Intro - GCD

Week1

GCD

- An algorithm to find the greatest common divisor of two positive integers m and n, $m \geq n$.

GCD

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GCD

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  1. Take the smaller number n.

GCD

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- **A naïve solution – Described *informally* as follows.**
  1. **Take the smaller number n.**
  2. **For each number k, n ≥k≥1, in descending order, do the following.**
     1. If k divides m and n, then k is the gcd of m and n

GCD

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- **This will compute gcd correctly, but is VERY slow (think about large numbers m and n).**
- **There is a faster way…**

GCD Algorithm - Intuition

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To find gcd of 8 and 6. Consider rods of length 8 and 6. Measure the longer with the shorter. Take the remainder if any. Repeat the process until the longer can be exactly measured as an integer multiple of the shorter.

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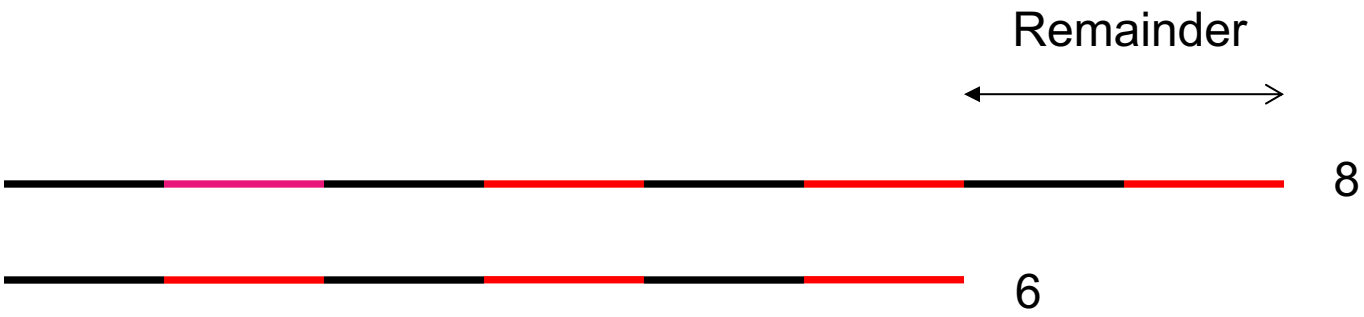

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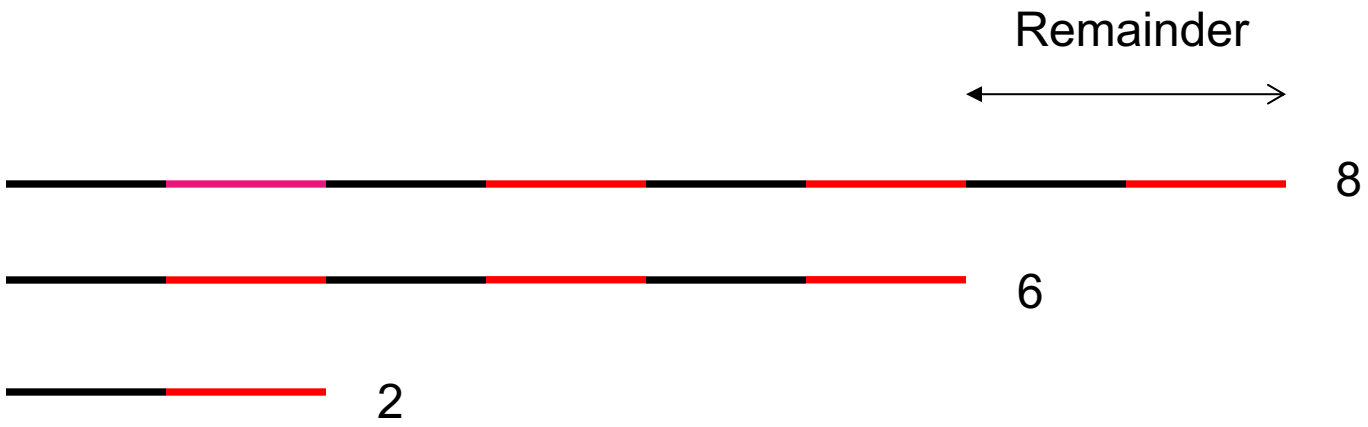

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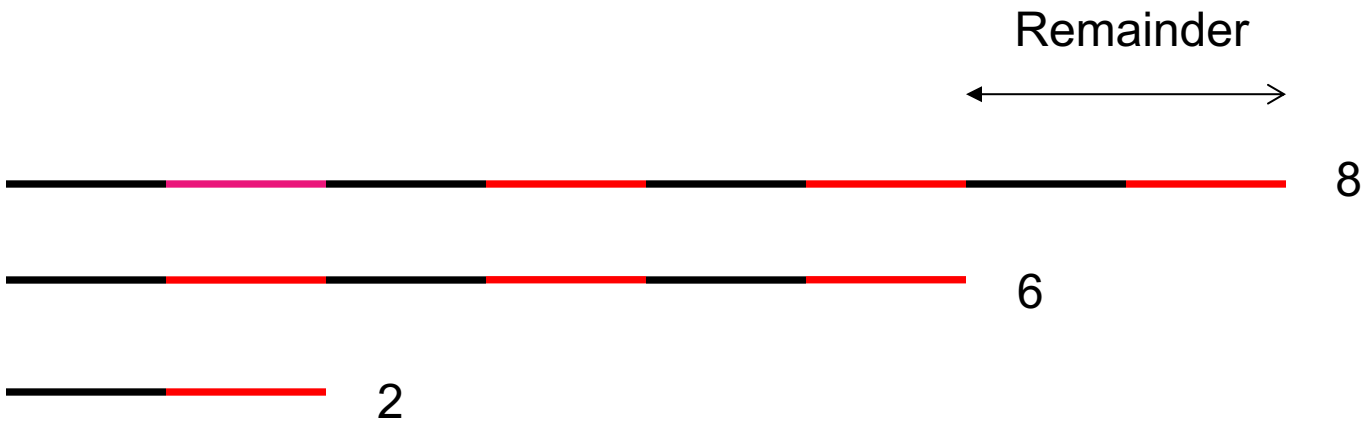


Gcd(8, 6) = 2.

GCD Algorithm - Intuition

GCD Algorithm - Intuition

102

21

GCD Algorithm - Intuition

102

102 mod 21 = 18

21

18

GCD Algorithm - Intuition

102

102 mod 21 = 18

21

21 mod 18 = 3

18

3

Gcd (102, 21) = 3

Euclid's method for gcd

Euclid's algorithm (step-by-step method for calculating gcd) is based on the following simple fact.

Suppose a > b. Then the gcd of a and b is the same as the gcd of b and the remainder of a when divided by b.

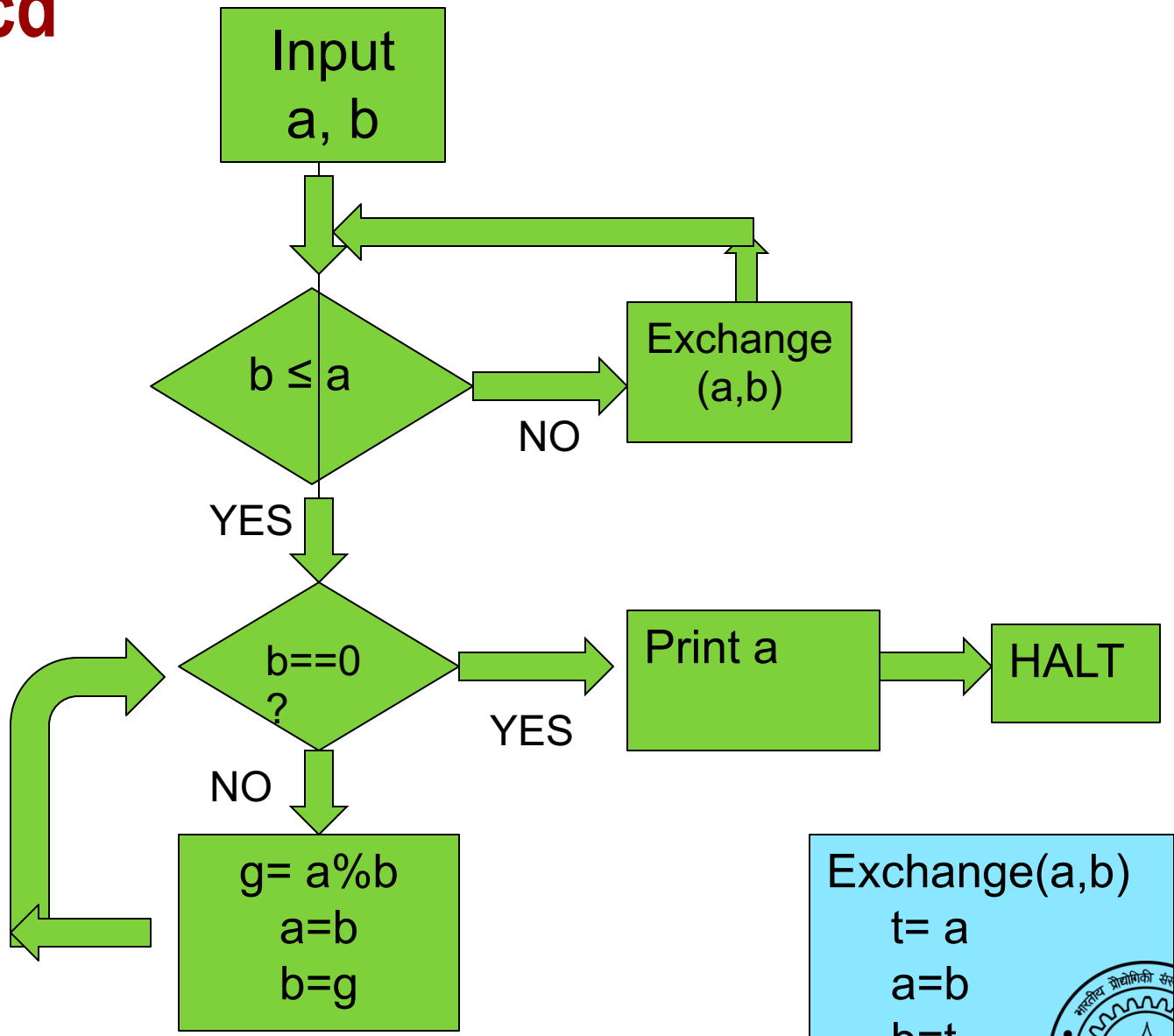
gcd(a,b) = gcd (b, a % b)

