Module 8.9 : Early stopping

Other forms of regularization

- L_2 regularization
- Dataset augmentation
- Parameter Sharing and tying
- Adding Noise to the inputs
- Adding Noise to the outputs
- Early stopping
- Ensemble methods
- Dropout

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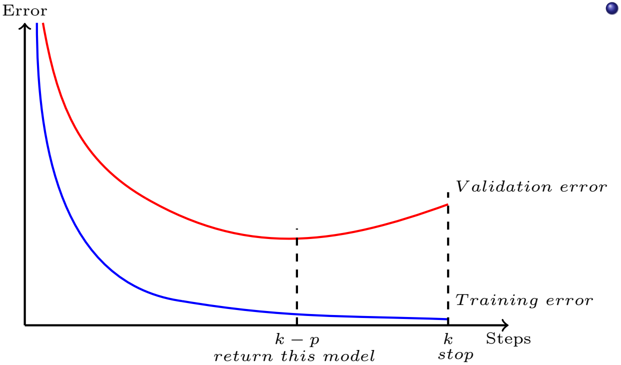
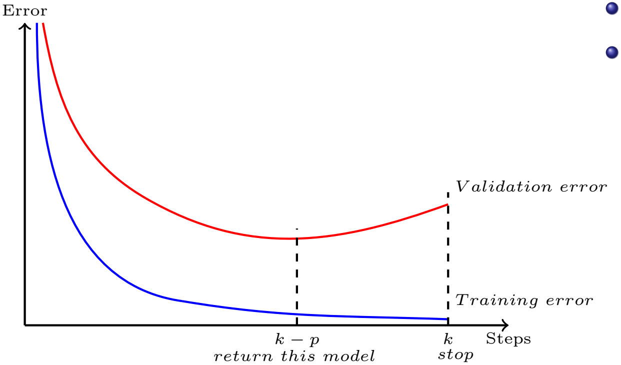
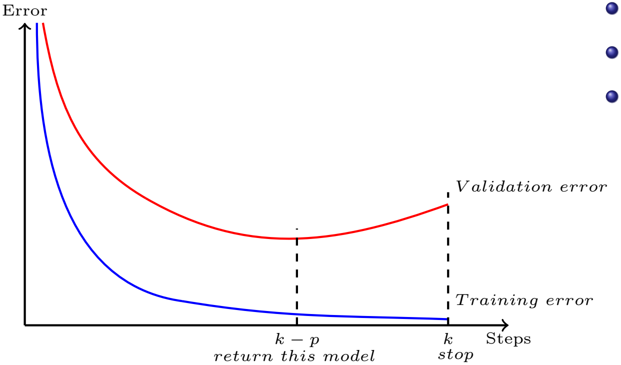


- Track the validation error



- Track the validation error
- Have a patience parameter p



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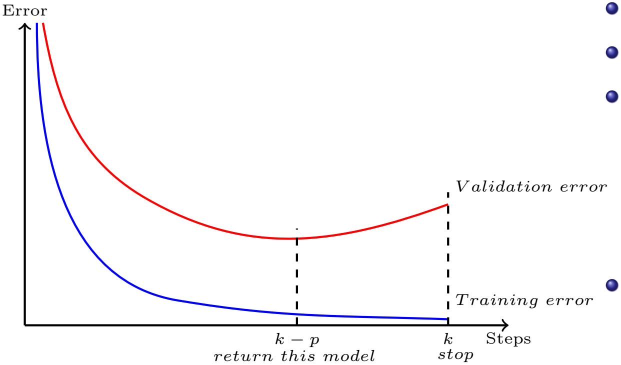
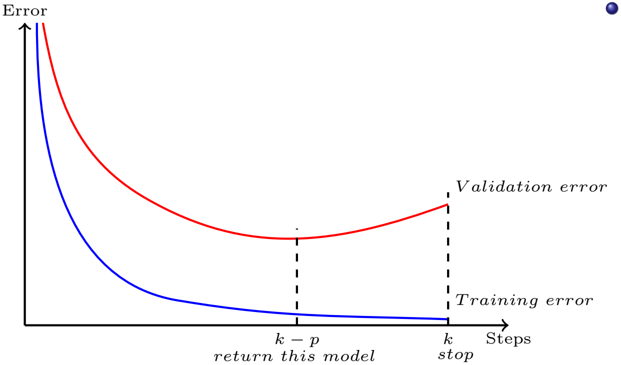
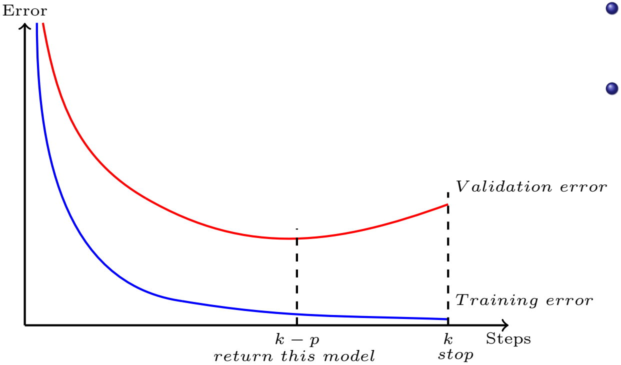
- Have a patience parameter p
- If you are at step k and there was no improvement in validation error in the previous p steps then stop training and return the model stored at step $k-p$



