The Theory of Relativity

1. THE SPECIAL THEORY

In 1905, his *annum mirabilis*, Einstein revolutionized physics with, among other things, his special theory of relativity. With it he completely overturned our concepts of space and time with two seemingly innocuous, almost "obvious," fundamental laws, or postulates that we call, for utter lack of imagination, Einstein's first postulate and Einstein's second postulate. Despite their apparent simplicity, these two postulates are incredibly profound and rich in consequences, and they conceal paradoxes that defy our deepest intuition.

1.1. Einstein's first postulate

1.1.1. Statement

The laws of physics are the same in any coordinate system moving at a constant velocity. We call this the "special" theory of relativity because it is restricted to the special case when the coordinate system is not accelerating.

1.1.2. Illustration

Pretend that we are moving along in a boxcar with no windows, so we can't see out. If we drop a ball it falls straight to the floor. A balloon rises straight to the ceiling, and stays there. There is no way we can tell if we are moving. In fact, the earth is just like this boxcar, at least to the extent that the earth is moving at a constant velocity relative to the sun, relative to the galaxy, relative to the ... No experiment done on the earth will detect the absolute motion of the earth. This statement is so profoundly true that there is, therefore, no meaning to the notion of absolute motion. All motion is relative to other bodies, and this is why we call it the "theory of relativity."

1.2. Einstein's second principle

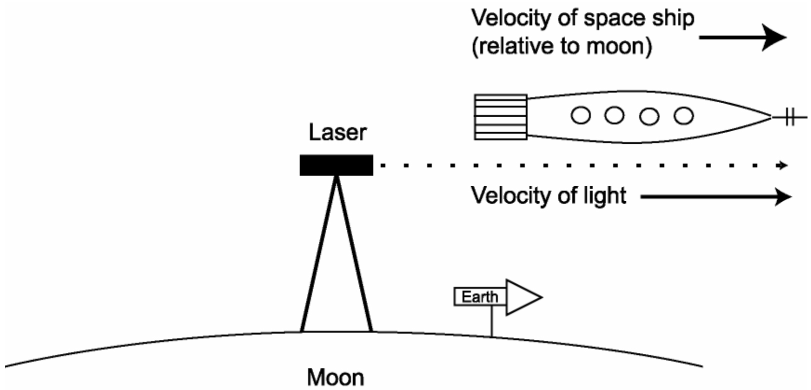
Einstein's second postulate is even easier than the first. Some would even say that it is included in the first, but this is where the trouble begins.

1.2.1. Statement

The speed of light is the same for all observers.

1.2.2. Illustration

Just to show you how quickly the problems arise, lets consider what happens if we shine a laser at a passing space ship:



We see the light moving at the velocity c as it passes the space ship. Since the space ship is moving at the velocity v, the *relative* velocity is $(c-v)$. That is, if we measure the time for the light to go the distance L from the back of the ship to the front, the time will be $L/(c-v)$. But on the space ship they must see the light go past at the velocity c. That is, the time for the light to go from the back to the front of the space ship must be L/c, not $L/(c-v)$. How can this possibly happen?

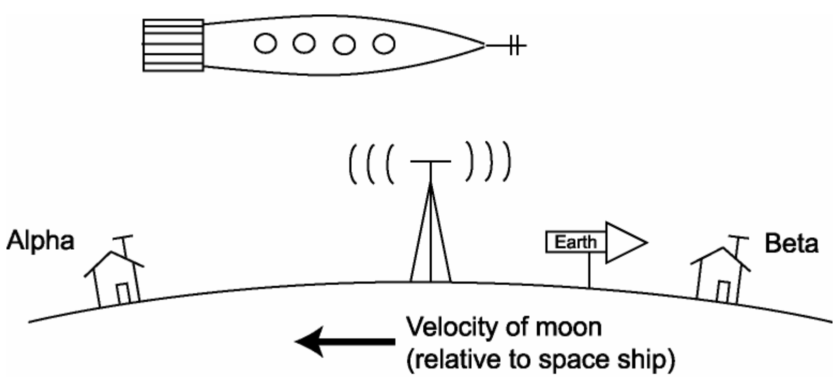
1.3. The problem of time

1.3.1. Simultaneity

The problem lies in our concept of time. We think that time is the same for everyone. But, it just isn't like that. In fact, it is possible that among two events, the event that happens first for one observer may happen last for another. Let's begin with the problem of simultaneity.