To see this consider division of a by b

a = bq + r

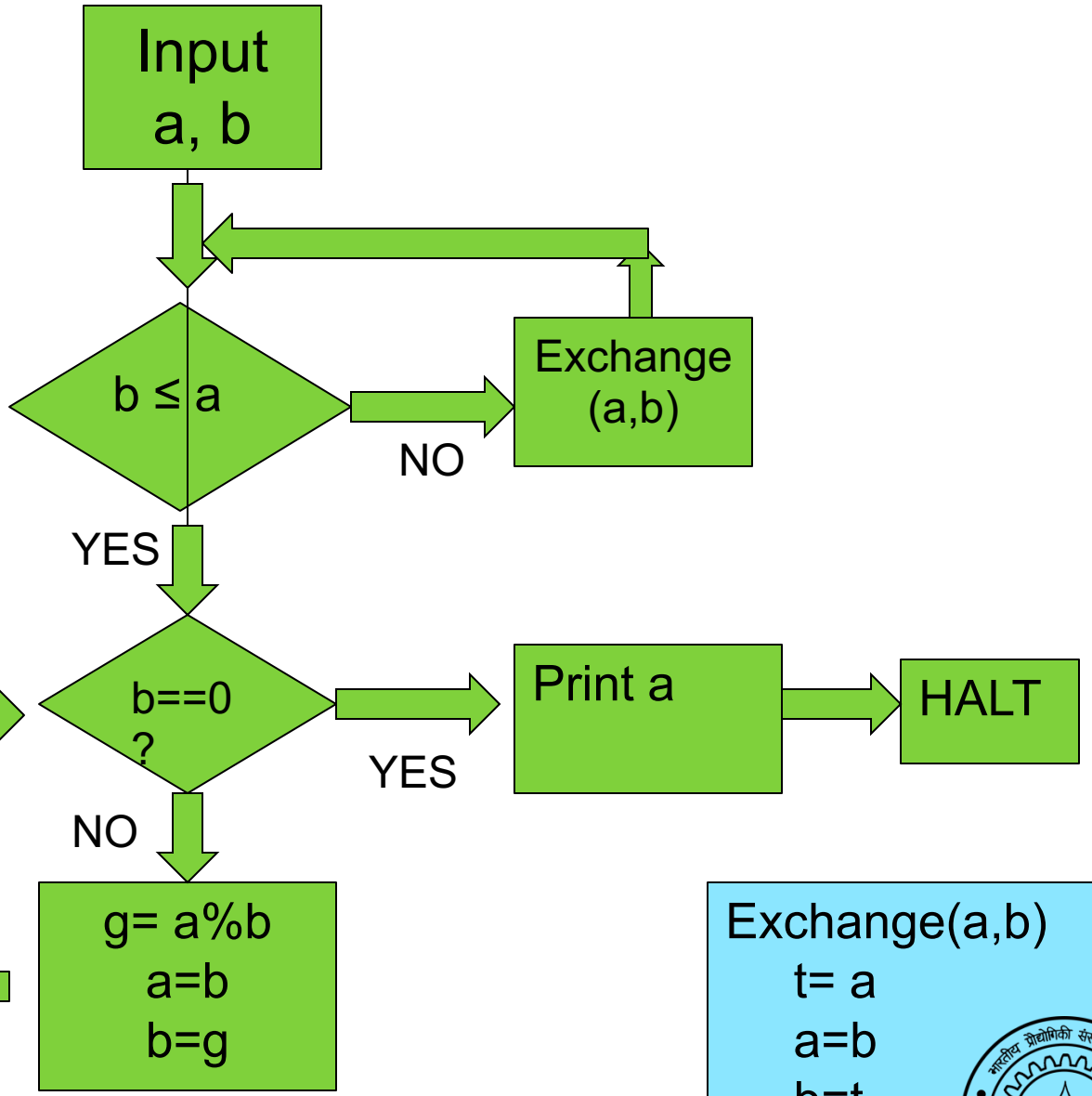
Euclid's gcd

a,b,g are variables. Variables "store" exactly one value at a time.



Exchange(a,b)
t= a
a=b
b=t

Euclid's gcd



a,b,g are variables. Variables "store" exactly one value at a time.

a%b is the remainder when a is divided by b.
Eg. 8%3 is 2

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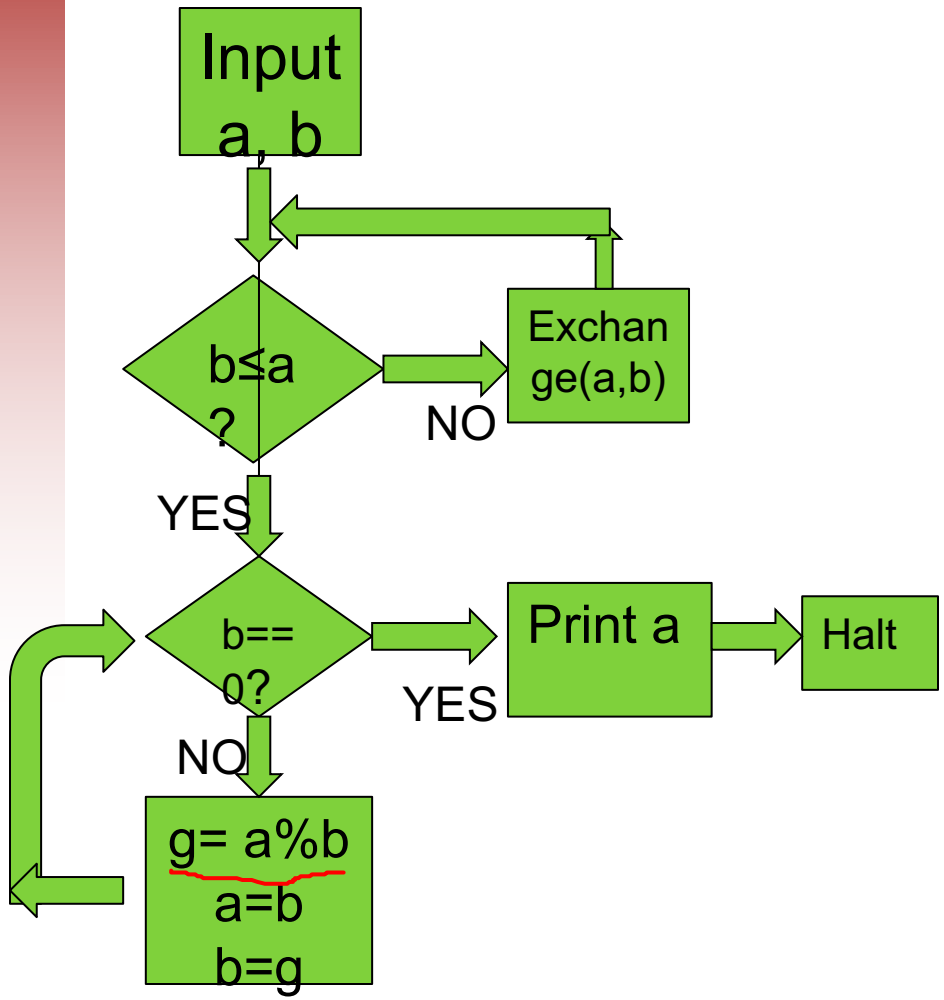
Variables and Assigning them



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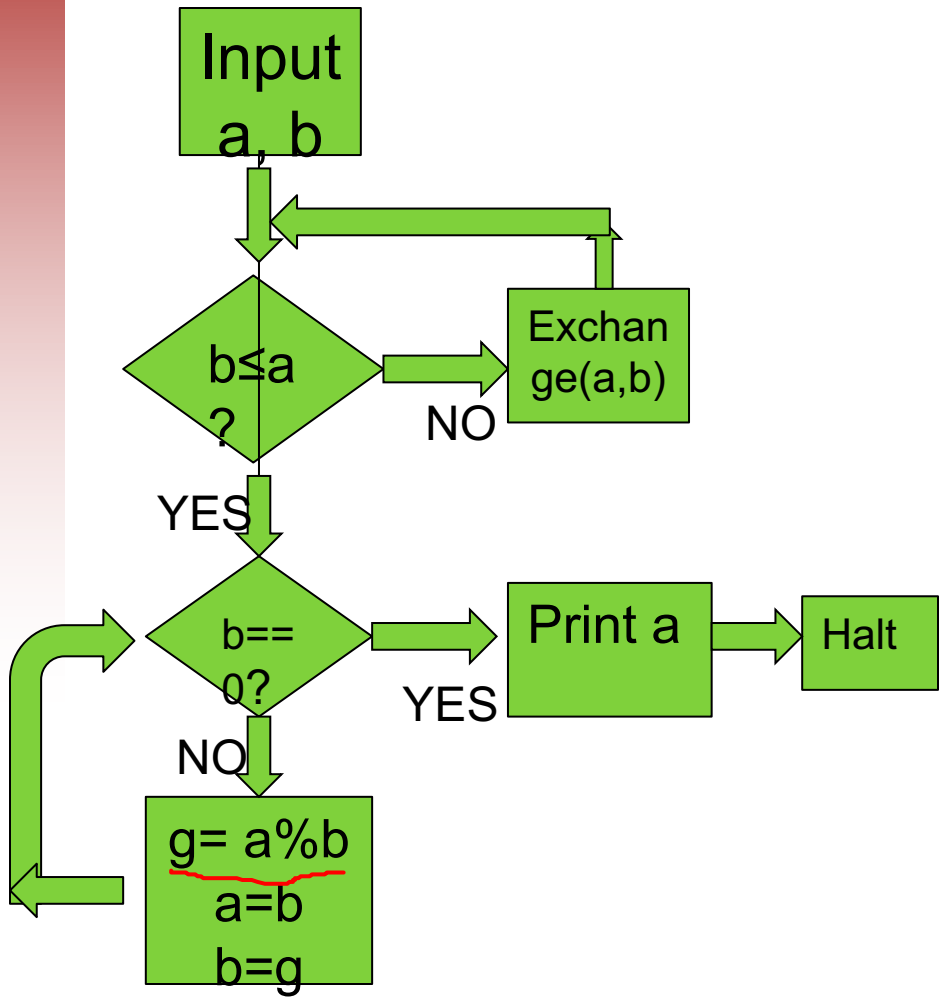

- Concept of variable: a name for a box.

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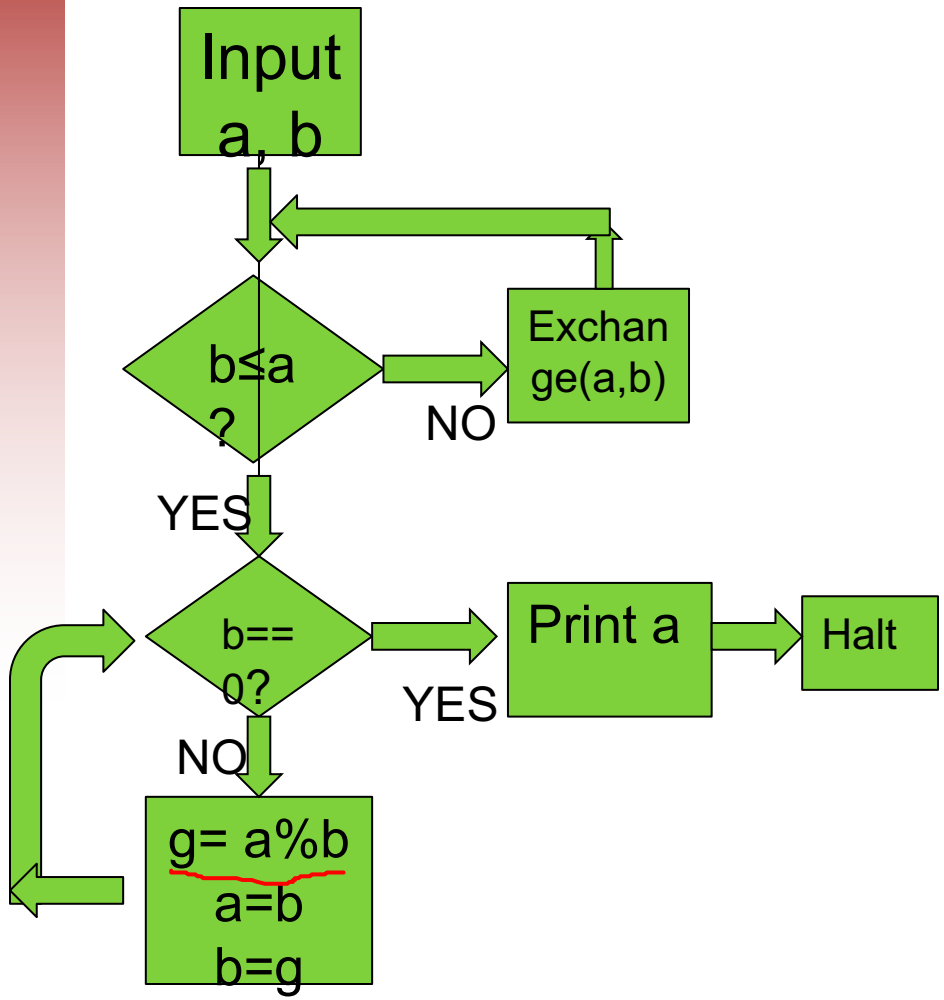

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a 5 3 b

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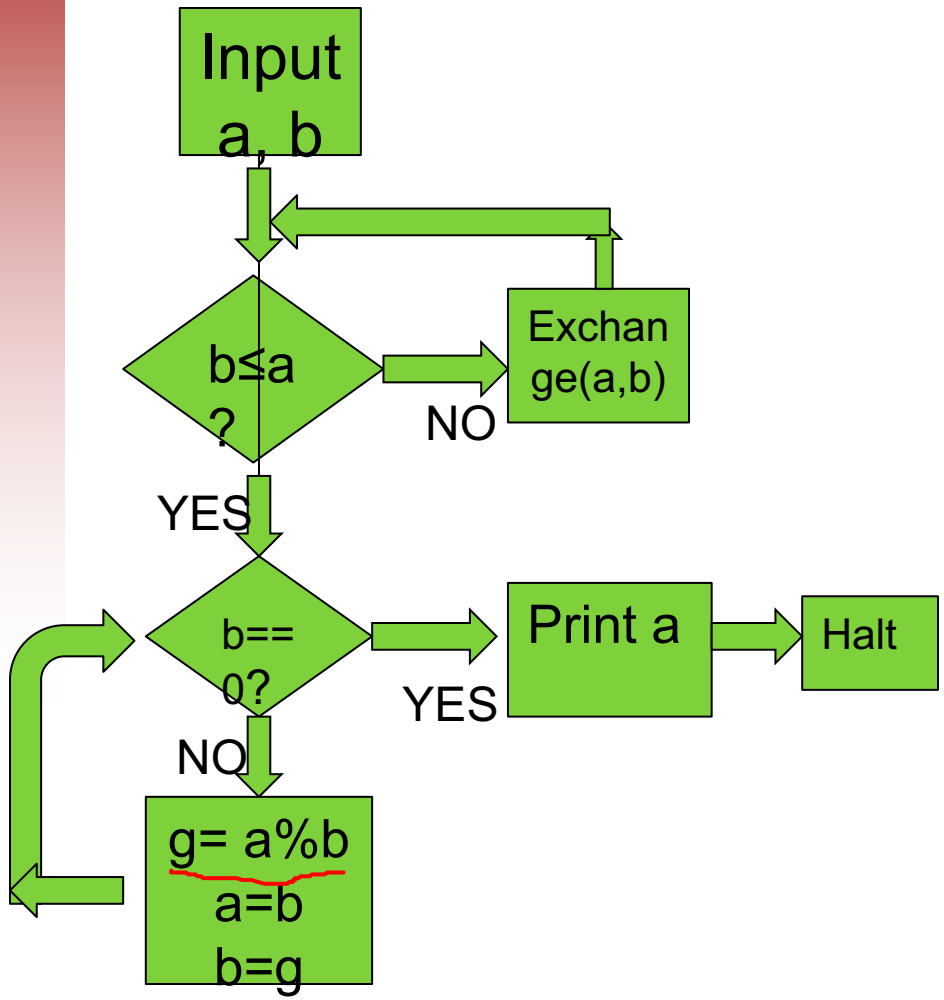

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- Assignment a = b replaces whatever is stored in a by what is stored in b.

