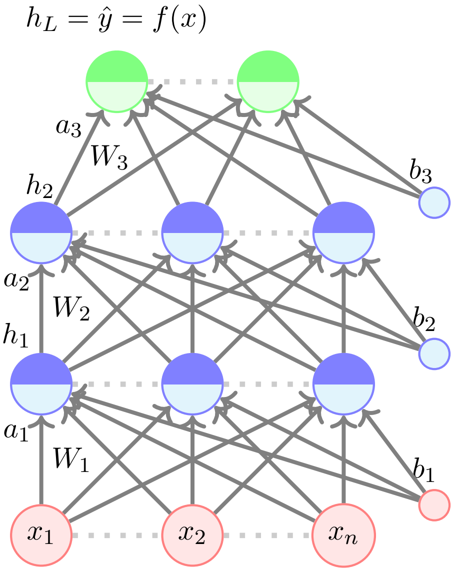
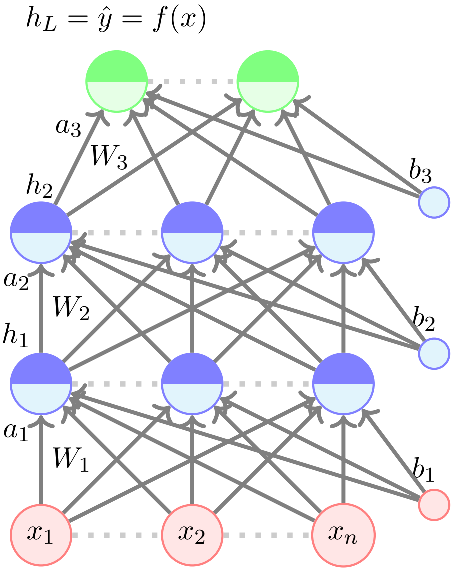
Module 4.2: Learning Parameters of Feedforward Neural Networks (Intuition)

The story so far...

- We have introduced feedforward neural networks
- We are now interested in finding an algorithm for learning the parameters of this model



- Recall our gradient descent algorithm



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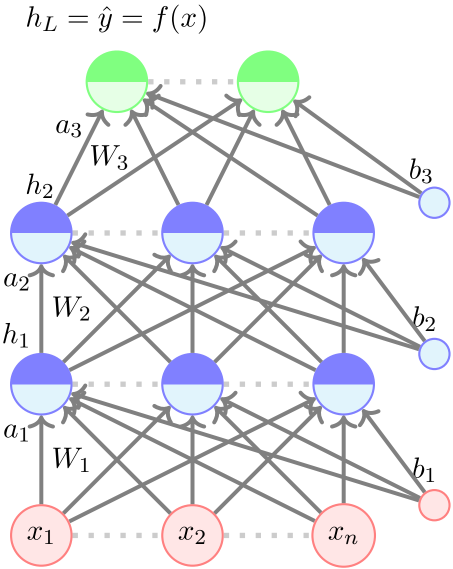
Algorithm: gradient_descent()

$t \leftarrow 0$;
$max\_iterations \leftarrow 1000$;
$Initialize \quad w_0, b_0$;
while $t{+}{+} < max\_iterations$ **do**
  $w_{t+1} \leftarrow w_t - \eta \nabla w_t$;
  $b_{t+1} \leftarrow b_t - \eta \nabla b_t$;
end



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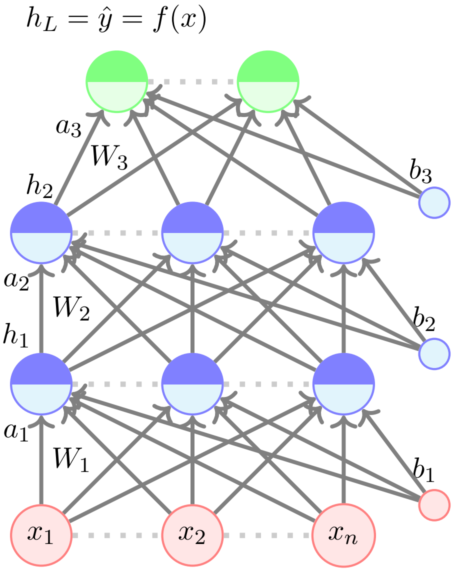

