

Chapter 1

The Algorithm

This section walks through the algorithm step by step, so as to explain which methods are used and how they are implemented. The idea is that the code from step S00..on can only accept one well defined structure of data. In earlier versions the approach was to write code that would adapt to different types of data automatically. All of this extra adaptivity turned out to visually and structurally clog the code more than it did offer much of a benefit. The concept was therefor reversed. `S00_prep_data` can be altered to produce required output. Yet, there should be no need to adapt any of the later steps in any way. All input parameters are to be set in `INPUT.m` and `INPUTxxx.m`.

1.1 Step S00: Prepare Data

```
function S00_prep_data
```

Before the actual eddy detection and tracking is performed, SSH-, latitude- and longitude-data is extracted from the given data at desired geo-coordinate bounds and saved as structures in the form needed by the next step (S01). This step also builts the file `window.mat` via `GetWindow3` which saves geometric information about the input and output data as well as a cross-referencing index-matrix which is used to reshape all *cuts* to the user defined geo-coordinate-geometry. The code can handle geo-coordinate input that crosses the longitudinal seam of the input data. E.g. say the input data comes in matrices that start and end on some (not necessarily strictly meridional) line straight across the Pacific and it is the Pacific only that is to be analyzed for eddies, the output maps are stitched accordingly. In the zonally continuous case i.e. the full-longitude case, an *overlap* in x-direction across the *seam-meridian* of the chosen map is included so that contours across the seam can be detected and tracked across it. One effect is that eddies in proximity to the seam can get detected twice at both zonal ends of the maps. The redundant *ghost*-eddies get filtered out in `S05_track_eddies`.

1.2 Step S01b: Find Mean Rossby Radii and Phase Speeds

```
function S01b_BruntVaisRossby
```

This function...

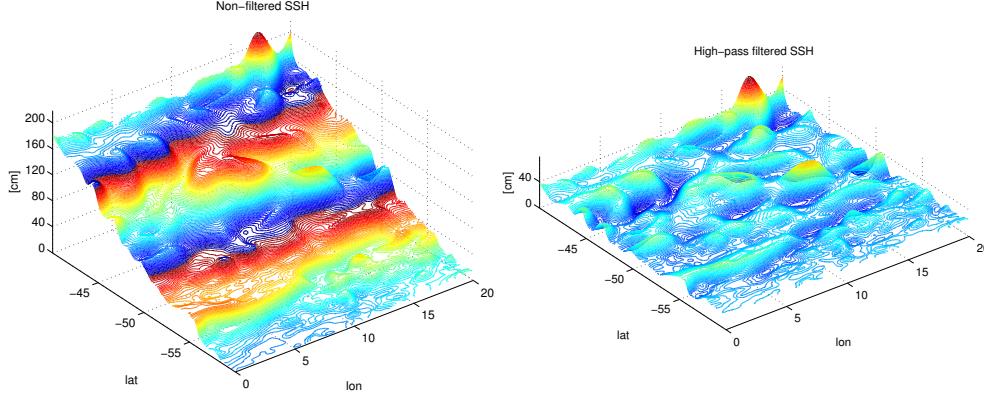


Figure 1.1: SSH with mean over time subtracted.

- – ...calculates the pressure $P(z, \phi)$ in order to...
 - ...calculate the Brunt-Väisälä-Frequency according to $N^2(S, T, P, \phi) = -\frac{g(\phi)}{P} \frac{\partial \rho(S, T, P)}{\partial z}$ in order to...
- – ...integrate the Rossby-Radius $L_R = \frac{1}{\pi f} \int_H N dz$ and ...
 - apply the long-Rossby-Wave dispersion relation to found L_R to estimate Rossby-Wave phase-speeds $c = -\frac{\beta}{k^2 + (1/L_r)^2} \approx -\beta L_R^2$

The 3-dimensional matrices (S and T) are cut zonally into slices which then get distributed to the threads. This allows for matrix operations for all calculations which would otherwise cause memory problems due to the immense sizes of the 3d-data ¹.

1.3 Step S02: Calculate Geostrophic Parameters

```
function S02_infer_fields
```

This step reads the cut SSH data from `S00_prep_data` to perform 2 steps:

1. Calculate a mean over time of $SSH(y, x)$.
2.
 - use one of the files' geo-information to determine f , β , g and the ratio g/f .
 - calculate geostrophic fields from SSH gradients.
 - calculate deformation fields (vorticity, divergence, stretch and shear) via the fields supplied by the last step.
 - calculate O_w .
 - Subtract the mean from step 1 from each $SSH(t)$ to filter out persistent SSH-gradients e.g. across the Gulf-Stream.

