# Module 4.1: Feedforward Neural Networks (a.k.a. multilayered network of neurons)

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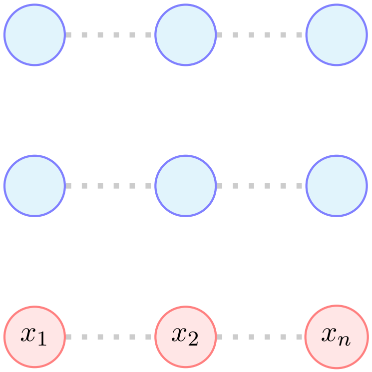

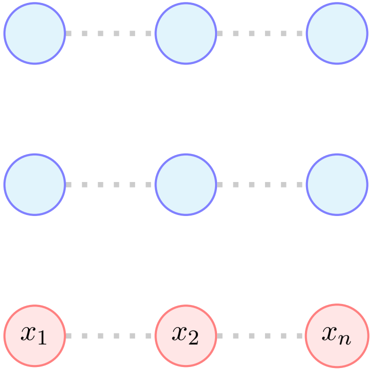


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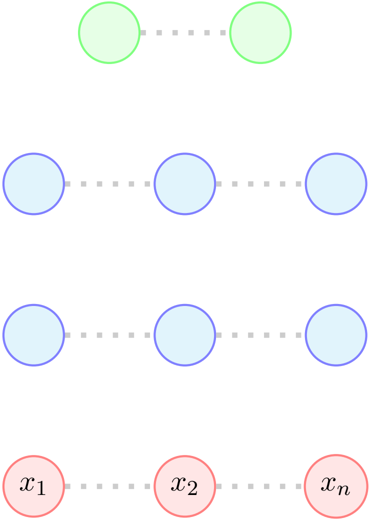


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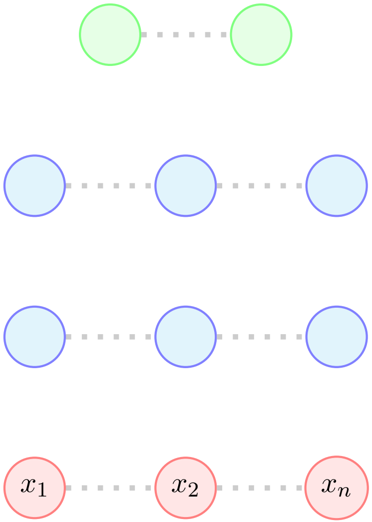


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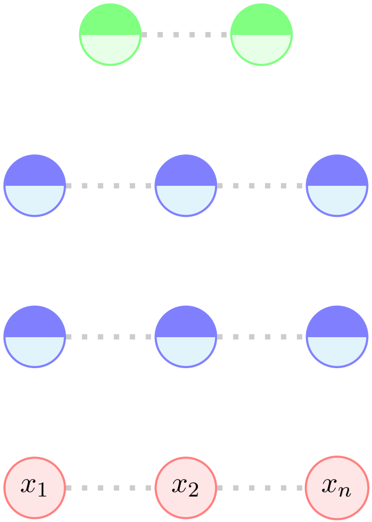


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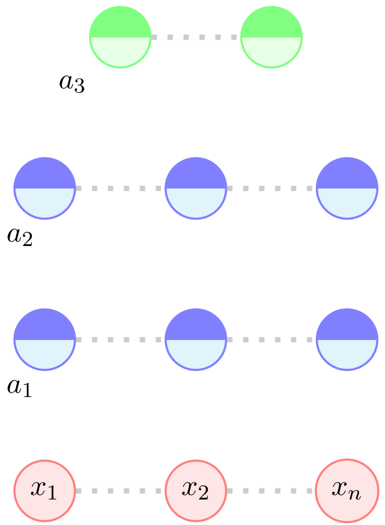


