Module 4

Push Down Automata

Regular Grammar - Type-3 → FSM

This can process only Finite Amount of Infoⁿ.

Context Free Grammar - Type-2 → Push Down Automata

This can process Infinite Amount of Infoⁿ.

This can be done using stack.

In stack — 2 opⁿ

push & pop.

push → insertion

pop → deletion

PDA is used to recognise CFG.

In PDA 3 components are used

1. Input Tape - Input String. (processed)
2. Finite Control - push/pop. (state can be changed)
3. Stack - Elements (to maintain)

Push Down Automata also can be called as $\boxed{\text{FSM + STACK}}$

↓

5 Tuples + 2 tuples.

total $\boxed{7 \text{ Tuples}}$.

Subject

Defn of PDA.
can be defined as 7 Tuple.

$(Q, \Sigma, \delta, \Gamma, q_0, z_0, F)$

Q - No. of states.
Σ - Alphabet with input symbols.
δ - Transition functions
Triple $\delta(q, a, x)$ $\{(q_n, \alpha), \ldots\}$
q → current state
a → input to be processed
x → Top of stack.
where q_n → new state
α → Top of stack to be replaced of x.
→ string of stack symbol.

Γ - Stack symbols.
q_0 - Initial state
z_0 - Initial top of stack.
F - Final State

Example 1

Construct a PDA for the lang.

$L = \{ a^n b^n \mid n \geq 1 \}$

→ $L = \{ab, aabb, aaabbb, \ldots\}$

$\delta(q, a, x) \rightarrow \delta(q_n, \alpha)$

- q → current state
- a → input
- x → top of stack
- q_n → new state
- α → string of stack symbols in stack.

for every transition fu^n

- a – push
- b – pop(a)
- unchanged

3 op^n can be done

(q_n, ϵ) – pop

a	a	a	b	b	b	ε

└→ Initially –

$\delta(q_0, a, z_0) = (q_0, az_0)$

z_0 ← Top.

└→ get out from stack and the push

a ← Top.
z_0

→ $\delta(q_0, a, a) = (q_0, aaz_0)$

az_0 out

a push so aaz_0.

push from right to left.

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a	← Top
a	
Z_0	

3rd a

$\delta(q_0, a, a) = (q_0, aa\,\cancel{Z_0})$ aZ_0

pop - a and push a

a	← Top
a	
a	
Z_0	

Now, b.

$\delta(q_0, b, a) = (q_1, \epsilon)$

pop(a)

a	← Top
a	
Z_0	

next b.

$\delta(q_1, b, a) = (q_1, \epsilon)$

pop(a)

a	← Top.
Z_0	

next b

$\delta(q_1, b, a) = (q_1, \epsilon)$

pop(a)

Z_0	← Top.

$\delta(q_1, \epsilon, Z_0) = (q_2, \epsilon)$

pop Z_0

empty stack

So, $Q = \{q_0, q_1, q_2\}$

$F = \{q_2\}$

$\Gamma = (Z_0, a)$

q_0 = initial state

Z_0 = Top of the stack

$\Sigma = \{a, b\}$

PDA

$P = \{Q, \Sigma, \Gamma, \delta, q_0, Z_0, F\}$

$P = \{\{q_0, q_1, q_2\}, \{a, b\}, \{Z_0, a\}, \delta, \{q_0\}, \{Z_0\}, \{q_2\}\}$

