Module 2.5: Perceptron Learning Algorithm

- We will now see a more principled approach for learning these weights and threshold but before that let us answer this question...

- We will now see a more principled approach for learning these weights and threshold but before that let us answer this question...
- Apart from implementing boolean functions (which does not look very interesting) what can a perceptron be used for ?

- We will now see a more principled approach for learning these weights and threshold but before that let us answer this question...
- Apart from implementing boolean functions (which does not look very interesting) what can a perceptron be used for ?
- Our interest lies in the use of perceptron as a binary classifier. Let us see what this means...

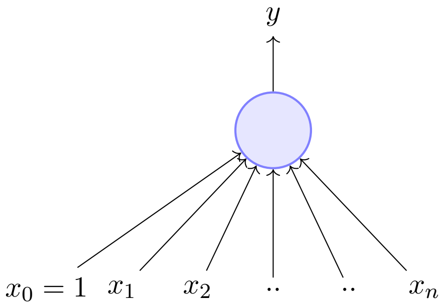
- Let us reconsider our problem of deciding whether to watch a movie or not

- Let us reconsider our problem of deciding whether to watch a movie or not
- Suppose we are given a list of m movies and a label (class) associated with each movie indicating whether the user liked this movie or not : binary decision

- Let us reconsider our problem of deciding whether to watch a movie or not
- Suppose we are given a list of m movies and a label (class) associated with each movie indicating whether the user liked this movie or not : binary decision
- Further, suppose we represent each movie with n features (some boolean, some real valued)

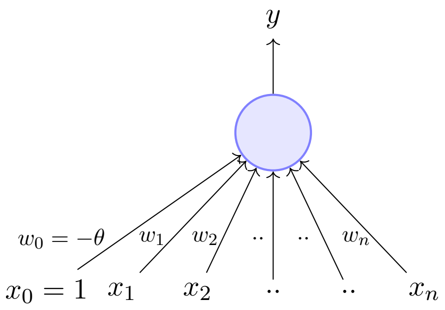
- Let us reconsider our problem of deciding whether to watch a movie or not
- Suppose we are given a list of m movies and a label (class) associated with each movie indicating whether the user liked this movie or not : binary decision
- Further, suppose we represent each movie with n features (some boolean, some real valued)

$$x_1 = isActorDamon$$
$$x_2 = isGenreThriller$$
$$x_3 = isDirectorNolan$$
$$x_4 = imdbRating(scaled\ to\ 0\ to\ 1)$$
$$\ldots \qquad \ldots$$
$$x_n = criticsRating(scaled\ to\ 0\ to\ 1)$$



$$x_1 = isActorDamon$$
$$x_2 = isGenreThriller$$
$$x_3 = isDirectorNolan$$
$$x_4 = imdbRating(scaled\ to\ 0\ to\ 1)$$
$$\ldots \qquad \ldots$$
$$x_n = criticsRating(scaled\ to\ 0\ to\ 1)$$

- Let us reconsider our problem of deciding whether to watch a movie or not
- Suppose we are given a list of m movies and a label (class) associated with each movie indicating whether the user liked this movie or not : binary decision
- Further, suppose we represent each movie with n features (some boolean, some real valued)
- We will assume that the data is linearly separable and we want a perceptron to learn how to make this decision



$x_1 = isActorDamon$

$x_2 = isGenreThriller$

$x_3 = isDirectorNolan$

$x_4 = imdbRating(scaled\ to\ 0\ to\ 1)$

... ...

$x_n = criticsRating(scaled\ to\ 0\ to\ 1)$

- Let us reconsider our problem of deciding whether to watch a movie or not
- Suppose we are given a list of m movies and a label (class) associated with each movie indicating whether the user liked this movie or not : binary decision
- Further, suppose we represent each movie with n features (some boolean, some real valued)
- We will assume that the data is linearly separable and we want a perceptron to learn how to make this decision
- In other words, we want the perceptron to find the equation of this separating plane (or find the values of $w_0, w_1, w_2, .., w_m$)

Algorithm: Perceptron Learning Algorithm

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize **w** randomly;

while $!convergence$ **do**

end

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize **w** randomly;

while !*convergence* **do**

end

//the algorithm converges when all the inputs are classified correctly

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize **w** randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

end

//the algorithm converges when all the inputs are classified correctly

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize **w** randomly;

while !*convergence* **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \sum_{i=0}^{n} w_i * x_i < 0$ **then**

  end

end

//the algorithm converges when all the inputs are classified correctly

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \sum_{i=0}^{n} w_i * x_i < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \sum_{i=0}^{n} w_i * x_i < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \sum_{i=0}^{n} w_i * x_i \geq 0$ **then**

  end

end

//the algorithm converges when all the inputs are classified correctly

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1$;

$N \leftarrow inputs \quad with \quad label \quad 0$;

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

    Pick random $\mathbf{x} \in P \cup N$;

    if $\mathbf{x} \in P \quad and \quad \sum_{i=0}^{n} w_i * x_i < 0$ **then**

        $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

    end

    if $\mathbf{x} \in N \quad and \quad \sum_{i=0}^{n} w_i * x_i \geq 0$ **then**

        $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

    end

end

//the algorithm converges when all the inputs are classified correctly

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \sum_{i=0}^{n} w_i * x_i < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \sum_{i=0}^{n} w_i * x_i \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

- Why would this work ?

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \sum_{i=0}^{n} w_i * x_i < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \sum_{i=0}^{n} w_i * x_i \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

- Why would this work ?
- To understand why this works we will have to get into a bit of Linear Algebra and a bit of geometry...

