Module 4.9: Derivative of the activation function

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Logistic function

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\begin{aligned}
g(z) &= \sigma(z) \\
&= \frac{1}{1+e^{-z}}
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g'(z) = (-1)\frac{1}{(1+e^{-z})^2}\frac{d}{dz}(1+e^{-z})
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g'(z) &= (-1)\frac{1}{(1+e^{-z})^2}\frac{d}{dz}(1+e^{-z}) \\
&= (-1)\frac{1}{(1+e^{-z})^2}(-e^{-z})
\end{aligned}
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&= (-1)\frac{1}{(1+e^{-z})^2}(-e^{-z}) \\
&= \frac{1}{1+e^{-z}}\left(\frac{1+e^{-z}-1}{1+e^{-z}}\right)
\end{aligned}
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&= \frac{1}{1+e^{-z}}\left(\frac{1+e^{-z}-1}{1+e^{-z}}\right) \\
&= g(z)(1-g(z))
\end{aligned}$$

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&= \frac{1}{1+e^{-z}}\left(\frac{1+e^{-z}-1}{1+e^{-z}}\right) \\
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\end{aligned}
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tanh

$$
\begin{aligned}
g(z) &= \tanh(z) \\
&= \frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}
\end{aligned}
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&= g(z)(1-g(z))
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tanh

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\begin{aligned}
g(z) &= \tanh(z) \\
&= \frac{e^z - e^{-z}}{e^z + e^{-z}} \\
g'(z) &= \frac{\left(\begin{array}{r}(e^z+e^{-z})\frac{d}{dz}(e^z-e^{-z}) \\ -(e^z-e^{-z})\frac{d}{dz}(e^z+e^{-z})\end{array}\right)}{(e^z+e^{-z})^2}
\end{aligned}
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Logistic function

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tanh

$$\begin{aligned} g(z) &= \tanh(z) \\ &= \frac{e^z-e^{-z}}{e^z+e^{-z}} \\ g'(z) &= \frac{\left(\begin{aligned}(e^z+e^{-z})\tfrac{d}{dz}(e^z-e^{-z}) \\ -(e^z-e^{-z})\tfrac{d}{dz}(e^z+e^{-z})\end{aligned}\right)}{(e^z+e^{-z})^2} \\ &= \frac{(e^z+e^{-z})^2-(e^z-e^{-z})^2}{(e^z+e^{-z})^2} \end{aligned}$$

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tanh

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\begin{aligned}
g(z) &= \tanh(z) \\
&= \frac{e^z - e^{-z}}{e^z + e^{-z}} \\
g'(z) &= \frac{\left(\begin{array}{c}(e^z+e^{-z})\frac{d}{dz}(e^z-e^{-z}) \\ \quad - (e^z-e^{-z})\frac{d}{dz}(e^z+e^{-z})\end{array}\right)}{(e^z+e^{-z})^2} \\
&= \frac{(e^z+e^{-z})^2 - (e^z-e^{-z})^2}{(e^z+e^{-z})^2} \\
&= 1 - \frac{(e^z-e^{-z})^2}{(e^z+e^{-z})^2}
\end{aligned}
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&= \frac{1}{1+e^{-z}}\left(\frac{1+e^{-z}-1}{1+e^{-z}}\right) \\
&= g(z)(1-g(z))
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&= \frac{e^z - e^{-z}}{e^z + e^{-z}} \\
g'(z) &= \frac{\left(\begin{array}{c}(e^z+e^{-z})\frac{d}{dz}(e^z-e^{-z}) \\ -(e^z-e^{-z})\frac{d}{dz}(e^z+e^{-z})\end{array}\right)}{(e^z+e^{-z})^2} \\
&= \frac{(e^z+e^{-z})^2-(e^z-e^{-z})^2}{(e^z+e^{-z})^2} \\
&= 1-\frac{(e^z-e^{-z})^2}{(e^z+e^{-z})^2} \\
&= 1-(g(z))^2
\end{aligned}
$$