- Track the validation error
- Have a patience parameter p
- If you are at step k and there was no improvement in validation error in the previous p steps then stop training and return the model stored at step $k - p$
- Basically, stop the training early before it drives the training error to 0 and blows up the validation error



- Very effective and the mostly widely used form of regularization



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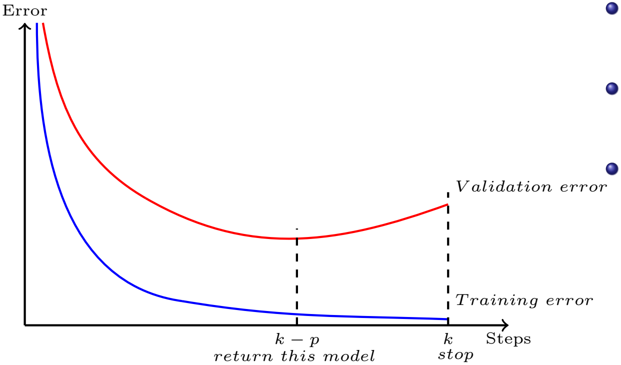

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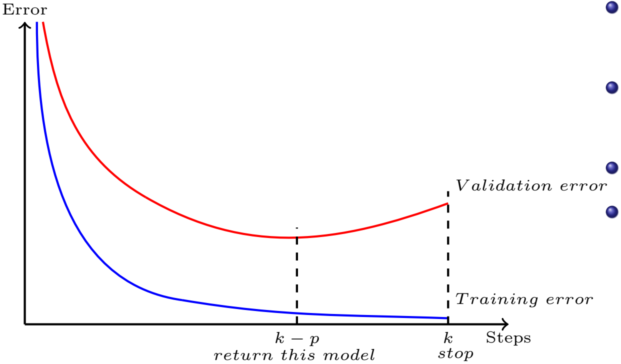
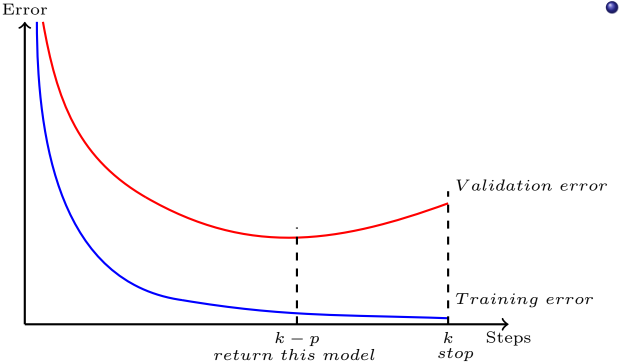
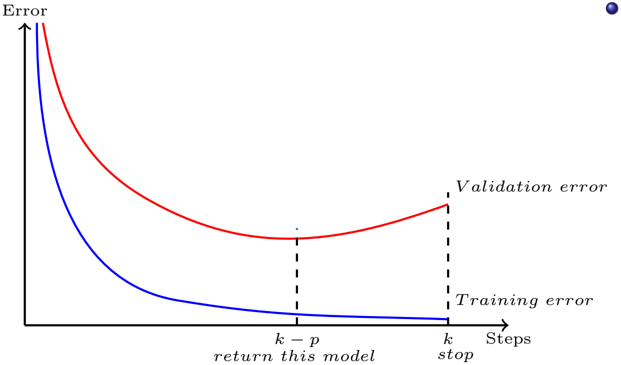
- How does it act as a regularizer ?



