Module 3.1: Sigmoid Neuron

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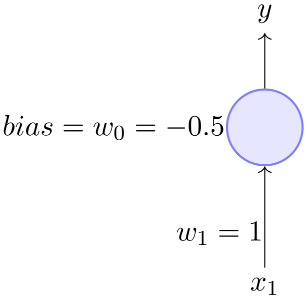
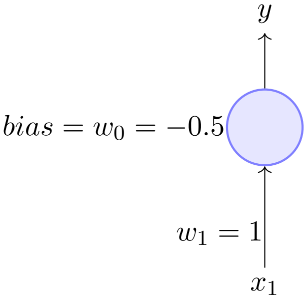
- Before answering the above question we will have to first graduate from ***perceptrons*** to ***sigmoidal neurons*** ...

Recall

- A perceptron will fire if the weighted sum of its inputs is greater than the threshold (-w_0)



- The thresholding logic used by a perceptron is very harsh !



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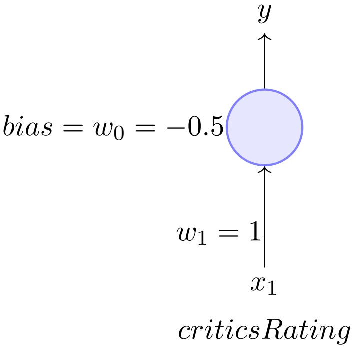

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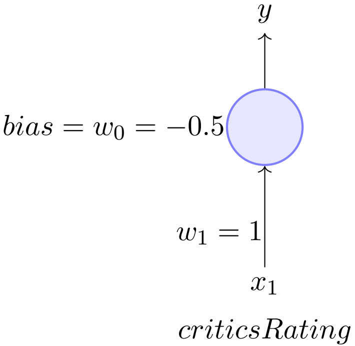

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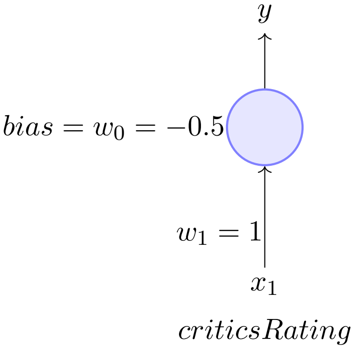

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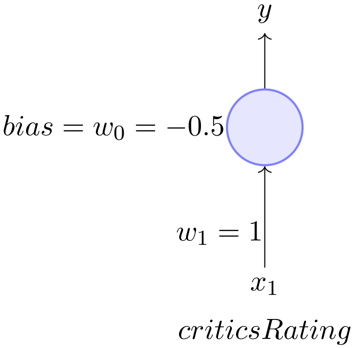

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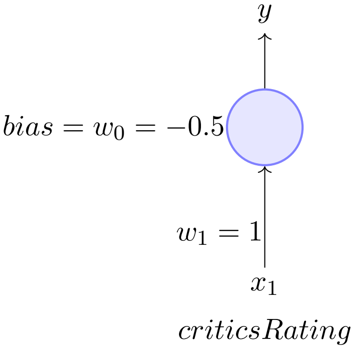

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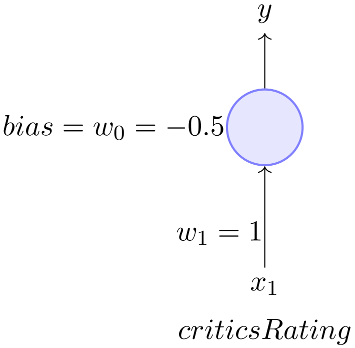

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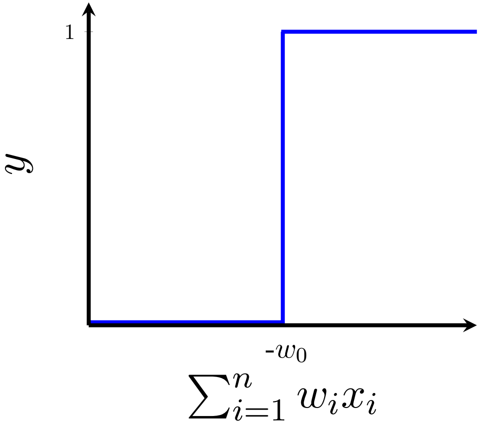
- What about a movie with $criticsRating = 0.49$? (dislike)
- It seems harsh that we would like a movie with rating 0.51 but not one with a rating of 0.49

- This behavior is not a characteristic of the specific problem we chose or the specific weight and threshold that we chose



