1

a: amplitude

$\mathfrak{r}$: *Gaussian radius* i.e. twice Gaussian *std*

H: Gaussian amplitude

A: determined area

r: determined radius

with $A = \pi r^2$ and $V = 2\pi\sigma^2$

$$H - a = H \exp\left(-\frac{A}{2\pi\left(\mathfrak{r}/2\right)^2}\right) \tag{1}$$

$$\tag{2}$$

and

$$H - a = H \exp\left(-\frac{A}{2\pi\left(\mathfrak{r}/2\right)^2}\right) \tag{3}$$

$$1 - a/H = \exp\left(-\frac{2A}{\pi\mathfrak{r}^2}\right) \tag{4}$$

$$\pi\mathfrak{r}^2 = \left(-\frac{2A}{\ln\left(1 - a/H\right)}\right) \tag{5}$$

$$\mathfrak{r} = \sqrt{-\frac{2A}{\pi\ln\left(1 - a/H\right)}} \tag{6}$$

$$\tag{7}$$

with $\gamma = -\frac{1}{2\sigma^2}$

$$g(x, y) = h \exp\left(\gamma\left(y^2 + x^2\right)\right) \tag{8}$$

$$V = \int_{-r}^{r}\int_{-r}^{r} g \;\mathrm{d}x\;\mathrm{d}y \tag{9}$$

$$
\begin{aligned}
\lambda(r) &= \frac{V}{A^{1.5}} && (10)\\
&= \frac{\int_{-r}^{r}\int_{-r}^{r} g \,\mathrm{d}x \,\mathrm{d}y}{(\pi r^2)^{3/2}} && (11)\\
&= \left(r\sqrt{\pi}\right)^{-3} \int_{-r}^{r}\int_{-r}^{r} g \,\mathrm{d}x \,\mathrm{d}y && (12)\\
&= \left(r\sqrt{\pi}\right)^{-3} \int_{-r}^{r} \left[\frac{g}{2\gamma x}\right]_{-r}^{r} \mathrm{d}y && (13)\\
&= \left(r\sqrt{\pi}\right)^{-3} \int_{-r}^{r} \frac{\exp\left(\gamma\left(y^2+r^2\right)\right)}{2\gamma r} - \frac{\exp\left(\gamma\left(y^2+r^2\right)\right)}{-2\gamma r} \,\mathrm{d}y && (14)\\
&= \left(r\sqrt{\pi}\right)^{-3} \int_{-r}^{r} \exp\left(\gamma\left(y^2+r^2\right)\right)\left(\frac{1}{\gamma r}\right) \,\mathrm{d}y && (15)\\
&= \left(\gamma r^4 \pi^{3/2}\right)^{-1} \int_{-r}^{r} g(r,y) \,\mathrm{d}y && (16)\\
&= \left(\gamma r^4 \pi^{3/2}\right)^{-1} \int_{-r}^{r} g(r,y) \,\mathrm{d}y && (17)\\
&= h\left(\gamma r^4 \pi^{3/2}\right)^{-1} \frac{\exp\left(\gamma\left(y^2+r^2\right)\right)}{\gamma r} && (18)\\
&= h\left(\gamma^2 r^5 \pi^{3/2}\right)^{-1} g(r,r) && (19)\\
&= h^2 \pi^{2/3} \frac{\exp\left(\gamma\left(2r^2\right)\right)}{\gamma^2 r^5} && (20)
\end{aligned}
$$

$$
= \frac{\exp\left(2r^2\right)}{r^5} \qquad (21)
$$