Example 2

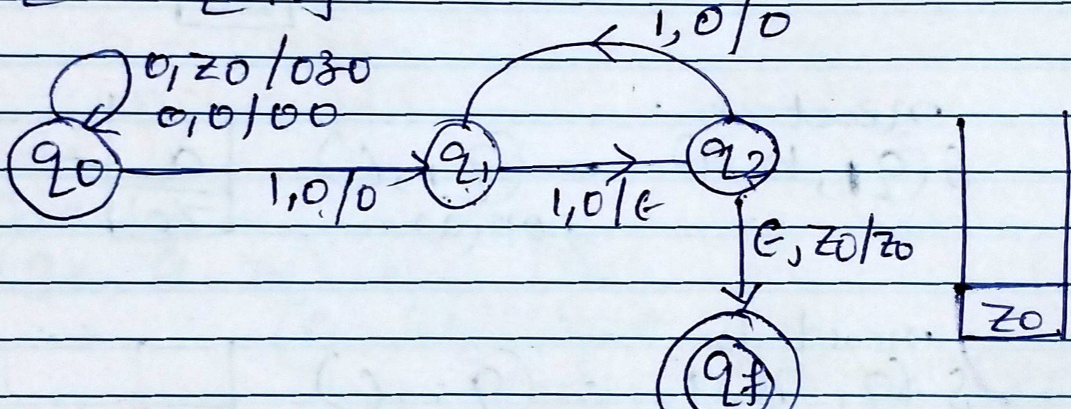
Construct a PDA for the language $L = \{0^n 1^{2n}, n > 0\}$

$\rightarrow$ $L = \{011, 001111, 000111111, \ldots\}$

q_0 - initial state

Σ - $\{0, 1\}$

0	0	1	1	1	1	ϵ



Final PDA

q_0 - initial state

q_f - Final state

Q - $\{q_0, q_1, q_2, q_f\}$

Σ - $\{0, 1\}$

Z_0 - initial element in stack.

Γ - $\{Z_0, 0\}$

Transition δ

$q_0, 0, Z_0 \rightarrow q_0, 0Z_0$

$q_0, 0, 0 \rightarrow q_0, 0\,0Z_0$

$q_0, 1, 0 \rightarrow q_1, 0\,0Z_0$

$q_1, 1, 0 \rightarrow q_2, \epsilon$

$q_2, 1, 0 \rightarrow q_1, 0Z_0$

$q_1, 1, 0 \rightarrow q_2, Z_0$

$q_2, \epsilon, Z_0 \rightarrow q_f, Z_0$

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Example 3

Construct a PDA for given language $L = \{a^n b^m c^n \mid n \geq 1, m \geq 1\}$

$\Longrightarrow$ $n, m = 1$ $\quad m=1, n=2$ $\quad m=2, n=2$

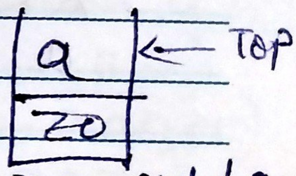
$L = \{abc,\ aabcc,\ aabbcc,$
$aaabbccc, \ldots\}$
$m=2, n=3$

- Condn – no. of a's & no. of c's are same

so, a – push(a)
b – unchange
c – pop(a)

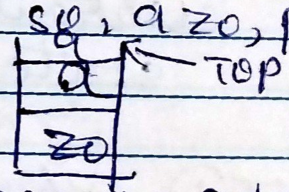
string - $\boxed{a|a|b|c|c|\epsilon}$

$\delta(q_0, a, z_0) = \delta(q_0, az_0)$



pop zo, add a
so, azo, push.

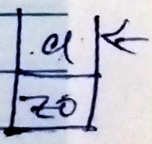
$\delta(q_0, a, a) = \delta(q_0, aa\,z_0)$



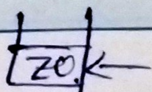
pop a, add a
so, aa, push

$\delta(q_0, b, a) = \delta(q_1, aaz_0)$ unchanged
stack is unchanged because i/p is b

$\delta(q_1, c, a) = \delta(q_2, \epsilon)$ pop(a)



$\delta(q_2, c, a) = \delta(q_2, \epsilon)$
pop(a)



$\delta(q_2, \epsilon, z_0) = (q_3, \epsilon)$ empty stack.
pop(zo)

Final PDA

$P = \{Q, \Sigma, \Gamma, \delta, q_0, z_0, F\}$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{a, z_0\}$

$q_0 = \{q_0\}$

$z_0 = \{z_0\}$

$F = \{q_3\}$

δ is already calculated

Instantaneous Description (ID) of PDA.