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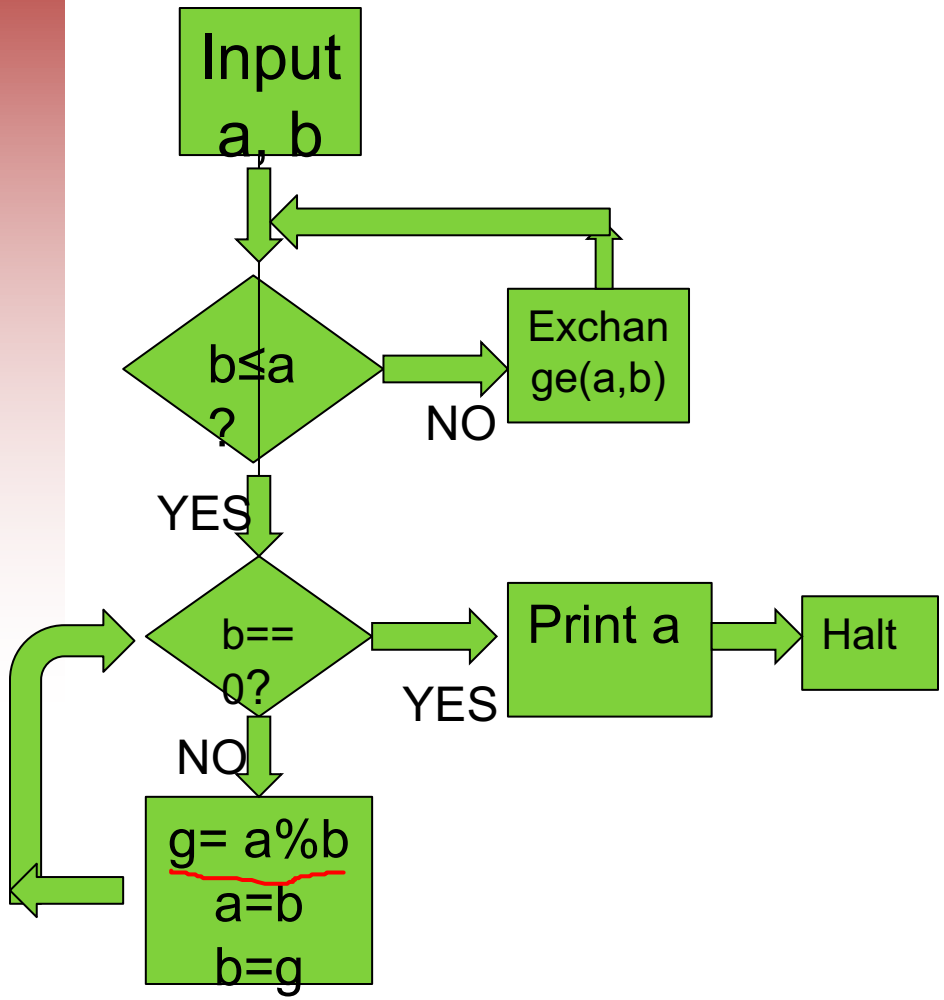

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a | 5 | 3 | b

- Assignment a = b replaces whatever is stored in a by what is stored in b.
- After a = b

a | 3 | 3 | b

Sequential assignments

```
g = a%b;
a = b;
b = g;
```

- Semi-colons give a sequential order in which to apply the statements.