- Consider two vectors $\mathbf{w}$ and $\mathbf{x}$

- Consider two vectors $\mathbf{w}$ and $\mathbf{x}$

$$\mathbf{w} = [w_0, w_1, w_2, ..., w_n]$$
$$\mathbf{x} = [1, x_1, x_2, ..., x_n]$$

- Consider two vectors $\mathbf{w}$ and $\mathbf{x}$

$$\mathbf{w} = [w_0, w_1, w_2, ..., w_n]$$

$$\mathbf{x} = [1, x_1, x_2, ..., x_n]$$

$$\mathbf{w} \cdot \mathbf{x} = \mathbf{w^T x} = \sum_{i=0}^{n} w_i * x_i$$

- Consider two vectors $\mathbf{w}$ and $\mathbf{x}$

$$\mathbf{w} = [w_0, w_1, w_2, ..., w_n]$$
$$\mathbf{x} = [1, x_1, x_2, ..., x_n]$$
$$\mathbf{w} \cdot \mathbf{x} = \mathbf{w^T x} = \sum_{i=0}^{n} w_i * x_i$$

- We can thus rewrite the perceptron rule as

- Consider two vectors $\mathbf{w}$ and $\mathbf{x}$

$$\mathbf{w} = [w_0, w_1, w_2, ..., w_n]$$
$$\mathbf{x} = [1, x_1, x_2, ..., x_n]$$
$$\mathbf{w} \cdot \mathbf{x} = \mathbf{w^T x} = \sum_{i=0}^{n} w_i * x_i$$

- We can thus rewrite the perceptron rule as

$$\begin{aligned} y &= 1 \quad if \quad \mathbf{w^T x} \geq 0 \\ &= 0 \quad if \quad \mathbf{w^T x} < 0 \end{aligned}$$

- Consider two vectors $\mathbf{w}$ and $\mathbf{x}$

$$\mathbf{w} = [w_0, w_1, w_2, ..., w_n]$$
$$\mathbf{x} = [1, x_1, x_2, ..., x_n]$$
$$\mathbf{w} \cdot \mathbf{x} = \mathbf{w^T x} = \sum_{i=0}^{n} w_i * x_i$$

- We can thus rewrite the perceptron rule as

$$\begin{aligned} y &= 1 \quad if \quad \mathbf{w^T x} \geq 0 \\ &= 0 \quad if \quad \mathbf{w^T x} < 0 \end{aligned}$$

- We are interested in finding the line $\mathbf{w^T x} = 0$ which divides the input space into two halves

- Consider two vectors $\mathbf{w}$ and $\mathbf{x}$

$$\mathbf{w} = [w_0, w_1, w_2, ..., w_n]$$
$$\mathbf{x} = [1, x_1, x_2, ..., x_n]$$
$$\mathbf{w} \cdot \mathbf{x} = \mathbf{w^T x} = \sum_{i=0}^{n} w_i * x_i$$

- We can thus rewrite the perceptron rule as

$$\begin{aligned} y &= 1 \quad if \quad \mathbf{w^T x} \geq 0 \\ &= 0 \quad if \quad \mathbf{w^T x} < 0 \end{aligned}$$

- We are interested in finding the line $\mathbf{w^T x} = 0$ which divides the input space into two halves
- Every point ($\mathbf{x}$) on this line satisfies the equation $\mathbf{w^T x} = 0$

- Consider two vectors $\mathbf{w}$ and $\mathbf{x}$

$$\mathbf{w} = [w_0, w_1, w_2, ..., w_n]$$
$$\mathbf{x} = [1, x_1, x_2, ..., x_n]$$
$$\mathbf{w} \cdot \mathbf{x} = \mathbf{w^T x} = \sum_{i=0}^{n} w_i * x_i$$

- We can thus rewrite the perceptron rule as

$$y = 1 \quad if \quad \mathbf{w^T x} \geq 0$$
$$= 0 \quad if \quad \mathbf{w^T x} < 0$$

- We are interested in finding the line $\mathbf{w^T x} = 0$ which divides the input space into two halves
- Every point $(\mathbf{x})$ on this line satisfies the equation $\mathbf{w^T x} = 0$
- What can you tell about the angle (α) between $\mathbf{w}$ and any point $(\mathbf{x})$ which lies on this line ?

- Consider two vectors **w** and **x**

$$\mathbf{w} = [w_0, w_1, w_2, ..., w_n]$$
$$\mathbf{x} = [1, x_1, x_2, ..., x_n]$$
$$\mathbf{w} \cdot \mathbf{x} = \mathbf{w^T x} = \sum_{i=0}^{n} w_i * x_i$$

- We can thus rewrite the perceptron rule as

$$\begin{aligned} y &= 1 \quad if \quad \mathbf{w^T x} \geq 0 \\ &= 0 \quad if \quad \mathbf{w^T x} < 0 \end{aligned}$$

- We are interested in finding the line $\mathbf{w^T x} = 0$ which divides the input space into two halves
- Every point ($\mathbf{x}$) on this line satisfies the equation $\mathbf{w^T x} = 0$
- What can you tell about the angle (α) between **w** and any point ($\mathbf{x}$) which lies on this line ?
- The angle is 90° ($\because cos\alpha = \frac{w^T x}{||w||||x||} = 0$)

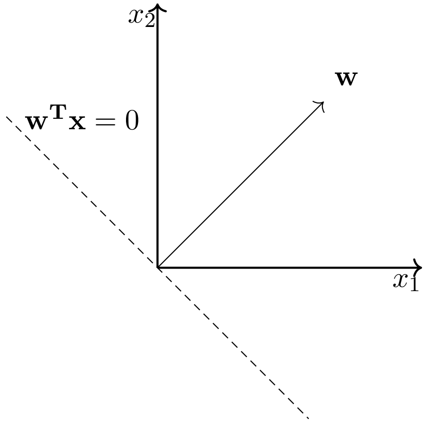
- Consider two vectors $\mathbf{w}$ and $\mathbf{x}$

$$\mathbf{w} = [w_0, w_1, w_2, ..., w_n]$$
$$\mathbf{x} = [1, x_1, x_2, ..., x_n]$$
$$\mathbf{w} \cdot \mathbf{x} = \mathbf{w^T x} = \sum_{i=0}^{n} w_i * x_i$$

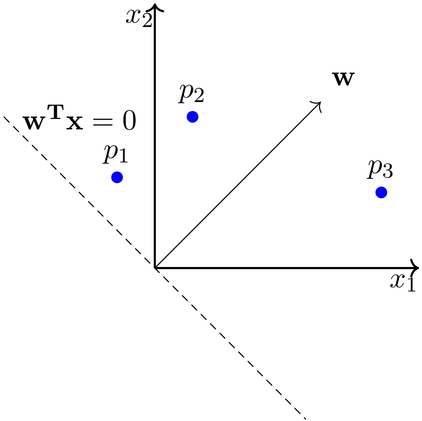
- We can thus rewrite the perceptron rule as

$$\begin{aligned} y &= 1 \quad if \quad \mathbf{w^T x} \geq 0 \\ &= 0 \quad if \quad \mathbf{w^T x} < 0 \end{aligned}$$