- Very effective and the mostly widely used form of regularization
- Can be used even with other regularizers (such as L_2)
- How does it act as a regularizer ?
- We will first see an intuitive explanation and then a mathematical analysis



- Recall that the update rule in SGD is :-



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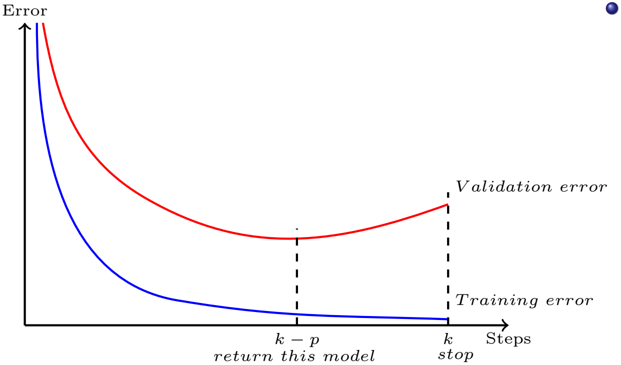

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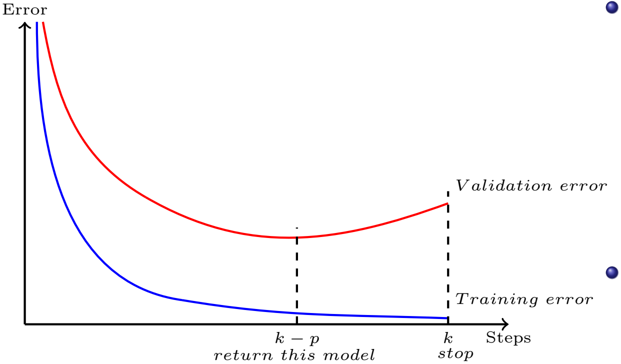

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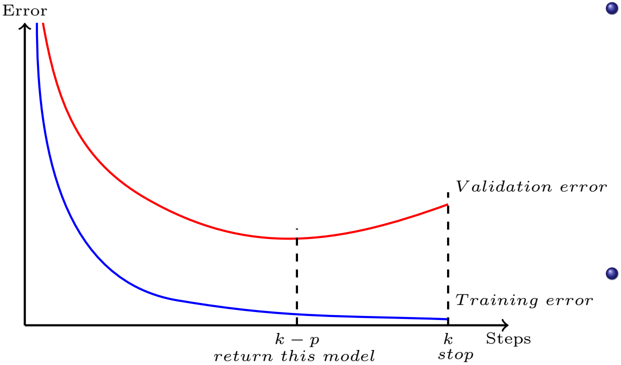
- Let τ be the maximum value of $\nabla \omega_i$ then



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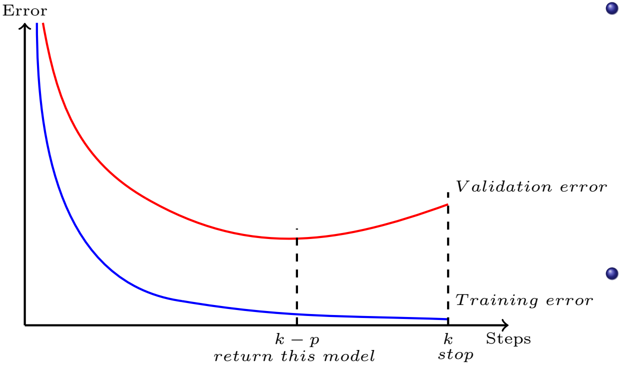

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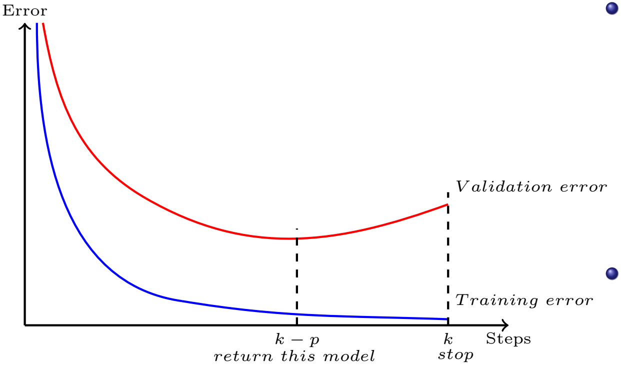

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- Thus, t controls how far ω_t can go from the initial ω_0
- In other words it controls the space of exploration

