Algorithmen in der Direkten Anwendung

NK

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Was ist ein *Eddy*?

detection and tracking-Algorithmus

Bewegunsgleichungen

$$\rho \frac{D\mathbf{u}}{Dt} = -\, 2\mathbf{\Omega} \times \mathbf{u} - \quad \nabla p \quad + \nabla \cdot \mathbf{T} \quad + \quad \rho \mathbf{g}$$
$$-\, Coriolis - Druckgrad. + Reibung + Gravi. \tag{1}$$

$$\frac{Dm}{Dt} = 0 \tag{2}$$

Approximationen/Manipulationen

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  Reynolds averaging

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- Horizontale Geschwindigkeiten viel grer als vertikale
  $w = 0$
- vertikal integrieren
  $\int_H \mathbf{u} \, \mathrm{d}z = \mathbf{U}$

Viereckiges flaches Honigbecken

brig bleibt...

$$\rho \frac{D\mathbf{u}}{Dt} = -\,2\boldsymbol{\Omega} \times \mathbf{u} - \quad \nabla p \qquad + \nabla \cdot \mathbf{T} \quad + \quad \rho \mathbf{g}$$

$$\Downarrow$$

$$0 = -\,(f_0 + \beta y)\,\mathbf{z} \times \mathbf{U} - gH\boldsymbol{\nabla} h + \tau_s - \tau_b + A\boldsymbol{\nabla}^2\mathbf{U} \tag{3}$$

$$\frac{D\mathbf{U}}{Dt} = 0 \tag{4}$$

Curl Operation

$$0 = \nabla \times \left[\quad -\mathbf{f} \times \mathbf{U} - gH\nabla h + \tau_s - \tau_b + A\nabla^2\mathbf{U} \quad \right]$$

Curl Operation

$$
\begin{aligned}
0 =& \boldsymbol{\nabla} \times \left[\quad -\mathbf{f}(y) \times \mathbf{U} - gH\boldsymbol{\nabla} h + \tau_s - \tau_b + A\boldsymbol{\nabla}^2\mathbf{U} \quad \right] \\
=& \qquad - \beta V + \boldsymbol{\nabla} \times (\boldsymbol{\tau}_\mathbf{s} - \boldsymbol{\tau}_\mathbf{b}) + A\boldsymbol{\nabla} \times \boldsymbol{\nabla}^2\mathbf{U}
\end{aligned}
$$

Stromfunktion

$$\mathbf{U} = \breve{\boldsymbol{\nabla}}\psi(x, y) \tag{5}$$

soll heien...

$$U = -\frac{\partial\psi}{\partial y}$$
$$V = \frac{\partial\psi}{\partial x}$$

Stromfunktion

$$0 = -\beta V + \boldsymbol{\nabla} \times (\boldsymbol{\tau}_\mathbf{s} - \boldsymbol{\tau}_\mathbf{b}) + A\boldsymbol{\nabla} \times \boldsymbol{\nabla}^2\mathbf{U}$$

$$\Downarrow$$

$$0 = -\beta\frac{\partial\psi}{\partial x} + \boldsymbol{\nabla} \times (\boldsymbol{\tau}_\mathbf{s} - \boldsymbol{\tau}_\mathbf{b}) + A\boldsymbol{\nabla}^4\psi \tag{6}$$

weitere Approximationen

- Bodenreibung proportional zu $\mathbf{U}$
  $\boldsymbol{\tau}_{\mathbf{b}} = R\mathbf{U}$

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  $\boldsymbol{\tau_b} = R\mathbf{U}$
- keine laterale Reibung!
  $A\boldsymbol{\nabla}^4\psi = 0$

weitere Approximationen

- Bodenreibung proportional zu $\mathbf{U}$
  $\boldsymbol{\tau_b} = R\mathbf{U}$
- keine laterale Reibung!
  $A\boldsymbol{\nabla}^4\psi = 0$
- Windstress sinusoidale Funktion von y
  $\boldsymbol{\tau_s} = F\cos(\pi y/B)$

weitere Approximationen

$$0 = -\beta\frac{\partial\psi}{\partial x} + \boldsymbol{\nabla}\times(\boldsymbol{\tau}_\mathbf{s} - \boldsymbol{\tau}_\mathbf{b}) + A\boldsymbol{\nabla}^4\psi$$

$$\Downarrow$$

$$\begin{aligned} 0 &= -\beta\frac{\partial\psi}{\partial x} - R\boldsymbol{\nabla}\times\mathbf{U} - F\boldsymbol{\nabla}\times\cos(\pi y/B) \\ &= -\beta\frac{\partial\psi}{\partial x} - R\boldsymbol{\nabla}^2\psi + \frac{F\pi}{B}\sin(\pi y/B) \\ R\boldsymbol{\nabla}^2\psi + \beta\frac{\partial\psi}{\partial x} &= F\alpha\sin(\alpha y) \end{aligned} \tag{7}$$

...mit $\alpha = \frac{\pi}{B}$

Analytische Lsung mglich

Inhomogene partielle Differentialgleichung 2er Ordnung.

$$R\nabla^2\psi + \beta\frac{\partial\psi}{\partial x} = F\alpha\sin(\alpha y)$$

Numerische Lsung

- variable Beckenform
- variabler Windstress

$$R\nabla^2\psi + \beta\frac{\partial\psi}{\partial x} = W(x, y) \quad (8)$$

Jacobi-Methode

Annahme: $\psi^{k=1} = \mathit{rand}$ berall

$$R\nabla^2\psi_{i,j}^{k} + \beta\frac{\partial}{\partial x}\psi_{i,j}^{k} - W_{i,j} = \Phi_{i,j}^{k} \tag{9}$$

$$R\nabla^2\psi_{i,j}^{k+1} + \beta\frac{\partial}{\partial x}\psi_{i,j}^{k+1} - W_{i,j} = 0 \tag{10}$$

a

a

a