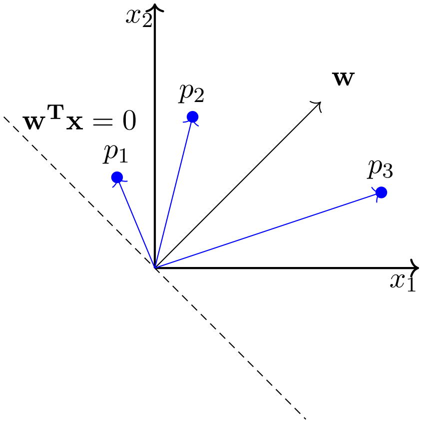
- We are interested in finding the line $\mathbf{w^T x} = 0$ which divides the input space into two halves
- Every point ($\mathbf{x}$) on this line satisfies the equation $\mathbf{w^T x} = 0$
- What can you tell about the angle (α) between $\mathbf{w}$ and any point ($\mathbf{x}$) which lies on this line ?
- The angle is 90° ($\because cos\alpha = \frac{w^T x}{||w||||x||} = 0$)
- Since the vector $\mathbf{w}$ is perpendicular to every point on the line it is actually perpendicular to the line itself


x_2
$\mathbf{w}$
$\mathbf{w^T x} = 0$
x_1

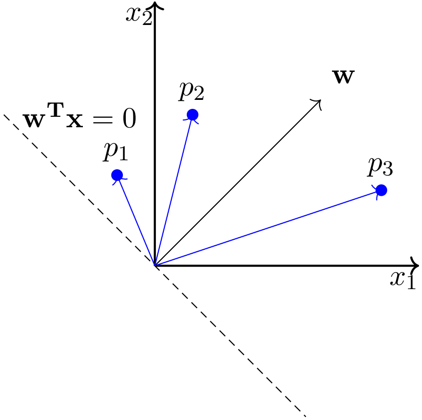
- Consider some points (vectors) which lie in the positive half space of this line (*i.e.*, $\mathbf{w^T x} \geq 0$)



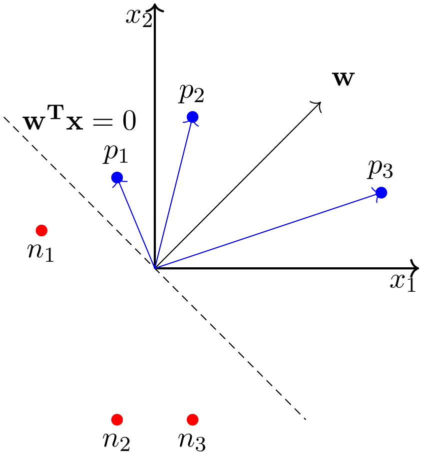
- Consider some points (vectors) which lie in the positive half space of this line (*i.e.*, $\mathbf{w}^{\mathbf{T}}\mathbf{x} \geq 0$)
- What will be the angle between any such vector and $\mathbf{w}$?



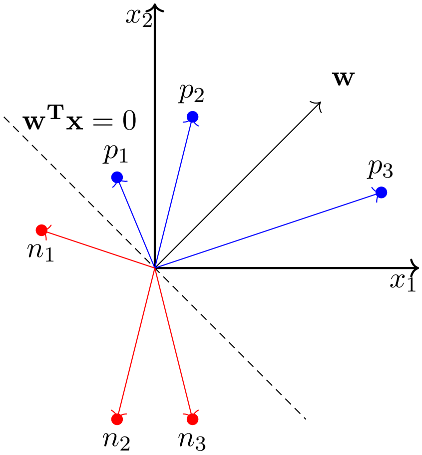
- Consider some points (vectors) which lie in the positive half space of this line (*i.e.*, $\mathbf{w^T x} \geq 0$)
- What will be the angle between any such vector and $\mathbf{w}$? Obviously, less than $90°$



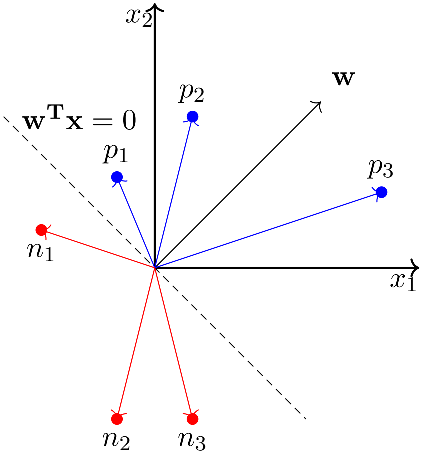
- Consider some points (vectors) which lie in the positive half space of this line (*i.e.*, $\mathbf{w^T x} \geq 0$)
- What will be the angle between any such vector and $\mathbf{w}$? Obviously, less than 90°
- What about points (vectors) which lie in the negative half space of this line (*i.e.*, $\mathbf{w^T x} < 0$)



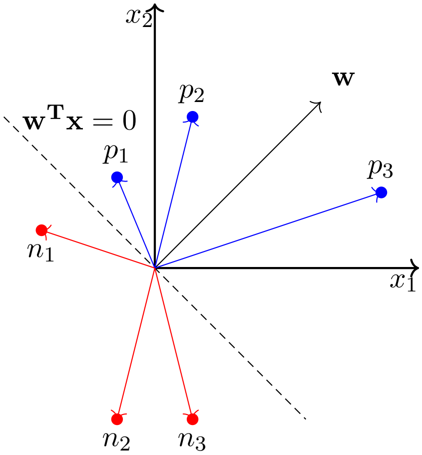
- Consider some points (vectors) which lie in the positive half space of this line (*i.e.*, $\mathbf{w}^{\mathbf{T}}\mathbf{x} \geq 0$)
- What will be the angle between any such vector and $\mathbf{w}$? Obviously, less than $90°$
- What about points (vectors) which lie in the negative half space of this line (*i.e.*, $\mathbf{w}^{\mathbf{T}}\mathbf{x} < 0$)
- What will be the angle between any such vector and $\mathbf{w}$?



