

# GATE 2022 IN 14

EE23BTECH11065 - prem sagar

## Question:

The output of the system  $y(t)$  is related to its input  $x(t)$  according to the relation  $y(t) = x(t) \sin(2\pi t)$ . This system is

- (A) Linear and time-variant
- (B) Non-Linear and time-invariant
- (C) Linear and time-invariant
- (D) Non-linear and time-variant

## Solution:

Symbol	Value	Description
$x(t)$		input signal
$y(t)$	$x(t) \sin(2\pi t)$	output signal
$\tau$		Time delay

TABLE 1  
INPUT PARAMETERS

Delay in input  $x(t)$ :

$$y_1(t) = x(t - \tau) \sin(2\pi t) \quad (10)$$

Delay in output  $y(t)$ :

$$y(t - \tau) = x(t - \tau) \sin(2\pi(t - \tau)) \quad (11)$$

$$y_2(t) = x(t - \tau) \sin(2\pi(t - \tau)) \quad (12)$$

$$y_1(t) \neq y_2(t) \quad (13)$$

$\therefore$  it is time variant

From Table 1

$$ax_1(t) + bx_2(t) \leftrightarrow ay_1(t) + by_2(t) \quad (1)$$

Applying Fourier transform:

$$\mathcal{F}(y_1(t)) = \mathcal{F}(x_1(t) \sin(2\pi t)) \quad (2)$$

$$Y_1(f) = X_1(f) * \frac{1}{2j} [\delta(f + 1) - \delta(f - 1)] \quad (3)$$

$$\mathcal{F}(y_2(t)) = \mathcal{F}(x_2(t) \sin(2\pi t)) \quad (4)$$

$$Y_2(f) = X_2(f) * \frac{1}{2j} [\delta(f + 1) - \delta(f - 1)] \quad (5)$$

$$aY_1(f) + bY_2(f) = aX_1(f) * \frac{1}{2j} [\delta(f + 1) - \delta(f - 1)] + bX_2(f) * \frac{1}{2j} [\delta(f + 1) - \delta(f - 1)] \quad (6)$$

$\therefore$  satisfies principle of superposition

$$ky(t) \leftrightarrow kx(t) \quad (7)$$

$$\mathcal{F}(y_1(t)) = \mathcal{F}(x_1(t) \sin(2\pi t)) \quad (8)$$

$$kY_1(f) = kX_1(f) * \frac{1}{2j} [\delta(f + 1) - \delta(f - 1)] \quad (9)$$