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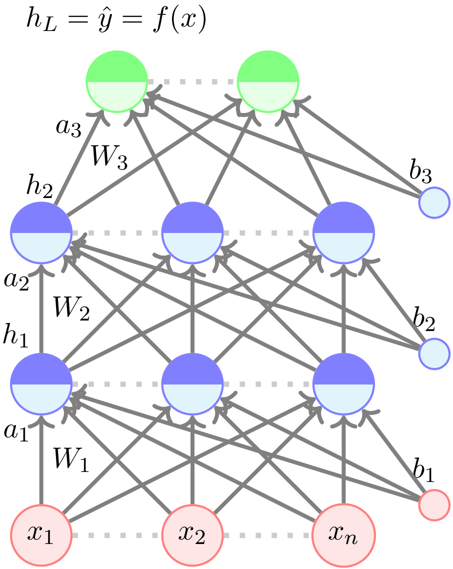
Algorithm: gradient_descent()

$t \leftarrow 0;$
$max\_iterations \leftarrow 1000;$
$Initialize \quad \theta_0 = [w_0, b_0];$
while $t{++} < max\_iterations$ **do**
 $\theta_{t+1} \leftarrow \theta_t - \eta \nabla \theta_t;$
end



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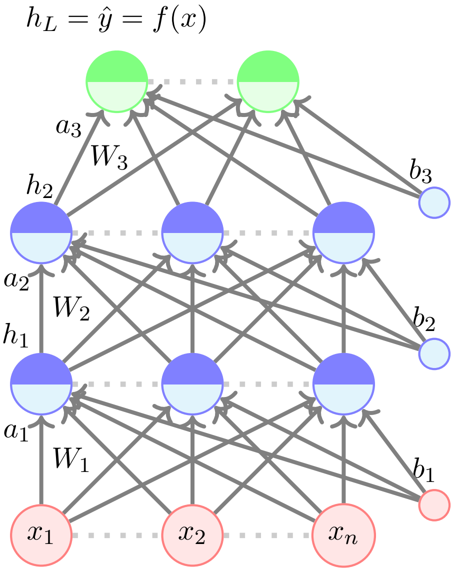
- where $\nabla \theta_t = \left[\frac{\partial \mathscr{L}(\theta)}{\partial w_t}, \frac{\partial \mathscr{L}(\theta)}{\partial b_t}\right]^T$



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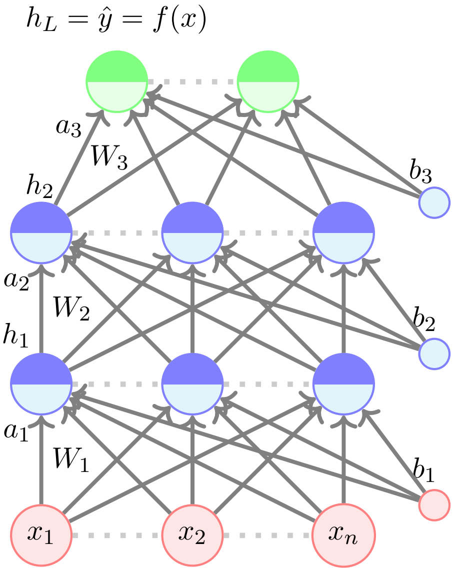
- Now, in this feedforward neural network, instead of $\theta = [w, b]$ we have $\theta = W_1, W_2, .., W_L, b_1, b_2, .., b_L$