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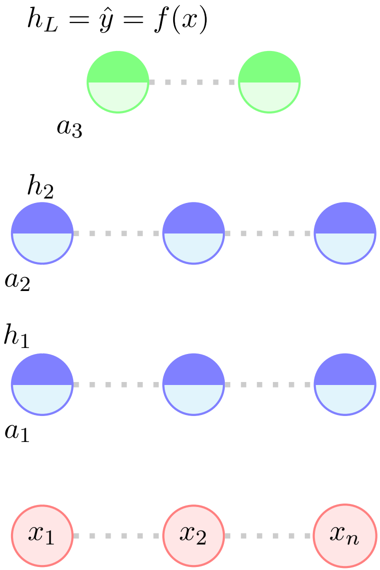


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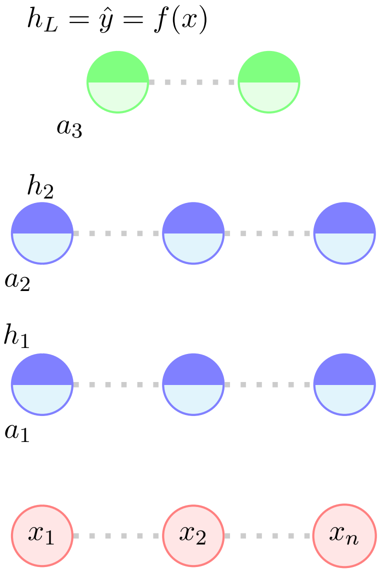


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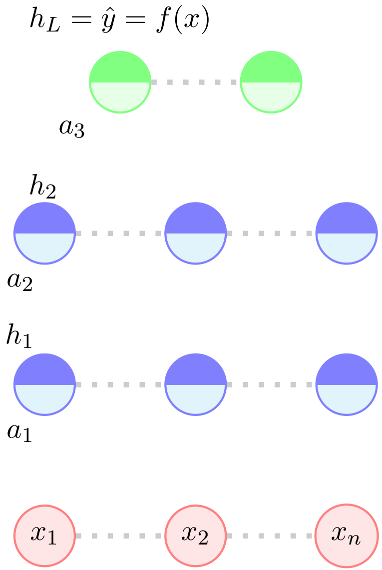


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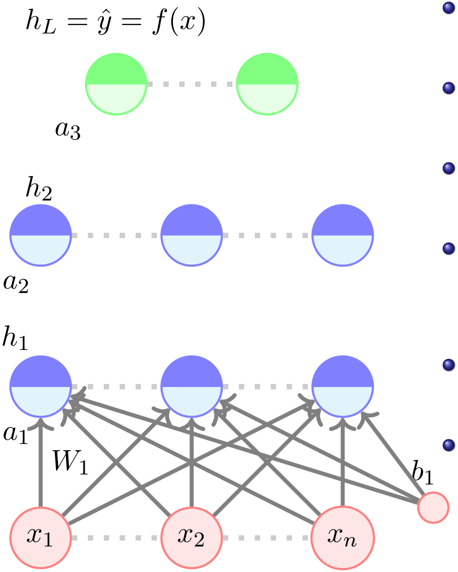


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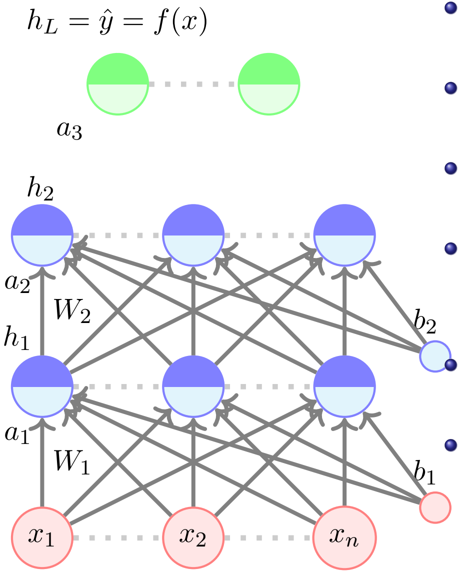


