Module 4.1: Feedforward Neural Networks (a.k.a. multilayered network of neurons)

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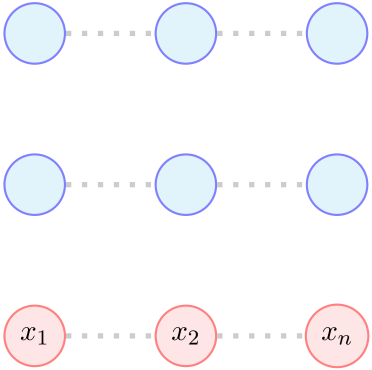

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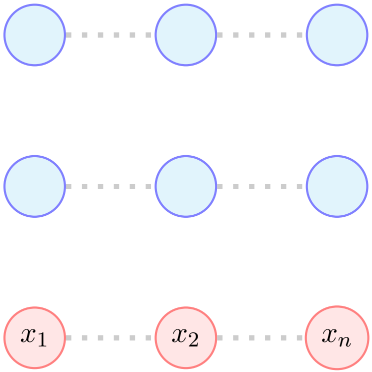

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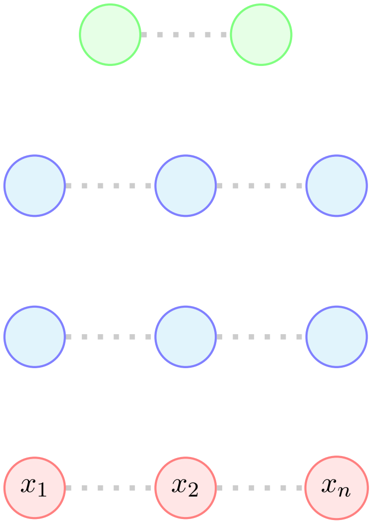

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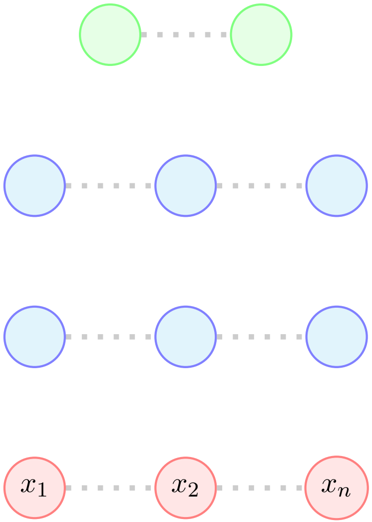

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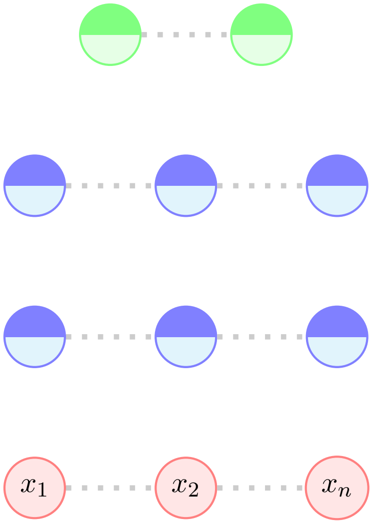

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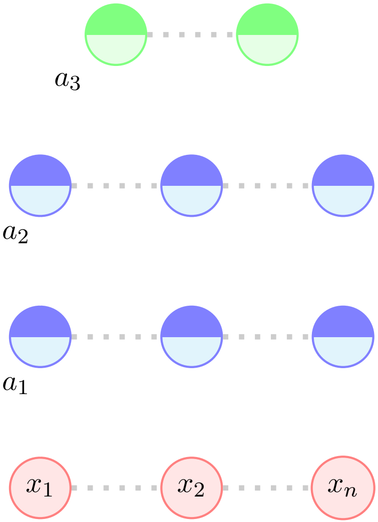

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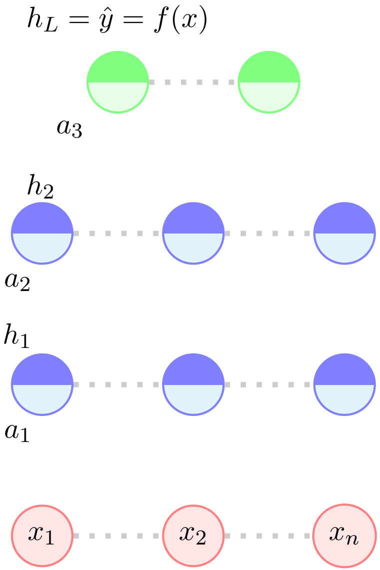

