Module 8.9 : Early stopping

Other forms of regularization

- l_2 regularization
- Dataset augmentation
- Parameter Sharing and tying
- Adding Noise to the inputs
- Adding Noise to the outputs
- Early stopping
- Ensemble methods
- Dropout

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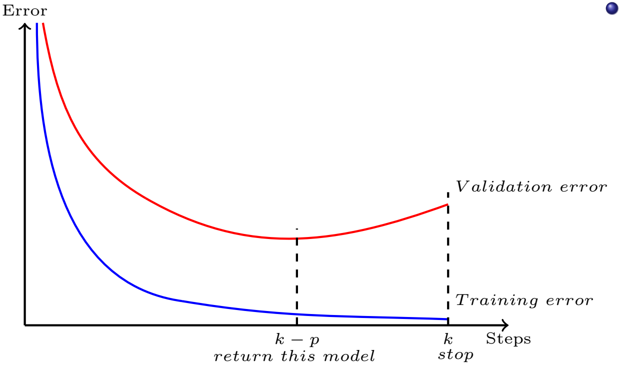
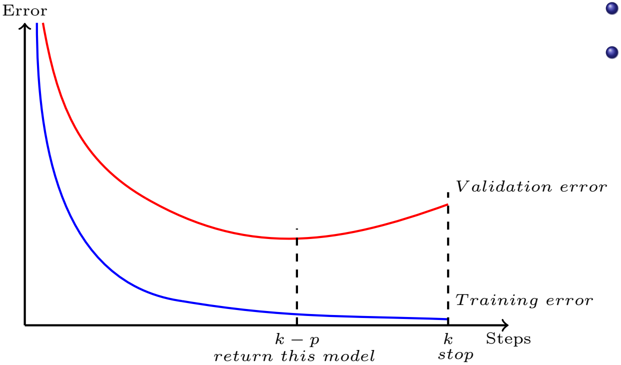


- Track the validation error



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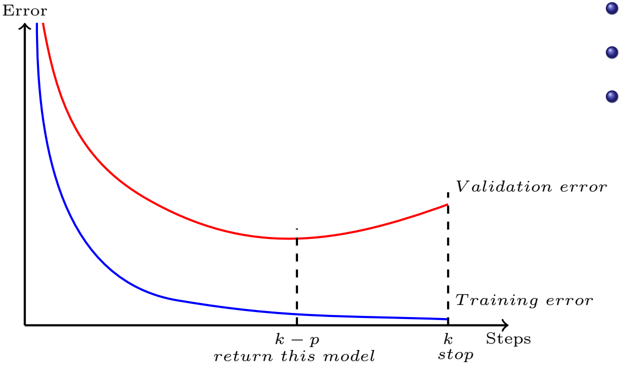

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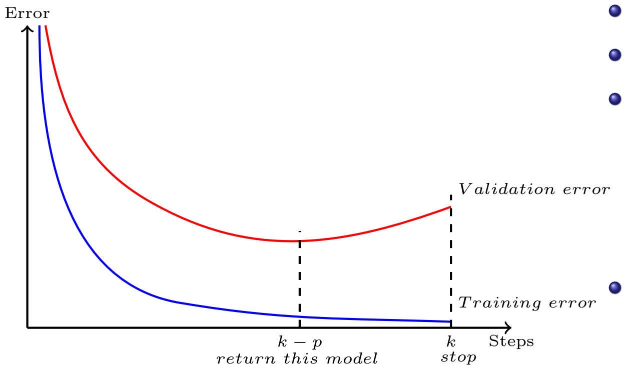

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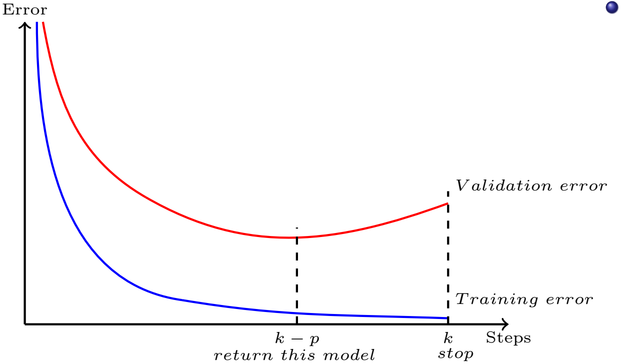
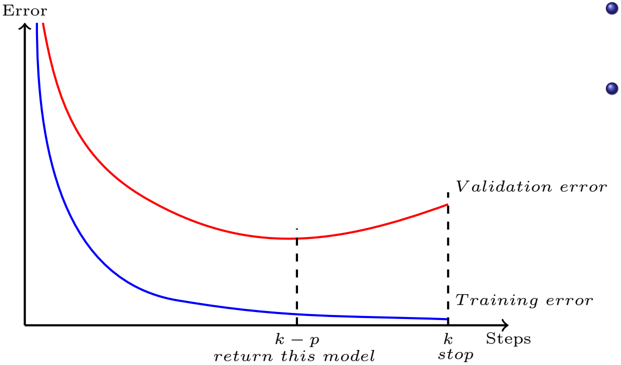
- Have a patience parameter p
- If you are at step k and there was no improvement in validation error in the previous p steps then stop training and return the model stored at step $k - p$
- Basically, stop the training early before it drives the training error to 0 and blows up the validation error