Sequential assignments

```
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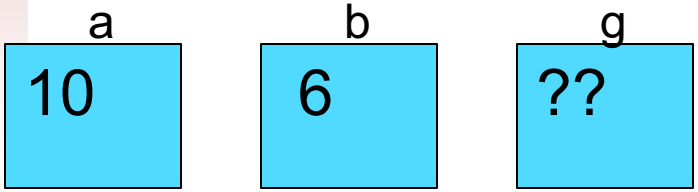

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- We have 3 variables: a, b, g. This gives us three boxes. Initially, a is 10, b is 6 and g is undefined.

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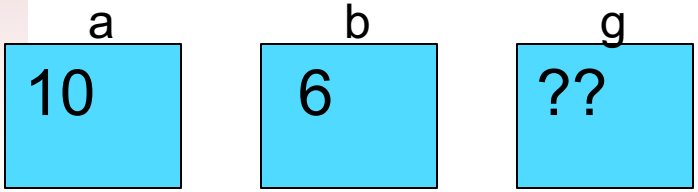

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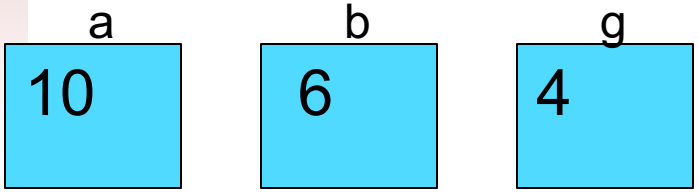
After g = a %b



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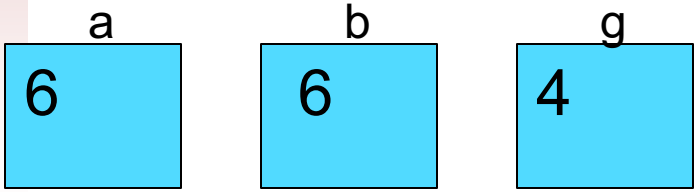

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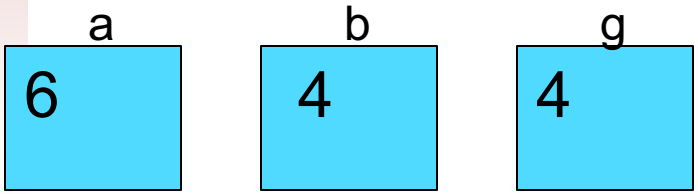

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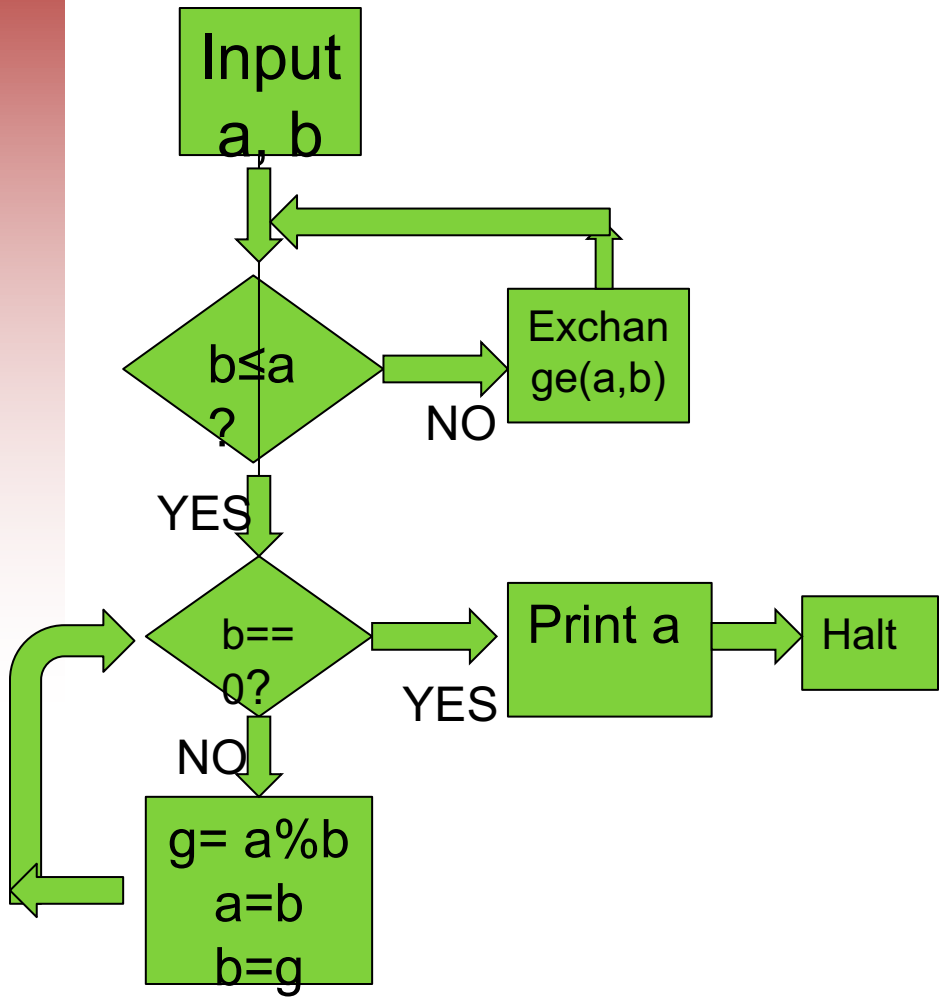
Running the program



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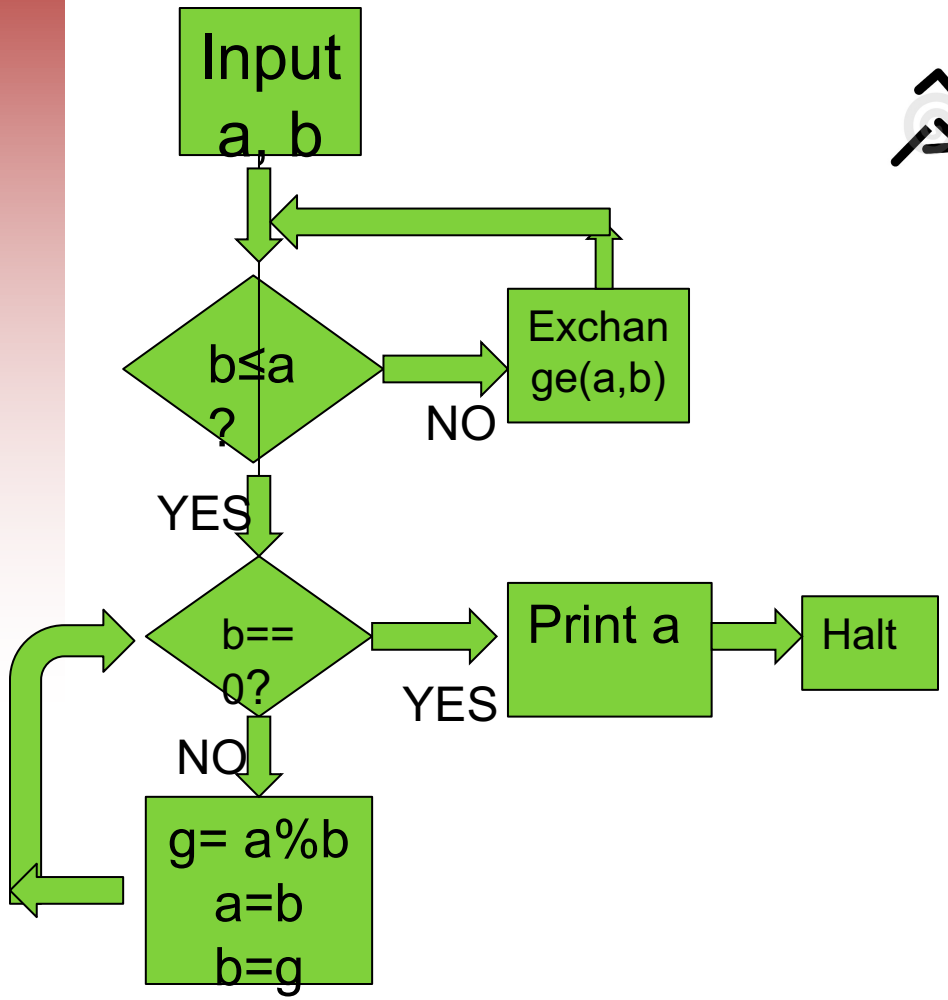

