Module 8.7 : Adding Noise to the inputs

Other forms of regularization

- l_2 regularization
- Dataset augmentation
- Parameter Sharing and tying
- Adding Noise to the inputs
- Adding Noise to the outputs
- Early stopping
- Ensemble methods
- Dropout

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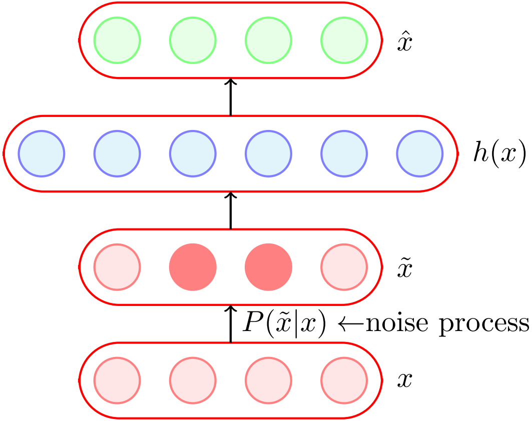
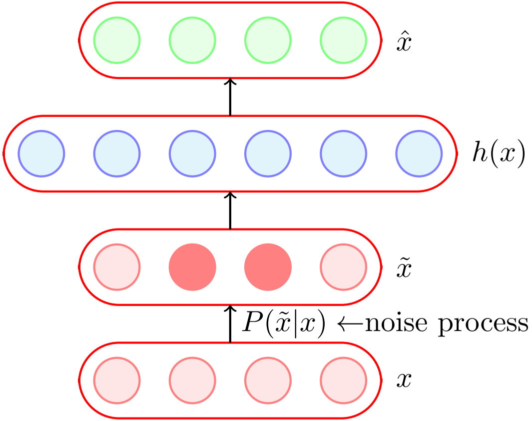
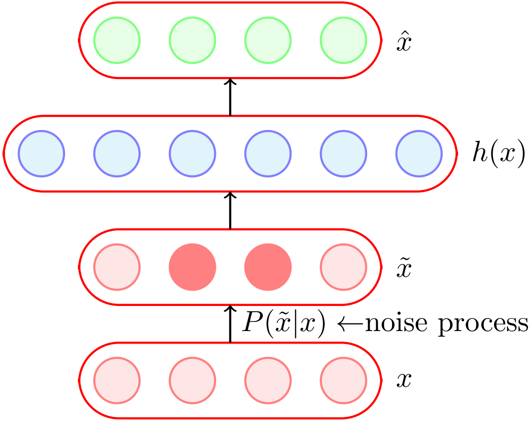

$\hat{x}$
$h(x)$
$\tilde{x}$
$P(\tilde{x}|x)$ ←noise process
x

- We saw this in Autoencoder





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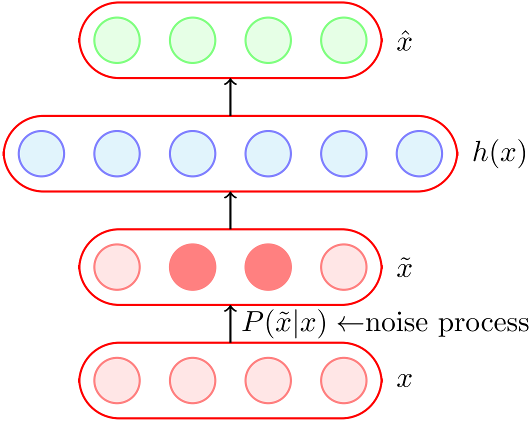
- We can show that for a simple input output neural network, adding Gaussian noise to the input is equivalent to weight decay (L_2 regularisation)



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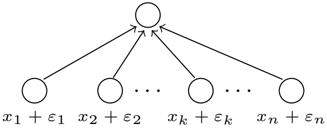
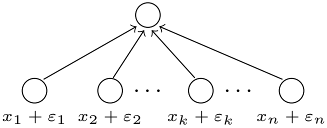
- We can show that for a simple input output neural network, adding Gaussian noise to the input is equivalent to weight decay (L_2 regularisation)
- Can be viewed as data augmentation



$$\varepsilon \sim \mathcal{N}(0, \sigma^2)$$



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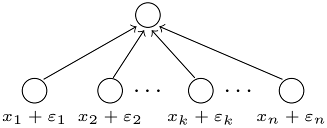
$$\widetilde{x}_i = x_i + \varepsilon_i$$



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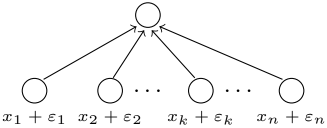

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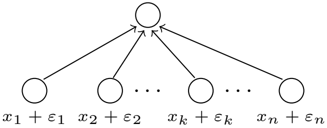

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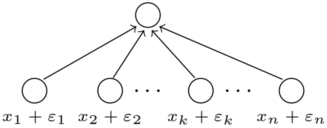
\end{aligned}$$



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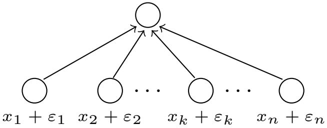

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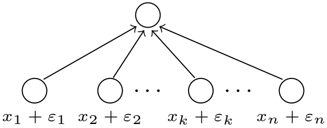
We are interested in $E[(\widetilde{y} - y)^2]$



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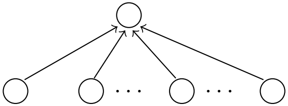

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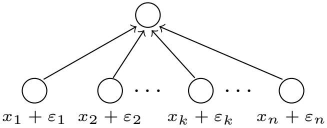

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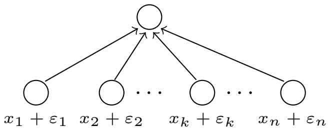

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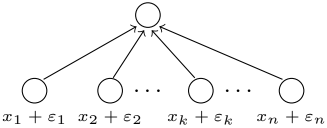
$$= E\left[(\widehat{y} - y)^2\right] + 0 + E\left[\sum_{i=1}^{n} w_i^2 \varepsilon_i^2\right]$$

($\because$ ε_i is independent of ε_j and ε_i is independent of ($\widehat{y}$-y))



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$$= (E\left[(\widehat{y} - y)^2\right] + \sigma^2 \sum_{i=1}^{n} w_i^2$$

(same as L_2 norm penalty)