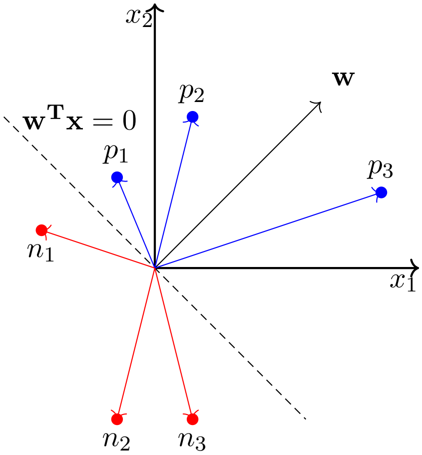
- Consider some points (vectors) which lie in the positive half space of this line (*i.e.*, $\mathbf{w^T x} \geq 0$)
- What will be the angle between any such vector and $\mathbf{w}$? Obviously, less than 90°
- What about points (vectors) which lie in the negative half space of this line (*i.e.*, $\mathbf{w^T x} < 0$)
- What will be the angle between any such vector and $\mathbf{w}$? Obviously, greater than 90°



- Consider some points (vectors) which lie in the positive half space of this line (*i.e.*, $\mathbf{w^T x} \geq 0$)
- What will be the angle between any such vector and $\mathbf{w}$? Obviously, less than 90°
- What about points (vectors) which lie in the negative half space of this line (*i.e.*, $\mathbf{w^T x} < 0$)
- What will be the angle between any such vector and $\mathbf{w}$? Obviously, greater than 90°
- Of course, this also follows from the formula ($cos\alpha = \frac{w^T x}{||w||||x||}$)



- Consider some points (vectors) which lie in the positive half space of this line (*i.e.*, $\mathbf{w^T x} \geq 0$)
- What will be the angle between any such vector and $\mathbf{w}$? Obviously, less than 90°
- What about points (vectors) which lie in the negative half space of this line (*i.e.*, $\mathbf{w^T x} < 0$)
- What will be the angle between any such vector and $\mathbf{w}$? Obviously, greater than 90°
- Of course, this also follows from the formula ($cos\alpha = \frac{w^T x}{||w||||x||}$)
- Keeping this picture in mind let us revisit the algorithm



Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in P$ if $\mathbf{w}.\mathbf{x} < 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is greater than 90°

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w.x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w.x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in P$ if $\mathbf{w.x} < 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is greater than $90°$ (but we want α to be less than $90°$)

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w.x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w.x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in P$ if $\mathbf{w.x} < 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is greater than 90° (but we want α to be less than 90°)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} + \mathbf{x}$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w.x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w.x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in P$ if $\mathbf{w.x} < 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is greater than 90° (but we want α to be less than 90°)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} + \mathbf{x}$

$$cos(\alpha_{new}) \propto \mathbf{w_{new}}^T\mathbf{x}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

Pick random $\mathbf{x} \in P \cup N$;

if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**

$\mathbf{w} = \mathbf{w} + \mathbf{x}$;

end

if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**

$\mathbf{w} = \mathbf{w} - \mathbf{x}$;

end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in P$ if $\mathbf{w}.\mathbf{x} < 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is greater than 90° (but we want α to be less than 90°)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} + \mathbf{x}$

$$\begin{aligned} cos(\alpha_{new}) &\propto \mathbf{w_{new}}^T\mathbf{x} \\ &\propto (\mathbf{w} + \mathbf{x})^T\mathbf{x} \end{aligned}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$
$N \leftarrow inputs \quad with \quad label \quad 0;$
Initialize $\mathbf{w}$ randomly;
while $!convergence$ **do**
  Pick random $\mathbf{x} \in P \cup N$;
  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**
    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;
  end
  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**
    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;
  end
end
//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in P$ if $\mathbf{w}.\mathbf{x} < 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is greater than 90° (but we want α to be less than 90°)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} + \mathbf{x}$

$$\begin{aligned} cos(\alpha_{new}) &\propto \mathbf{w_{new}}^T\mathbf{x} \\ &\propto (\mathbf{w}+\mathbf{x})^T\mathbf{x} \\ &\propto \mathbf{w}^T\mathbf{x} + \mathbf{x}^T\mathbf{x} \end{aligned}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w.x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w.x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in P$ if $\mathbf{w.x} < 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is greater than 90° (but we want α to be less than 90°)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} + \mathbf{x}$

$$\begin{aligned} cos(\alpha_{new}) &\propto \mathbf{w_{new}}^T\mathbf{x} \\ &\propto (\mathbf{w}+\mathbf{x})^T\mathbf{x} \\ &\propto \mathbf{w}^T\mathbf{x} + \mathbf{x}^T\mathbf{x} \\ &\propto cos\alpha + \mathbf{x}^T\mathbf{x} \end{aligned}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w.x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w.x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in P$ if $\mathbf{w.x} < 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is greater than $90°$ (but we want α to be less than $90°$)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} + \mathbf{x}$

$$\begin{aligned} cos(\alpha_{new}) &\propto \mathbf{w_{new}}^T\mathbf{x} \\ &\propto (\mathbf{w} + \mathbf{x})^T\mathbf{x} \\ &\propto \mathbf{w}^T\mathbf{x} + \mathbf{x}^T\mathbf{x} \\ &\propto cos\alpha + \mathbf{x}^T\mathbf{x} \\ cos(\alpha_{new}) &> cos\alpha \end{aligned}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in P$ if $\mathbf{w}.\mathbf{x} < 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is greater than 90° (but we want α to be less than 90°)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} + \mathbf{x}$

$$\begin{aligned} cos(\alpha_{new}) &\propto \mathbf{w_{new}}^T\mathbf{x} \\ &\propto (\mathbf{w} + \mathbf{x})^T\mathbf{x} \\ &\propto \mathbf{w}^T\mathbf{x} + \mathbf{x}^T\mathbf{x} \\ &\propto cos\alpha + \mathbf{x}^T\mathbf{x} \\ cos(\alpha_{new}) &> cos\alpha \end{aligned}$$

- Thus α_{new} will be less than α and this is exactly what we want

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w.x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w.x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in N$ if $\mathbf{w.x} \geq 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is less than $90°$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w.x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w.x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in N$ if $\mathbf{w.x} \geq 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is less than 90° (but we want α to be greater than 90°)