1.3.2. Gedanken experiment

Consider the following "gedanken experiment" (thought experiment; Einstein loved gedanken experiments):



On the moon, Buffy and Bubba, representating the lunar colonies Alpha and Beta, are having a green-cheese eating contest. When the winner is declared, the news is radioed to the folks back home in Alpha and Beta, which are equidistant from the site of the contest. Each colony receives the news, which travels at the speed of light, at exactly the same time. Right? Of course. But Hilda and Wolfgang, who are passing the moon in their space ship on their way back to earth after a vacation, watch the events on the moon and come to a different conclusion. Since, as they view it, the moon is moving to the left, the news reaches Beta, which is moving toward the contest, before it reaches Alpha. Right? Of course.

Who is right?

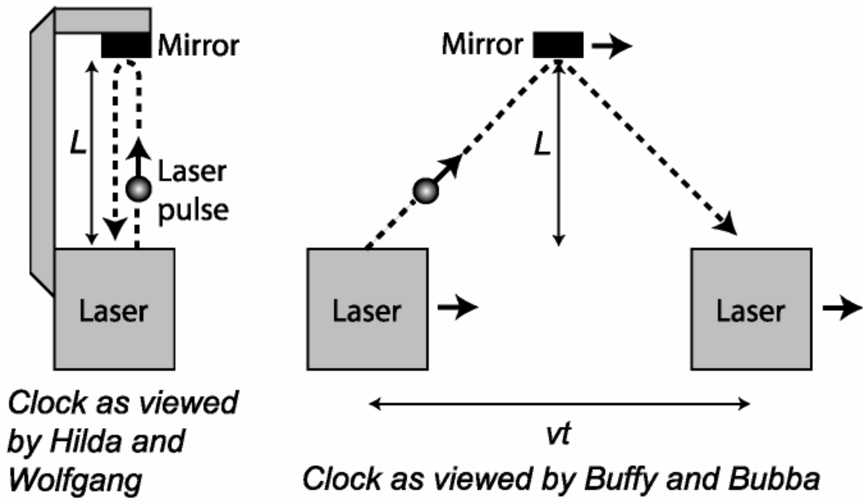
Well, they both are. There is no absolute meaning to the concept of simultaneity. In fact, let's check in with Edgar and Eloise, who are in a space ship going the opposite direction from Wolfie and Hildie, just starting their vacation. As viewed by E and E, the moon is going the opposite direction and the news reaches Alpha before it reaches Beta. We can't even get agreement on which of two events (the news arriving at Alpha and at Beta) occurred first. In fact, there is really no absolute meaning to simultaneity. Time just doesn't work that way, although the effects are usually so small that you never noticed it.

You may (you should!) wonder what has happened to cause and effect. For example, if event A causes event B, what happens if someone else observes events A and B to occur in the reverse order. This is similar to the logical difficulty that occurs when people travel back in time. Can Dr. No travel back in time and kill his mother so that he himself is never born? Well, of course not, regardless of whether you liked "Back to the Future" or not. In fact, the theory of relativity does not violate causality. Two events can be reordered in time by other observers only if the events happen so far apart in distance and so close together in time that neither light nor anything else (which must travel slower than light) can get from the first event to the second. Therefore, event A cannot have any influence on (or cause) event B, and the principle of causality is not violated. Clearly, the two events called "news reaching Alpha" and "news reaching Beta" are too far apart in distance to be connected by a single light pulse. It takes two light pulses to reach the two events, so it is OK that they can be reordered in time by different observers. One of these two events can never cause or influence the other.

