Module 8.3 : True error and Model complexity

Using Stein's Lemma (and some trickery) we can show that

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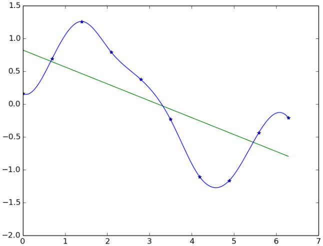
- Yes, indeed a complex model will be more sensitive to changes in observations whereas a simple model will be less sensitive to changes in observations
- Hence, we can say that
  true error = empirical train error + small constant + Ω(model complexity)

- Let us verify that indeed a complex model is more sensitive to minor changes in the data



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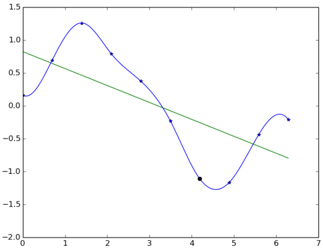

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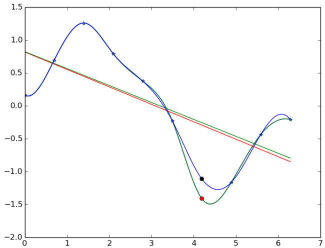

- Let us verify that indeed a complex model is more sensitive to minor changes in the data
- We have fitted a simple and complex model for some given data
- We now change one of these data points
- The simple model does not change much as compared to the complex model

- Hence while training, instead of minimizing the training error $\mathscr{L}_{train}(\theta)$ we should minimize

$$\min_{w.r.t\ \theta} \mathscr{L}_{train}(\theta) + \Omega(\theta) = \mathscr{L}(\theta)$$

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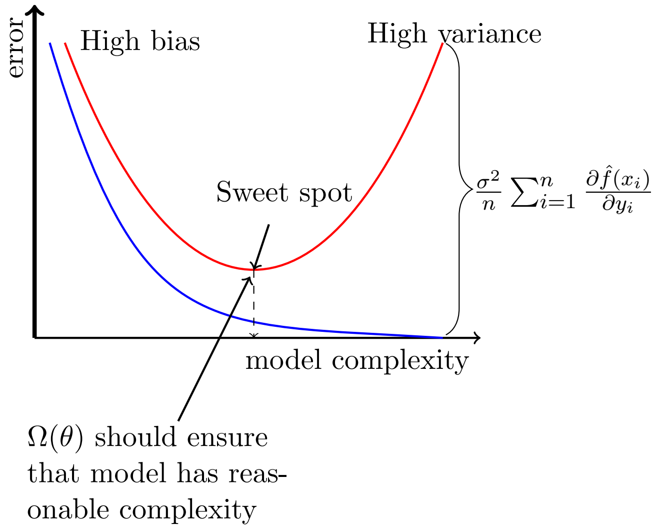
- This is the basis for all regularization methods
- We can show that l_1 regularization, l_2 regularization, early stopping and injecting noise in input are all instances of this form of regularization.



error
High bias
High variance
$\frac{\sigma^2}{n} \sum_{i=1}^{n} \frac{\partial \hat{f}(x_i)}{\partial y_i}$
Sweet spot
model complexity
$\Omega(\theta)$ should ensure that model has reasonable complexity

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- Hence we need some form of regularization.

Different forms of regularization

- l_2 regularization

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- Dataset augmentation

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- Dropout