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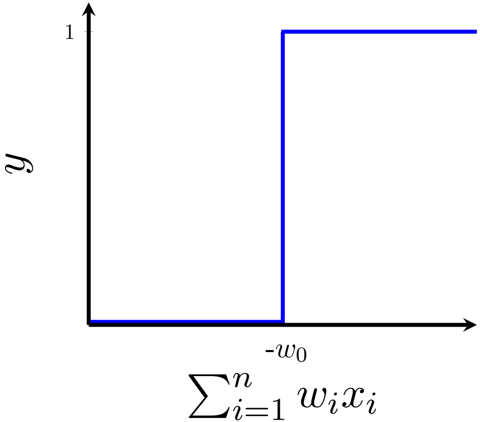

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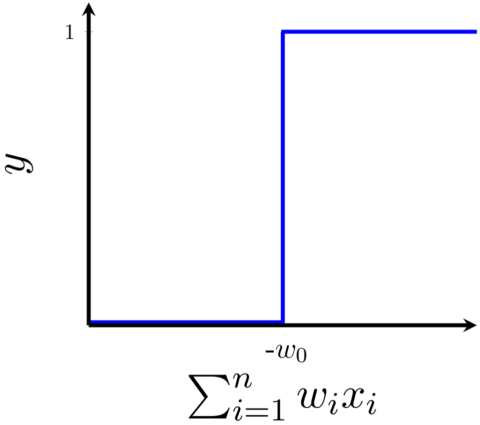

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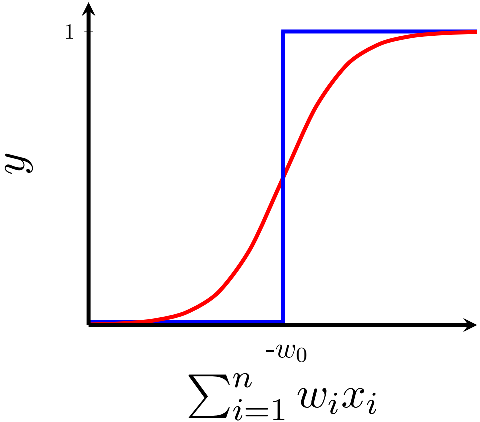
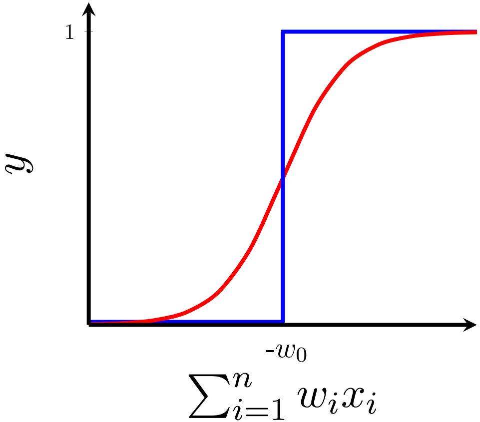
- There will always be this sudden change in the decision (from 0 to 1) when $\sum_{i=1}^{n} w_i x_i$ crosses the threshold ($-w_0$)
- For most real world applications we would expect a smoother decision function which gradually changes from 0 to 1

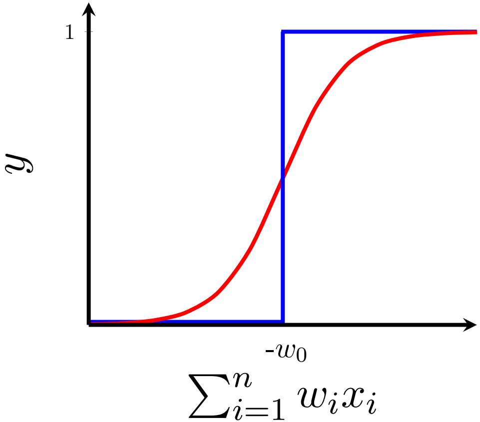


- Introducing sigmoid neurons where the output function is much smoother than the step function