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in N$ if $\mathbf{w}.\mathbf{x} \geq 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is less than 90° (but we want α to be greater than 90°)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} - \mathbf{x}$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in N$ if $\mathbf{w}.\mathbf{x} \geq 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is less than 90° (but we want α to be greater than 90°)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} - \mathbf{x}$

$$cos(\alpha_{new}) \propto \mathbf{w_{new}}^T\mathbf{x}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in N$ if $\mathbf{w}.\mathbf{x} \geq 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is less than $90°$ (but we want α to be greater than $90°$)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} - \mathbf{x}$

$$\begin{aligned} cos(\alpha_{new}) &\propto \mathbf{w_{new}}^T\mathbf{x} \\ &\propto (\mathbf{w} - \mathbf{x})^T\mathbf{x} \end{aligned}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in N$ if $\mathbf{w}.\mathbf{x} \geq 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is less than $90°$ (but we want α to be greater than $90°$)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} - \mathbf{x}$

$$\begin{aligned} cos(\alpha_{new}) &\propto \mathbf{w_{new}}^T\mathbf{x} \\ &\propto (\mathbf{w} - \mathbf{x})^T\mathbf{x} \\ &\propto \mathbf{w}^T\mathbf{x} - \mathbf{x}^T\mathbf{x} \end{aligned}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in N$ if $\mathbf{w}.\mathbf{x} \geq 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is less than 90° (but we want α to be greater than 90°)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} - \mathbf{x}$

$$\begin{aligned} cos(\alpha_{new}) &\propto \mathbf{w_{new}}^T\mathbf{x} \\ &\propto (\mathbf{w} - \mathbf{x})^T\mathbf{x} \\ &\propto \mathbf{w}^T\mathbf{x} - \mathbf{x}^T\mathbf{x} \\ &\propto cos\alpha - \mathbf{x}^T\mathbf{x} \end{aligned}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$

$N \leftarrow inputs \quad with \quad label \quad 0;$

Initialize $\mathbf{w}$ randomly;

while $!convergence$ **do**

  Pick random $\mathbf{x} \in P \cup N$;

  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**

    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;

  end

  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**

    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;

  end

end

//the algorithm converges when all the inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in N$ if $\mathbf{w}.\mathbf{x} \geq 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is less than $90°$ (but we want α to be greater than $90°$)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} - \mathbf{x}$

$$\begin{aligned} cos(\alpha_{new}) &\propto \mathbf{w_{new}}^T\mathbf{x} \\ &\propto (\mathbf{w} - \mathbf{x})^T\mathbf{x} \\ &\propto \mathbf{w}^T\mathbf{x} - \mathbf{x}^T\mathbf{x} \\ &\propto cos\alpha - \mathbf{x}^T\mathbf{x} \\ cos(\alpha_{new}) &< cos\alpha \end{aligned}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$
$N \leftarrow inputs \quad with \quad label \quad 0;$
Initialize $\mathbf{w}$ randomly;
while $!convergence$ **do**
  Pick random $\mathbf{x} \in P \cup N$;
  if $\mathbf{x} \in P \quad and \quad \mathbf{w.x} < 0$ **then**
    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;
  end
  if $\mathbf{x} \in N \quad and \quad \mathbf{w.x} \geq 0$ **then**
    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;
  end
end
//the algorithm converges when all the inputs are classified correctly

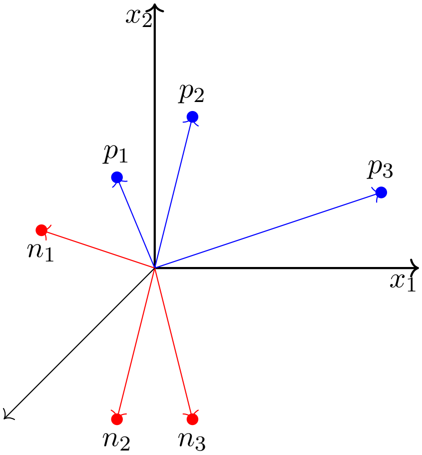
$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in N$ if $\mathbf{w.x} \geq 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is less than 90° (but we want α to be greater than 90°)
- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} - \mathbf{x}$

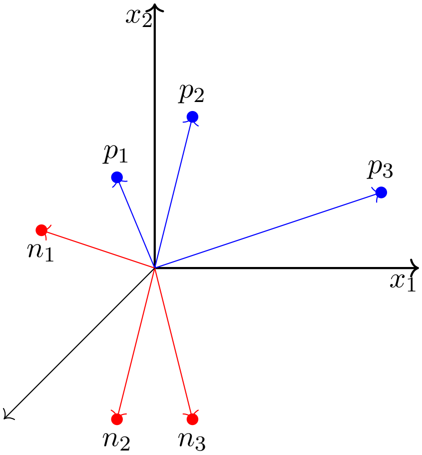
$$\begin{aligned} cos(\alpha_{new}) &\propto \mathbf{w_{new}}^T\mathbf{x} \\ &\propto (\mathbf{w} - \mathbf{x})^T\mathbf{x} \\ &\propto \mathbf{w}^T\mathbf{x} - \mathbf{x}^T\mathbf{x} \\ &\propto cos\alpha - \mathbf{x}^T\mathbf{x} \\ cos(\alpha_{new}) &< cos\alpha \end{aligned}$$

- Thus α_{new} will be greater than α and this is exactly what we want

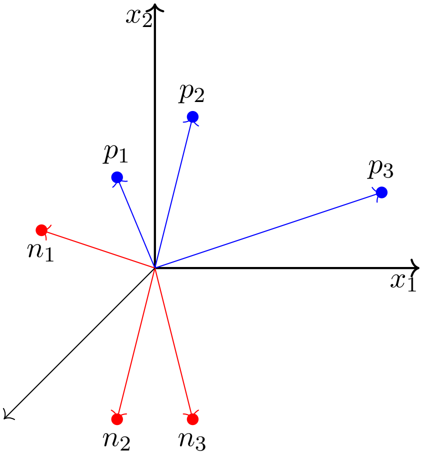
- We will now see this algorithm in action for a toy dataset



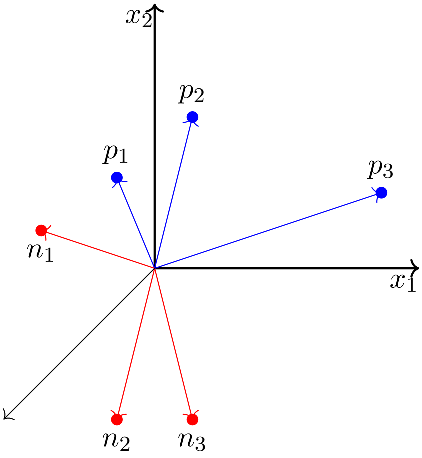
- We initialized $\mathbf{w}$ to a random value