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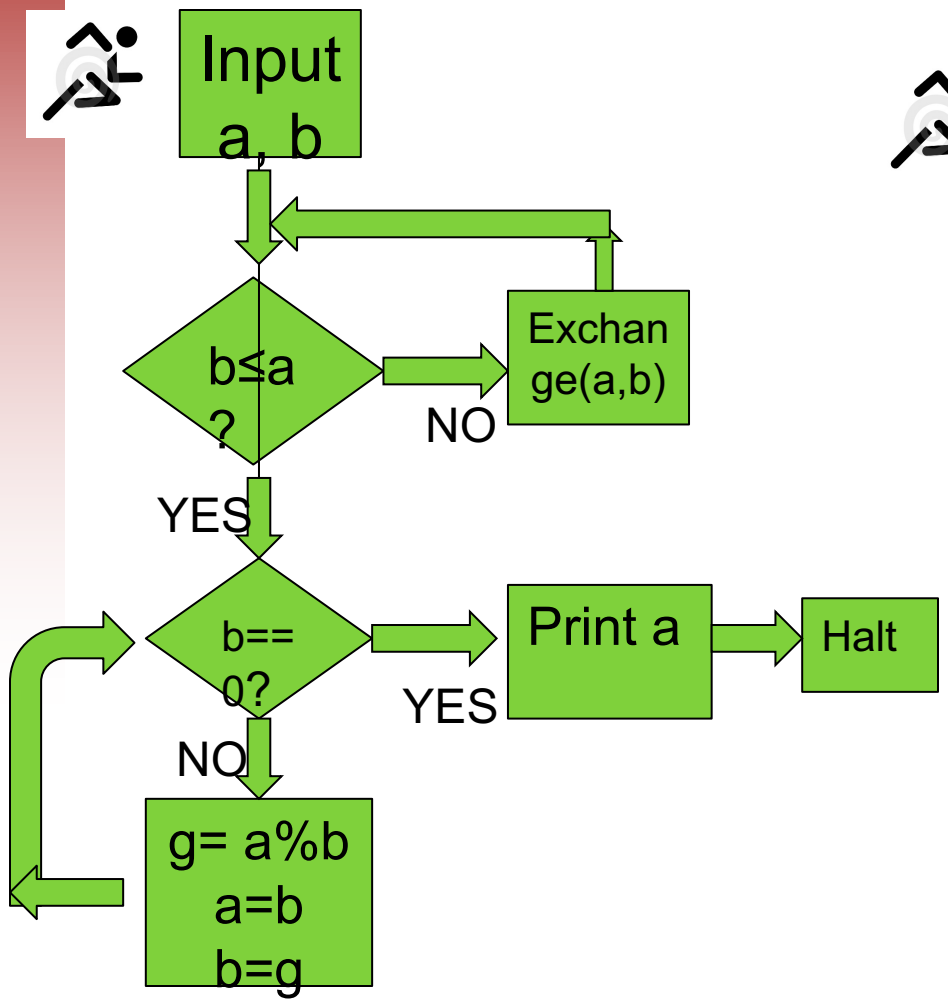
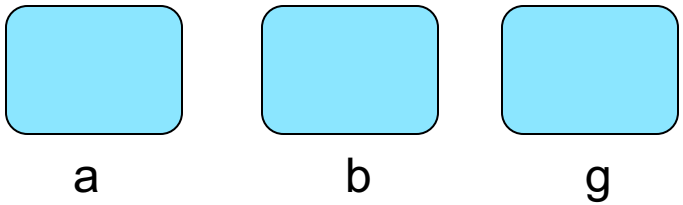


Program counter. At the next step to be executed. Initially at beginning.

State of the program is variables : boxes with names.



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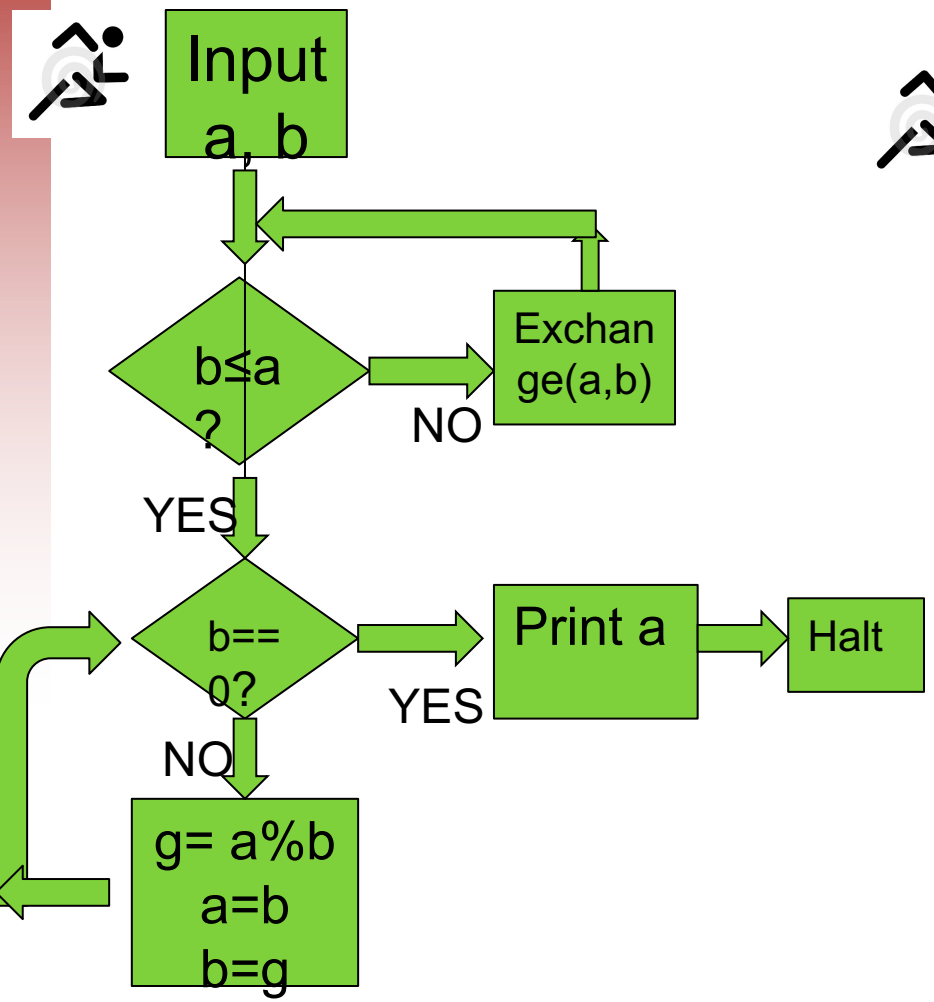

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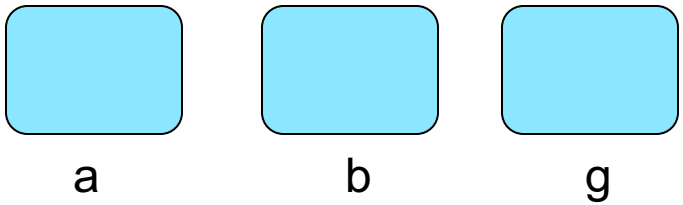

Now let us start running the flowchart. One step at a time.

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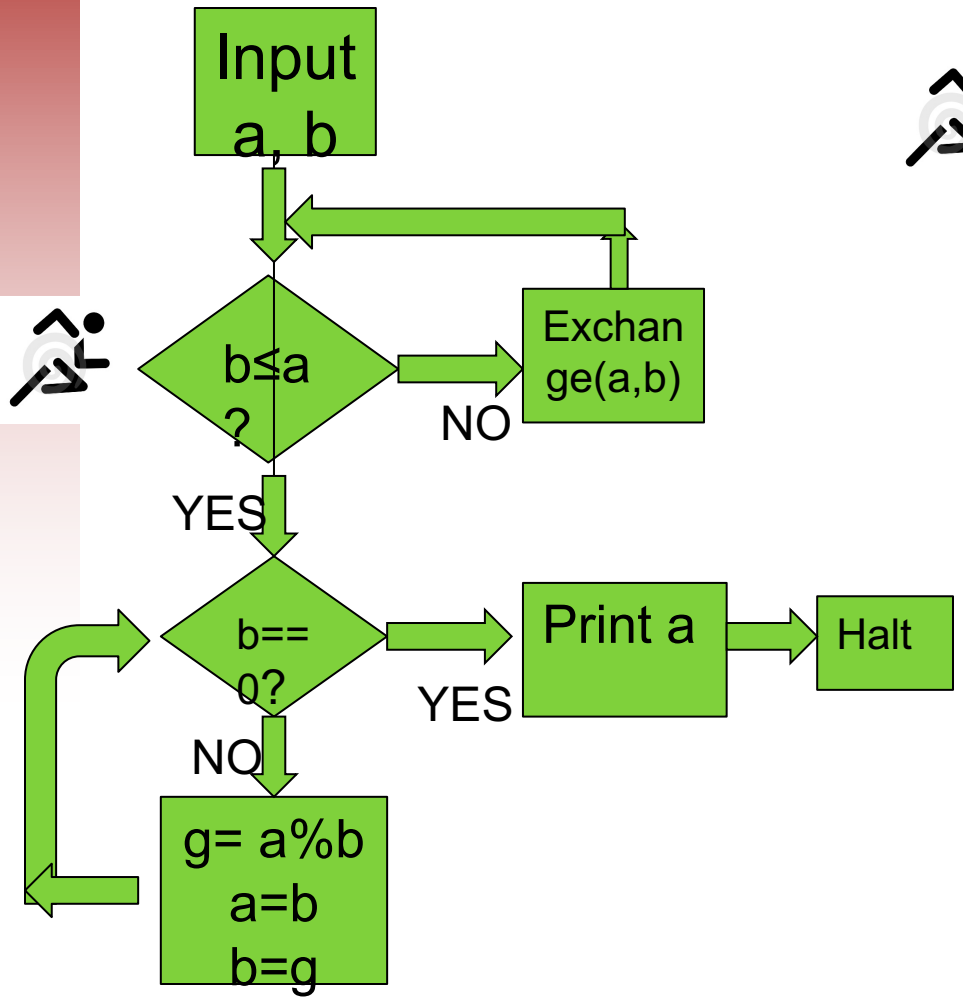

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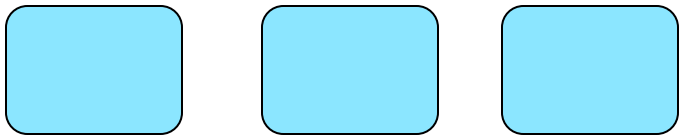

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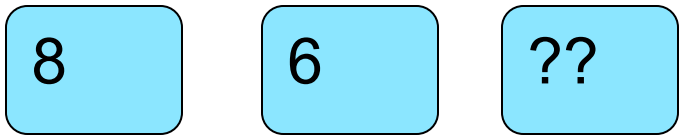
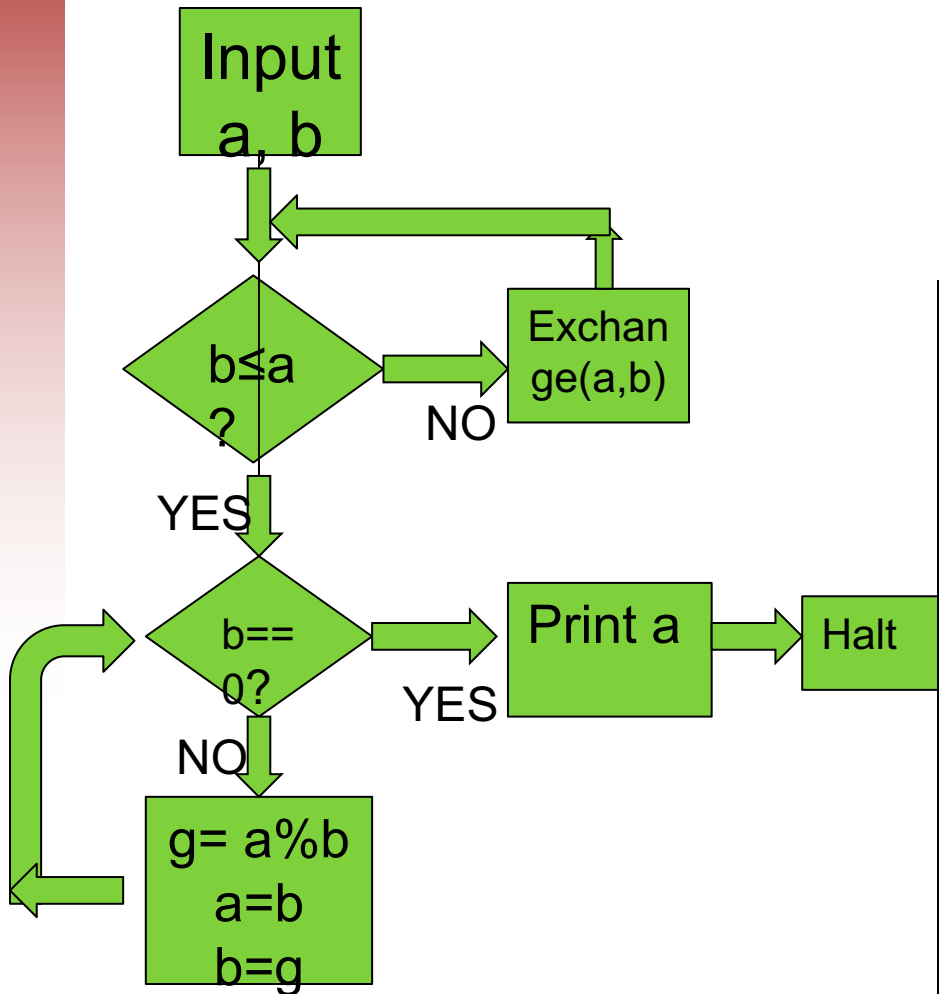
1. After input step:



Tracing the execution



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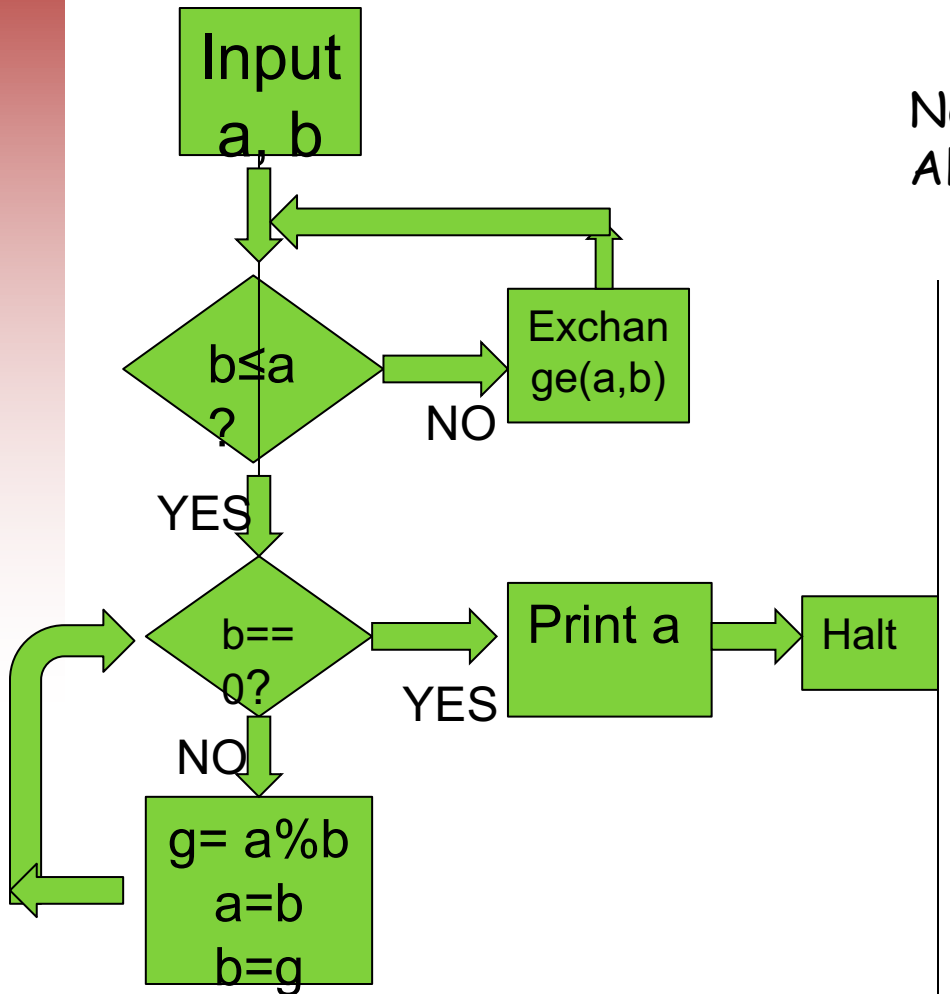
