# 1
# *Introduction*

The main purpose of this study is to investigate the dynamics of mesoscale ocean eddies on a global scale, *i.e.* to provide a statistical census on horizontal scale, lifetime and zonal drift speeds. By virtue of the geostrophic character of the scales of concern, such vortices implicate a local upheaval/depression of density surfaces, usually also including the sea surface [1]. The resultant *hills* and *valleys* in surface anomaly can be resolved by combining multiple satellite-altimetry signals (see fig. 1.1). One motivation of this study is to investigate whether the resolutions in space and time of such altimeter-derived products suffice to successfully track individual eddies over long periods of time and to precisely determine their horizontal extent and drift speed. The detection/tracking/analyzing procedure of individual eddies is done globally via an automated parallelized computer-program. To analyze the effects of different time/space-resolutions, a finer-grid SSH-product of a modern ocean-circulation model is subjected to the algorithm as well. Due to the inherently technical character of the matter, large parts of this text are dedicated to details of the algorithm [2]. Oceanographic results are treated in the results- and discussion-chapters. This chapter discusses the physics of mesoscale geostrophic turbulence and introduces a handful of relevant historical papers. Since focus is on horizontal scales, translational speeds and the comparison of results between the Aviso-altimetry product and SSH-data from the POP ocean model, sections generally focus on either of these

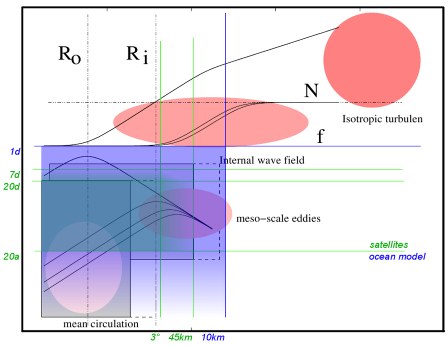


Figure 1.1: Resolutions for model vs satellite. Modified version from **?**.

[1] As in theory, baroclinic eddies have most of their energy in the first (surface-intensified) baroclinic mode (**?**).

[2] see the **??**.

three topics.

INTUITIVELY any translative motion of a vortex should stem from an asymmetry of forces as in an imperfectly balanced gyroscope wobbling around and translating across a table. The main effects that cause a quasi-geostrophic ocean eddy to translate laterally can be explained rel. easy heuristically:

Drift Speed 1.0.0.1: Lateral Density Gradient

Consider a mean layer-thickness gradient $\frac{\partial h}{\partial x} > 0$ somewhere in the high northern latitudes and a geostrophic, positive density anomaly within that layer. In other words, a high-pressure vortex or an anti-cyclonic eddy with length scale $L \approx \mathrm{L_R}$. Next consider a parcel of water adjacent to the eddy's northern flank of initially zero relative vorticity that is being entrained by the eddy. As the clockwise rotating eddy advects the parcel towards its eastern side, the water-column comprising said fluid will be stretched vertically as it is advected towards larger depths. In order to maintain total vorticity a small new relative-vorticity term is introduced via term $C$ in equation (**??**). Since the vorticity budget is dominated by the planetary component, this new term has sign of $f$ *i.e.* **positive**. The opposite effect holds for a parcel advected towards the western side. Then, vortex *squeezing* leads to a new **negative** relative-vorticity term. Hence water masses on both sides of the thickness gradient acquire rotation that slowly pushes the eddy in the direction $-f \times \frac{\partial h}{\partial x}$ (in this case south). Note that since vorticity is dominated by the planetary component, the rotational sense of the eddy is irrelevant here. I.e. water columns stretched [squeezed], will always lead to new $\omega$ with sign of $f$ $[-f]$.

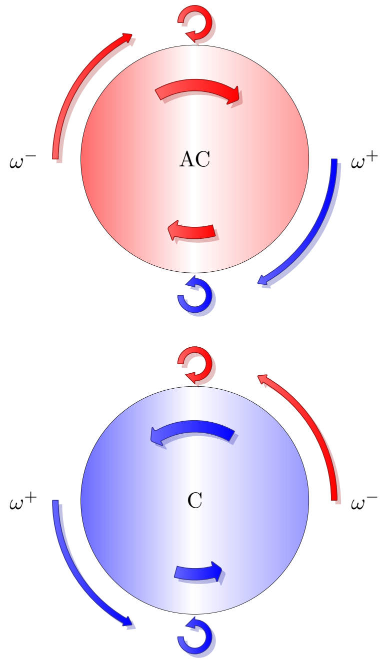



Figure 1.2: Bottom [Top]: Northern hemisphere [anti]cyclone. Blue [red] color indicates presence/production of positive [negative] relative vorticity. Advection of adjacent water masses leads to a westward drift, irrespective of the eddy's sign (see drift speed box 1.0.0.2). Inside, the discrepancy in swirl strength between north and south requires another (smaller) zonal drift term, which is eastward [westward] for [anti]cyclones.

