

Module 3.1: Sigmoid Neuron

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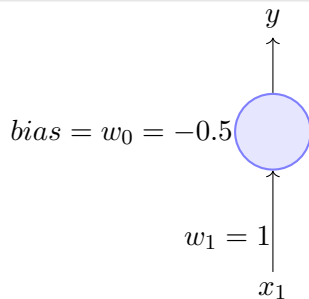
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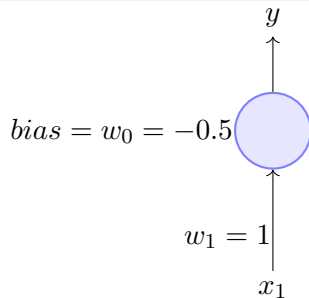
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- Before answering the above question we will have to first graduate from *perceptrons* to *sigmoidal neurons* ...

Recall

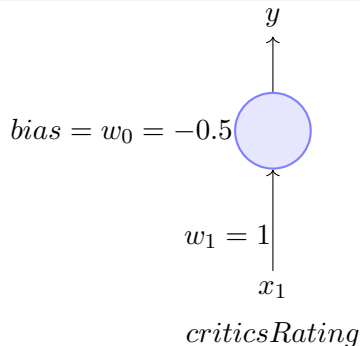
- A perceptron will fire if the weighted sum of its inputs is greater than the threshold ($-w_0$)



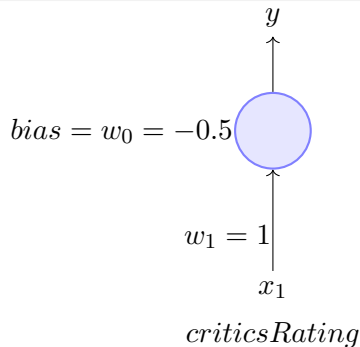
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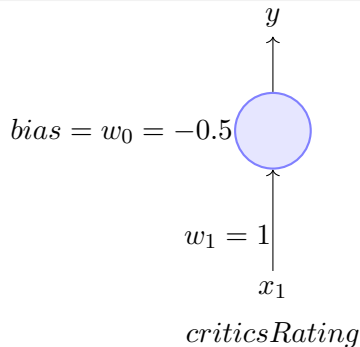
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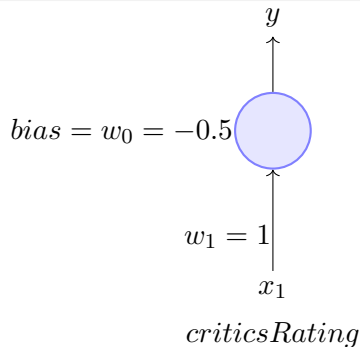
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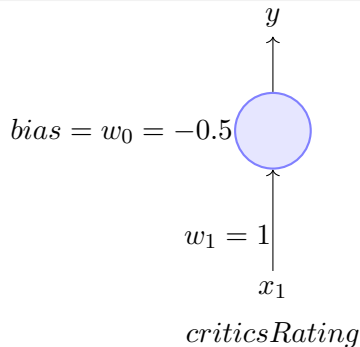
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- If the threshold is 0.5 ($w_0 = -0.5$) and $w_1 = 1$ then what would be the decision for a movie with $criticsRating = 0.51$?



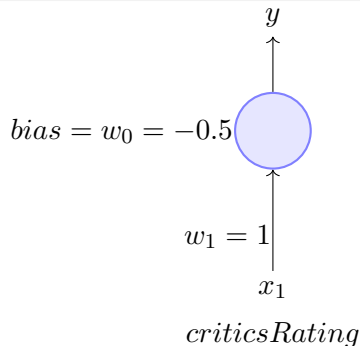
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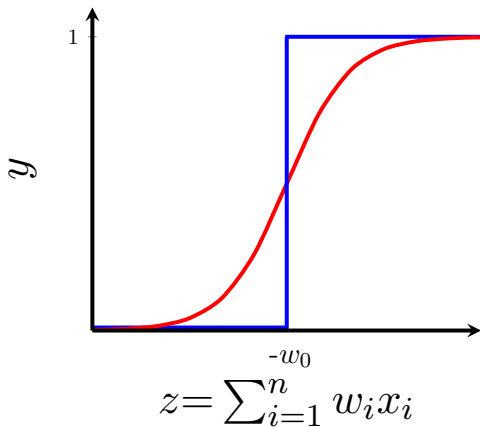


