Algorithmen in der Direkten Anwendung

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Bewegunsgleichungen

$$\rho \frac{D\mathbf{u}}{Dt} = -2\boldsymbol{\Omega} \times \mathbf{u} - \nabla p + \nabla \cdot \mathbf{T} + \rho \mathbf{g}$$
$$- \textit{Coriolis} - \textit{Druckgrad.} + \textit{Reibung} + \textit{Gravi.} \quad (1)$$

$$\frac{Dm}{Dt} = 0 \quad (2)$$

Approximationen/Manipulationen

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- Horizontale Geschwindigkeiten viel grer als vertikale
  $w = 0$
- vertikal integrieren
  $\int_H \mathbf{u} \, \mathrm{d}z = \mathbf{U}$

Viereckiges flaches Honigbecken

brig bleibt...

$$\rho \frac{D\mathbf{u}}{Dt} = -\,2\boldsymbol{\Omega} \times \mathbf{u} - \quad \nabla p \qquad + \nabla \cdot \mathbf{T} \quad + \quad \rho \mathbf{g}$$

$$\Downarrow$$

$$0 = -\,(f_0 + \beta y)\,\hat{\mathbf{z}} \times \mathbf{U} - gH\boldsymbol{\nabla} h + \tau_s - \tau_b + A\boldsymbol{\nabla}^2\mathbf{U} \tag{3}$$

$$\frac{D\mathbf{U}}{Dt} = 0 \tag{4}$$

Curl Operation

$$0 = \boldsymbol{\nabla} \times \left[\quad -\mathbf{f} \times \mathbf{U} - gH\boldsymbol{\nabla} h + \tau_s - \tau_b + A\boldsymbol{\nabla}^2\mathbf{U} \quad \right]$$

Curl Operation

$$
\begin{aligned}
0 =& \boldsymbol{\nabla} \times \left[\quad -\mathbf{f}(y) \times \mathbf{U} - gH\boldsymbol{\nabla} h + \tau_s - \tau_b + A\boldsymbol{\nabla}^2\mathbf{U} \quad \right] \\
= & \qquad - \beta V + \boldsymbol{\nabla} \times (\boldsymbol{\tau}_\mathbf{s} - \boldsymbol{\tau}_\mathbf{b}) + A\boldsymbol{\nabla} \times \boldsymbol{\nabla}^2\mathbf{U}
\end{aligned}
$$

Stromfunktion

$$\mathbf{U} = \underset{\neg}{\boldsymbol{\nabla}} \psi(x, y) \tag{5}$$

soll heien...

$$U = -\frac{\partial \psi}{\partial y}$$
$$V = \quad \frac{\partial \psi}{\partial x}$$

Stromfunktion

$$0 = -\beta V + \boldsymbol{\nabla} \times (\boldsymbol{\tau}_\mathbf{s} - \boldsymbol{\tau}_\mathbf{b}) + A\boldsymbol{\nabla} \times \boldsymbol{\nabla}^2 \mathbf{U}$$

$$\Downarrow$$

$$0 = -\beta \frac{\partial \psi}{\partial x} + \boldsymbol{\nabla} \times (\boldsymbol{\tau}_\mathbf{s} - \boldsymbol{\tau}_\mathbf{b}) + A\boldsymbol{\nabla}^4 \psi \qquad (6)$$

weitere Approximationen

- Bodenreibung proportional zu $\mathbf{U}$
  $\boldsymbol{\tau}_\mathbf{b} = R\mathbf{U}$

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  $\boldsymbol{\tau_b} = R\mathbf{U}$
- keine laterale Reibung!
  $A\boldsymbol{\nabla}^4\psi = 0$

weitere Approximationen

- Bodenreibung proportional zu $\mathbf{U}$
  $\boldsymbol{\tau}_{\mathbf{b}} = R\mathbf{U}$
- keine laterale Reibung!
  $A\boldsymbol{\nabla}^4\psi = 0$
- Windstress sinusoidale Funktion von y
  $\boldsymbol{\tau}_{\mathbf{s}} = F\cos\left(\pi y/B\right)$

weitere Approximationen

$$0 = -\beta\frac{\partial\psi}{\partial x} + \boldsymbol{\nabla}\times(\boldsymbol{\tau}_{\mathbf{s}} - \boldsymbol{\tau}_{\mathbf{b}}) + A\boldsymbol{\nabla}^4\psi$$

$$\Downarrow$$

$$\begin{aligned} 0 =& -\beta\frac{\partial\psi}{\partial x} - R\boldsymbol{\nabla}\times\mathbf{U} - F\boldsymbol{\nabla}\times\cos(\pi y/B) \\ =& -\beta\frac{\partial\psi}{\partial x} - R\boldsymbol{\nabla}^2\psi + \frac{F\pi}{B}\sin(\pi y/B) \\ R\boldsymbol{\nabla}^2\psi + \beta\frac{\partial\psi}{\partial x} =& F\alpha\sin(\alpha y) \end{aligned} \tag{7}$$

...mit $\alpha = \frac{\pi}{B}$

Analytische Lsung mglich

Inhomogene partielle Differentialgleichung 2er Ordnung.

$$R\nabla^2\psi + \beta\frac{\partial\psi}{\partial x} = F\alpha\sin(\alpha y)$$

Numerische Lsung

- variable Beckenform
- variabler Windstress

$$R\nabla^2\psi + \beta\frac{\partial\psi}{\partial x} = W(x, y) \tag{8}$$

Jacobi-Methode

Annahme: $\psi^{k=1} = \mathit{rand}$ berall

$$R\nabla^2\psi^k_{i,j} + \beta\frac{\partial}{\partial x}\psi^k_{i,j} - W_{i,j} = \Phi_{i,j} \tag{9}$$

$$R\nabla^2\psi^{k+1}_{i,j} + \beta\frac{\partial}{\partial x}\psi^{k+1}_{i,j} - W_{i,j} = 0 \tag{10}$$

a

a

a