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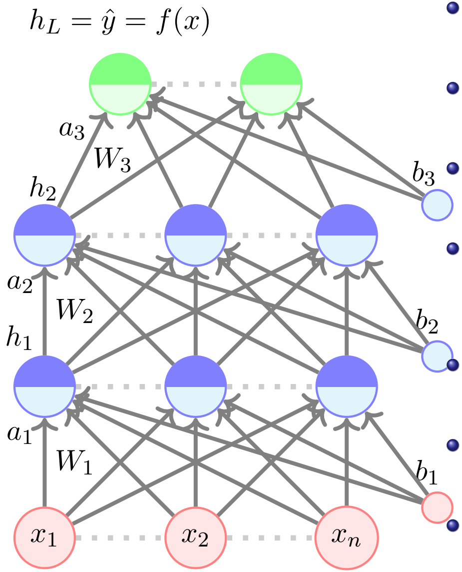


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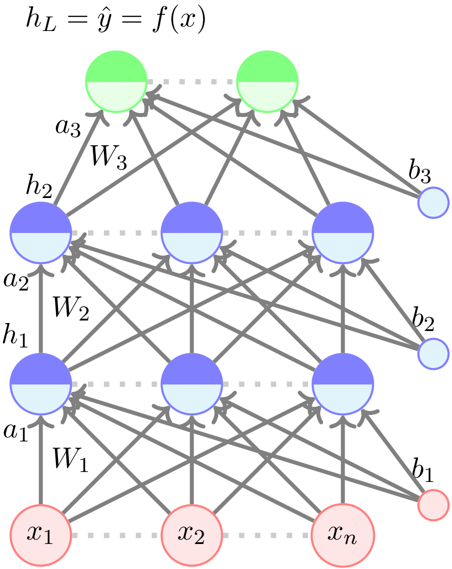
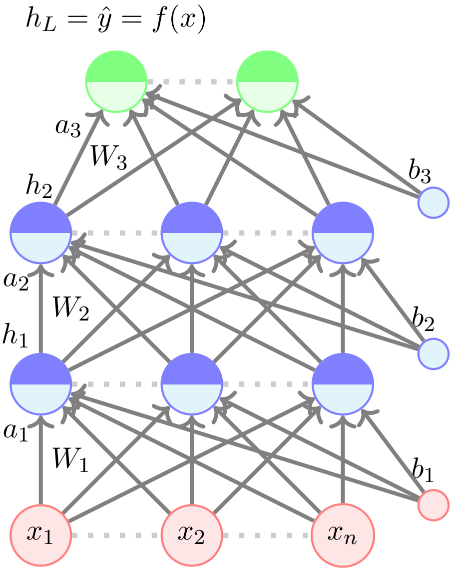



- The pre-activation at layer $i$ is given by

$$a_i(x) = b_i + W_i h_{i-1}(x)$$

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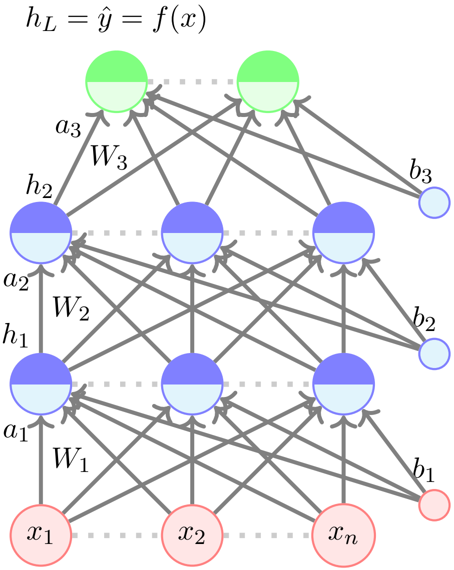


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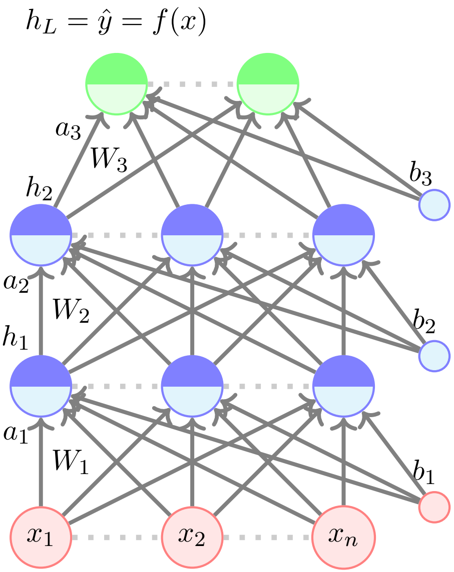


