11.9.3.3

EE23BTECH11065 - prem sagar

Question:
The 5th,8th and 11th terms of a GP are p,q and s respectively .show that

$$q^2 = ps$$

solution:
Given,

$$x(5) = p \tag{1}$$
$$x(8) = q \tag{2}$$
$$x(11) = s \tag{3}$$

let first term of a GP= a
common ratio of GP=r
we know,

$$\text{nth term of a GP} = x(n) = a \cdot r^n, \text{if} n \geq 0 \tag{4}$$
$$\text{so 5th term of GP}(x(5)) = a \cdot r^5 = p \tag{5}$$
$$\text{8th term of GP}(x(8)) = a \cdot r^8 = q \tag{6}$$
$$\text{11th term of GP}(x(11)) = a \cdot r^{11} = s \tag{7}$$
$$x(8) \cdot x(8) = a \cdot r^8 \cdot a \cdot r^8 \tag{8}$$
$$= a^2 \cdot r^{16} \tag{9}$$
$$x(5) \cdot x(11) = a \cdot r^5 \cdot a \cdot r^{11} \tag{10}$$
$$= a^2 \cdot r^{16} \tag{11}$$
$$x(8)^2 = x(5) \cdot x(11) \tag{12}$$

so,

$$p = a \cdot r^5 \tag{13}$$
$$q = a \cdot r^8 \tag{14}$$
$$s = a \cdot r^{11} \tag{15}$$
$$q^2 = p \cdot s \tag{16}$$

hence proved

symbol	value	description
$x(5)$	$a \cdot r^5 = p$	5th term of GP
$x(8)$	$a \cdot r^8 = q$	8th term of GP
$x(11)$	$a \cdot r^{11} = s$	11th term of GP

$$u(n) = \begin{cases} 1, & \text{if} n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$x(n) = a \cdot r^n \cdot u(n) \tag{17}$$

from (17)

$$x(n) = \begin{cases} a \cdot r^n & \text{if} n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$x(n) \xleftrightarrow{Z} X(Z) \tag{18}$$
$$X(Z) = \sum_{n=-\infty}^{\infty} x(n) \cdot Z^{-n} \tag{19}$$

using (19)

$$= \sum_{n=-\infty}^{\infty} a \cdot r^n \cdot u(n) \cdot z^{-n} \tag{20}$$
$$= a \sum_{n=-\infty}^{\infty} r^n \cdot u(n) \cdot z^{-n} \tag{21}$$
$$= a \sum_{n=0}^{\infty} r^n \cdot z^{-n} \tag{22}$$
$$\text{sum of infinite terms in G.P} = \frac{a}{1-r} \tag{23}$$

from (23)

$$= a \cdot \frac{1}{1 - r \cdot z^{-1}} \tag{24}$$
$$= \frac{a}{1 - r \cdot z^{-1}} \tag{25}$$

R.O.C$\rightarrow |z| > r$

symbol	value	description
$x(n)$	$a \cdot r^n$	nth term of GP
$X(Z)$	$\frac{a}{1-r\cdot z^{-1}}$	Z transform of x(n)
$u(n)$		unit step function