- Very effective and the mostly widely used form of regularization



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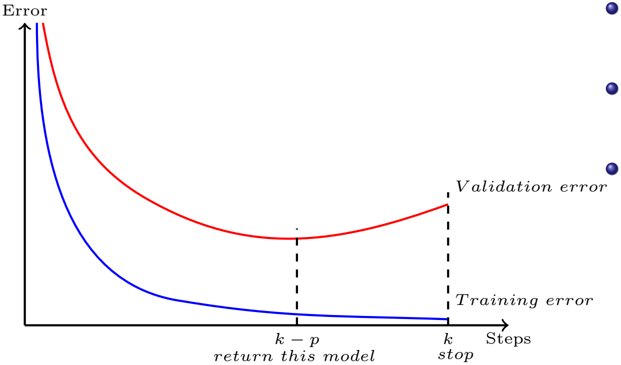

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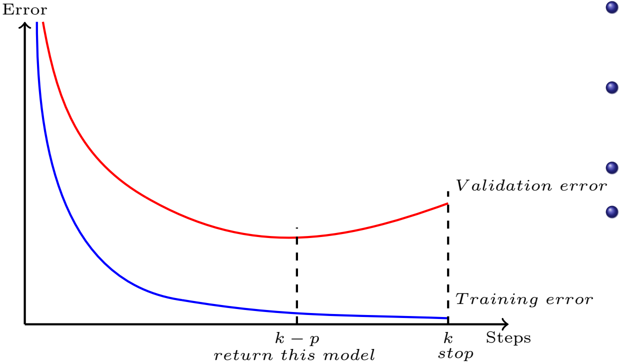

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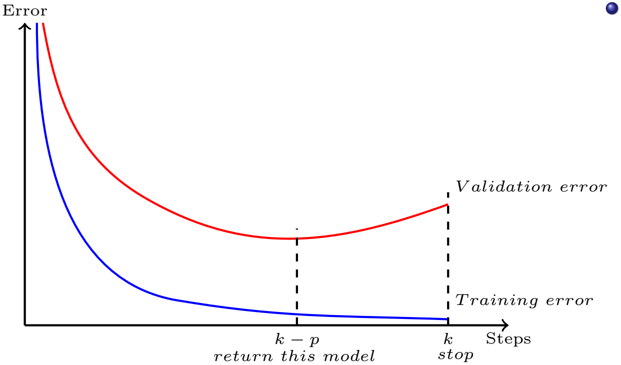
- How does it act as a regularizer ?
- We will first see an intuitive explanation and then a mathematical analysis



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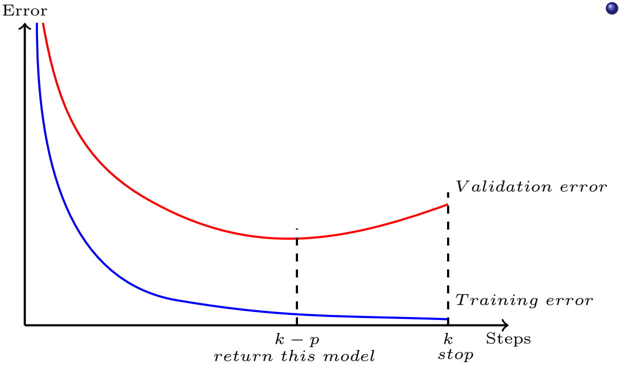

