TUTORIAL-4

Q.1) $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a \geq 1;\ b \geq 1$

On comparing,

$a = 3,\ b = 2,\ f(n) = n^2$

$c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\because f(n) > n^c$

$\therefore T(n) = O(n^2)$

Q2) $T(n) = 4T(n/2) + n^2$

$a \geq 1,\ b > 1$

$a = 4,\ b = 2,\ f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$T(n) = \Theta(n^2 \log_2 n)$

Q.3) $T(n) = T(n/2) + 2^n$

$a = 1$

$b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

Q.4) $T(n) = 2^n T(n/2) + n^n$

$a = 2^n,\ b = 2,\ f(n) = n^n$

$c = \log_b a = \log_2 2^n = n$

$n^c = n^n$

$f(n) = n^c$

$T(n) = \Theta(n^2 \log_2 n)$

Q.5) $T(n) = 16T(n/4) + n$

$a = 16,\ b = 4,\ f(n) = n$

$c = \log_4 16 = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

Q.6) $T(n) = 2T(n/2) + n\log n$

$a = 2,\ b = 2,\ f(n) = n\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n\log n > n$

$f(n) > n^c$

$T(n) = O(n\log n)$

Q.6) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2,\ b = 4,\ f(n) = n^{0.5}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = O(n^{0.51})$

Q.7) $T(n) = 2T(n/2) + n/\log n$

$a = 2,\ b = 2,\ f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\frac{n}{\log n} < n$

$f(n) < n^c$

$\therefore T(n) = O(n)$

Q.9) $T(n) = 0.5\,T(n/2) + 1/n$

$a = 0.5,\ b = 2$

$a \geq 1$ but here a is 0.5,

So we cannot apply Master's theorem.

Q.11) $T(n) = 4T(n/2) + \log n$

$a = 4,\ b = 2,\ f(n) = \log n$

$c = \log_2 4 = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

Q.13) $T(n) = 3T(n/2) + n$

$a = 3\ b = 2,\ f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.5489}$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^{1.5849})$

Q.15) $T(n) = 4T(n/2) + n$

$a = 4,\ b = 2$

$c = \log_b a = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = O(n^2)$

Q.17) $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3$

$c = \log_b a = 1$

$f(n) = n/2$

$n^c = n$

$f(n) < n^c$

$\therefore T(n) = O(n)$

Q.10) $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

Q.12) $T(n) = \text{sqrt}(n)\,T(n/2) + \log n$

$a = \sqrt{n},\ b = 2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\frac{1}{2}\log_2 n < \log(n)$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(\log n)$

Q.14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3,\ b = 3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$f(n) < n^c$

$\therefore T(n) = O(n)$

Q.16) $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$f(n) = n\log n > n^c$

$\therefore T(n) = O(n\log n)$

Q.18) $T(n) = 6T(n/3) + n^2\log n$

$a = 6,\ b = 3,\ f(n) = n^2\log n$

$c = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

$f(n) > n^c$

$\therefore T(n) = O(n^2\log n)$

Q.19) $T(n) = 4T(n/2) + n/\log n$

$a = 4,\ b = 2,\ f(n) = \frac{n}{\log n}$

$c = \log_b a = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$\therefore T(n) = O(n^2)$

Q.20) $T(n) = 64T(n/8) - n^2 \log n$

$a = 64,\ b = 8$

$c = 2$

$n^c = n^2$

$f(n) > n^c$

$\therefore T(n) = O(n^2 \log n)$

Q.21) $T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$c = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$f(n) > n^c$

$\therefore T(n) = O(n^2)$

Q.22) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1,\ b = 2,\ f(n) = n(2 - \cos n)$

$c = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$\therefore T(n) = O(n(2 - \cos n))$