1.4. Time dilation

1.4.1. Gedanken experiment

Let's do another gedanken experiment. This time we put a simple (in concept, at least) clock on the space ship with Wolfie and Hildie. The clock sends a short laser pulse up to a mirror, and when it strikes the mirror and returns the clock ticks once and sends out the next pulse. If the distance to the mirror is L, the round-trip distance traveled by the laser pulse is $l = 2L$. If c is the (universal) velocity of light, the clock ticks once in the time $\Delta t = l / c = 2L / c$. But what do Buffy and Bubba think of this?



As they see it, the light makes a triangular trip up and down as the laser and the mirror move to the right at the velocity v. The total distance the light travels in one tick is found from Pythagoras' theorem:

$$l'^2 = (2L)^2 + v^2 \Delta t'^2$$

But light travels at the velocity c, so the time for the clock to tick once is $\Delta t' = l'/c$. The moving clock goes too slow (that is, it is observed by Buffy and Bubba to take too long to tick) by the factor

$$\frac{\Delta t'}{\Delta t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

But this isn't just a case of a clock going too slow. The clock is just fine. In fact, everything on the moving space ship is going too slow. Wolfie and Hildie's hearts beat too slow, and they are aging too slowly. At least, according to Buffy and Bubba. Wolfie and Hildie don't see anything wrong. There is no experiment they can do to detect their motion, after all.

1.4.2. The clock paradox

Should Bubba and Buffy be jealous that Wolfie and Hildie are getting old slower? Not at all. After all, Wolfie and Hilda are not enjoying the extra time. Their thoughts, their days, everything is going slower for them. They don't experience any extra time. In fact, when you think of it, Wolfie and Hildie see a clock belonging to Buffy and Bubba going too slow compared with their clock. After all, they see themselves as stationary and Buffy and Bubba moving past them on a (very large) space ship. So they see Buffy and Bubba getting old slower than they are. Each one sees the other's clock as moving slower than their own! This is known as time dilation; time on a moving space ship is observed to be stretched out. Once again, this is not a problem caused by bad clocks. This is the nature of time itself!

How can this happen? The paradox is resolved when we consider how the comparison is done. When Buffy and Bubba watch Wolfie and Hildie's clock, they do it (or can do it, and all methods are equivalent) by watching one clock on the space ship as it passes close to two clocks in two separate places on the moon. When Wolfie and Hildie do the comparison, they watch one moon clock as it goes past two separate clocks on their space ship. It turns out that since the two pairs of observers can't agree on simultaneity, they have set their clocks incorrectly (relative to one another, in some sense) when they moved them into place for the comparisons. In any event, there is no paradox since the same clocks are not being used in the two measurements.

1.4.3. The twin paradox

There is one way to get around the problem of multiple clocks. Let's do another gedanken experiment. This time we have two twins. One twin gets in a space ship and flies to alpha centauri and back at high speed. Her twin brother on earth knows that her clock and her life processes move slower than his. When she returns to earth he is not surprised to see that she is younger than he is. But shouldn't she see the same thing? That is, since he (on earth) was moving relative to her space ship, shouldn't she see that he is younger? Well, the correct answer is that she is younger than he is. The symmetry of the situation is broken because she had to accelerate to fly away, decelerate to turn around at alpha centauri, and then accelerate and decelerate again to return home. Therefore, her clocks behave differently. After all, Einstein's postulates apply only to coordinate systems travelling at a constant velocity.

Actually, this gedanken experiment has already been done. In very careful experiments, two atomic clocks were flown around the earth in opposite directions. When they returned to the original laboratory and were compared with "stay-at-home" clocks, they were slower (younger) by about a tenth of a microsecond, just the amount Einstein would have predicted (actually, the effects of gravity had to be taken into account since the planes were flying high above the earth; more about this later)!

1.5. Length contraction

1.5.1. The effect

It should come as no surprise, by now, that not only is time not absolute, space isn't either. Lengths measured in the direction of relative motion will be different. In fact, the length of an object in motion appears shorter than when the object is stationary. This is called length contraction.

