# Module 8.7 : Adding Noise to the inputs

## Other forms of regularization

- $L_2$ regularization
- Dataset augmentation
- Parameter Sharing and tying
- Adding Noise to the inputs
- Adding Noise to the outputs
- Early stopping
- Ensemble methods
- Dropout

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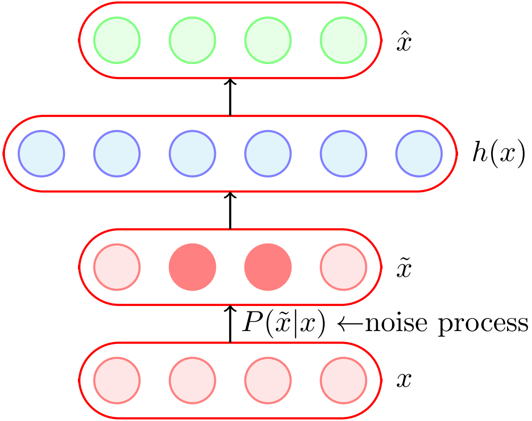
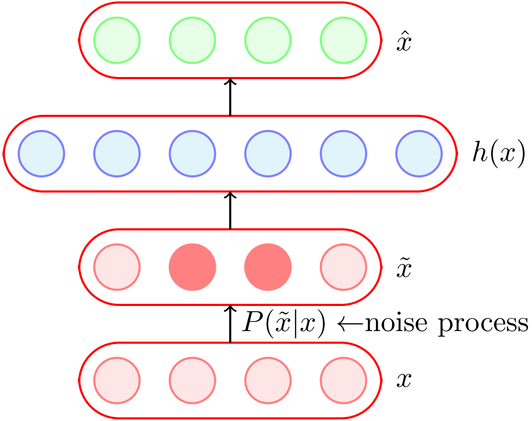
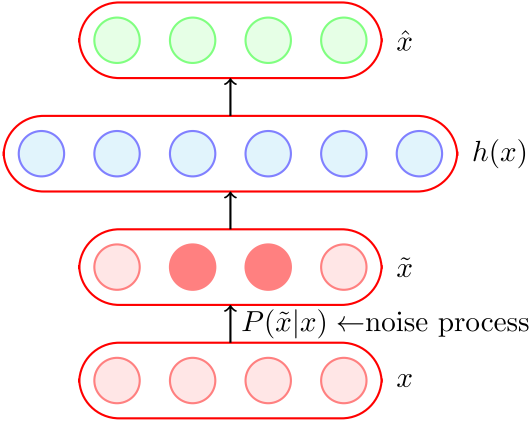
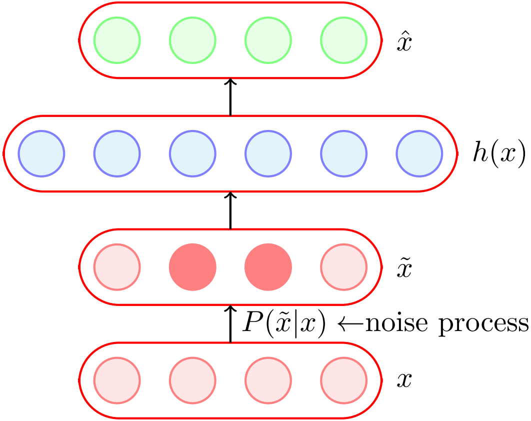

$\hat{x}$
$h(x)$
$\tilde{x}$
$P(\tilde{x}|x)$ ←noise process
$x$

- We saw this in Autoencoder






- We saw this in Autoencoder
- We can show that for a simple input output neural network, adding Gaussian noise to the input is equivalent to weight decay ($L_2$ regularisation)

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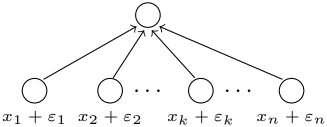
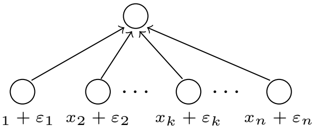
- We can show that for a simple input output neural network, adding Gaussian noise to the input is equivalent to weight decay ($L_2$ regularisation)
- Can be viewed as data augmentation

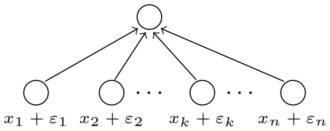



$$\varepsilon \sim \mathcal{N}(0, \sigma^2)$$




$$\varepsilon \sim \mathcal{N}(0, \sigma^2)$$

$$\widetilde{x_i} = x_i + \varepsilon_i$$

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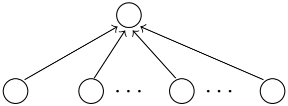


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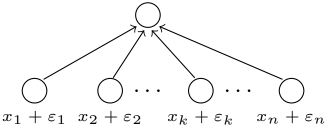


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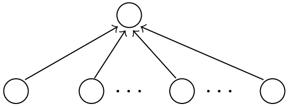
\end{aligned}$$

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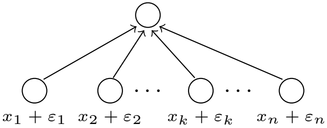


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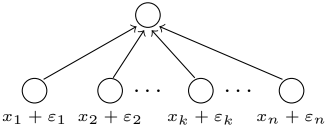
We are interested in $E[(\widetilde{y} - y)^2]$

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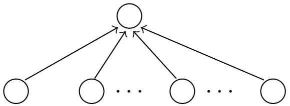


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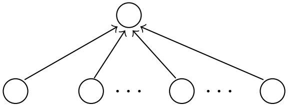


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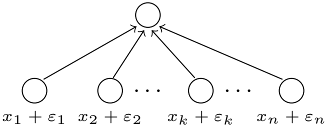


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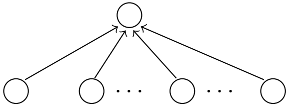
$$= E\left[(\widehat{y} - y)^2\right] + 0 + E\left[\sum_{i=1}^{n} w_i^2 \varepsilon_i^2\right]$$

($\because$ $\varepsilon_i$ is independent of $\varepsilon_j$ and $\varepsilon_i$ is independent of ($\widehat{y}$-$y$) )

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$$= (E\left[(\widehat{y} - y)^2\right] + \boxed{\sigma^2 \sum_{i=1}^{n} w_i^2}$$ (same as $L_2$ norm penalty)