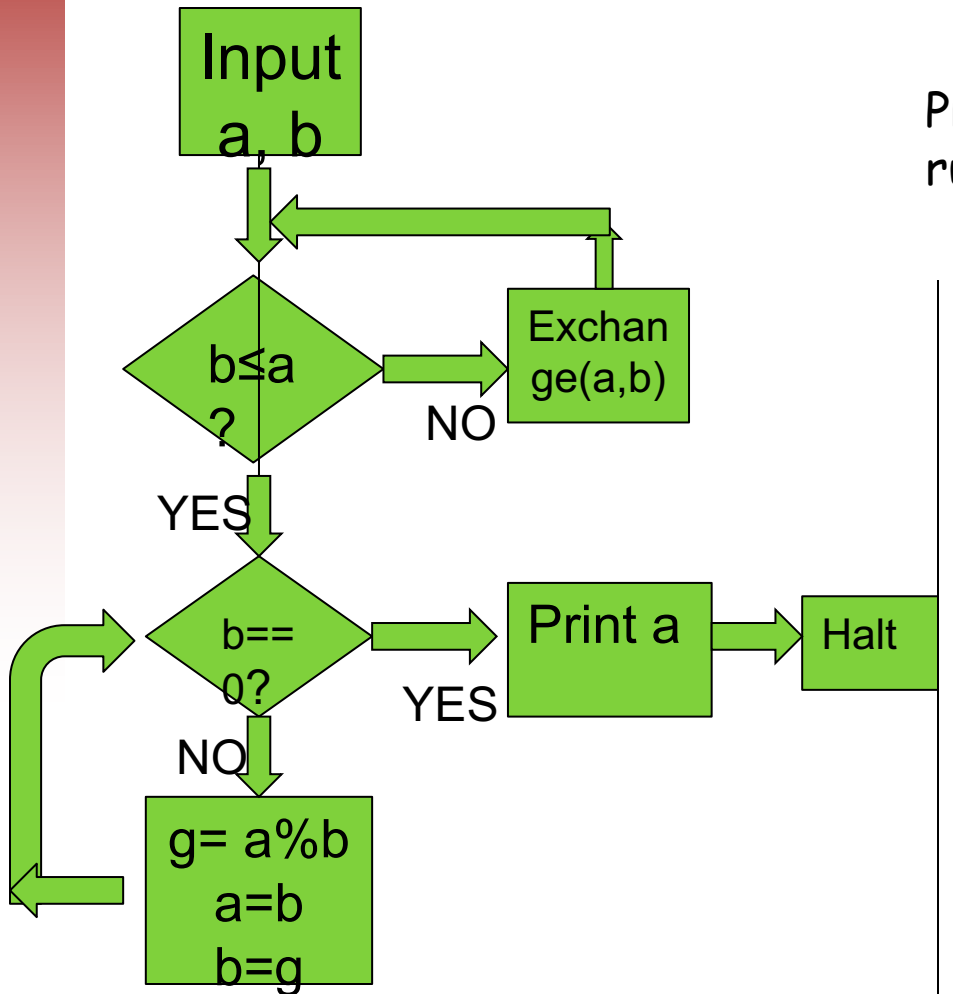


Now let us start running the flowchart.
Always one box at a time.

Tracing the execution



Program Counter is at the next step to be run

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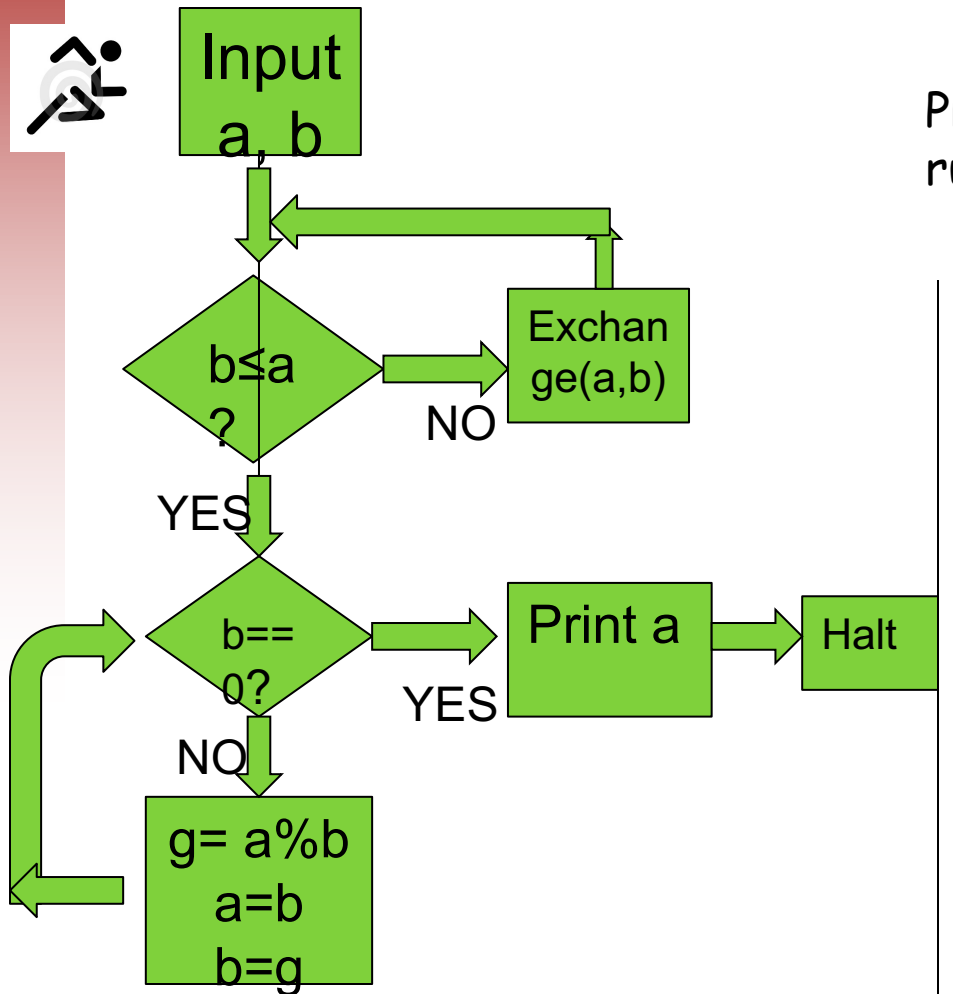

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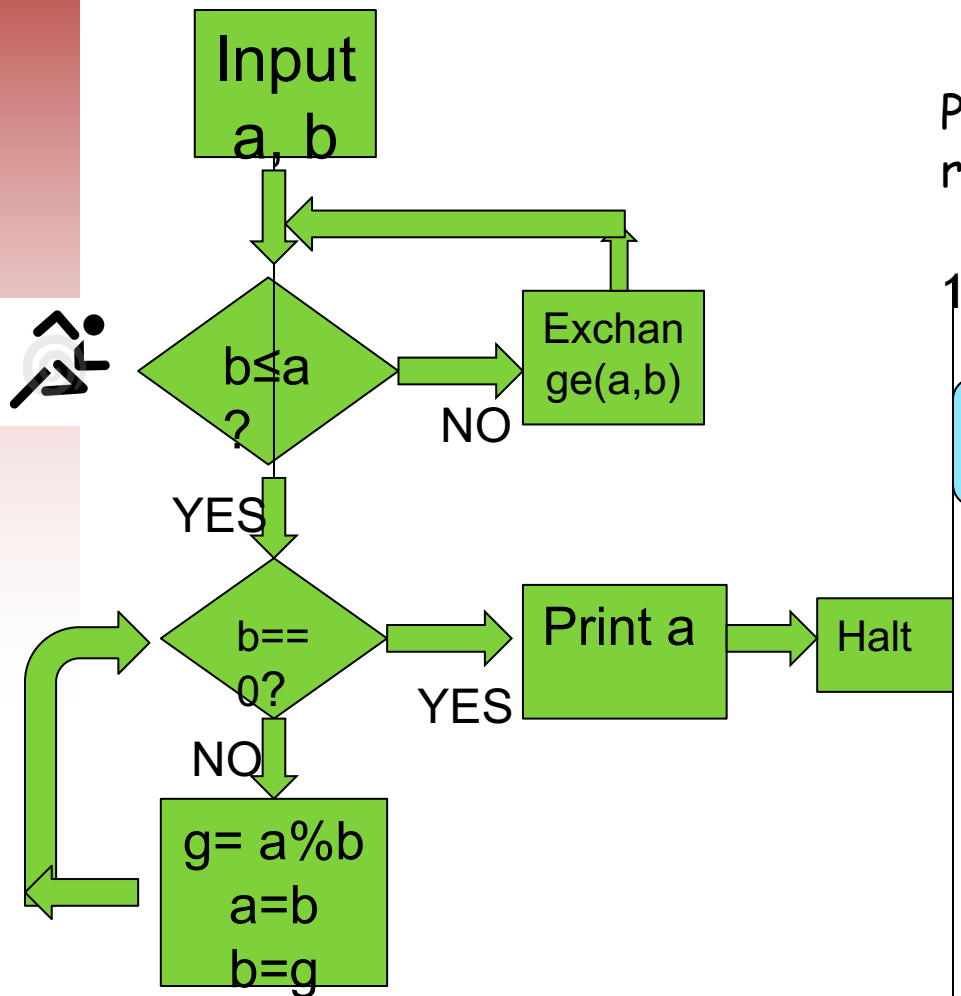

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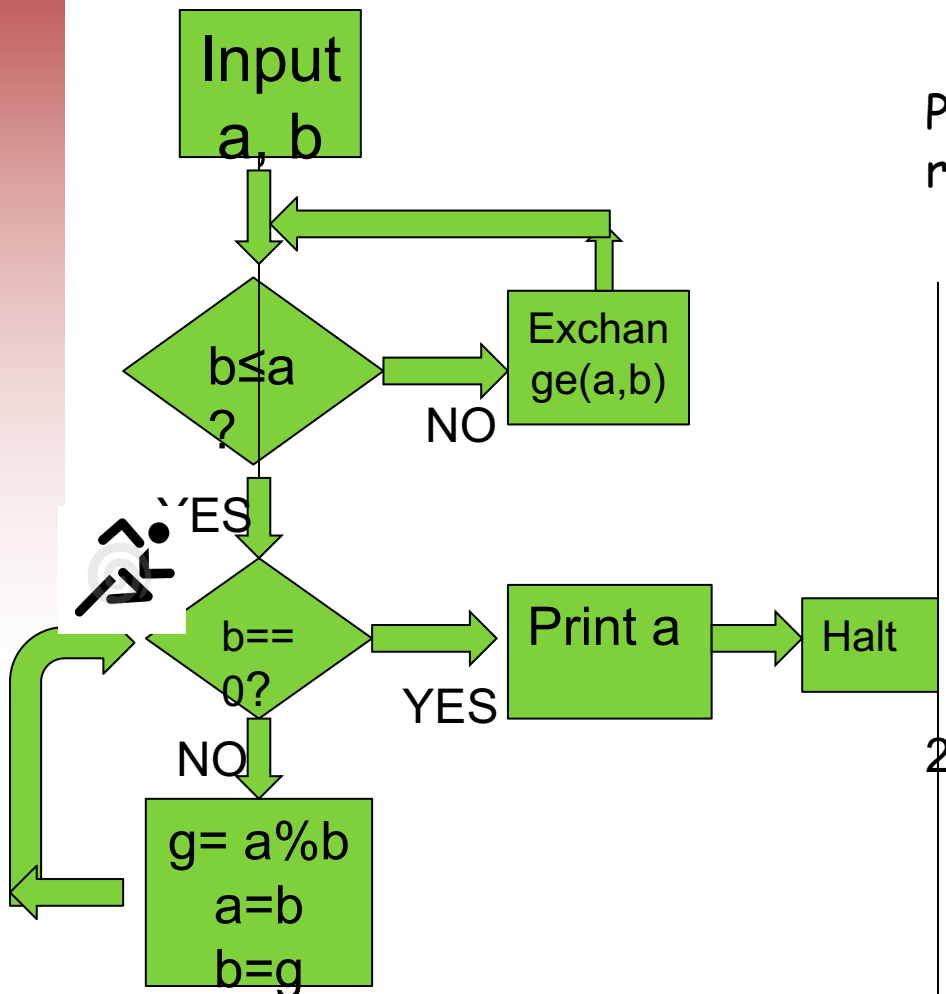
1. After input step:

8	6	??
a	b	g

Tracing the execution

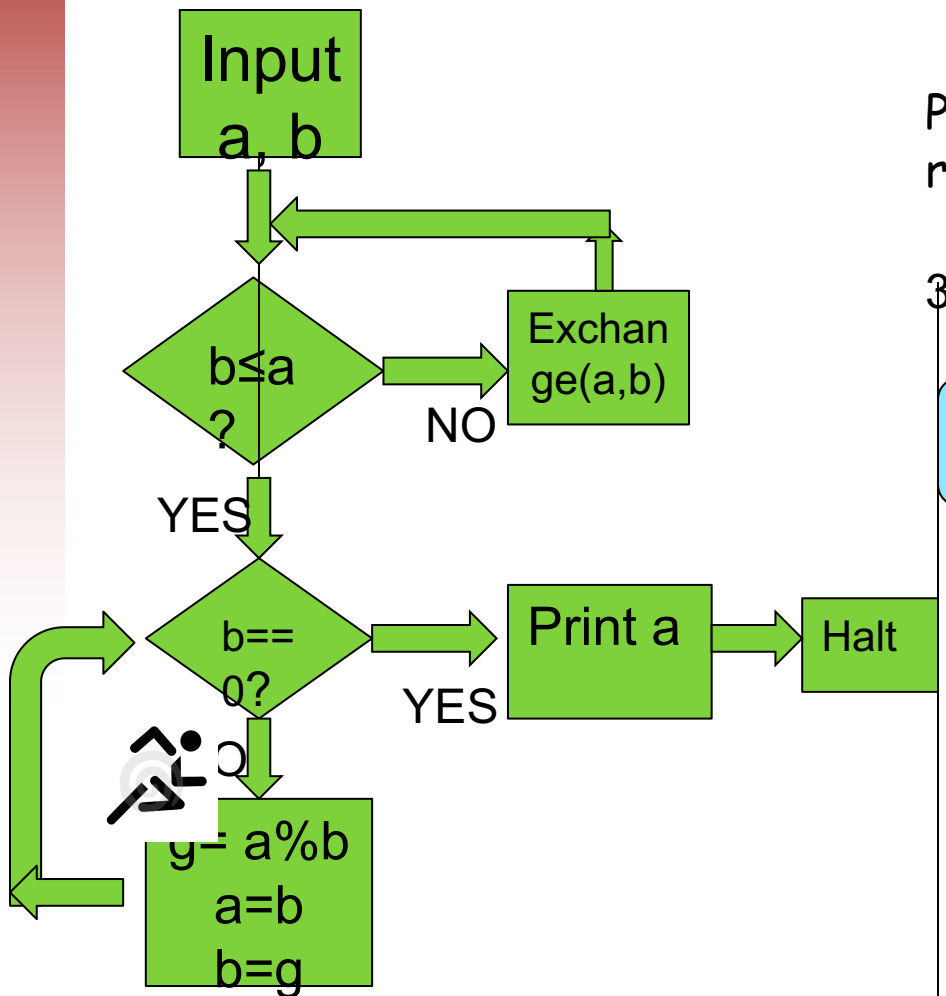


Program Counter is at the next step to be run

2. Test b < a? YES

8	6	??
a	b	g

Tracing the execution

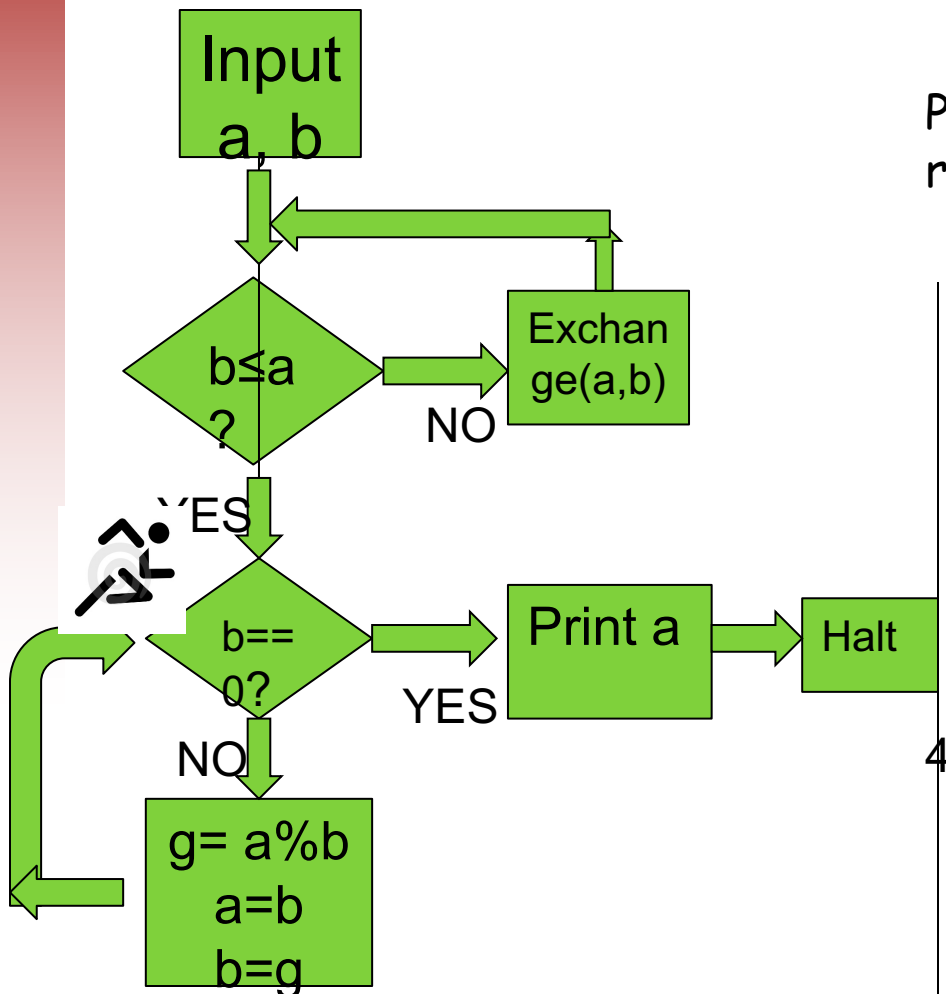


Program Counter is at the next step to be run

3. Test b==0? NO

a	b	g
8	6	??

Tracing the execution

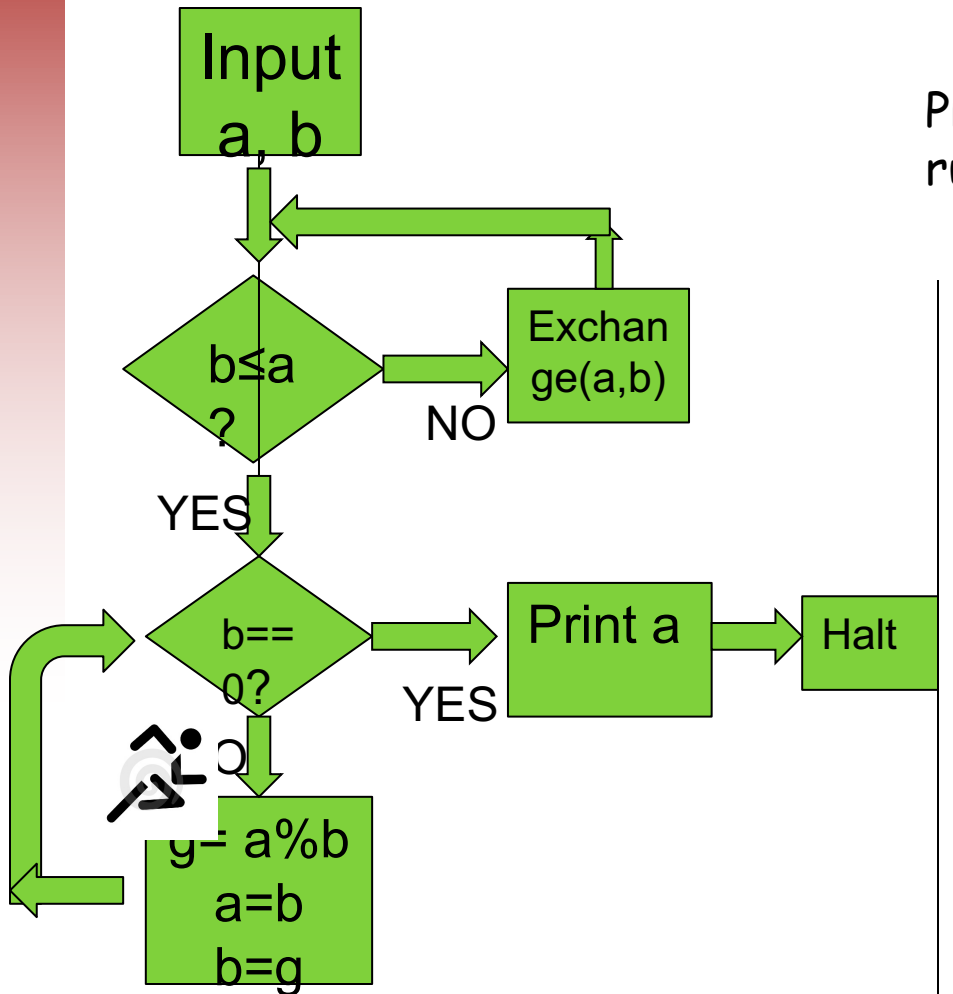


Program Counter is at the next step to be run