- Introducing sigmoid neurons where the output function is much smoother than the step function
- Here is one form of the sigmoid function called the logistic function

$$y = \frac{1}{1 + e^{-(w_0 + \sum_{i=1}^{n} w_i x_i)}}$$



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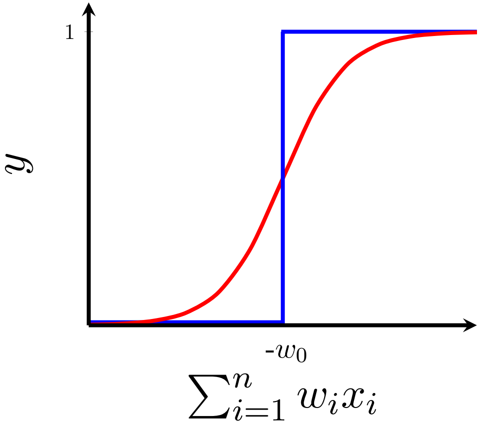

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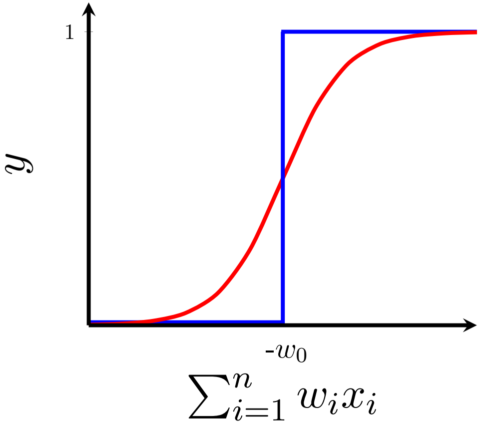

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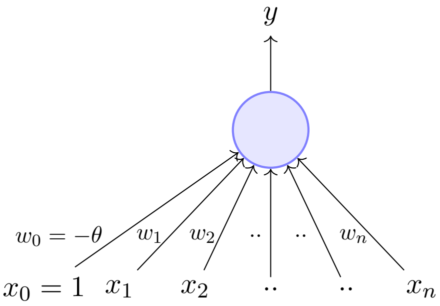
- Instead of a like/dislike decision we get the probability of liking the movie

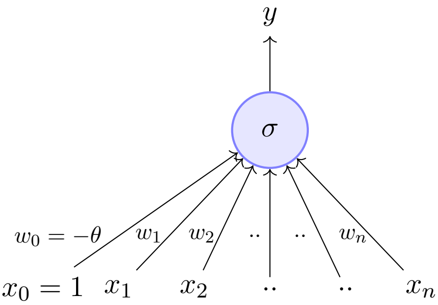
Perceptron



$$y = 1 \quad if \sum_{i=0}^{n} w_i * x_i \geq 0$$

$$= 0 \quad if \sum_{i=0}^{n} w_i * x_i < 0$$

Sigmoid (logistic) Neuron



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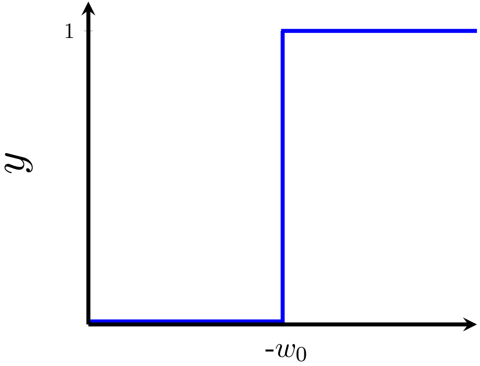

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Not smooth, not continuous (at $w0$), **not differentiable**

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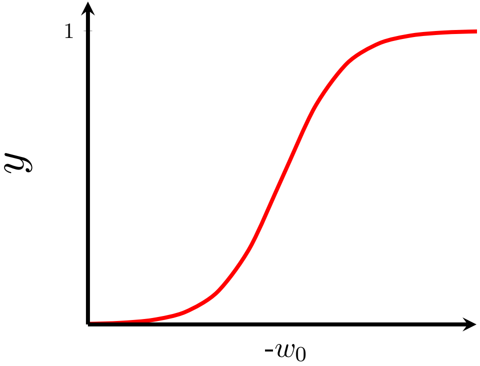

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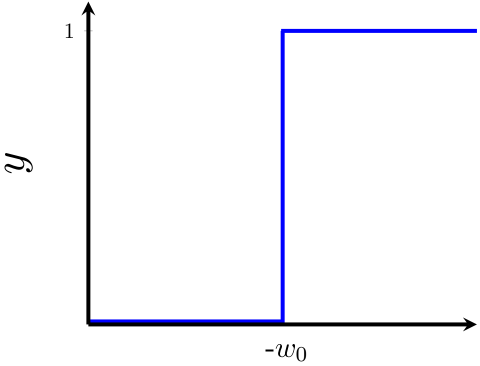