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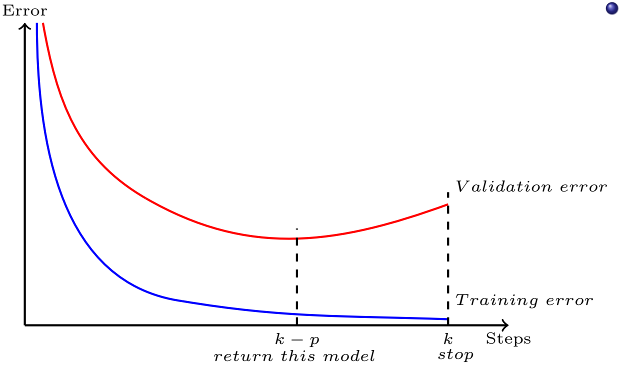

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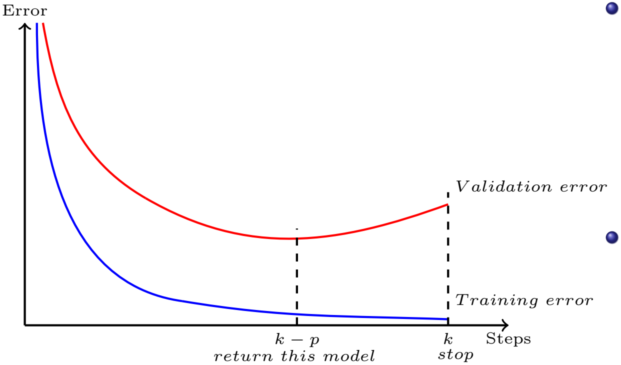

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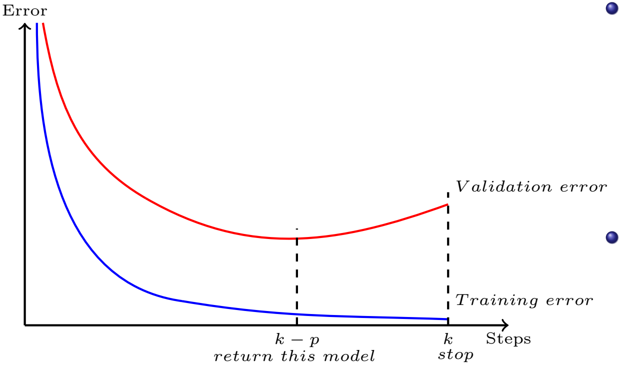

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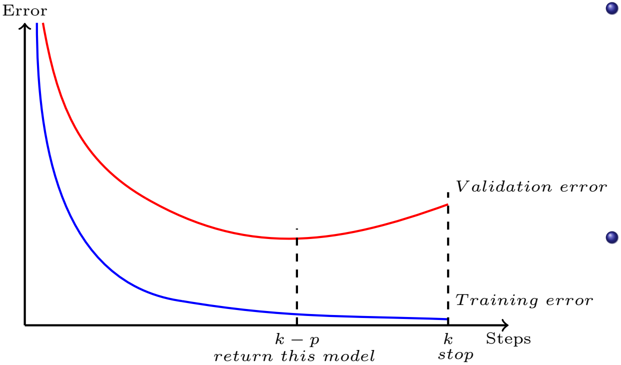

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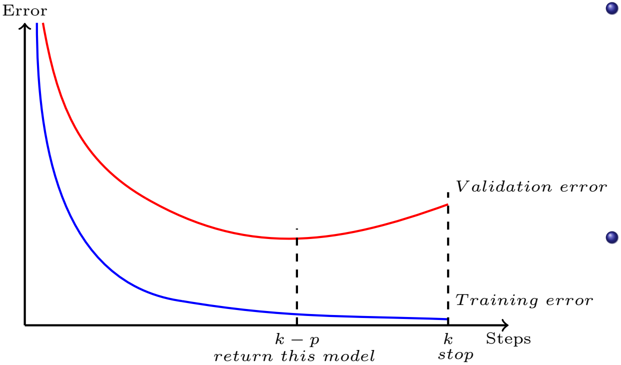

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- In other words it controls the space of exploration

We will now see a mathematical analysis of this

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- We observe that $\omega_t = \tilde{\omega}$, if we choose ε,t and α such that

$$(I - \varepsilon\Lambda)^t = (\Lambda + \alpha I)^{-1}\alpha$$

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- However if a parameter is not important ($\frac{\partial \mathscr{L}(\theta)}{\partial \omega}$ is small) then its updates will be small and the parameter will not be able to grow large in 't' steps
- Early stopping will thus effectively shrink the parameters corresponding to less important directions (same as weight decay).