GATE 2022 IN 14

EE23BTECH11065 - prem sagar

Question:
The output of the system y(t) is related to its input x(t) according to the relation $y(t) = x(t)\,sin(2\pi t)$.This system is

(A) Linear and time-variant
(B) Non-Linear and time-invariant
(C) Linear and time-invariant
(D) Non-linear and time-variant

Solution:

Symbol	Value	Description
$x(t)$		input signal
$y(t)$	$x(t)\,sin(2\pi t)$	output signal
τ		Time delay

TABLE 1
INPUT PARAMETERS

From Table 1

$$y_1(t) \leftrightarrow x_1(t) \tag{1}$$
$$y_2(t) \leftrightarrow x_2(t) \tag{2}$$
$$ay_1(t) + by_2(t) \leftrightarrow ax_1(t) + bx_2(t) \tag{3}$$
$$ay_1(t) + by_2(t) = (ax_1(t) + bx_2(t))\,sin(2\pi t) \tag{4}$$

$\therefore$ satisfies principle of superposition

$$ky(t) \leftrightarrow kx(t) \tag{5}$$
$$ky(t) = k(x(t)\,sin(2\pi t)) \tag{6}$$

$\therefore$ satisfies principle of homogenity
$\therefore$ it is linear

Delay in input x(t):

$$y_1(t) = x(t-\tau)\,sin(2\pi t) \tag{7}$$

Delay in output y(t):

$$y(t-\tau) = x(t-\tau)\,sin(2\pi(t-\tau)) \tag{8}$$
$$y_2(t) = x(t-\tau)\,sin(2\pi(t-\tau)) \tag{9}$$
$$y_1(t) \neq y_2(t) \tag{10}$$

$\therefore$ it is time variant
$\therefore$ (A) linear and time variant