# TUTORIAL-6

**Q1** What do you mean by Minimal Spanning Tree? What are the applications of MST?

MST is a subset of edges of a connected edge-weighted undirected graph that connects all the vertices together without any cycles and with minimum possible edge weighted.

**APPLICATIONS:-**

1) Consider n stations, are to be linked using a communication network and laying of communication link between any two stations involves a cost. The ideal solution would be to extract a subgraph termed as minimum cost spanning tree.
2) Designing LAN.
3) Highways or railways b/w cities.
4) Laying pipelines connecting offshore drilling sites, refineries & consumer markets.

**Q2)** Analyze time and space complexity of Prim, Kruskal, Dijkstra's, and Bellman-Ford.

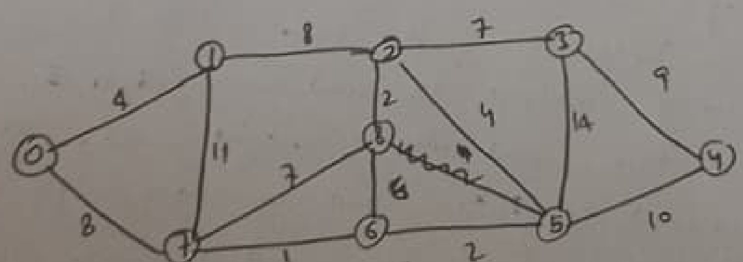
**Prim's**
TC - $O(|E| \log |V|)$
SC - $O(|V|)$

**Kruskal's**
TC - $O(|E| \log |E|)$
SC - $O(|V|)$

**Dijkstra's:-**
TC - $O(V^2)$
SC - $O(V^2)$

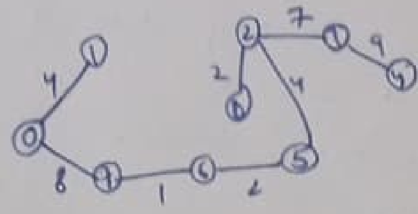
**Bellman-Ford's:-**
TC - $O(VE)$
SC - $O(E)$

**Q3)** Apply Kruskal's and Prim's Algorithms on given graph to compare MST & its weight.



**Kruskal's**

| U | V | W | |
|---|---|---|---|
| 6 | 7 | 1 | ✓ |
| 5 | 6 | 2 | ✓ |
| 2 | 8 | 2 | ✓ |
| 0 | 1 | 4 | ✓ |
| 2 | 5 | 4 | ✓ |
| 6 | 8 | 6 | ✗ |
| 2 | 3 | 7 | ✓ |
| 7 | 8 | 7 | ✗ |
| 0 | 7 | 8 | ✓ |
| 1 | 2 | 8 | ✗ |
| 4 | 3 | 9 | ✓ |
| 4 | 5 | 10 | ✗ |
| 1 | 7 | 11 | ✗ |
| 3 | 5 | 14 | ✗ |



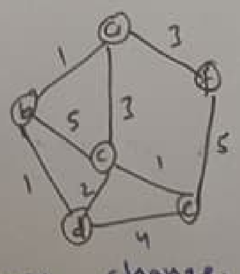

Weight = 1+2+2+4+4+7+8+9
= 37

~~Kruskal's~~ **Prim's:-** Weight = 4+8+2+4+~~1~~+7+9+3
= 37

**Q.4)** Given a direted weighted graph. You are also given the shortest path from a source vertex 's' to a destination vertex 't'. ~~Does~~ the shortest path remain same in following cases:

i) If weight of every edge is increased by 10 units.

ii) If weight of every edge is multiplied by 10 units.

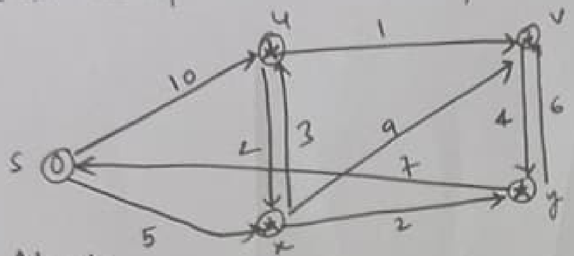


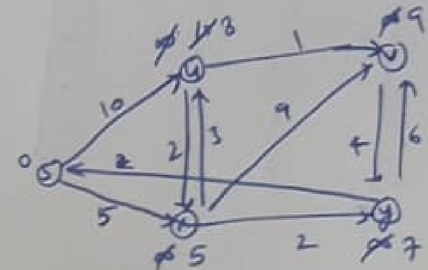

(i) The shortest path may change. The reason is that there maybe different no. of edges in different paths from 's' to 't'.

eg:- Let the shortest path of weight 15 and has edges 5. Let there be another path with 2 edges and total weight 25. The weight of shortest path is increased by 5*10 and becomes 15 + 50. weight of other path is increased by 2*10 and becomes 25 + 20. So, the shortest path changes to other path with weight as 45.

(ii) If we multiply all edges' weights by 10, the shortest path remains the same. The weights of all paths from 's' to 't' gets multiplied by same unit. The number of edges or path doesn't matter.

Q5) Apply Dijkstra's and Bellman Ford Algorithms on graph given right side to compare shortest path to all nodes from node s.



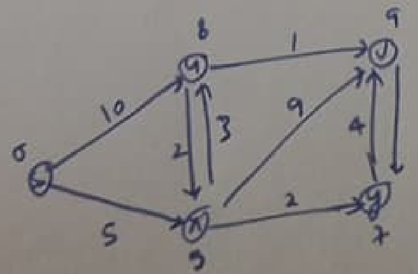

**Dijkstra's Algo:-**

| Node | Shortest dist. from source node |
|---|---|
| u | 8 |
| x | 5 |
| v | 9 |
| y | 7 |

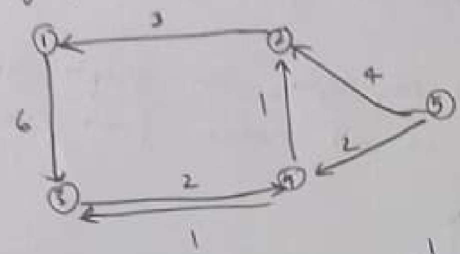
**Bellman Ford's Algo:-**

| | s | u | v | x | y |
|---|---|---|---|---|---|
| 1st - | 0 | ~~0~~ 10 | ∞ | ~~0~~ 5 | ∞ |
| 2nd - | 0 | 10 | ~~0~~ 11 | 5 | ∞ |
| 3rd - | 0 | ~~10~~ 8 | ~~11~~ 9 | 5 | ~~∞~~ 7 |
| 4th - | 0 | 8 | 9 | 5 | 7 |

Graph does not have -ve cycle.



Q6) Apply all pair shortest path algorithm - Floyd Warshall on below mentioned graph. Also, analyze space and time complexity of it. 4.




$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & \infty \end{bmatrix} \end{array}$$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 3 & 1 & 1 & 0 & \infty \\ 6 & 4 & 12 & 2 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 3 & 1 & 1 & 0 & \infty \\ 6 & 4 & 12 & 2 & 0 \end{bmatrix} \end{array}$$

$TC = O(|V|^3)$

$SC = O(|V|^2)$

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