1.4 Step S03: Find Contours

```
function S03_contours
```

The sole purpose of this step is to apply MATLAB's `contourc.m` function to the SSH

¹E.g. the pop data has dimensions $42 \times 3600 \times 1800$.

data. It simply saves one file per time-step with all contour indices appended into one vector². The contour intervals are determined by the user defined increment and range from the minimum- to the maximum of given SSH data.

The function `initialise.m`, which is called at the very beginning of every step, here has the purpose of rechecking the *cuts* for consistency and correcting the time-steps accordingly (i.e. when files are missing). `initialise.m` also distributes the files to the threads i.e. parallelization is in time dimension.

1.5 Step S04: Filter Eddies

```
function S04_filter_eddies
```

Since indices of all possible contour lines at chosen levels are available at this point, it is now time to subject each and every contour to a myriad of tests to decide whether it qualifies as the outline of an eddy as defined by the user input threshold parameters.

Reshape for Filtering and Correct out of Bounds Values

```
function eddies2struct
function CleanEddies
```

In the first step the potential eddies are transformed to a more sensible format, that is, a structure `Eddies(EddyCount)` where `EddyCount` is the number of all contours. The struct has fields for level, number of vertices, exact i.e. interpolated coordinates and rounded integer coordinates.

The interpolation of `contourc.m` sometimes creates indices that are either smaller than 0.5 or larger than $N + 0.5$ ³ for contours that lie along a boundary. After rounding, this seldomly leads to indices of either 0 or $N + 1$. These values get set to 1 and N respectively in this step.

Descent/Accend Water Column and Apply Checks

```
function walkThroughContsVertically
```

The concept of this step is a direct adaption of the algorithm described by ?. It is split into two steps, one for anti-cyclones and one for cyclones. Consider e.g. the anti-cyclone situation. Since all geostrophic anti-cyclones are regions of relative high pressure, all anti-cyclones⁴ effect an elevated `SSH` i.e. a *hill*. The algorithm ascends the full range of `SSH` levels where contours were found. Consider an approximately Gaussian shaped AC that has a peak `SSH` of say 5 increments larger than the average surrounding waters. As the algorithm approaches the sea surface from below, it will eventually run into contours that are closed onto themselves and that encompass the AC. At first these contours might be very large and encompass not only one but several ACs and likely also cyclones, but as the algorithm continues upwards found contour will get increasingly circular, describing some outer *edge* of the AC. Once the contour and its interior pass all of the tests the algorithm will decide that an AC was found and write it and all its parameters to disk. The AC's region i.e. the interior of the contour will be flagged from here on. Hence any inner contour further up the water column will not pass the tests. Once all AC's are found for a given time-step, the `SSH` flags get reset and the entire procedure is repeated only this time *descending* the `SSH`-range to find cyclones. The tests for cyclones and anti-cyclones are identical

²see the MATLAB documentation.