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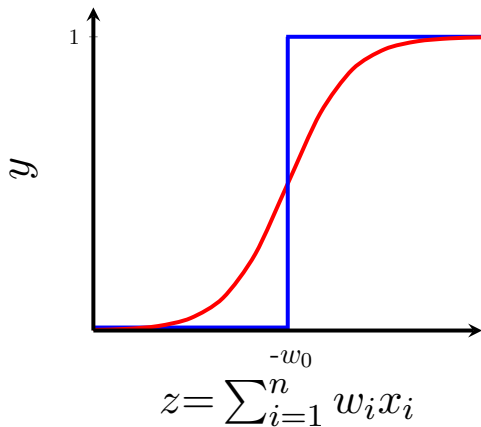


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- It seems harsh that we would like a movie with rating 0.51 but not one with a rating of 0.49

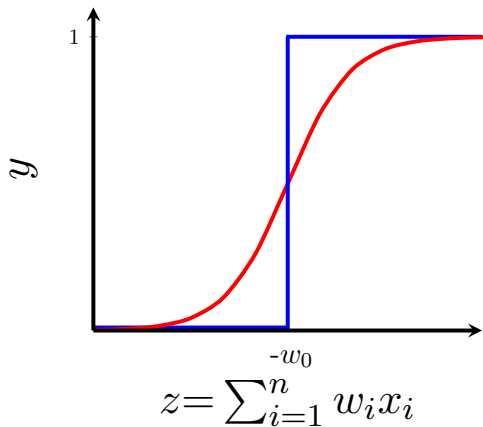
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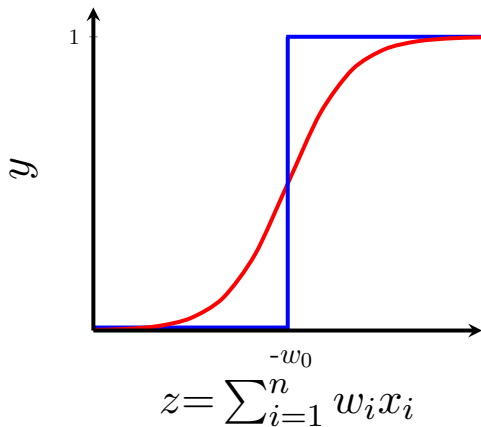


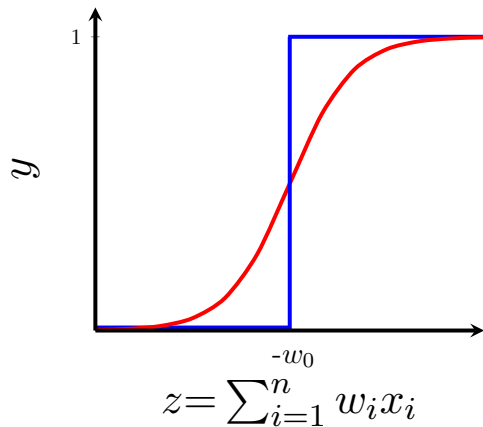
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- There will always be this sudden change in the decision (from 0 to 1) when $\sum_{i=1}^n w_i x_i$ crosses the threshold ($-w_0$)
- For most real world applications we would expect a smoother decision function which gradually changes from 0 to 1