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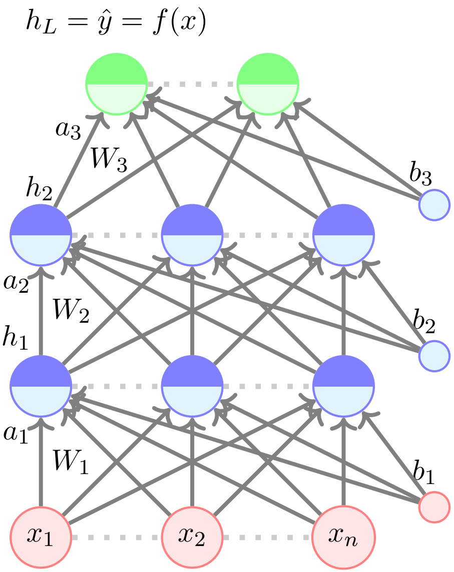


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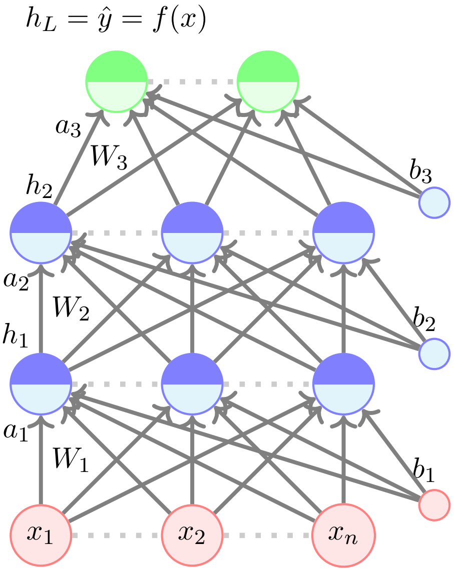


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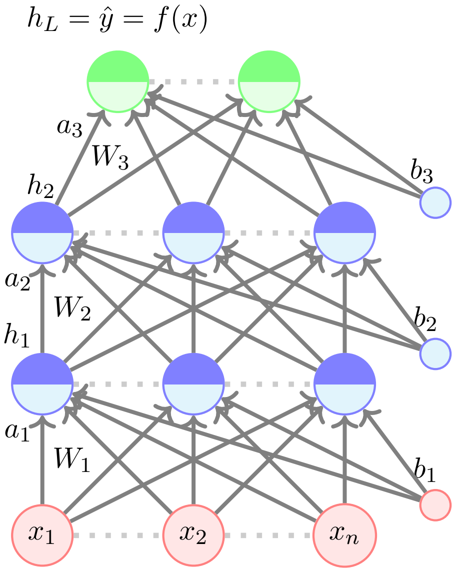
- To simplify notation we will refer to $a_i(x)$ as $a_i$ and $h_i(x)$ as $h_i$

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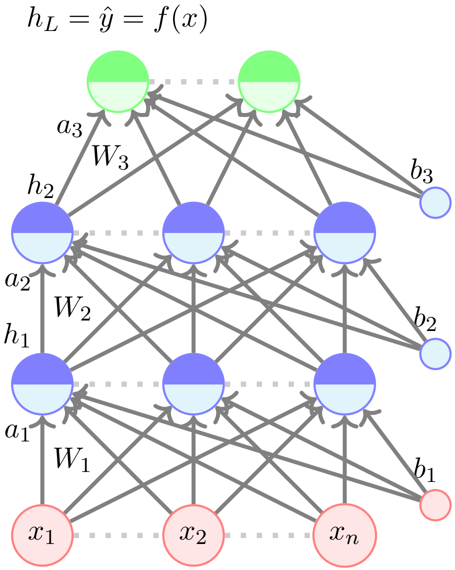
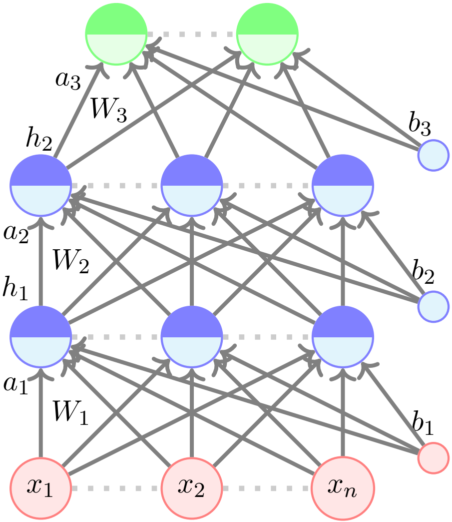