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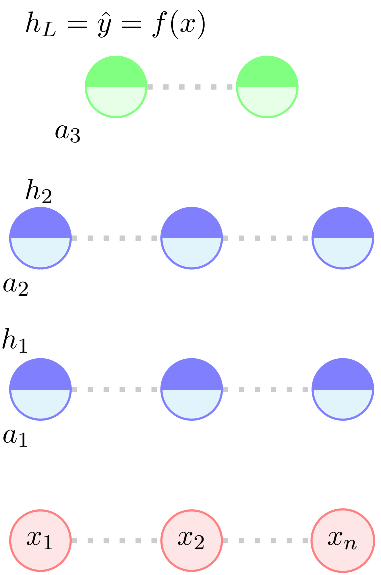

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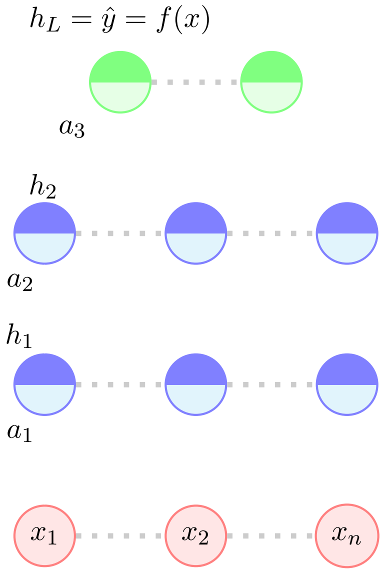

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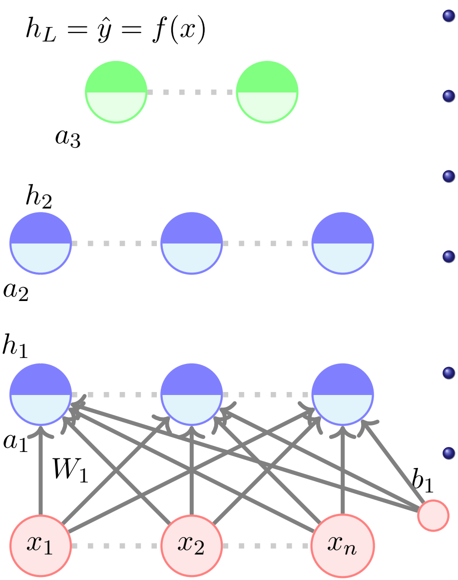

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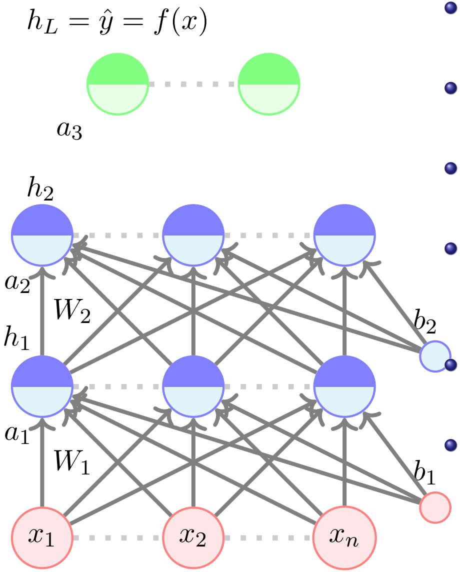

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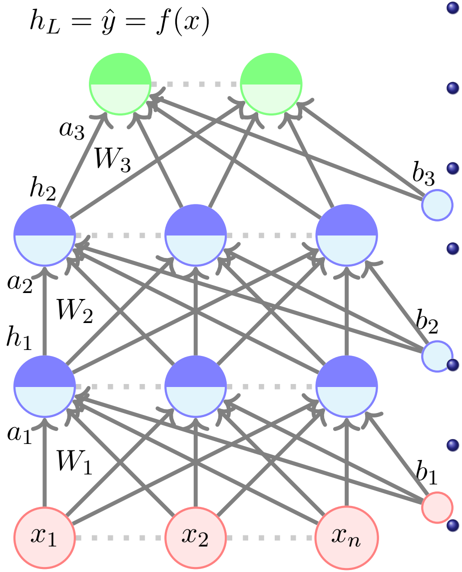