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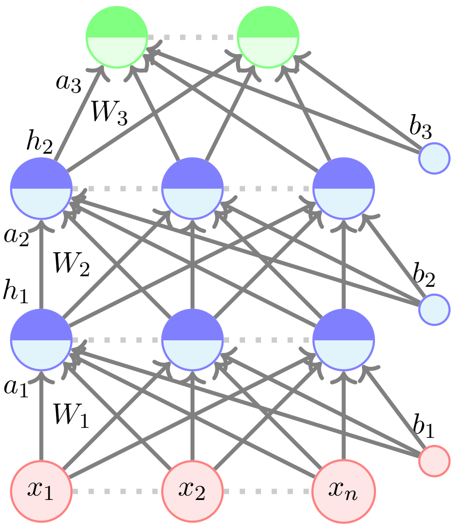
- We can still use the same algorithm for learning the parameters of our model

$h_L = \hat{y} = f(x)$



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Algorithm: gradient_descent()

$t \leftarrow 0$;
$max\_iterations \leftarrow 1000$;
$Initialize \quad \theta_0 = [W_1^0, ..., W_L^0, b_1^0, ..., b_L^0]$;
while $t{+}{+} < max\_iterations$ **do**
  $\theta_{t+1} \leftarrow \theta_t - \eta \nabla \theta_t$;
end

- where $\nabla \theta_t = \left[\frac{\partial \mathscr{L}(\theta)}{\partial w_t}, \frac{\partial \mathscr{L}(\theta)}{\partial b_t}\right]^T$
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$$\begin{bmatrix} \frac{\partial \mathscr{L}(\theta)}{\partial W_{111}} & \cdots & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \end{bmatrix}$$

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$$\begin{bmatrix} \frac{\partial \mathscr{L}(\theta)}{\partial W_{111}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{11n}} \\ & & \\ & & \\ & & \end{bmatrix}$$

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$$\begin{bmatrix} \frac{\partial \mathscr{L}(\theta)}{\partial W_{111}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{11n}} \\ \frac{\partial \mathscr{L}(\theta)}{\partial W_{121}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{12n}} \\ \vdots & \vdots & \vdots \\ \frac{\partial \mathscr{L}(\theta)}{\partial W_{1n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{1nn}} \end{bmatrix}$$

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\frac{\partial \mathscr{L}(\theta)}{\partial W_{111}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{11n}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{211}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{21n}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,11}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,1k}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,1k}} \\
\frac{\partial \mathscr{L}(\theta)}{\partial W_{121}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{12n}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{221}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{22n}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,21}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,2k}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,2k}} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{\partial \mathscr{L}(\theta)}{\partial W_{1n1}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{1nn}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{2n1}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{2nn}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,n1}} & \dots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,nk}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,nk}}
\end{bmatrix}$$

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\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{\partial \mathscr{L}(\theta)}{\partial W_{1n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{1nn}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{2n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{2nn}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,n1}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,nk}} & \frac{\partial \mathscr{L}(\theta)}{\partial W_{L,nk}} & \frac{\partial \mathscr{L}(\theta)}{\partial b_{1n}} & \cdots & \frac{\partial \mathscr{L}(\theta)}{\partial b_{Ln}}
\end{bmatrix}$$

- $\nabla\theta$ is thus composed of
  $\nabla W_1, \nabla W_2, \ldots \nabla W_{L-1} \in \mathbb{R}^{n\times n}, \nabla W_L \in \mathbb{R}^{n\times k}$,
  $\nabla b_1, \nabla b_2, \ldots, \nabla b_{L-1} \in \mathbb{R}^n$ and $\nabla b_L \in \mathbb{R}^k$

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- How to choose the loss function $\mathscr{L}(\theta)$?

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- How to compute $\nabla\theta$ which is composed of $\nabla W_1, \nabla W_2, ..., \nabla W_{L-1} \in \mathbb{R}^{n\times n}, \nabla W_L \in \mathbb{R}^{n\times k}$,
  $\nabla b_1, \nabla b_2, ..., \nabla b_{L-1} \in \mathbb{R}^n$ and $\nabla b_L \in \mathbb{R}^k$?