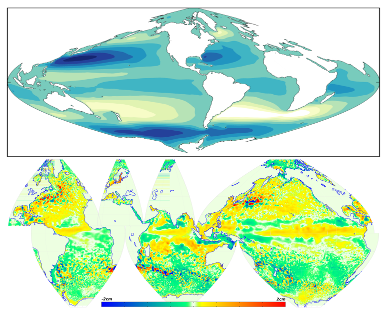


Figure 1.3: top: Stommel's equation $\mathrm{F}_{bottom} - \mathrm{F}_{surface} = -V\beta$ with constant eddy viscosity. bottom: POP eddy-resolving model snapshot with SSH mean of one year subtracted.

Drift Speed 1.0.0.2: Planetary Lift

Assume now that $\beta L$ be comparable or larger even than $f + \omega$ from the previous example. Then, independent of layer-thickness, all fluid adjacent to the eddy on its northern and southern flanks will be transported meridionally, thereby be tilted with respect to $\mathbf{\Omega}$ and hence acquire relative vorticity to compensate. All fluid transported north [south] will balance the increase in planetary vorticity with a decrease [increase] in relative vorticity. This is again independent of the eddy's sense and in this case also independent of hemisphere since $\frac{\partial f}{\partial y} = \beta > 0$ for all latitudes. The result is that small negative vortices to the northern and small positive vortices to the southern flank of eddies will push them west. This effect is directly analogous to the *conveyer-belt*-like westward creeping of Rossby waves. Theory shows that at first approximation it is indeed equivalent (see *e.g.* section 1.0.2).

Drift Speed 1.0.0.3: Eddy-Internal $\beta$-Effect

In the later case clearly particles within the vortex undergo a change in planetary vorticity as well. Or from a different point of view, since orbital speed $u_o$ is proportional to $-\nabla p/f$, and noting that the pressure gradient is the driving force here and hence fix at first approximation, particles drifting north [south] will be subjected to an increase [decrease] in coriolis force irrespective of eddy sense. Yet, due to conservation of mass, $u_o$ cannot increase [decrease] when moving north [south]. The center of volume must hence be shifted west for an anti-cyclone and east for a cyclone. Or the other way around: The only way to compensate for the discrepancy in Coriolis acceleration north and south, whilst maintaining constant eddy-relative particle speed, is to superimpose a zonal drift velocity so that net particle velocities achieve symmetric Coriolis acceleration. Or yet from another perspective: As particles within an anti-cyclone move north on its western side, they are subjected to an increase in Coriolis force, no longer entirely balanced by the pressure gradient. Hence there will be a tendency for water to *climb up* the thickness gradient, effectively drawing water from outside into the eddy thereby creating a negative pressure anomaly to the western side of the eddy. Water flowing south on the eastern side undergoes a decrease in Coriolis force, thereby creating a relaxation of the pressure gradient. In other words, the eddy effectively *pulls* itself west on its western side and *pushes* itself west on its eastern side. In the case of a cyclone the increase in $f$ as water flows north on its eastern side pushes the negative density anomaly east and water on its western side, relaxing the (negative in x-direction) pressure gradient pulls in the thicker layer to the west, thereby pushing the cyclone east.

THE following discusses a handful of selected historical papers that are concerned with either the theory of mesoscale eddies or with the detection/tracking of eddies from SSH data.

### 1.0.1 *Waves and Turbulence on a $\beta$-Plane* [3]

Rhines investigated the effect of the $\beta$-plane on the inverse energy cascade of quasi-2-dimensional atmospheric and oceanic turbulence. At constant $f$, energy should be cascaded to ever-larger scales until halted by the scale of the domain. This is clearly not the case, as no storm has ever grown to global scale. The presence of a meridional restoring force creates a critical scale beyond which the *turbulent migration of the dominant scale nearly ceases ...*. Rossby waves are excited which would in theory eventually give way to alternating zonal jets of width $\frac{U}{\beta}$. This scale was later coined the Rhines Scale $L_\beta$.

[3] Rhines, Peter B. 1974. Waves and turbulence on a beta-plane. *J. Fluid Mech.*, **69**(03), 417

### 1.0.2 *Westward Motion of Mesoscale Eddies* [4]

**?** already noted that the $\beta$-effect causes a mass-imbalance in planetary vortices that, if not met by an asymmetry in shape must lead to westward propagation.

