10.5.3.19

EE23BTECH11065 - prem sagar

Question:
200 logs are stacked in the following manner .20 logs in the bottom row ,19 in the next row ,18 in the row next to it and so on(see Fig 5.5).In how many rows are the 200 logs placed and how many logs are in the top row.

Solution:

Symbol	Value	Description
$x(0)$	20	first term of AP
d	−1	common difference
$x(n)$		$(x(0)+nd)\,u(n)$
$y(n)$	200	

TABLE 1
INPUT PARAMETERS

From Table 1:
Applying Z transform:

$$\implies X(Z) = \frac{20}{1-z^{-1}} - \frac{z^{-1}}{\left(1-z^{-1}\right)^2}, |z| > 1 \quad (1)$$

$$y(n) = x(n) * u(n) \quad (2)$$

$$\implies Y(Z) = X(Z)\,U(Z) \quad (3)$$

$$= \frac{20}{\left(1-z^{-1}\right)^2} - \frac{z^{-1}}{\left(1-z^{-1}\right)^3}, |z| > 1 \quad (4)$$

Using contour integration to find inverse Z transformation

$$y(n) = \frac{1}{2\pi j}\oint_C Y(Z)\,z^{n-1}\,dz \quad (5)$$

$$= \frac{1}{2\pi j}\oint_C \left(\frac{20}{\left(1-z^{-1}\right)^2} - \frac{z^{-1}}{\left(1-z^{-1}\right)^3}\right) z^{n-1} dz \quad (6)$$

we can observe only that there is only a 3 times repeated pole at z=1,

$$y(n) = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \quad (7)$$

$$= \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\,\frac{\left(20z^{n+2}-21z^{n+1}\right)}{(z-1)^3}\right) \quad (8)$$

$$= \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left(20z^{n+2}-21z^{n+1}\right) \quad (9)$$

$$= \frac{(n+1)(40-n)}{2} \quad (10)$$

since number of logs=200

$$200 = \frac{(n+1)(40-n)}{2} \quad (11)$$

$$n = 24, 15 \quad (12)$$

for n=24

$$x(24) = 20 - 24 \quad (13)$$

$$= -4 \quad (14)$$

but logs can't be negative
for n=15

$$x(15) = 20 - 15 \quad (15)$$

$$= 5 \quad (16)$$

so number of rows=15
number of logs=5

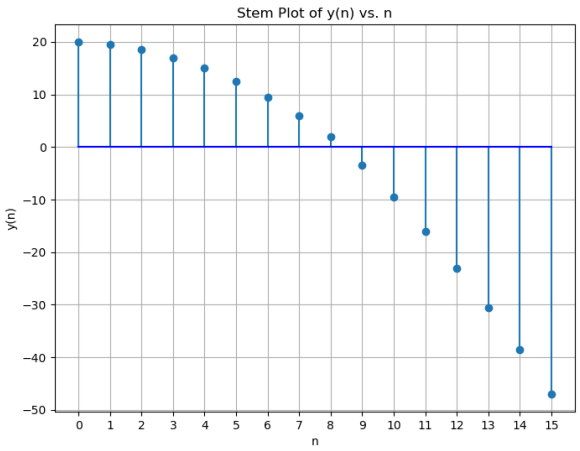

Fig. 1. plot of y(n) v/s n