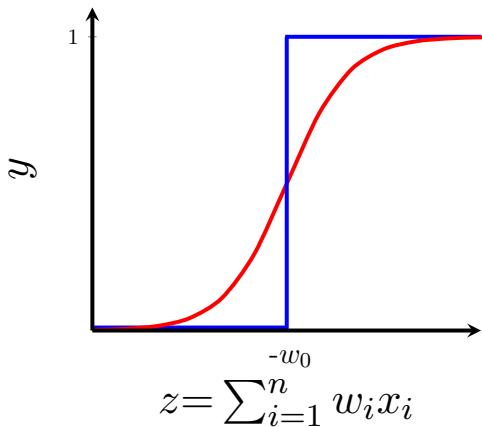
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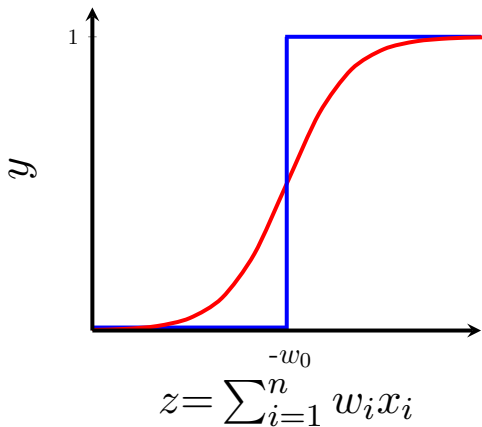
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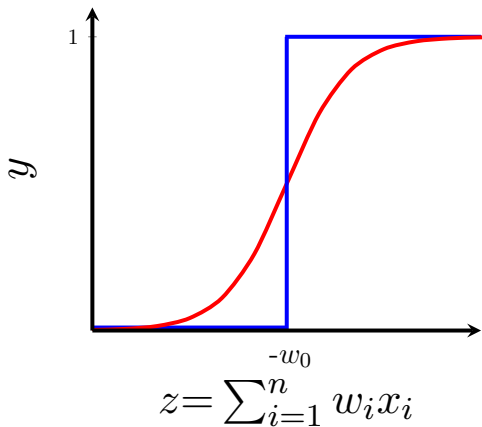
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- Also the output y is no longer binary but a real value between 0 and 1 which can be interpreted as a probability

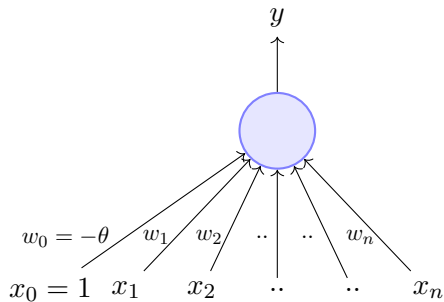


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- Instead of a like/dislike decision we get the probability of liking the movie

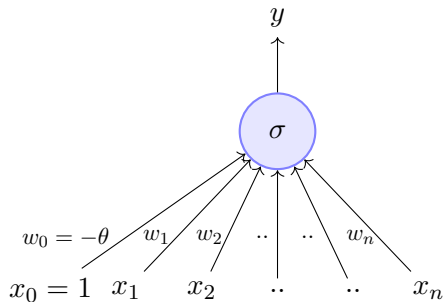
Perceptron



$$y = 1 \quad \text{if} \quad \sum_{i=0}^n w_i * x_i \geq 0$$

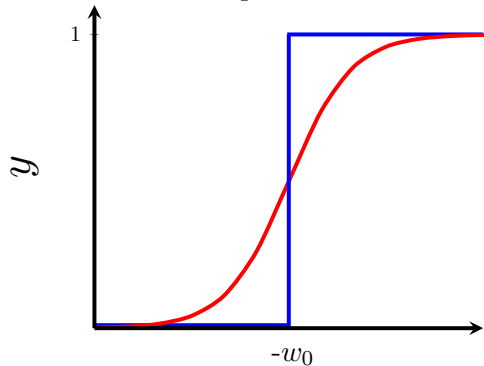
$$= 0 \quad \text{if} \quad \sum_{i=0}^n w_i * x_i < 0$$

Sigmoid (logistic) Neuron



$$y = \frac{1}{1 + e^{-(\sum_{i=0}^n w_i x_i)}}$$

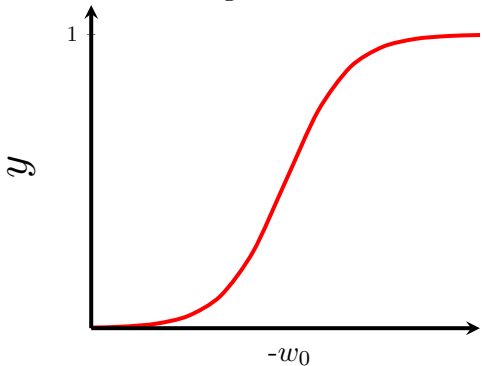
Perceptron



$$z = \sum_{i=1}^n w_i x_i$$

Not smooth, not continuous (at w_0), **not differentiable**

Sigmoid Neuron



$$z = \sum_{i=1}^n w_i x_i$$

Smooth,
continuous, **differentiable**