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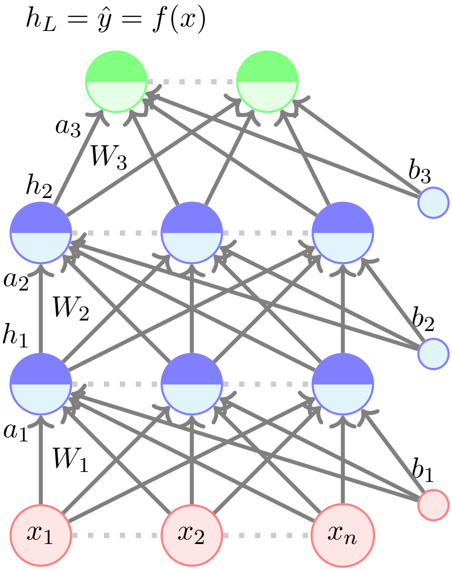
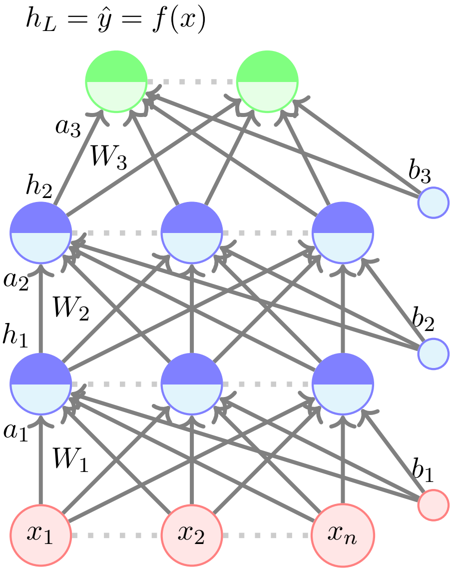
- $W_L \in \mathbb{R}^{n \times k}$ and $b_L \in \mathbb{R}^k$ are the weight and bias between the last hidden layer and the output layer ($L = 3$ in this case)



- The pre-activation at layer i is given by

$$a_i(x) = b_i + W_i h_{i-1}(x)$$



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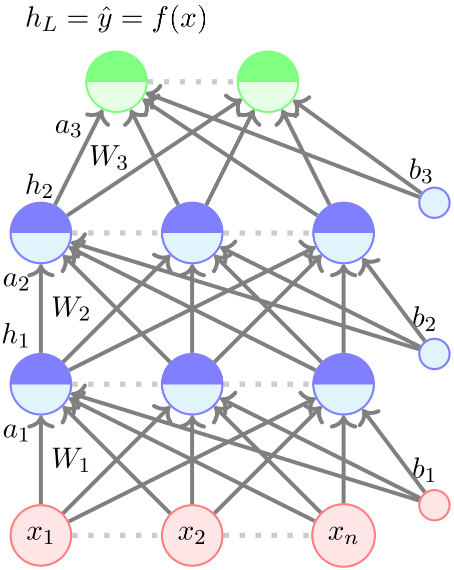

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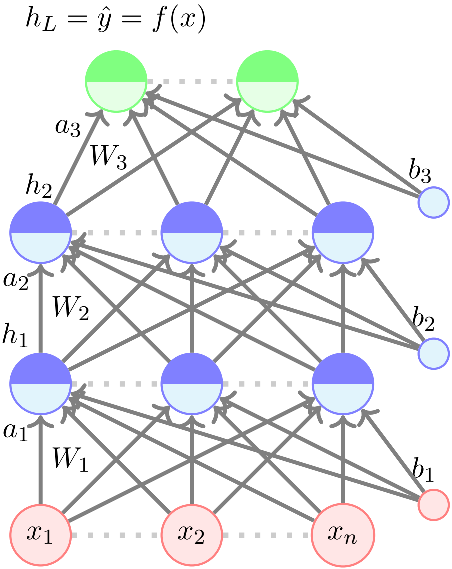