³where N is the domain size

⁴henceforth abbreviated AC

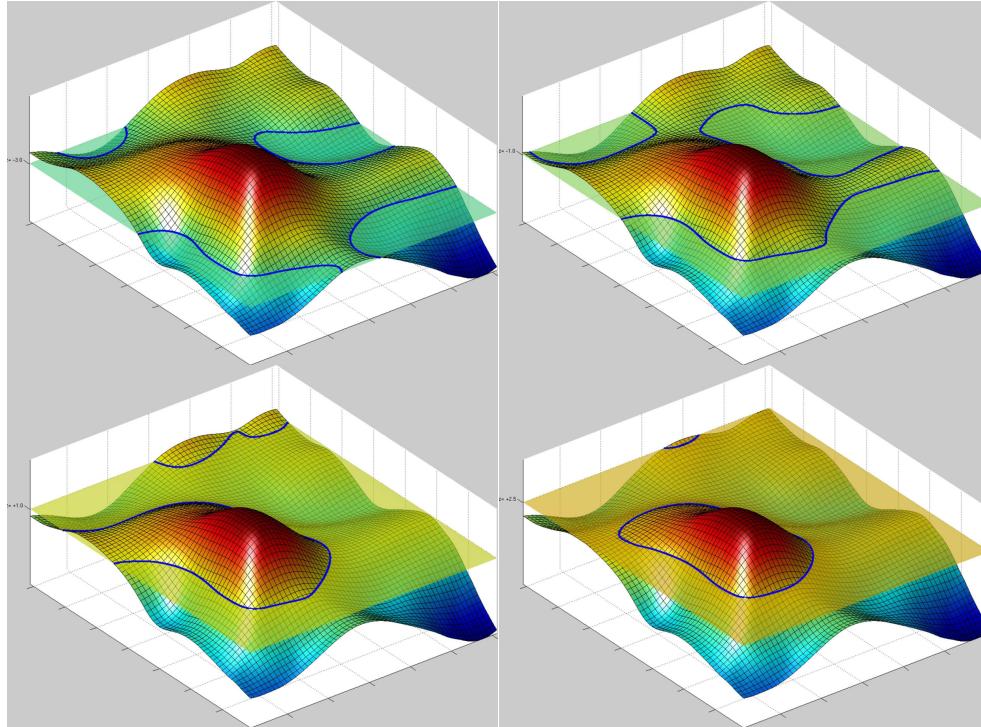


Figure 1.2: The detection method.

except for a factor -1 where applicable. In the following the most important steps of the analysis are outlined.

NaN-Check Contour

```
function CR_RimNan
The first and most efficient test is to check whether indices of the contour are already flagged. Contours within an already found eddy get thereby rejected immediately.
```

Sub-Window

```
function get_window_limits
function EddyCut_init
```

For further analysis a sub-domain around the eddy is cut out of the SSH data. These functions determine the indices of that window and subtract the resultant offset for the contour indices.

Logical Mask of Eddy Interiour

```
function EddyCut_mask
```

Basically this function creates a [flood-fill](#) logical mask of the eddy-interior. This is by far the most calculation intensive part of the whole filtering procedure. A lot more time was wasted on attempting to solve this problem more efficiently than time could have been saved would said attempts have been successful. The current solution is basically just MATLAB's `imfill.m`, which was also used in the very first version of 09/2013. EDIT: `imfill.m` was replaced by using `inpoly.m` to determine

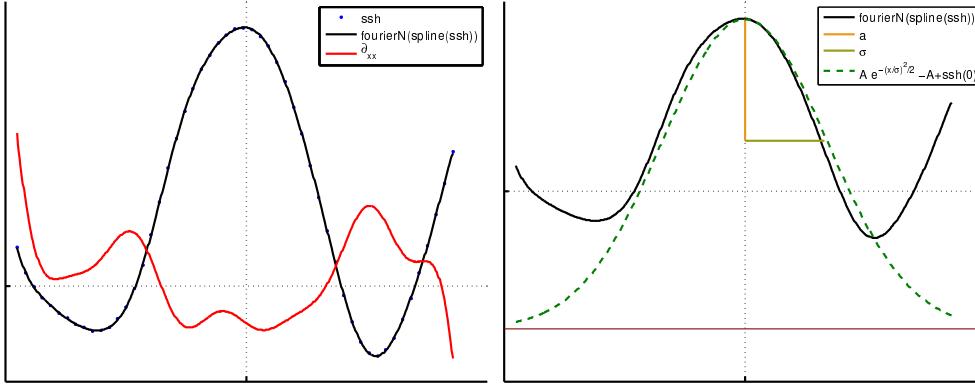


Figure 1.3: Left: Fourier-fit of an eddy from POP-SSH-data and the 2nd differential thereof. Right: Theoretical Gauss-Shape built from the resulting *standard-deviation* i.e. σ and amplitude.

which indices lie within the contour-polygon. This method seems to be more exact at determining whether the inside-part of one grid cell (with respect to the smooth, spline-interpolated contour) is larger than the outside part or not.

Sense

```
function CR_sense
```

All of the interior SSH values must lie either above or below current contour level, depending on whether anti-cyclones or cyclones are sought.

Area

```
function getArea
```

The main goal here is to determine the area encompassed by the exact coordinates of the contour. It does so via MATLAB's polyarea function. This area is not related to the scale σ that is determined in [section][5][1]1.5. It is however the relevant scale for the determination of the isoperimetric quotient in [section][5][1]1.5.

If the respective switch is turned on, this function also checks that the area of found contour does not surpass a given threshold which in turn is a function of L_R . Since L_R gets very small in high latitudes a lower bound on the L_R used here should be set as well. This is especially important for the southern ocean where L_R gets very small while the strong meso-scale turbulence of the Antarctic circumpolar current results in an abundance of relatively large eddies as far south as $60^\circ S$ and beyond.