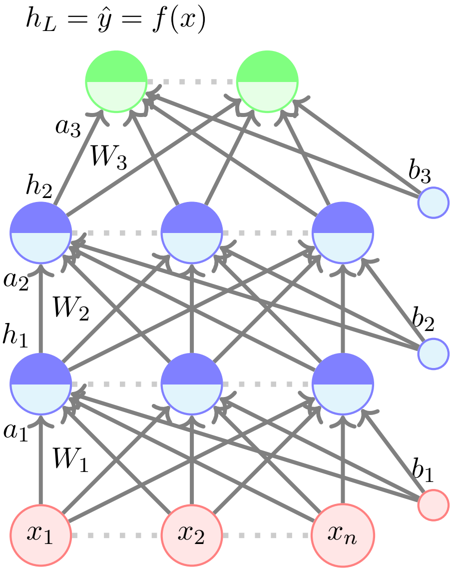

- **Data:** $\{x_i, y_i\}_{i=1}^N$

$h_L = \hat{y} = f(x)$




- **Data:** $\{x_i, y_i\}_{i=1}^N$
- **Model:**

$$\hat{y}_i = f(x_i) = O(W^3 g(W^2 g(W^1 x + b_1) + b_2) + b_3)$$

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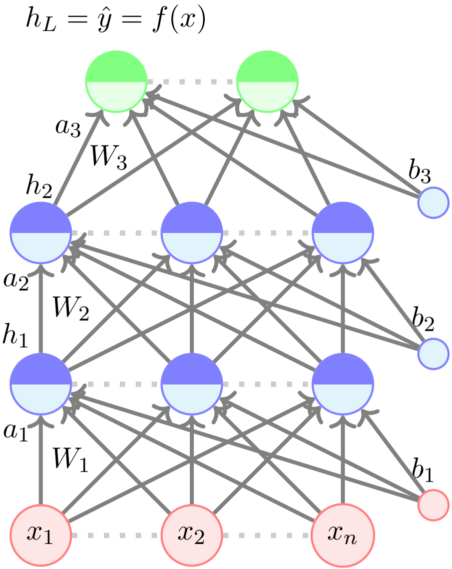
  $\theta = W_1, .., W_L, b_1, b_2, ..., b_L (L = 3)$

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- **Algorithm:** Backpropagation

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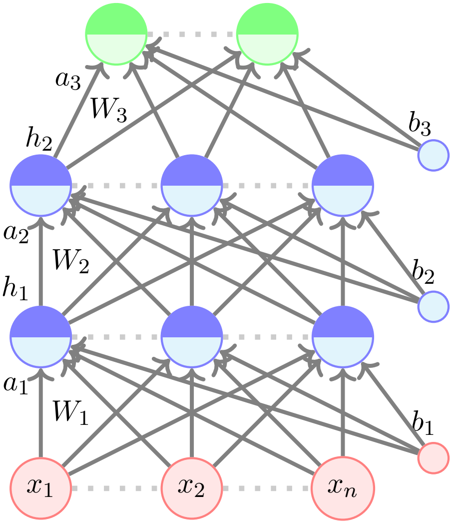


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- **Algorithm:** Backpropagation
- **Objective/Loss/Error function:** Say,

$$min \ \frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$$

$$\textit{In general, } min \ \mathscr{L}(\theta)$$

where $\mathscr{L}(\theta)$ is some function of the parameters