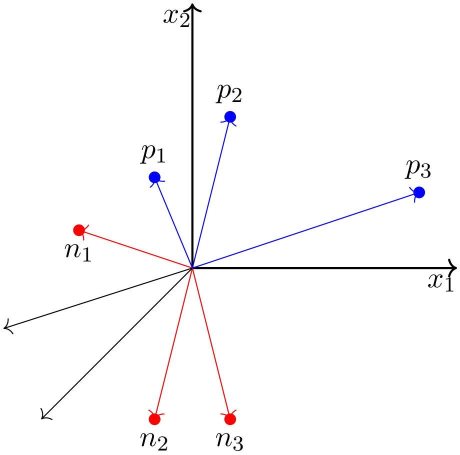
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)



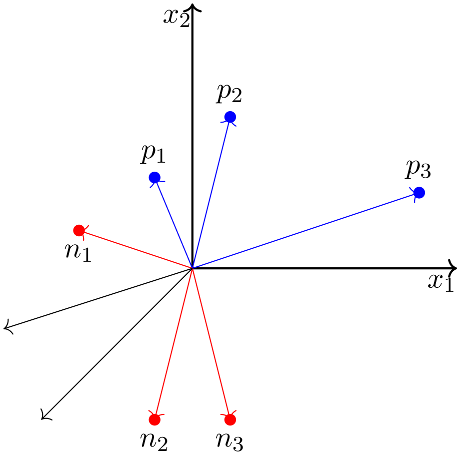
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points



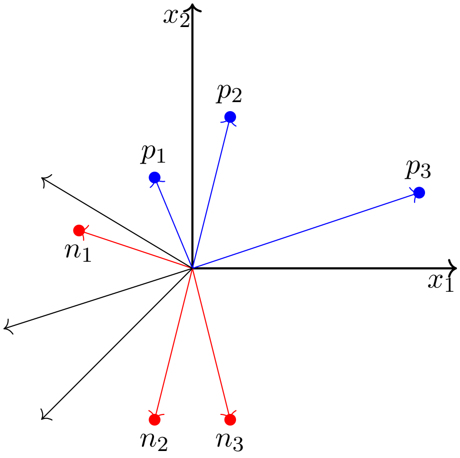
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_1), apply correction $\mathbf{w} = \mathbf{w} + \mathbf{x}$ $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



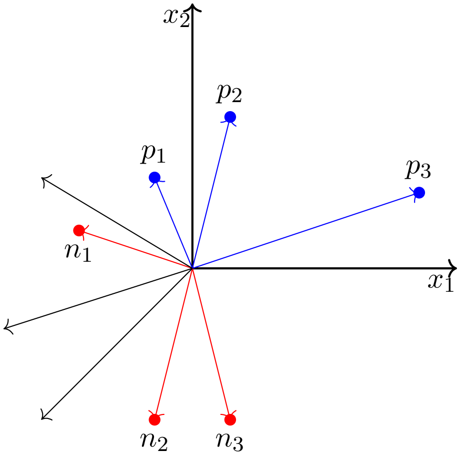
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_1), apply correction $\mathbf{w} = \mathbf{w} + \mathbf{x}$ $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



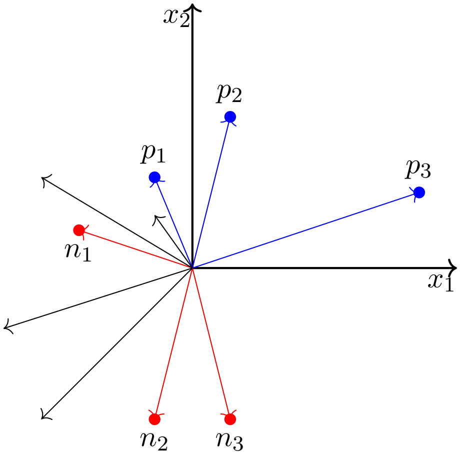
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_2), apply correction $\mathbf{w} = \mathbf{w} + \mathbf{x}$ $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



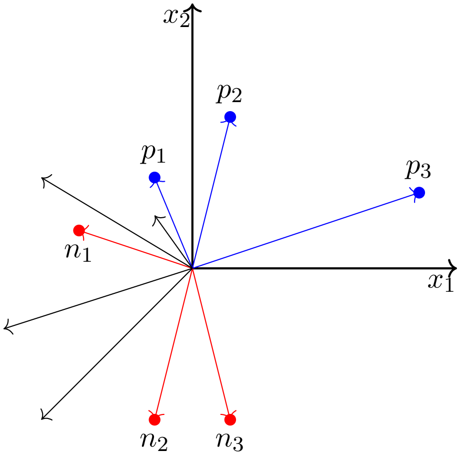
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_2), apply correction $\mathbf{w} = \mathbf{w} + \mathbf{x}$ $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



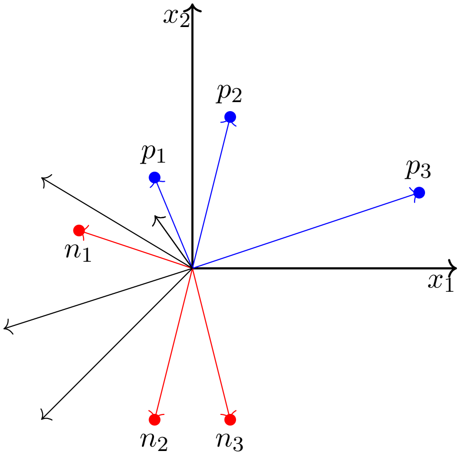
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_1), apply correction $\mathbf{w} = \mathbf{w} - \mathbf{x}$ $\because$ $\mathbf{w} \cdot \mathbf{x} \geq \mathbf{0}$ (you can check the angle visually)