Cirumference

```
function EddyCircumference
```

Circumference e.g. line-length described by the contour. This is the other parameter needed for [section][5][1]1.5. This is however neither related to the actual eddy scale determined in [section][5][1]1.5.

Shape

```
function CR_Shape
```

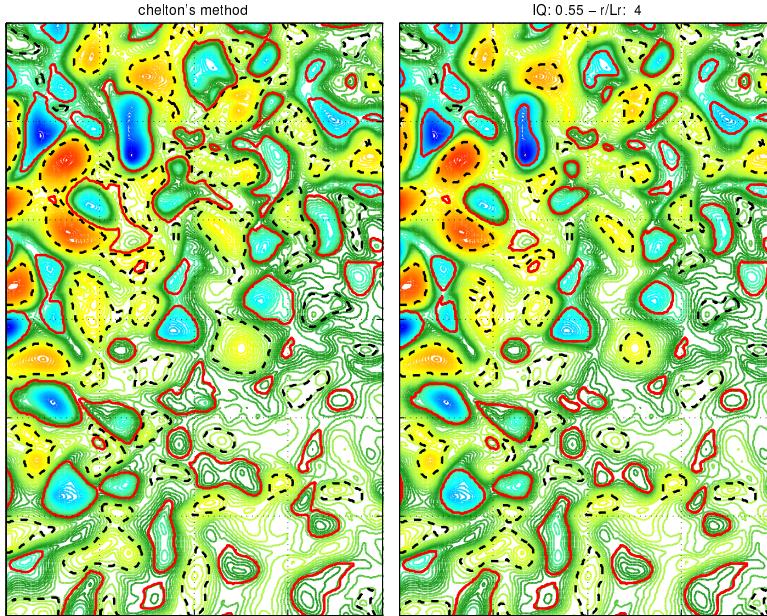


Figure 1.5: TODO

This is the crucial part of deciding whether the object is *round enough*. A perfect vortex with $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ is necessarily a circle. The problem is that eddies get formed, die, merge, run into obstacles, get asymmetrically advected etc. To successfully track them it is therefore necessary to allow less circle-like shapes whilst still avoiding to e.g. count 2 semi merged eddies as one. This is achieved by calculating the [isoperimetric quotient](#), defined as the ratio of a ring's area to the area of a circle with equal circumference. ? use a similar method. They require:

The distance between any pair of points within the connected region must be less than a specified maximum [?].

While this method clearly avoids overly elongated shapes it allows for stronger deformation within its distance bounds.

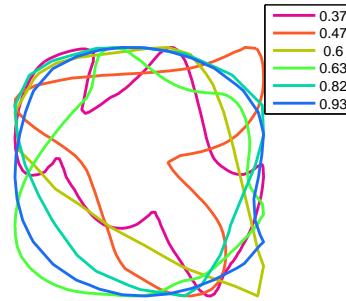


Figure 1.4: Different values of the isoperimetric quotient.

compare both,
show pictures

Amplitude

```
function CR_AmpPeak
```

This function determines the amplitude i.e. the maximum of the absolute difference between SSH and current contour level and the position thereof as well as the amplitude relative to the mean SSH value of the eddy interior as done by ?. The amplitude is then tested against the user-given threshold. The function also creates a matrix

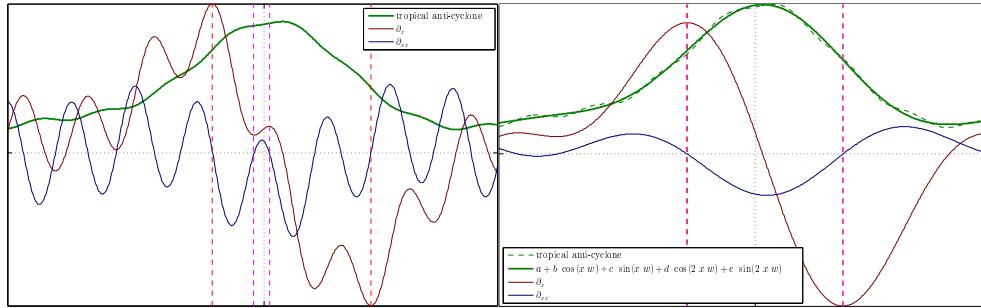


Figure 1.6: A flat wobbly low-latitude eddy resulting in multiple zero-crossings of its ∇^2 . The problem is solved by differentiating the profile's Fourier-Series-fit instead.

with current contour level shifted to zero and all values outside of the eddy set to zero as well.