**?** derived that the $\beta$-effect results in a net meridional force on the integrated mass of the vortex, which in balance with the Coriolis acceleration shoves cyclones eastward and anti-cyclones westward. They also explained how displaced water outside the eddy's perimeter causes a much stronger westward component, with the result that all eddies propagate westward irrespective or rotational sense.

The westward drift was also derived in various forms by *e.g.* **???**.

[4] Cushman-Roisin, Benoit, Tang, Benyang, & Chassignet, Eric P. 1990. Westward motion of mesoscale eddies. *J. Phys. ...*, **20**(5), 758–768

THE paper by **?** is particularly helpful to understand where the two components of westward drift come from. By scaling the terms in the one-layer primitive equations by their respective dimensionless numbers, integrating the interface-displacement caused by the eddy over the eddy's domain and applying mass continuity they derive for the location $(X, Y)$ of an eddy's centroid[5]:

[5] $\langle\rangle \equiv \frac{1}{A}\int_A \, \mathrm{d}A$

$$\Pi X_{tt} - Y_t = \mathrm{L}_\mathrm{R}^1 T\beta \langle yv \rangle + L\frac{\beta}{f}\langle y\eta v\rangle \tag{1.1}$$

$$\Pi Y_{tt} - X_t = -\mathrm{L}_\mathrm{R}^1 T\beta \langle yu \rangle - L\frac{\beta}{f}\langle y\eta u\rangle \tag{1.2}$$

where $\Pi = 1/f_0 T$.

Hence, independent of balance of forces the eddy's center of mass describes inertial oscillations [6] on the $f$-plane, even in the absence of $\beta$. Using geostrophic values for $\boldsymbol{u}$ and returning to

[6] compare to *harmonic oscillator*

dimensional variables equation (1.1) can be cast into:

$$\begin{aligned}\frac{\partial X}{\partial t} &= -\frac{\beta g'}{f_0^2}\frac{\int_A H\eta \, \mathrm{d}A + \int_A \eta^2/2 \, \mathrm{d}A}{\int_A \eta \, \mathrm{d}A} \\ &= -\beta\left(\frac{NH}{f_0}\right)^2 - \frac{\int_A \eta^2/2 \, \mathrm{d}A}{\int_A \eta \, \mathrm{d}A} \\ &= \frac{\partial \omega_{long}}{\partial k} - u_{internal}\end{aligned} \tag{1.3}$$

THE first term of the RHS of equation (1.3) represents the *planetary lift* [7], which is identical to the zonal group velocity of long Rossby waves (**?**). The second term $u_{internal}$ represents the *eddy-internal β-effect* (see drift speed box 1.0.0.3). Note that the first term is always westwards, while the second has sign of $-\eta$, *i.e.* westward for anti-cyclones and eastward for cyclones and that the first is always much larger than the second.

[7] see drift speed box 1.0.0.2 from **??**

### *1.0.3 Early Altimeter Data*

THE advent of satellite altimetry, which Walter Munk called *the most successful ocean experiment of all time* (**?**), finally allowed for global-scale experimental investigations of oceanic planetary phenomena on long time- and spatial scales. Among others, **???** were the first to use satellite-data to present evidence for the existence of Rossby waves and their westward-migration in accord with theory. Surprisingly all of the observations found the phase speeds to be 1 to 1.5 times larger than what theory predicted. Several theories to explain the discrepancy were presented. E.g. **?** argued that the discrepancy was caused by mode-2-east-west-mean-flow velocities. Interestingly it appears that hitherto, the relevant altimeter signal was mainly associated with linear waves. Non-linearities are rarely mentioned in the papers of those years. Probably simply due to the fact that the turbulent character of much of the mesoscale variability was still obscured by the poor resolution of the first altimeter products.

### *1.0.4 SSH Altimeter Data* [8]

From the beginning of satellite altimetry **?** have invested tremendous effort to thoroughly analyze the data in terms of Rossby waves and geostrophic turbulence. At the time of the **?** paper only 3 years of Topex/Poseidon data alone had been available, which led them to interpret the data mainly in terms of Rossby waves. Once the merged Aviso T/P and ERS 1/2 (**?**) was released 7 years later, **?** presented a new analysis that was based on an automated eddy-tracking algorithm using the geostrophic Okubo-Weiss parameter [9,10]. For the first