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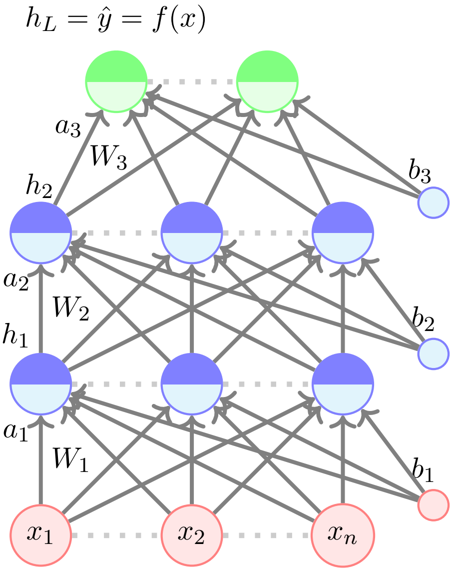

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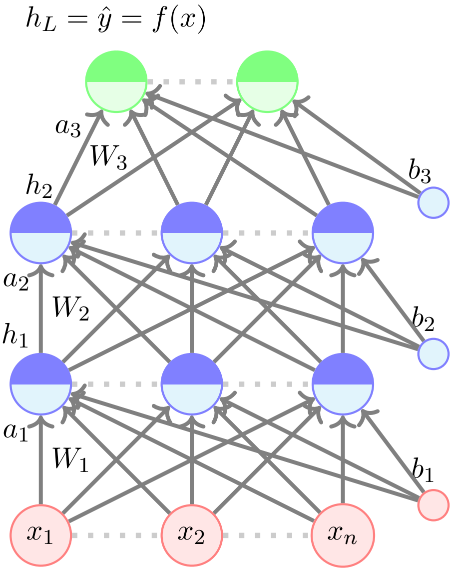

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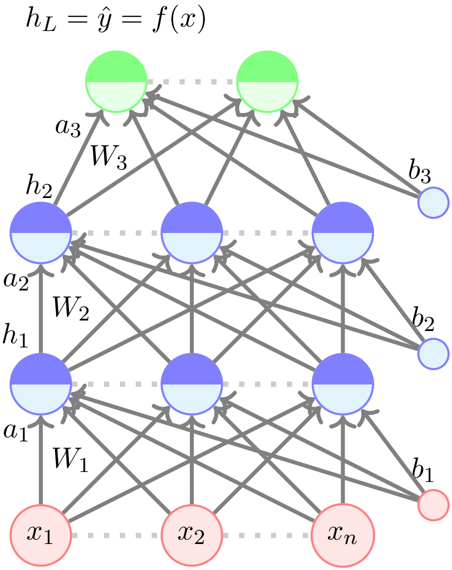
- To simplify notation we will refer to $a_i(x)$ as a_i and $h_i(x)$ as h_i



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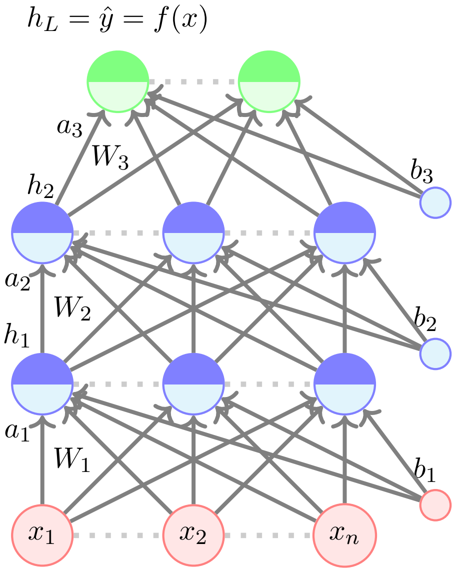
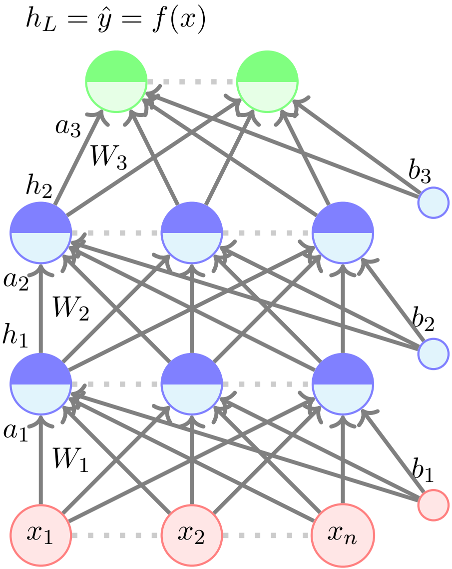