- used to check Acceptance of string

ID of PDA is represented by a Triple

(q, w, Γ)

current state — input string to be processed — content of the stack

ID is represented as $\vdash$ (Fallen T) called as Turnstile Notation

$\vdash$ single move

$\vdash^*$ Multiple / sequence of moves

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Example 1

$L = \{a^n b^n \mid n \geq 1\}$

→ $L = \{ab, aabb, aaabbb \ldots\}$

aaabbb

Write transition fu^n

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aaz_0)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$

Here, we have taken string aaabbbϵ and check it is accepted or not.

Represented as Triple

$(q_0, aaabbb, z_0) \vdash (q_0, aabbb, az_0)$

$\vdash (q_0, abbb, aaz_0)$

$\vdash (q_0, bbb, aaaz_0)$

$\vdash (q_1, bb, aaz_0)$ pop

$\vdash (q_1, b, az_0)$ pop

$\vdash (q_1, \epsilon, z_0)$ pop

$\vdash \boxed{(q_2, \epsilon)}$ pop

At the end we get stack is empty so, the string is accepted.

Acceptance of PDA can be
- Empty stack
- Final state

Example-2

$L = \{a^n b^m c^n \mid m, n \geq 1\}$

$\rightarrow L = \{abc, aabcc, aabbcc, aaabccc, \ldots\}$

write the transition fu^n

string aabcc

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aaz_0)$

$\delta(q_0, b, a) = (q_1, aaz_0)$

$\delta(q_1, c, a) = (q_2, \epsilon)$

$\delta(q_2, \epsilon, z_0) = (q_3, \epsilon)$

Now, $(q_0, aabcc, z_0) \vdash (q_0, abcc, az_0)$

$\vdash (q_0, bcc, aaz_0)$

$\vdash (q_1, cc, aaz_0)$

unchanged

$\vdash (q_2, c, az_0)$ pop

$\vdash (q_2, \epsilon, z_0)$ pop

$\vdash \boxed{(q_3, \epsilon)}$ pop

accepted

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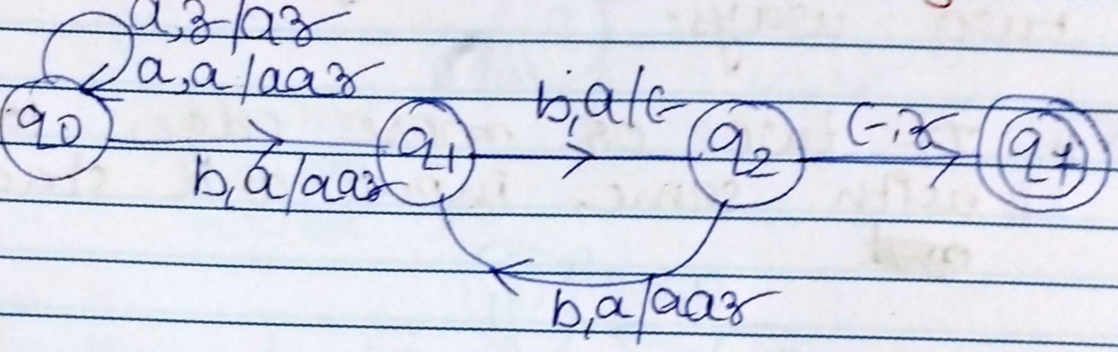
Deterministic Push Down Automata

DPDA :- A PDA is said to be deterministic if all derivations in the design has to give only single move.