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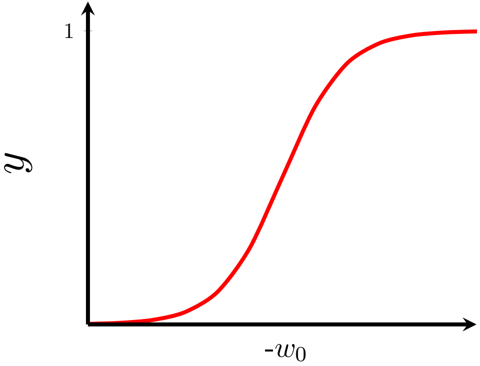

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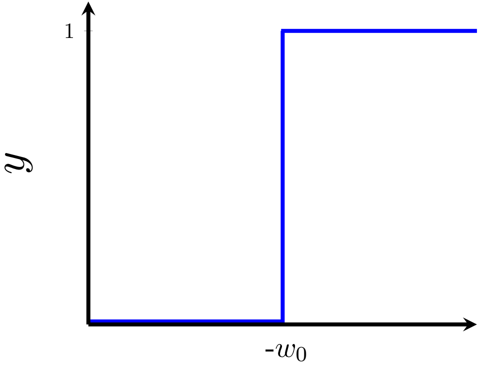

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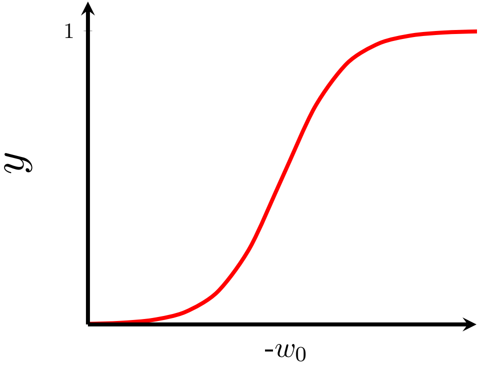

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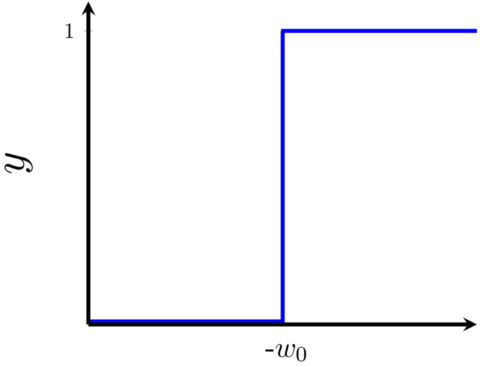

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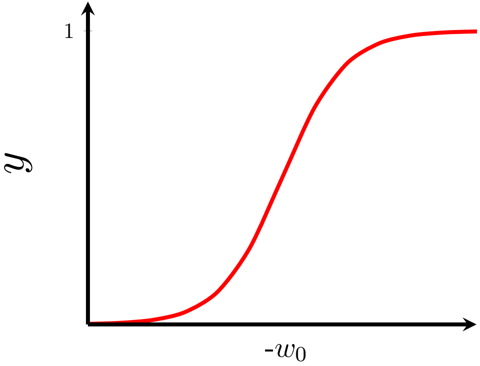

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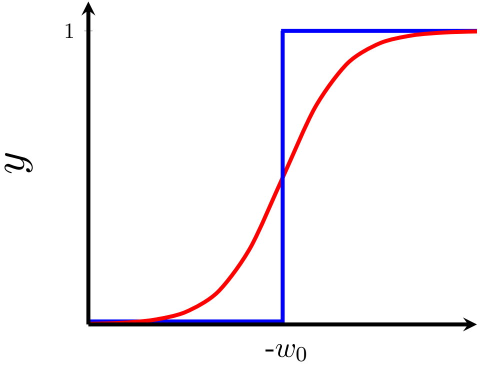

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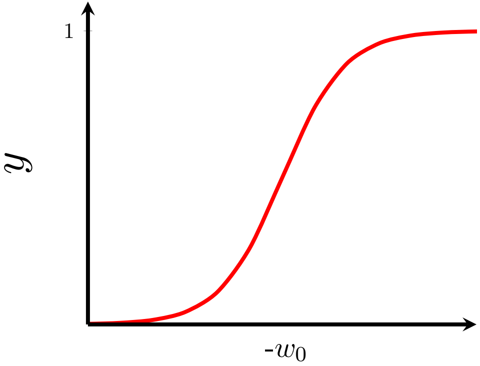

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