Chelton's Scales

```
function cheltStuff
```

? introduced 4 different eddy-scales.

1. The effective scale L_{eff} as the radius of a circle with its area equal to that enclosed by the contour.
2. The scale $L_e = L(z = e^{-1}a)$ where a as the amplitude with reference to the original contour and the z -axis zero-shifted to that contour. In other words the effective scale of the contour that is calculated at $1/e$ of the original amplitude.
3. The scale $L = L_e/\sqrt{2}$.
4. The scale L_s which is a direct estimate based on the contour of SSH within the eddy interior around which the average geostrophic speed is maximum. [?] It is hence conceptually the same as σ . This scale was not calculated here, as I could not think of an efficient simple way to estimate the area bounded by maximum geostrophic speed i.e. the zero-vorticity contour. To understand why this would be difficult to achieve see also sections... .

legend ref

ref to approp
sections eg noise
in vort etc

Profiles

```
function EddyProfiles
```

This step

- saves the meridional and zonal profiles of SSH, U and V through the eddy's peak and spanning the entire sub-domain as described in [section][5][1]1.5.
- creates spline functions from the ssh-profiles and uses them to interpolate the profiles onto 100-piece equi-distant coordinate vectors to build smooth interpolated versions of ssh-profiles in both directions.
- in turn uses the splined data to create smooth 4-term Fourier-series functions for the profiles.

Dynamic Scale (σ)

```
function EddyRadiusFromUV
```

The contour line that is being used to detect the eddy is not necessarily a good measure of the eddy's *scale* i.e. it doesn't necessarily represent the eddy's outline very well. This becomes very obvious when the area, as inferred by [section][5][1]1.5, is plotted over time for an already successfully tracked eddy. The result is not a smooth curve at all. This is so because at different time steps the eddy usually gets detected at different contour levels. Since its surrounding continuously changes and since the eddy complies with the testing-criteria the better the closer the algorithm gets to the eddy's peak value, the determined area of the contour jumps considerably between time steps. This is especially so for large flat eddies with amplitudes on the order of 1cm. If the contour increment is on that scale as well, the difference in contour-area between two time steps easily surpasses 100% and more. Since there is no definition for the *edge* of an eddy, it is here defined as the ellipse resulting from the meridional and zonal diameters that are the distances between first minimum and maximum orbital velocity, away from the eddy's peak in positive and negative y and x directions respectively. In the case of a meandering jet with a maximum flow speed at its center, that is shedding off an eddy, this scale corresponds to half the distance between two opposing center-points of the meander. It is also the distance at which a change in vorticity-polarity occurs and is thus assumed to be the most plausible dividing line between vortices.

Trying to determine the location where this sign change in vorticity occurs in the profiles turns out to be very tricky. What we seek are local extrema of the geostrophic speeds i.e. of the ssh-gradients h_x . In a perfect Gaussian-shaped eddy, these would simply correspond to the first local extrema of h_x away from the peak. In reality the eddies can be very wobbly with numerous local maxima and minima in the gradients of their flanks. One could argue, that it must be the largest extrema, as it is the highest geostrophic speeds that are sought. In practice⁵ multiple superimposed signals of different scales often create very strong gradients locally. But the main issue here is that one weak eddy adjacent to one strong eddy also has the stronger gradients of the stronger one within its domain so that simply looking for the fastest flow speeds along the profiles is insufficient. It is also not possible to restrict the cut domain to the extent of a single eddy only, because at

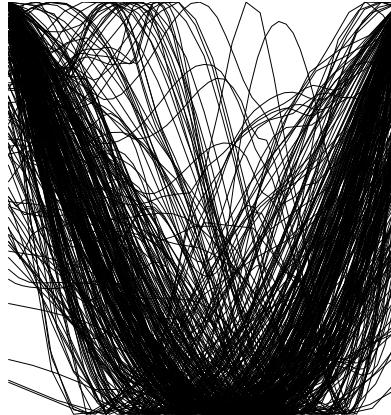


Figure 1.7: Zonal x - and z -normalized cyclone-profiles (early data ~ '13/12).