[8] Chelton, Dudley B., Schlax, Michael G., Samelson, Roger M., & de Szoeke, Roland a. 2007. Global observations of large oceanic eddies. *Geophys. Res. Lett.*, **34**(15), L15606; and Chelton, Dudley B., Schlax, Michael G., & Samelson, Roger M. 2011. Global observations of nonlinear mesoscale eddies. *Prog. Oceanogr.*, **91**(2), 167–216

[9] see **??**

[10] see **??**

time satellite data was resolved sufficiently fine to unveil the dominance of *blobby structures rather than latitudinally β-refracted continuous crests and troughs* that had hitherto been assumed to characterize the large-scale SSH topography. They presented results of a refined algorithm in their **?** paper, in which they abandoned the Okubo-Weiss concept and instead identified eddies via closed contours of SSH itself [11]. The improved algorithm and longer data record now allowed them to separate the non-linear eddy activity from the larger-scale Rossby waves. They find that the vast majority of extra-tropical westward propagating SSH variability does indeed consist of coherent, isolated, non-linear, mesoscale eddies that propagate about 25% slower [12] than the linear waves. Apart from this though they find little evidence for any dispersion in the signal, neither do they find evidence for significant meridional propagation, as should be found for Rossby waves (**?**, chapter 8.2.1). In agreement with **?**, they find this eddy-dominated regime to fade towards the equator, giving way to the characteristic Rossby wave profile. Almost all of their eddies propagate westwards. Those eddies that are advected eastwards by *e.g.* the ACC show significantly shorter life-times than those that are not. For more detail on their results and a discussion of the limitations of eddy-tracking via satellites (see **??**).

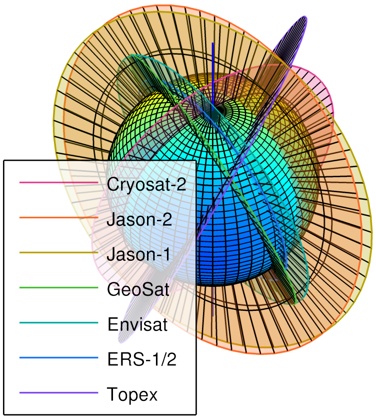
[11] note that geostrophic $O_W$ is a second derivative of SSH and thus exacerbates noise in the SSH data.

[12] pointing to dispersion.

**NOTE: i took out...**

- **??????????????**

| | POP | merged T/P - ERS-1 |
|---|---|---|
| dx | 0.1° | 0.25° |
| dt | $1d$ | $7d$ |
| $\log_{10} 2$ filter cutoff | – | 2° by 2° |
| z-levels | 42 | 1 |
| variables | SSH, S, T, u/v/w, tracers etc | SSH |
| pot. interpolation artifacts | – | yes |
| reality | no | yes |




THE latest Aviso SSH data from satellites features impressive accuracy, constancy and resolutions in both space and time. This is achieved by collecting all of the data from all of the altimeter-equipped satellites available at any given moment for any given coordinate. This conglomerate of highly inhomogeneous data is then subjected to state-of-the-art interpolation methods to produce a spatially and temporally coherent product. One satellite alone is not sufficient to adequately resolve mesoscale variability globally.

E.g. the Topex/Poseidon satellite had a ground repeat track orbit of 10 days and circled the earth in 112 minutes or $\approx 13$ times a day with a swath width of 5 km. Hence it drew $\sim 26$ 5-km-wide stripes onto the globe every day. The orbit's precession is such that this pattern is then repeated after 10 days, which means that at the equator only $10 \times 26 \times 5 = 1300$ km of the $2\pi \times 6371 = 40\,000$ km get covered, *i.e.* 3.25%. At every 10 d time-step, on average, effectively $(40000 - 1300)/26 = 1490$ km are left blank in-between swaths on the equator. This is why, no matter how fine the resolution within the swath at one moment in time may be, the spatial resolution is so coarse. The merged ERS-1/Topex-data as used by **?** has a time step of 7 days. Assuming eddy drift speeds of $u_e = \mathcal{O}\left(10^{-1}\right)$ m/s implies a distance traveled per time step of $L_{\delta t} \approx 60$ km. **?** estimate their effective spatial resolution as $\delta x \approx 40$ km. Eddies of smaller scale are not resolved.