Example 1 $L = a^n b^{2n} \quad n>0$

$L = \{abb, aabbbb, --\}$



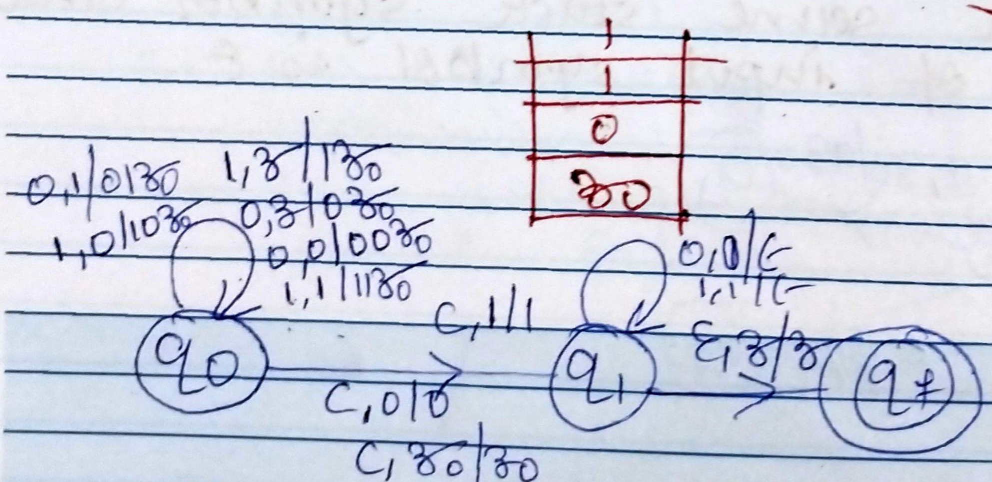
Example 2

$L = wcw^R$

$w = (0+1)^*$

$\Sigma = \{0,1,c\}$

$L = \{011c110,\ 001c100, ---\}$



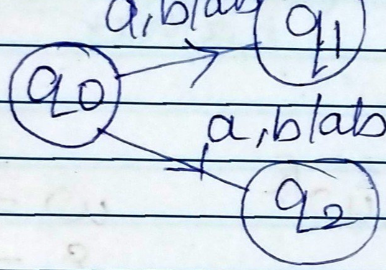
Non-Deterministic Push Down Automata

NPDA

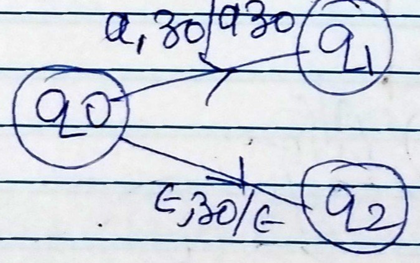
A PDA is nondeterministic with one transition we can take more than one move.

This can be done by following two ways.

1. If two or more edges labeled with same input and stack symbol ~~and~~



2. When a state have two edges with same stack symbol and one of input symbol is ϵ



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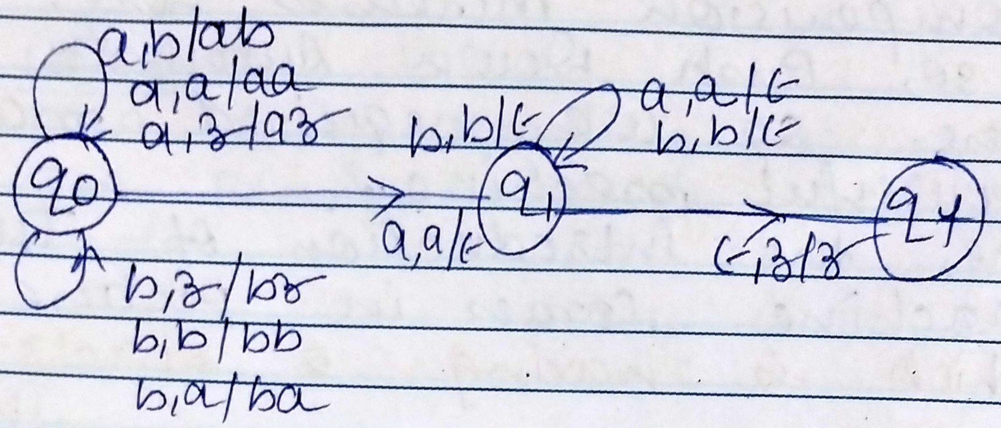
Example

$L = ww^R \quad w \in (a,b)^*$

$L = \{ abba, abbbba, \ldots \}$



Appln of PDA

1. For designing the parsing phase of a compiler (Syntax Ana.)
2. For implementation of stack appln.
3. For evaluating the arithmatic expressions.
4. For solving the Tower of Hanoi Problem