⁵especially for the high-resolution model data.

the point where the domain is selected, we do not know yet whether the detection algorithm *took bait* at the eddy's base or later close to the tip.

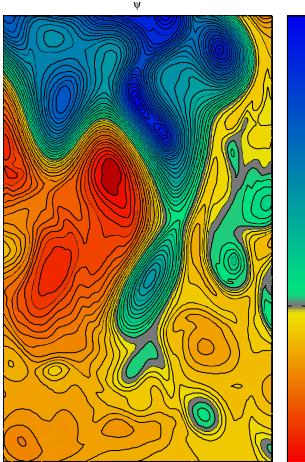


Figure 1.8: Stream function of a meandering jet shedding off a vortex. The line of strongest gradient i.e. fastest geostrophic speed later becomes the zero-vorticity-line at a theoretical distance σ from the center (Offset of Ψ is chosen arbitrarily).

The best method thus far seems to be to use the Fourier-series functions from [section][5][1]1.5 to determine the first extremata away from the eddy's peak. The Fourier order was chosen to be 4 by trial and error. The effect is that small-scale low-amplitude noise is avoided, allowing for more reliable determinations of $\nabla^2 h_{fou} = 0$

Once the zero crossings in all 4 directions are found, their mean is taken as the eddy's scale.

Dynamic Amplitude

```
function EddyAmp2Ellipse
```

As mentioned above, the contour that helps to detect the eddy is not representative of its extent. This is also true for the z -direction, for the same reasons. This function therefor takes an SSH-mean at indices of the ellipse created by the determined zonal and meridional *dynamical* diameters, and uses this as the basal value to determine a *dynamic* amplitude.

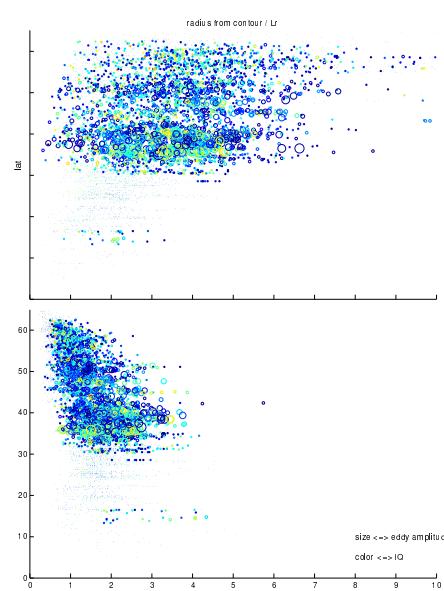


Figure 1.9: Eddies in the North-Atlantic. Y-axis: latitude. X-axis top: ratio of *radius of circle with equal area to that of found contour* to local Rossby-radius. X-axis

Center of Volume

```
function CenterOfVolume
```

Instead of using the geo-position of the eddy's peak in the tracking procedure, it was decided to instead use the center of the volume created by the basal shifted matrix from [section][5][1]1.5 i.e. *the center of volume of the dome (resp. valley) created by capping off the eddy at the contour level*. This method was chosen because from looking at animations of the tracking procedure it became apparent that, still using peaks as reference points, the eddy sometimes jumped considerably from one time step to the next if two local maxima existed within the eddy. E.g. in one time-step local maximum A might be just a little bit larger than local maximum B and one time-step later a slight shift of mass pushes local maximum B in pole position, creating a substantial jump in the eddy-identifying geo-position hence complicating the tracking procedure.

Geo Projection

```
function ProjectedLocations
```

An optional threshold on the distance an eddy is allowed to travel over one time-step is implemented in the tracking algorithm [section][5][1]1.5. This is a direct adaptation of the ellipse-based constraint described by ?. The maximum distance in western direction traveled by the eddy within one time-step is limited according to $x_{west} = \alpha c \delta t$ with c as the local long-Rossby-wave phase-speed and e.g. $\alpha = 1.75$. In eastern direction the maximum is fixed to a value of e.g. $x_{east} = 150\text{km}$. This value is also used to put a lower bound on x_{west} and for half the minor axis (y -direction) of the resultant ellipse.

This function builds a mask of eligible geo-coordinates with respect to the next time-step.