1.5.2. A paradox

Consider the following situation (sort of a gedanken experiment). Wolfie and Hilda want to land on the moon. To see if their space ship will fit in the garage on the moon, they fly by and check out the parking place. To them, it seems too small, since it's length contracted. To Buffy and Bubba, the rocket ship is length contracted, and appears to fit with room to spare. Who is right? They both are. The resolution of the paradox is, once again, to be found in the concept of simultaneity. The length of the space ship, as determined by Buffy and Bubba on the moon, depends on the position of the two ends of the ship at the same time as defined on the moon. The length of the parking place, as determined by Hildie and Wolfie, depends on the position of the ends of the parking space at the same time as defined by space-ship time. Since they don't agree on simultaneity, they don't agree on the lengths.

1.6. Mass and energy

Arguably the most famous equation in all of physics is Einstein's famous equation

$$E = mc^2$$

This is too technical to derive here, but it says that mass and energy are equivalent. Then, since light has energy, it has mass, and its motion is influenced by gravity. When an atomic bomb explodes, the energy that is given off is reflected in a loss of mass of the constituents of the bomb. In fact, the same is true of a flashlight that gives off energy in the form of light. Likewise, when matter and antimatter annihilate one another, energy in the amount given by Einstein's equation results, typically in the form of light (gamma rays). Conversely, two gamma rays can come together and pool their energy. This creates matter and antimatter (an electron and a positron) out of "nothing!"

2. THE GENERAL THEORY

Ten years after he proposed the special theory of relativity, Einstein completed the work and derived the general theory of relativity, which describes accelerating coordinate systems and gravity! Like the special theory, the general theory begins with a very simple statement, called Einstein's equivalence principle.

2.1. The equivalence principle

2.1.1. Statement

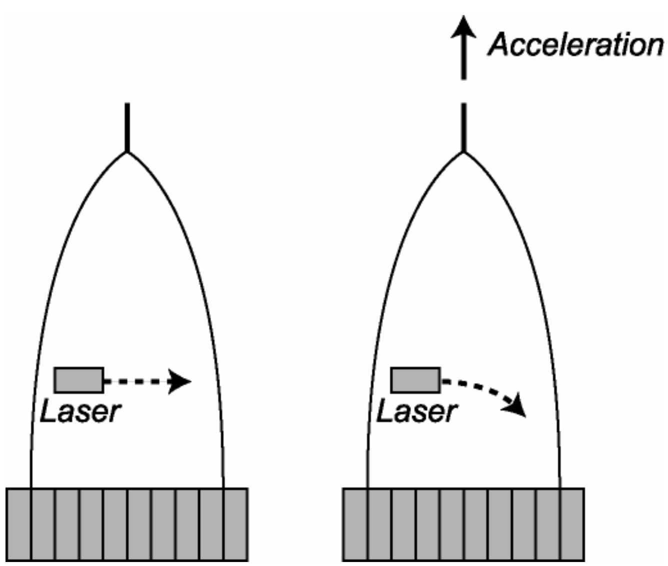
It is impossible to distinguish between gravity and acceleration.

2.1.2. An illustration

To illustrate the common sense of this principle, think of yourself in an elevator. If you drop a ball, it falls to the floor. If the elevator accelerates upward, your knees bend; and it feels as though you are heavier. If you drop a ball, it hits the floor sooner. Is this because the floor is accelerating toward the ball, or because you are in a stronger gravitational field? Except by looking out the window (if any), there is no experiment which can distinguish between these two possibilities. By the same token, if the elevator is in free fall, you will experience total weightlessness (until you hit the bottom of the elevator shaft!) This is indistinguishable from being stationary in no gravitational field.