We will now see a mathematical analysis of this

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- Let us see the derivation

- To prove: The below two equations are equivalent

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- ω_1 according to the first equation:

$$\begin{aligned}\omega_1 &= (I - \eta Q \Lambda Q^T)\omega_0 + \eta Q \Lambda Q^T \omega^* \\ &= \eta Q \Lambda Q^T \omega^*\end{aligned}$$

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- ω_1 according to the second equation:

$$\begin{aligned}\omega_1 &= Q(I - (I - \eta\Lambda)^1)Q^T w^* \\ &= \eta Q\Lambda Q^T w^*\end{aligned}$$

- Induction step: Let the two equations be equivalent for t^{th} step

$$\therefore \omega_t = (I + \eta Q \Lambda Q^T)\omega_{t-1} - \eta Q \Lambda Q^T \omega^*$$
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(using $\omega_t = Q[I - (I - \varepsilon\Lambda)^t]Q^T\omega^*$)

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(Opening this bracket)

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$$(\text{Opening this bracket})$$
$$= {\color{red}I} Q (I - (I - \eta \Lambda)^t) Q^T \omega^* - {\color{red}\eta Q \Lambda Q^T} Q (I - (I - \eta \Lambda)^t) Q^T \omega^* + \eta Q \Lambda Q^T \omega^*$$

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$$\omega_{t+1} = Q(I - (I - \eta\Lambda)^t)Q^T\omega^* - \eta Q\Lambda Q^T Q(I - (I - \eta\Lambda)^t)Q^T\omega^* + \eta Q\Lambda Q^T\omega^*$$

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&= Q\left[(I-(I-\eta\Lambda)^t) - \eta\Lambda(I-(I-\eta\Lambda)^t) + \eta\Lambda\right]Q^T\omega^*
\end{aligned}
$$

- Continuing

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\begin{aligned}
\omega_{t+1} &= Q(I - (I - \eta\Lambda)^t)Q^T\omega^* - \eta Q\Lambda Q^T Q(I - (I - \eta\Lambda)^t)Q^T\omega^* + \eta Q\Lambda Q^T\omega^* \\
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&= Q(I - (I - \eta\Lambda)^t)Q^T\omega^* - \eta Q\Lambda(I - (I - \eta\Lambda)^t)Q^T\omega^* + \eta Q\Lambda Q^T\omega^* \\
&= Q\big[(I - (I - \eta\Lambda)^t) - \eta\Lambda(I - (I - \eta\Lambda)^t) + \eta\Lambda\big]Q^T\omega^* \\
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\end{aligned}
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&= Q\Big[I-(I-\eta\Lambda)^t(I-\eta\Lambda)\Big]Q^T\omega^* \\
&= Q(I-(I-\eta\Lambda)^{t+1})Q^T\omega^*
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\omega_{t+1} &= Q(I-(I-\eta\Lambda)^t)Q^T\omega^* - \eta Q\Lambda Q^T Q(I-(I-\eta\Lambda)^t)Q^T\omega^* + \eta Q\Lambda Q^T\omega^* \\
&= Q(I-(I-\eta\Lambda)^t)Q^T\omega^* - \eta Q\Lambda(I-(I-\eta\Lambda)^t)Q^T\omega^* + \eta Q\Lambda Q^T\omega^* (\because Q^TQ = I) \\
&= Q(I-(I-\eta\Lambda)^t)Q^T\omega^* - \eta Q\Lambda(I-(I-\eta\Lambda)^t)Q^T\omega^* + \eta Q\Lambda Q^T\omega^* \\
&= Q\big[(I-(I-\eta\Lambda)^t) - \eta\Lambda(I-(I-\eta\Lambda)^t) + \eta\Lambda\big]Q^T\omega^* \\
&= Q\big[I-(I-\eta\Lambda)^t + \eta\Lambda(I-\eta\Lambda)^t\big]Q^T\omega^* \\
&= Q\big[I-(I-\eta\Lambda)^t(I-\eta\Lambda)\big]Q^T\omega^* \\
&= Q(I-(I-\eta\Lambda)^{t+1})Q^T\omega^*
\end{aligned}
$$

Hence, proved!

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- We observe that $\omega_t = \tilde{\omega}$, if we choose ε,t and α such that

$$(I - \varepsilon\Lambda)^t = (\Lambda + \alpha I)^{-1}\alpha$$

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- However if a parameter is not important ($\frac{\partial \mathscr{L}(\theta)}{\partial \omega}$ is small) then its updates will be small and the parameter will not be able to grow large in 't' steps
- Early stopping will thus effectively shrink the parameters corresponding to less important directions (same as weight decay).