1.6 Step S05: Track Eddies

Main Tracking Procedure

```
function function S04_track_eddies
```

Due to the the relatively fine temporal resolution (daily) of the model data, the tracking procedure for this case turns out to be much simpler than the one described by ?. There is really no need to project the new position of an eddy, as it generally does not travel further than its own scale

in one day. This means that one eddy can be unambiguously tracked from one time step to the next as long both time-steps agree on which eddy from the *other* time-step is located the least distance away. The algorithm therefore simply builds an arc-length-distance matrix between all old and all new eddies and then determines the minima of that matrix in both directions i.e. one array for the new with respect to the old, and one for the old with respect to the new set. This leads to the following possible situations:

- Old and new agree on a pair. I.e. old eddy O_a has a closest neighbour in the new set N_a and N_a agrees that O_a is the closest eddy from the old set. Hence the eddy is tracked. N_a is O_a at a later time.
- N_a claims O_a to be the closest, but N_b makes the same claim. I.e. two eddies from the new set claim one eddy from the old set to be the closest. In this situation the closer one is decided to be the old one at a later time-step and the other one must be a newly formed eddy.
- At this point all new eddies are either allocated to their respective old eddies or assumed to be *newly born*. The only eddies that have not been taken care of are those from the old set, that *lost* ambiguity claims to another old eddy, that was closer to the same claimed new eddy. I.e. there is no respective new eddy available which must mean that the eddy just *died*. In this case the entire track with all the information for each time step is saved as long as the track-length meets the threshold criterium. If it doesn't, the track is abandoned.

Improvements

The former is the core of the tracking algorithm. It is sufficient by itself as long as the temporal resolution is fine enough. The larger the time-step, the more ambiguities arise, which are attempted to be mitigated by flagging elements of the distance matrix not meeting certain thresholds:

- `function` `checkDynamicIdentity`

Consider the ambiguous case when there are two eddies N_a and N_b in sufficient proximity to eddy O_a . Let's assume O_a is a relatively solid eddy of rel. large scale with a steep slope i.e. large amplitude and that N_a

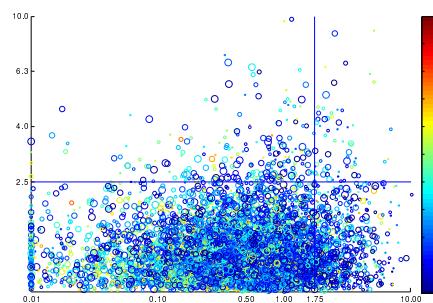


Figure 1.10: Each circle represents one eddy in the new time step. Y-axis: Maximum ratio to closest eddy in old set of either amplitude or σ , where 1 means *identical* and 2 means 100% difference. The threshold used for the final runs was 2. X-axis: Ratio of distance to closest eddy from old set divided by δt to local long-Rossby-wave phase-speed. Color-axis: Isoperimetric Quotient. Radius of circles: ratio of σ to local Rossby-radius. All eddies with said ratio larger than 10 are omitted. Note the obvious inverse correlation of scale to $|Q|$ (negative the total long- R).

is just a subtle blob of an eddy whilst N_b is somewhat similar to O_a but with only half the amplitude. The situation then is clear: N_b is the, apparently slowly dying, O_a at a later time, while N_a could either be a newly formed eddy, an old eddy with its respective representation in the old set something other than O_a , or even just temporary coincidental noise not representative of any significant meso-scale vortex. This interpretation should hold even when O_a sat right between the other two thereby being much closer to O_a than N_b was.

The purpose of this step is to make such decisions. It does so by comparing the *dynamic* versions of amplitude and scale (*ampToEllipse* and σ) between the time-steps. If either ratio from new to old⁶ surpasses a given threshold the pair is flagged as non-eligible. It is important to use the *dynamic* parameters rather than those stemming from the contour line, because as mentioned in [section][5][1]1.5 the contour line itself and the eddy's geometric *character* are hardly correlated at all. One eddy can get detected at different z -levels from one time-step to the next, resulting in completely different amplitudes, scales and shapes with respect to the contour.

The initial idea was, by assuming Gaussian shapes, to construct a single dimensionless number representing an eddy's geometrical character built upon the contour-related amplitude- and scale values only. Since we have no information about the vertical position of a given contour with respect to assumed Gauss bell, this problem turned out to be intrinsically under-determined and hence useless. The method eventually used, which checks amplitude and scale separately is again very similar to that described by ?. TODO HOW?