2.2. Gedanken experiment on the bending of light

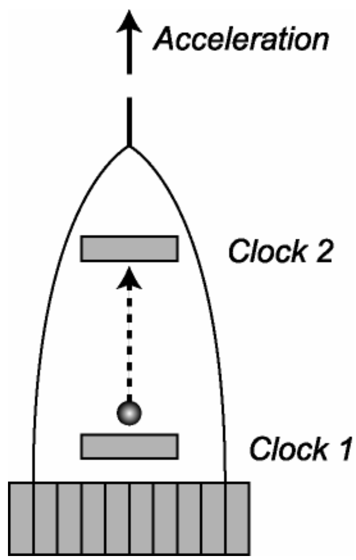
Let's do another gedanken experiment in the space ship. Assume, for now that the space ship is far from any gravitational field. Let's shine a laser across the space ship at a spot on the opposite wall:



Now let the space ship accelerate as we do the experiment. This time, as the laser beam travels across the space ship the space ship moves, so the spot hits the wall at a slightly lower point. But wait. Was the space ship accelerating, or was it in a gravitational field/ Since there is no way to know, we have to conclude that light bends in a gravitational field! Actually, it took Einstein's genius to recognize that the light travels in a straight line, in some sense (it travels along a geodesic in spacetime). It is spacetime itself that is curved by gravity. Figure that!

2.3. Gedanken experiment on time in a gravitational field

Let's do another experiment in our space ship. This time, let's try to synchronize two clocks, one on the floor and one on the ceiling:



We do this by sending out light pulses from the lower clock to the upper one. After some initial checking with the space ship at rest we find out that the two atomic clocks keep the same time. Now let the space ship accelerate upward. Every time the lower clock ticks, the upper clock moves away faster and it takes the light pulse longer to catch up to the upper clock. Therefore, every pulse from the lower clock arrives later than the one before, relative to the clock on the ceiling. An observer watching the upper clock will conclude that the lower clock is ticking more slowly than the upper one. But acceleration is indistinguishable from gravity, so the lower clock in a gravitational field appears to go more slowly than the higher one. That is, time itself moves more slowly deeper in a gravitational field!

Actually, this experiment has been done, several times. The comparison of the clocks in the twin paradox experiment described above actually included the effects of the earth's gravitational field. In addition, a careful experiment with two atomic time standards was conducted between the bottom and top of a laboratory at Harvard. And finally, the global positioning system that we use to navigate in our ships and cars would not be as accurate as it is without including relativistic corrections. This is the first commercial application of Einstein's general theory of relativity!

2.4. Black holes

2.4.1. Escape velocity of light

Let's do another gedanken experiment. If we Stand on the surface of the earth and shoot a bullet into the sky, it will fall back to earth unless it is going fast enough to escape the gravitational pull of the earth, as when a rocket is launched to probe Jupiter. The minimum energy to escape the earth is just the potential energy of the body at the earth's surface:

$$\tfrac{1}{2}mv^2 = \frac{mMG}{R},$$

so the escape velocity is

$$v = \sqrt{\frac{2MG}{R}}$$

Note that the mass of the rocket cancels out. What happens if the escape velocity is c, the speed of light, or higher? The short answer is that nothing escapes! Not even light. The massive, compact body producing this much gravity becomes a black hole.

Actually, we believe now that some black holes have been discovered. How can we "see" a black hole? Well, nothing escapes from it, but its gravity extends outside it and we can see that. For example, some stars appear to be orbiting invisible, massive bodies, which must be black holes. In fact, at the center of galaxies there appear to be super massive (but invisible) bodies sucking up all the surrounding galaxy! We "see" them by the radiation produced by the matter getting sucked in and by the motion of the rest of the galaxy around the massive center. Finally, we see the light from stars apparently being bent by the gravity from invisible bodies along the path from the star to us.

2.4.2. The event horizon, and the stopping of time

When a massive star explodes (called a supernova) and then collapses, it is believed that when its radius gets smaller than the value given by the equation above, and the star becomes a black hole. No light, or any information other than the gravity, can escape from that radius at which the escape velocity is the speed of light. We call this the "event horizon." The actual radius of the star (if this has any meaning) can be smaller, but we can see only to the event horizon. On the other hand, the gravitational field can suck the surrounding matter (and light) into the black hole, which grows, and grows, and…

Curiously (but probably not surprisingly, at this point), time stops at the event horizon. That is, the gravitational slowing of clocks stops them completely there. This is not the fault of a bad clock, even though any clock reaching this point is likely to be in bad shape. It is the distortion of space and time itself by gravity.