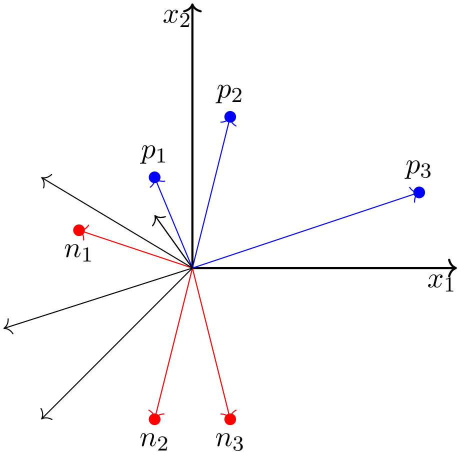
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_1), apply correction $\mathbf{w} = \mathbf{w} - \mathbf{x}$ $\because$ $\mathbf{w} \cdot \mathbf{x} \geq \mathbf{0}$ (you can check the angle visually)



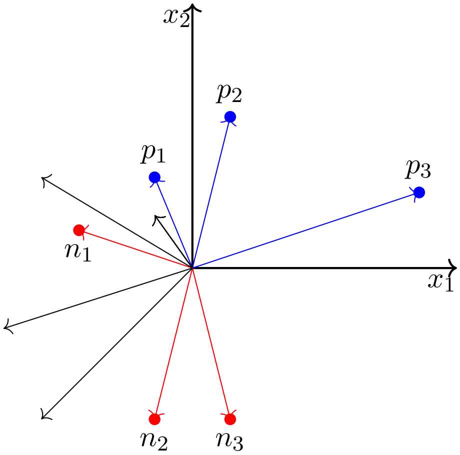
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_3), no correction needed $\because \mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



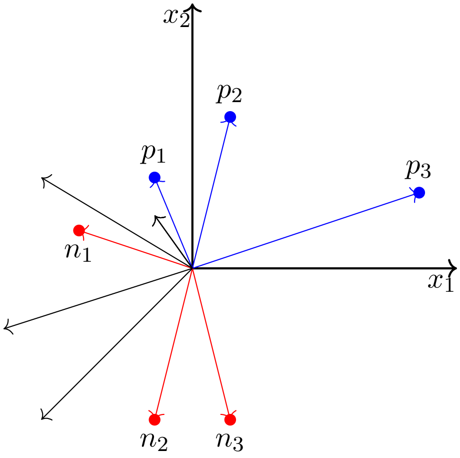
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_3), no correction needed $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



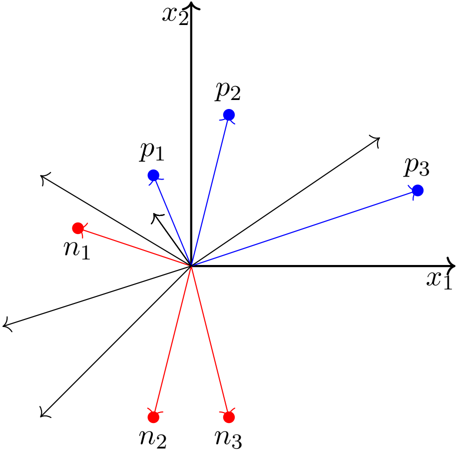
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_2), no correction needed $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



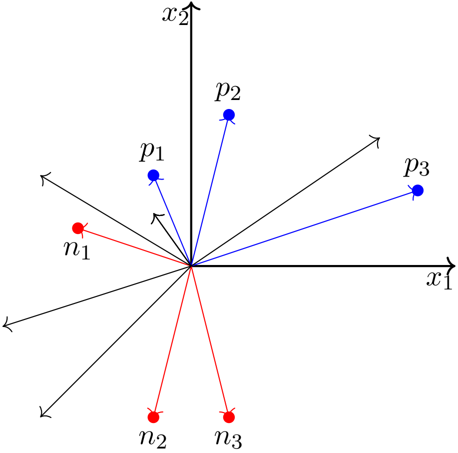
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_2), no correction needed $\because \mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



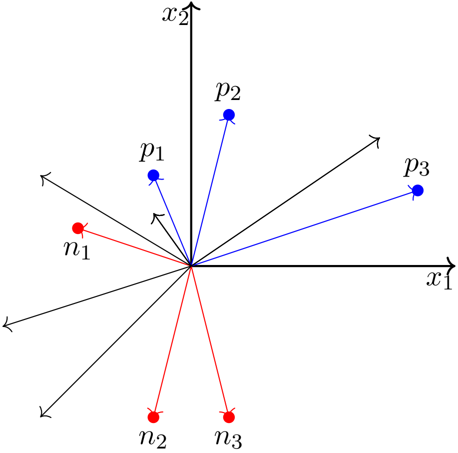
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_3), apply correction $\mathbf{w} = \mathbf{w} + \mathbf{x}$ $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



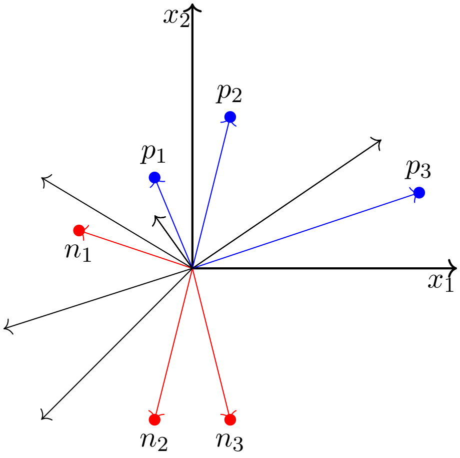
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_3), apply correction $\mathbf{w} = \mathbf{w} + \mathbf{x} \because \mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



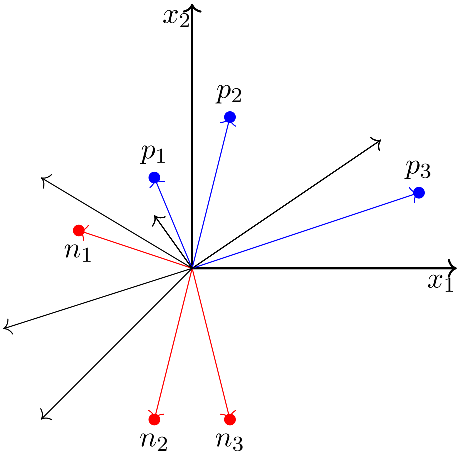
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_1), no correction needed $\because$ $\mathbf{w} \cdot \mathbf{x} \geq \mathbf{0}$ (you can check the angle visually)