4. g=a%b; a=b; b=g;

6	2	2
a	b	g

Tracing the execution

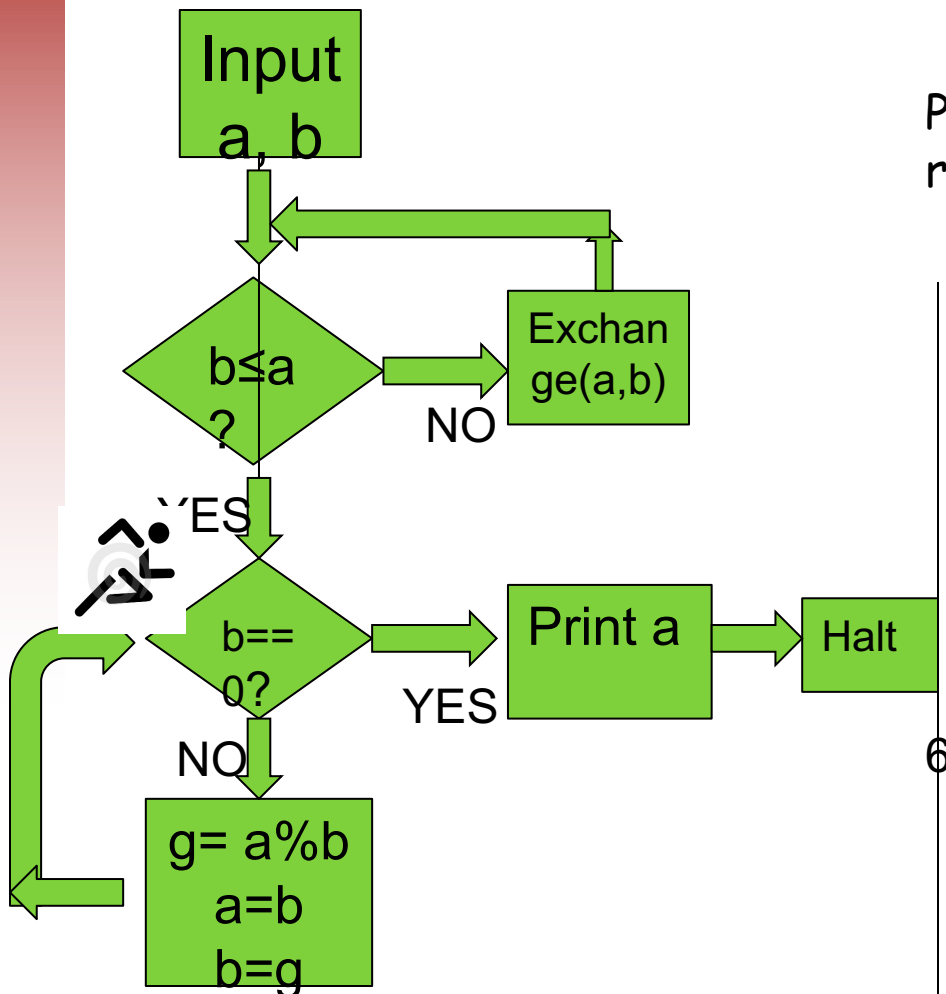


Program Counter is at the next step to be run

5. Test b==0? NO

a	b	g
6	2	2

Tracing the execution



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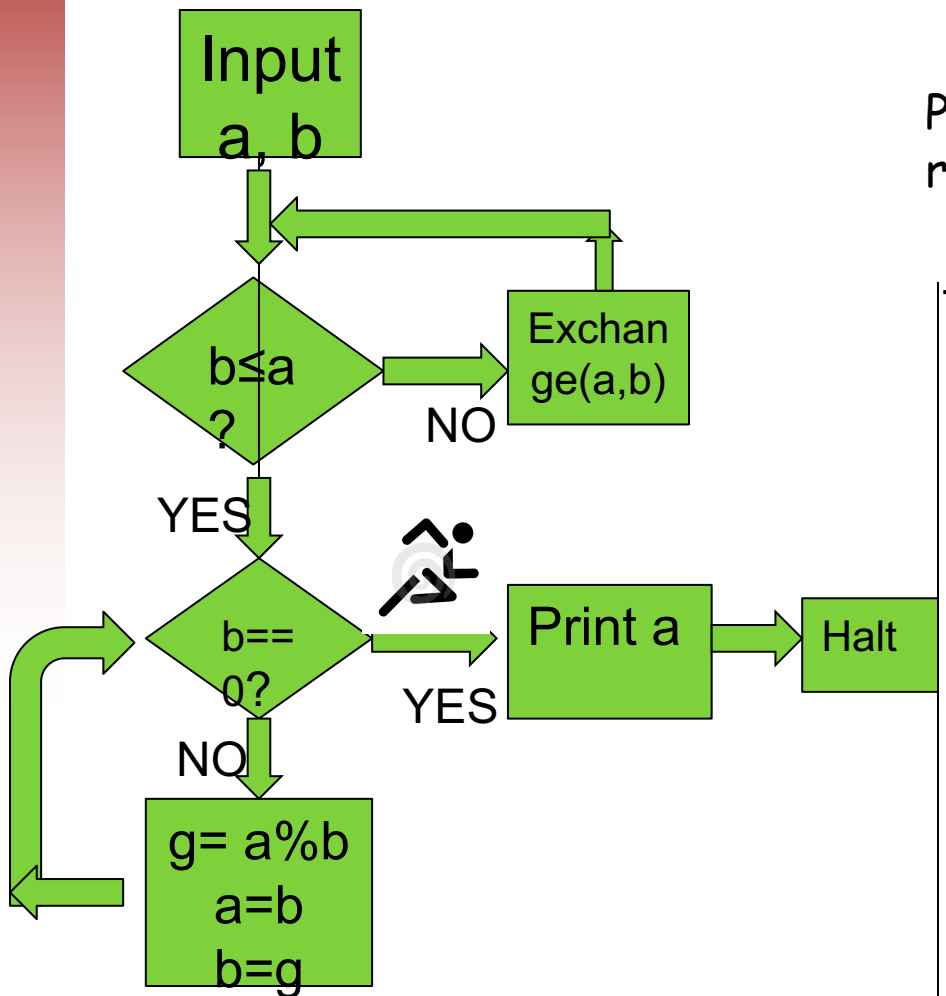
6. g=a%b; a=b; b=g;

a	b	g
2	0	0

Tracing the execution

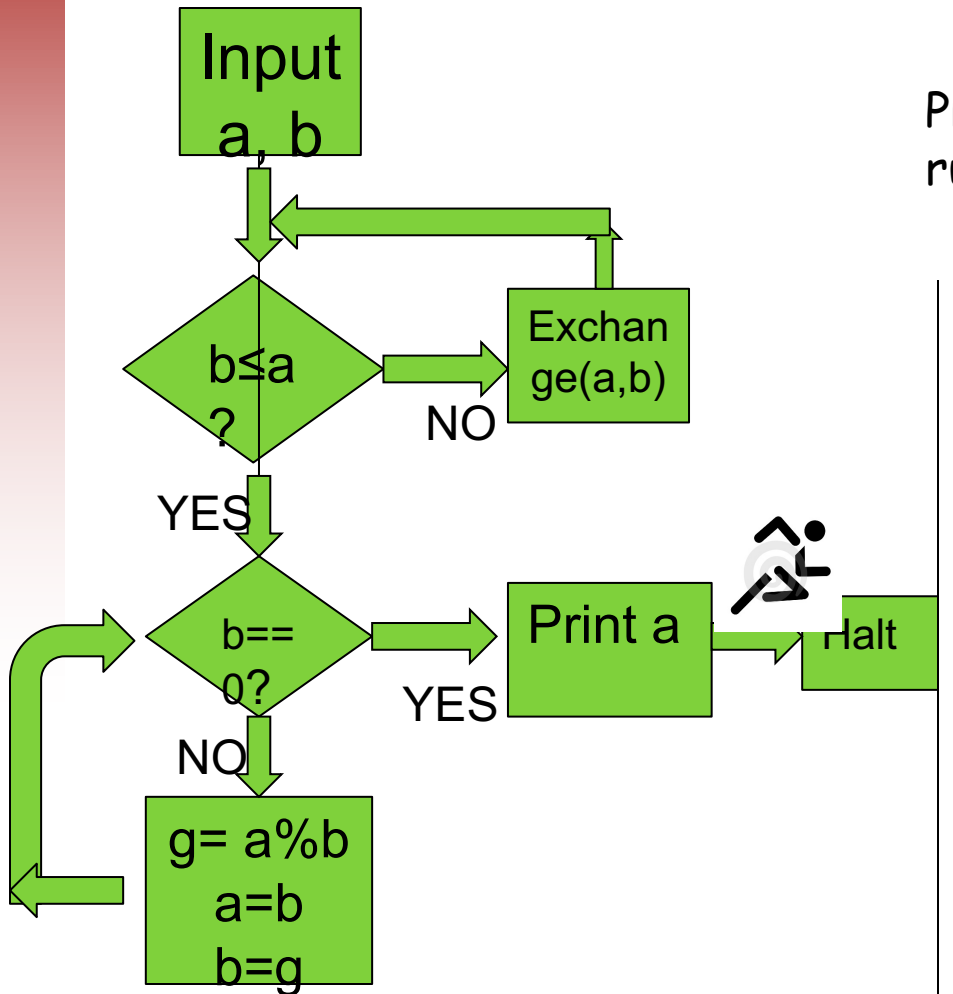


Program Counter is at the next step to be run

7. Test b==0? YES

a	b	g
2	0	0

Tracing the execution

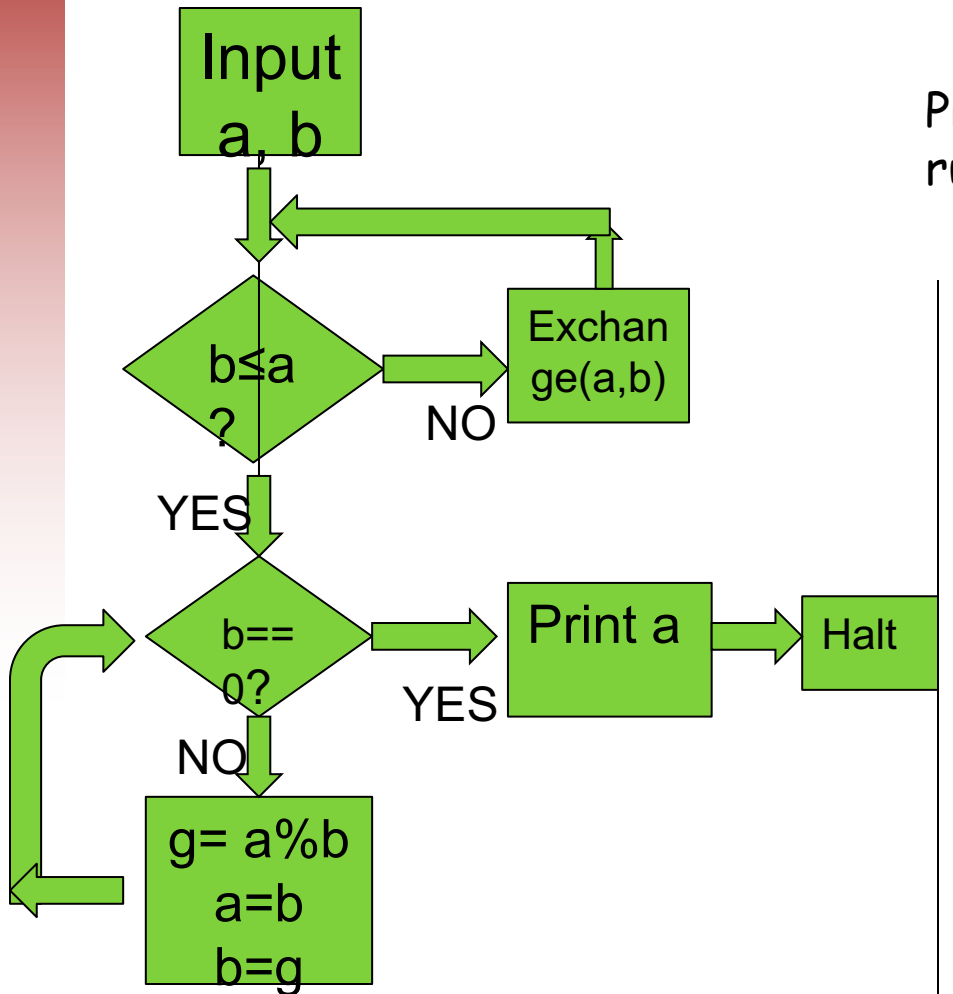


Program Counter is at the next step to be run

8. Print a

2

Tracing the execution



Program Counter is at the next step to be run

8. Print a

2

Multiple solutions and comparing them

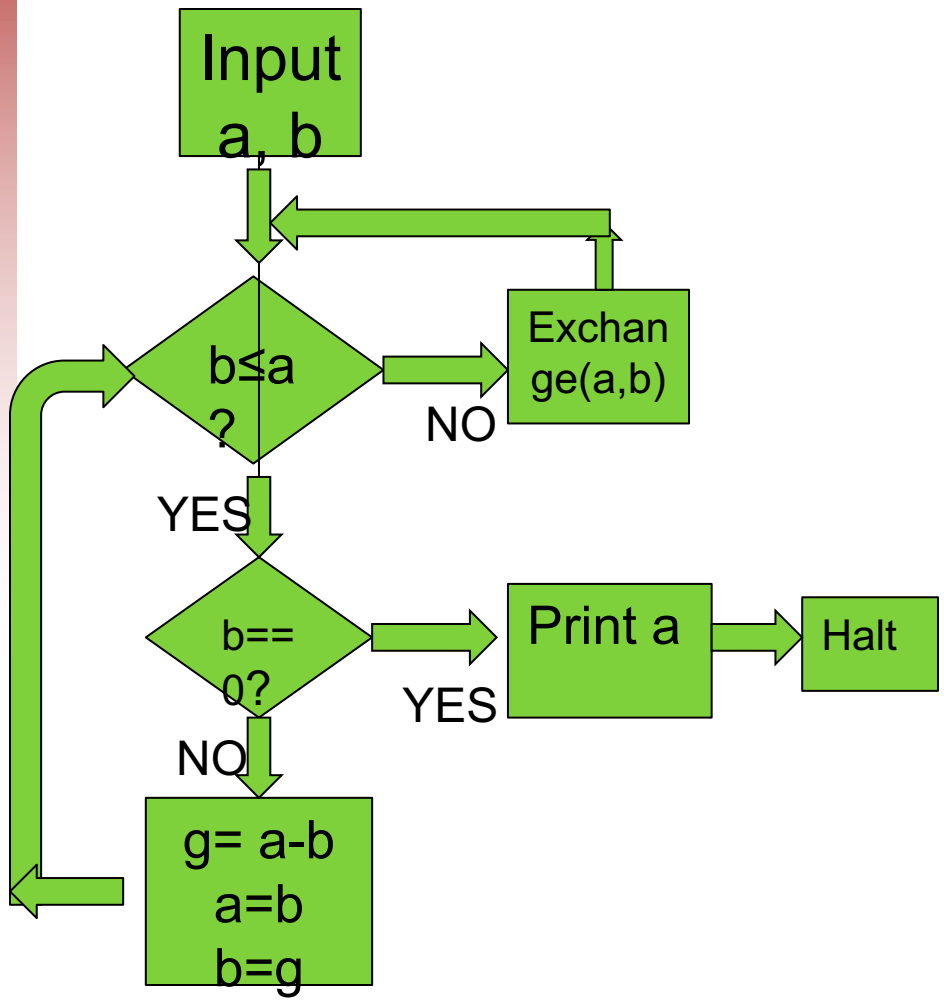
- How many times did we run in the loop? The fewer the better.

Multiple solutions and comparing them

- Multiple solutions are possible for the same problem.