- `function checkAmpAreaBounds`
TODO MI `function nanOutOfBounds`
TODO MI and MII

1.7 Step S06: Cross Reference Old to New Indices

```
function function S05_init_output_maps
```

The output Mercator-maps usually have different geometry from the input maps'. This step allocates all grid nodes of the input data to their respective nodes in the output map. Each output cell will then represent a mean (or median or std) of all input-nodes falling into that *square*.

1.7.1 Running the Code

The separate steps can be run all at once via `sall.m` or one by one, as long as they are started consecutively in the order indicated by their name (`s00..`, then `s01..`

⁶In order to compare in both directions equally: $\exp(\overline{\log(v_n/v_o)})$ where v is either amplitude or scale.

ifnextchar.etcetc.). S01b is not necessary though. Each step saves its own files which are then read by the next step. All output data is saved in the user given root-path from ???. This concept uses quite a lot of disk space and is also quite substantially slowed by all the reading and writing procedures. The benefits, on the other hand, are that debugging becomes much easier. If the code fails at some step, at least all the calculations up to that step are dealt with and do not need to be re-run. The concept also makes it easy to extend the code by further add-ons.

Chapter 2

Results

The short time-frame and limited computational resources allowed for only a few complete global runs over the available data. It was therefore critical to carefully choose which method/parameters to use in order to maximize the deducible insight from the results. For best comparability of the results with each other it was decided to agree on one complete set of parameters as a basis, which would then be altered at key parameters. The first run is an attempt to reproduce the results from ?, by setting the algorithm to be the most similar to the algorithm described by ?. The SSH-data for this run is therefore that of the Aviso product. This method will be called **MI**.

The second run is equivalent, except that this time the alternative **IQ**-based shape filtering method from TODO ref and the slightly different tracking-filter as described in TODO ref are used. This setting will be called **MII**. **MII** is then fed with 7-day time-step POP data as well.

To investigate what role space-resolution plays, the POP data was remapped to that of the Aviso data and fed to the **MI** method. Finally, to investigate the effects of resolution in time, an **MII**/3-day-time-step run over POP data was executed.

Start and end dates were fix for all runs as the intersection of availability of both data sets (see table [table][1][2]2.1 for details).

TODO lookup term used for krummes grid
TODO check tables for completeness

time frame	1994-01-05 till 2006-12-27
scope	full globe ($80S : 80N$ $180W : 180E$)
AVISO geometry	$641x1440$ true Mercator
POP geometry	$2400x3600$
contour step	0.01
thresholds	[all SI]
max σ/L_R	4
min L_R TODO	20e3
min IQ	0.55
min data points of an eddy	8
max(abs(rossby phase speed)) TODO	0.2
max amplitude TODO	0.01

Table 2.1: Fix parameters for all runs.

	MI	MII
shape threshold	The distance between any pair of points within the connected region must be less than a specified maximum.	The Isoperimetric Quotient must be at least a specified minimum.
Comparison of old to new eddy	ratio between new and old must lie between 0.25 and 2.5 for amplitude and area TODO ref	similar but with $\sqrt{\text{area}/\pi}$ instead of area and the lower threshold as reciprocal of the higher and vice versa. TODO ref

Table 2.2: The two methods **MI** and **MII**.

2.1 Method MI

The concepts used in this method are mostly based on the description of the algorithm described by [?] and all parameters are set accordingly. Basically **MI** is a modification of **MII** (which was completed first), with the aim to try to recreate the results from [?]. It differs from **MII** in the following:

- **detection** As mentioned in TODO ref, the approach by ? is to avoid overly elongated objects by demanding:
 - high latitudes
The maximum distance between any vertices of the contour must not be larger than 400km for $|\phi| > 25^\circ$.
 - low latitudes
The 400km -threshold increases linearly towards the equator to 1200km .
- **tracking** The other minor difference to **MII** is in the way the tracking algorithm flags eddy-pairs between time-steps as sufficiently similar to be considered successful tracking-candidates (see TODO ref). In this method an eddy B from time-step $k + 1$ is considered as a potential manifestations of an eddy A from time-step k as long as both - the ratio of amplitudes (with regard to the mean of SSH within the found contour) and the ratio of areas (interpolated versions as discussed in TODO ref) fall within a lower and an upper bound.

2.2 Method MII

Even though, in its core, directly inspired by [?], this method differs from **MII** and thus from the description by ? mainly in the way the shape of a found contour is deemed sufficiently eddy-like.

- **detection** IQ
- **tracking** $\max([\exp |\log R_\alpha| ; \exp |\log R_\sigma|])$

Chapter 3

TODO