

11.9.3.3

EE23BTECH11065 - prem sagar

Question:

The 5th, 8th and 11th terms of a GP are p, q and s respectively. show that $q^2 = ps$

solution:

Symbol	Value	Description
$x(5)$	p	$x(0)r^5$
$x(8)$	q	$x(0)r^8$
$x(11)$	s	$x(0)r^{11}$
$x(n)$		$x(0)r^n u(n)$
r	$\left(\frac{s}{p}\right)^{\frac{1}{6}}$	common ratio

TABLE 1
INPUT PARAMETERS

From Table 1:

$$\begin{aligned}
 q^2 &= x(0)r^8 x(0)r^8 \\
 &= x(0)^2 r^{16} \\
 ps &= x(0)r^5 x(0)r^{11} \\
 &= x(0)^2 r^{16} \\
 \Rightarrow q^2 &= ps
 \end{aligned}$$

now we will find r and x(0):

$$\begin{aligned}
 \frac{s}{p} &= \frac{x(0)r^{11}}{x(0)r^5} \\
 r &= \left(\frac{s}{p}\right)^{\frac{1}{6}} \\
 p &= x(0)\left(\frac{s}{p}\right)^{\frac{5}{6}} \\
 x(0) &= \frac{p^{\frac{11}{6}}}{s^{\frac{5}{6}}}
 \end{aligned}$$

Applying z-Transform:

$$X(z) = \frac{x(0)}{1 - rz^{-1}}, |z| > |r| \quad (10)$$

$$\Rightarrow X(z) = \frac{p^3}{p^{\frac{7}{6}}s^{\frac{5}{6}} - q^2z^{-1}}, |z| > \left|\left(\frac{q}{p}\right)^{\frac{1}{3}}\right| \quad (11)$$

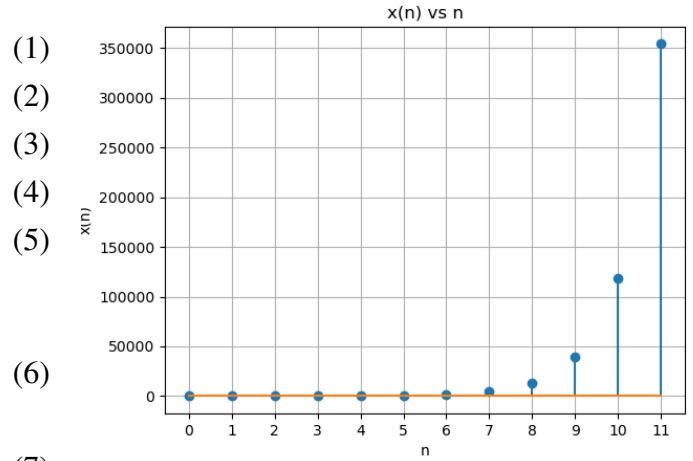


Fig. 1. plot $x(n)$ vs n $p = 486$, $q = 13122$, $s = 354294$, $r = 3$