- **Data:** $\{x_i, y_i\}_{i=1}^{N}$



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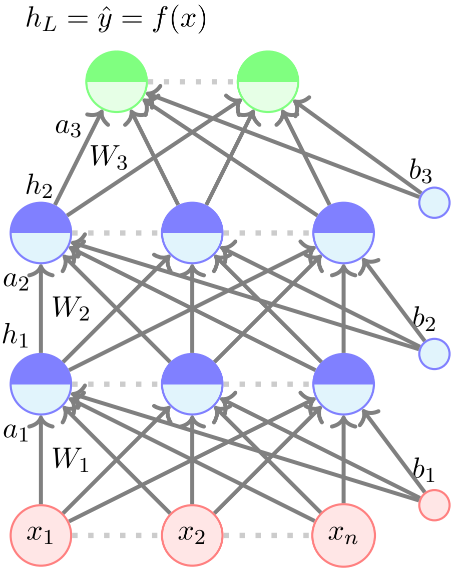

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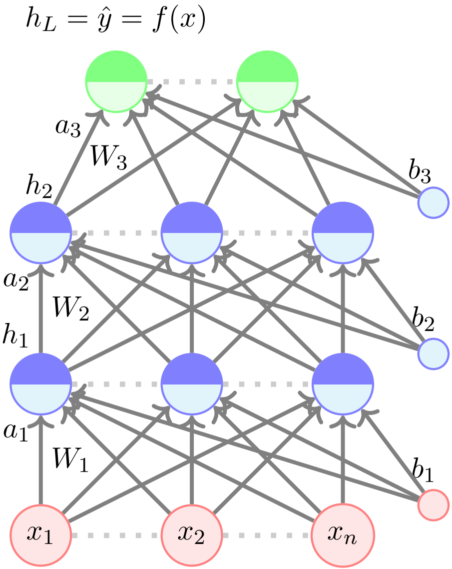

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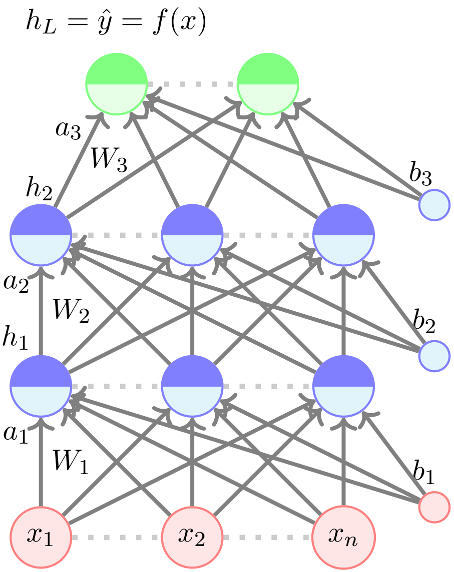

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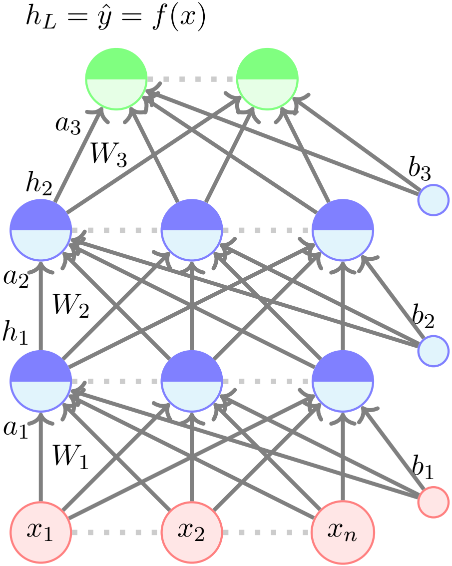

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- **Objective/Loss/Error function:** Say,

$$min \ \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{k} (\hat{y}_{ij} - y_{ij})^2$$

In general, $min \ \mathscr{L}(\theta)$

where $\mathscr{L}(\theta)$ is some function of the parameters