{11}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial b_{L1}} \\[2mm] \frac{\partial \mathscr{L}(\theta)}{\partial W_{121}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{12n}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{221}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{22n}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,21}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,2k}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,2k}} & \frac{\partial \mathscr{L}(\theta)}{\partial b_{12}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial b_{L2}} \\[2mm] \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\[2mm] \frac{\partial \mathscr{L}(\theta)}{\partial W_{1n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{1nn}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{2n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{2nn}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,nk}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,nk}} & \frac{\partial \mathscr{L}(\theta)}{\partial b_{1n}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial b_{Lk}} \end{bmatrix}$$

- $\nabla\theta$ is thus composed of
  $\nabla W_1, \nabla W_2, ... \nabla W_{L-1} \in \mathbb{R}^{n \times n}, \nabla W_L \in \mathbb{R}^{n \times k}$,
  $\nabla b_1, \nabla b_2, ..., \nabla b_{L-1} \in \mathbb{R}^n$ and $\nabla b_L \in \mathbb{R}^k$

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- How to choose the loss function $\mathscr{L}(\theta)$?

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- How to choose the loss function $\mathscr{L}(\theta)$?
- How to compute $\nabla\theta$ which is composed of
  $\nabla W_1, \nabla W_2, ..., \nabla W_{L-1} \in \mathbb{R}^{n \times n}, \nabla W_L \in \mathbb{R}^{n \times k}$
  $\nabla b_1, \nabla b_2, ..., \nabla b_{L-1} \in \mathbb{R}^n$ and $\nabla b_L \in \mathbb{R}^k$?