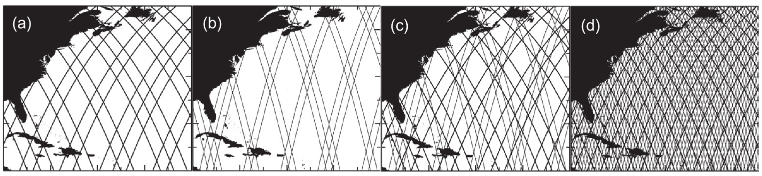



Figure 1.4: The ground track patterns for the 10-day repeat orbit of T/P and its successors Jason-1 and Jason-2 (thick lines) and the 35-day repeat orbit of ERS-1 and its successors ERS-2 and Envisat (thin lines). (a) The ground tracks of the 10-day orbit during a representative 7-day period; (b) The ground tracks of the 35-day orbit during the same representative 7-day period; (c) The combined ground tracks of the 10-day orbit and the 35-day orbit during the 7-day period; and (d) The combined ground tracks of the 10-day orbit and the 35-day orbit during the full 35 days of the 35-day orbit. (sic) (**?**)

TRACKING a single eddy from one time-step to the next is complicated by the sheer abundance of eddies at any given point in time and the fact that eddy activity is usually concentrated into regions of strong geostrophic turbulence. The ambiguities in matching the eddies from the old time-step to those of the new one might cause aliasing effects in the final statistics.

THE translational speeds ($\mathcal{O}\left(10^{1}\right)$ km/d) of eddies are not really the problem here, as they usually drift slow enough to not cover more than 1 grid node per 7 day time step. The issue are those areas where eddies are born, die and merge. According to **?**, instabilities within the ACC grow at rates of up to $1/(2\text{days})$, which means that at one time-step up to 3 eddies have emerged and equally many died for every eddy identified within such region. The ground-repeat-frequency of a satellite can of course not be set arbitrarily. Especially when the satellite is desired to cover as far north and south as possible, whilst still being subjected to just the right torque from the earth's variable gravitational field to precess at preferably a sun-synchronous frequency *i.e.* $360°/year$ (**?**). Neither can the satellite's altitude be chosen arbitrarily. If too low, the oblateness of the earth creates too much eccentricity in the orbit that can no longer be *frozen* [13]. Another problem could be potential inhomogeneity in the merged data in time dimension, since data of old and current missions are lumped together into one product. This is why **?** opted against the finest resolution available and instead went for a product that had the most satellites merged in unison for the longest period of time.

[13] minimizing undulating signals in altitude by choosing the right initial values (**?**)

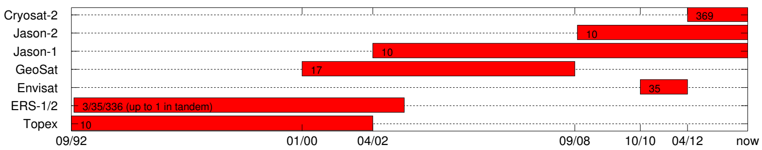



Figure 1.5: Length of mission. Numbers are orbit-period in days.

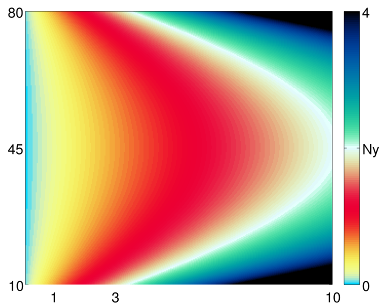



Figure 1.6: $n(\phi, \mu)$. Ny $\equiv 2$ *i.e.* the Nyquist frequency.

THE surface velocities inferred from altimetry are the geostrophic components only, which should suffice to *e.g.* determine the non-linearity and kinetic energy of an eddy for almost all regions, but less so for *e.g.* the western boundary currents.

**Box 1: Horizontal Resolution**

Assume Bu $= 1$, so that $L = NH/f$ and $NH = a/10d$ (corresponds to $L(\phi = 30°) = 100\text{km}$), a model resolution of $1°/\mu$ and that the eddy diameter is twice the Rossby radius. Then, how many grid notes $n$ fit into one eddy as a function of latitude?

$$
\begin{aligned}
n\delta x &= L \\
n \cos\phi \frac{2\pi}{360\mu} a &= \frac{2NH}{f} \\
n &= \frac{NH\mu\Omega}{\sin 2\phi} \frac{180}{\pi^2 a}
\end{aligned}
\tag{1.4}
$$

In this flat-bottom, constant $\rho_z$, Mercator-gridded model the worst eddy-resolution is interestingly at mid-latitude (see fig. 1.6).

The finer resolution of the POP data in space and time should certainly yield more precise results. It must be kept in mind though that by using the model data, what one analyses is of course just that - a *model*. Baroclinic geostrophic space/time scales depend crucially on *e.g.* the vertical density structure (see section 1.0.2, **?**), which is resolved only poorly in the model. A useful comparison among satellite/model results should hence be tricky.