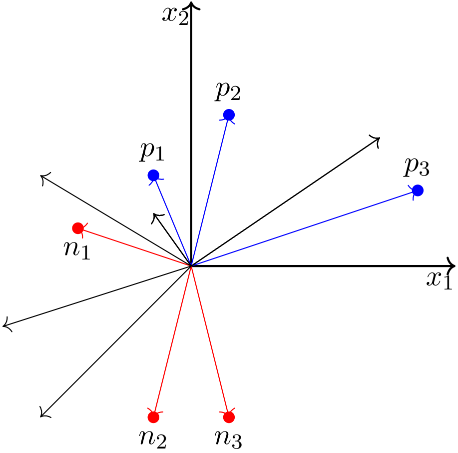
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_1), no correction needed $\because \mathbf{w} \cdot \mathbf{x} \geq \mathbf{0}$ (you can check the angle visually)



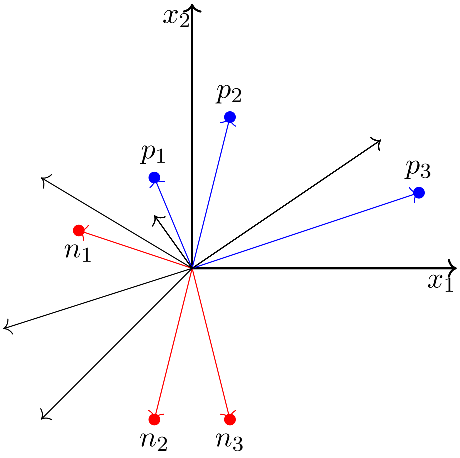
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_2), no correction needed $\because$ $\mathbf{w} \cdot \mathbf{x} \geq \mathbf{0}$ (you can check the angle visually)



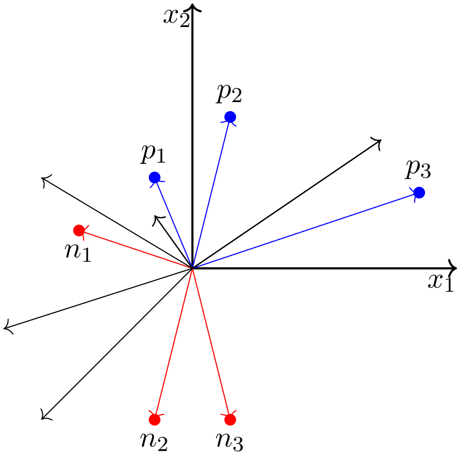
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_2), no correction needed $\because \mathbf{w} \cdot \mathbf{x} \geq \mathbf{0}$ (you can check the angle visually)



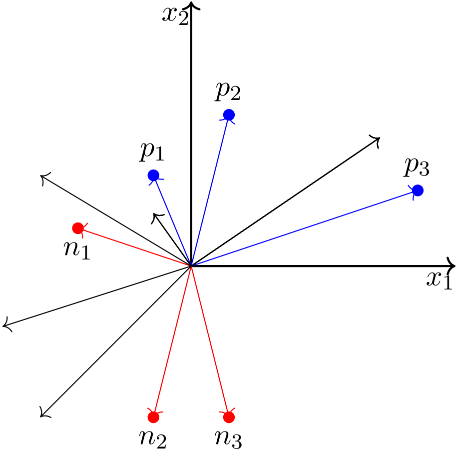
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_1), no correction needed $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



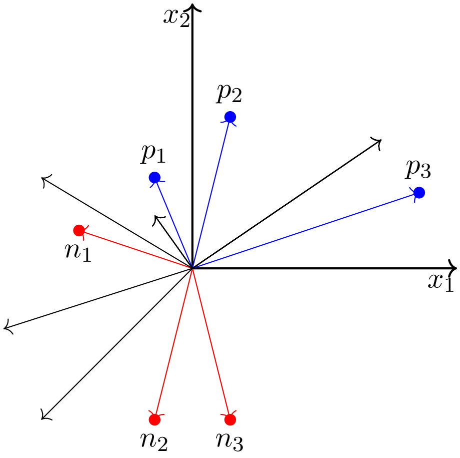
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_1), no correction needed $\because \mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



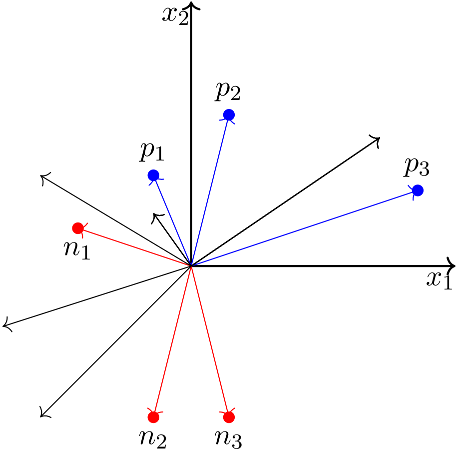
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_3), no correction needed $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



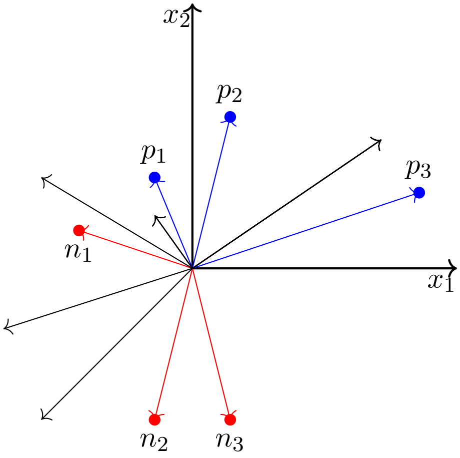
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_3), no correction needed $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



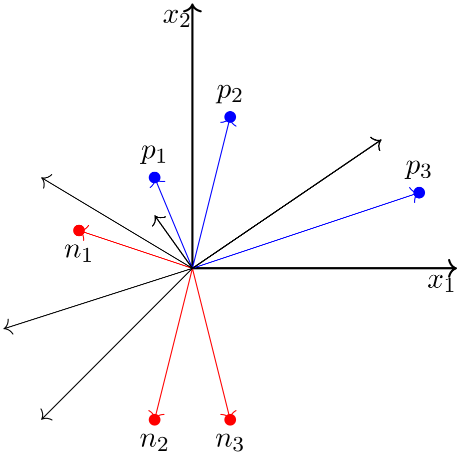
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_2), no correction needed $\because \mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_2), no correction needed $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)



- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_3), no correction needed $\because \mathbf{w} \cdot \mathbf{x} \geq \mathbf{0}$ (you can check the angle visually)



- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- The algorithm has converged