- How many times did we run in the loop? The fewer the better.

Multiple solutions and comparing them



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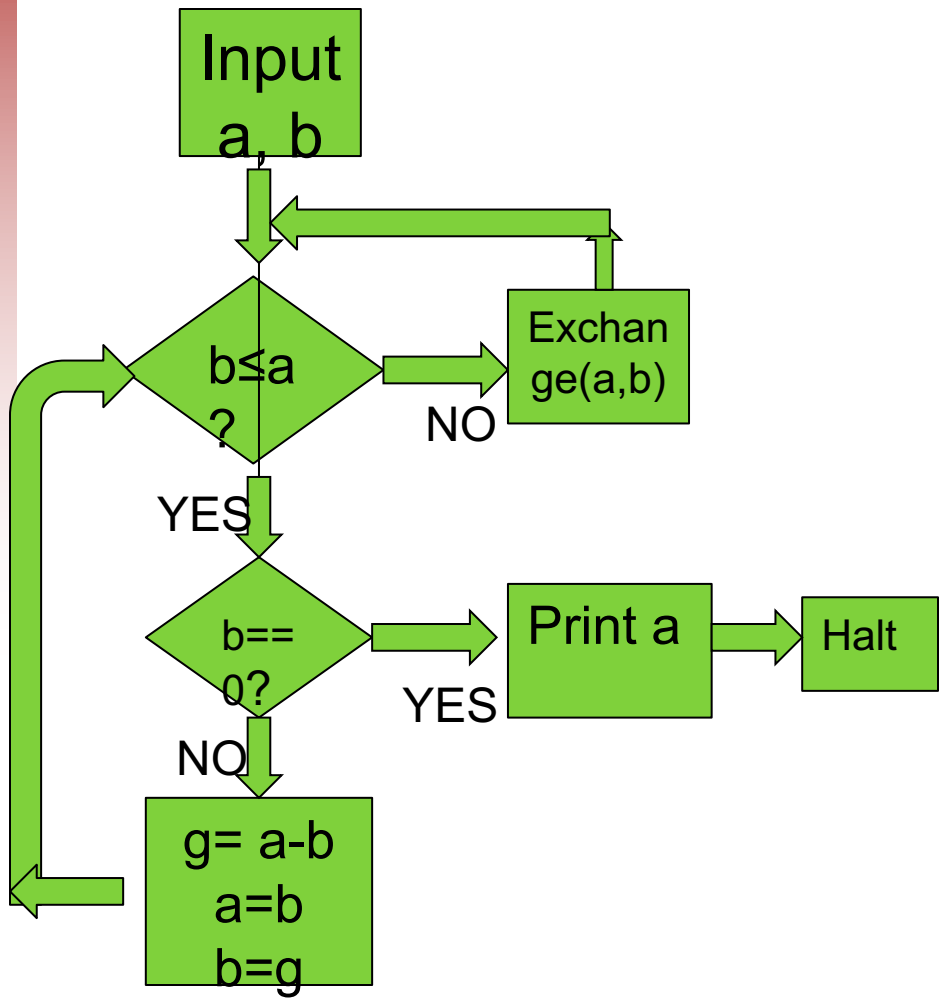
- Is the adjacent flowchart correct for gcd?
- How many times did we run in the loop? The fewer the better.

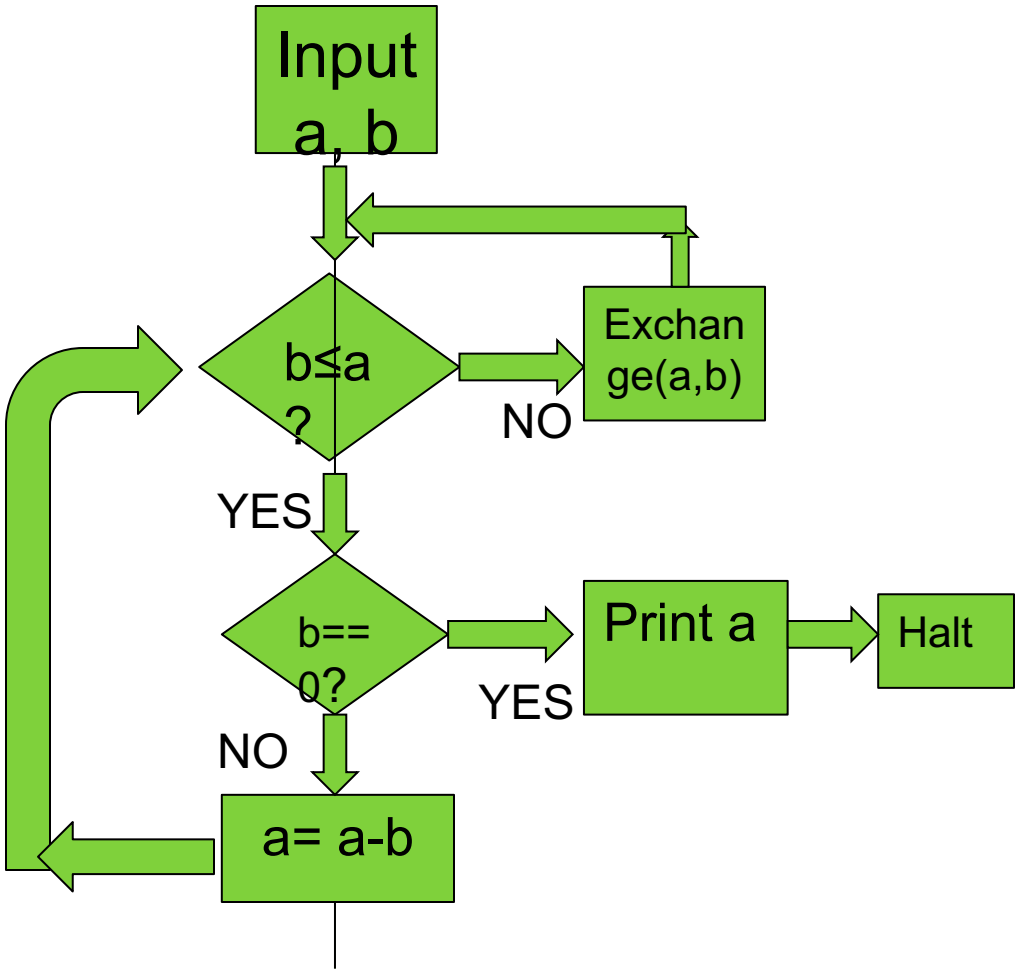
Multiple solutions and comparing them



- Multiple solutions are possible for the same problem.
- Is the adjacent flowchart correct for gcd?
- How many times did we run in the loop? The fewer the better.
- Is it seriously more: by an order of magnitude? Notion of complexity.

Another solution

- A (slower) alternative. How many times does the loop iterate?



Acknowledgments: This lecture slide is based on the material prepared by Prof. Sumit Ganguly, CSE, IIT Kanpur. The slide design is based on a template by Prof. Krithika Venkataramani.