11.9.3.3

EE23BTECH11065 - prem sagar

Question:
The 5th,8th and 11th terms of a GP are p,q and s respectively .show that $q^2 = ps$

solution:
let r be common ratio

Symbol	Value	Description
$x(5)$	p	$x(0)\,r^5$
$x(8)$	q	$x(0)\,r^8$
$x(11)$	s	$x(0)\,r^{11}$
$x(n)$		$x(0)\,r^n u(n)$

TABLE 1

INPUT PARAMETERS

From Table 1:

$$q^2 = x(0)\,r^8\,x(0)\,r^8 \tag{1}$$

$$= x(0)^2\,r^{16} \tag{2}$$

$$ps = x(0)\,r^5\,x(0)\,r^{11} \tag{3}$$

$$= x(0)^2\,r^{16} \tag{4}$$

$$\implies q^2 = ps \tag{5}$$

now we will find r and x(0):

$$\frac{s}{p} = \frac{x(0)\,r^{11}}{x(0)\,r^5} \tag{6}$$

$$r = \left(\frac{s}{p}\right)^{\frac{1}{6}} \tag{7}$$

$$p = x(0)\left(\frac{s}{p}\right)^{\frac{5}{6}} \tag{8}$$

$$x(0) = \frac{p^{\frac{11}{6}}}{s^{\frac{5}{6}}} \tag{9}$$

Applying z-Transform:

$$X(z) = \frac{x(0)}{1 - r\,z^{-1}} \ , |z| > |r| \tag{10}$$

$$\implies X(z) = \frac{p^3}{p^{\frac{7}{6}}s^{\frac{5}{6}} - q^2 z^{-1}} \ , |z| > \left|\left(\frac{q}{p}\right)^{\frac{1}{3}}\right| \tag{11}$$
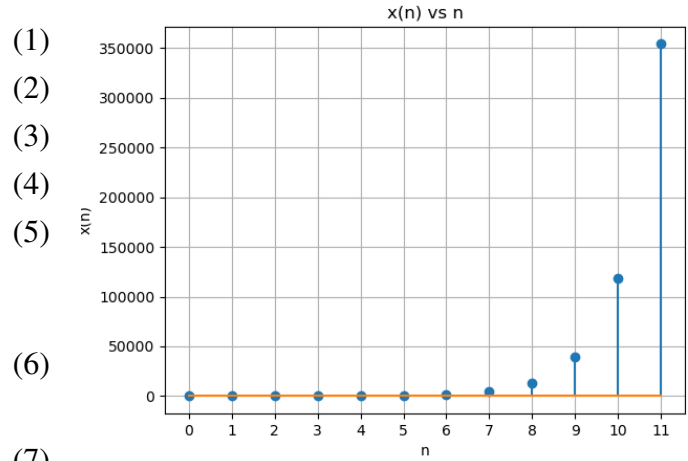

Fig. 1. plot x(n)vs n $p = 486$, $q = 13122$, $s = 354294$, $r = 3$