

# Assignment - 1

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**Question 1.1.7** Find the angles  $A, B, C$ , given that

$$\cos A \triangleq \frac{(\mathbf{B} - \mathbf{A})^T(\mathbf{C} - \mathbf{A})}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{C} - \mathbf{A}\|} \quad (1)$$

**solution:**

From the given values of  $\mathbf{A}, \mathbf{B}, \mathbf{C}$ ,

1) finding the value of angle  $A$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \quad (2)$$

and

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \quad (3)$$

and the values of norms

$$\|\mathbf{B} - \mathbf{A}\| = \sqrt{74} \quad (4)$$

$$\|\mathbf{C} - \mathbf{A}\| = \sqrt{32} \quad (5)$$

and doing by matrix multiplication we get,

$$\begin{aligned} (\mathbf{B} - \mathbf{A})^T(\mathbf{C} - \mathbf{A}) &= \begin{pmatrix} -4 \\ -4 \end{pmatrix} \\ &= -8 \end{aligned} \quad (6)$$

so

$$\cos A = \frac{-8}{\sqrt{74} \sqrt{32}} \quad (7)$$

$$= \frac{-1}{\sqrt{37}} \quad (8)$$

$$\Rightarrow A = \cos^{-1} \frac{-1}{\sqrt{37}} \quad (9)$$

2) Finding the value of angle  $B$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 1 \\ -11 \end{pmatrix} \quad (10)$$

and

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -7 \end{pmatrix} \quad (11)$$

also calculating the values of norms

$$\|\mathbf{C} - \mathbf{B}\| = \sqrt{122} \quad (12)$$

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{74} \quad (13)$$

and by doing matrix multiplication we get,

$$\begin{aligned} (\mathbf{C} - \mathbf{B})^T(\mathbf{A} - \mathbf{B}) &= \begin{pmatrix} 1 & -11 \end{pmatrix} \begin{pmatrix} 5 \\ -7 \end{pmatrix} \\ &= 82 \end{aligned} \quad (14)$$

so

$$\cos B = \frac{82}{\sqrt{74} \sqrt{122}} \quad (15)$$

$$= \frac{41}{\sqrt{2257}} \quad (16)$$

$$\Rightarrow B = \cos^{-1} \frac{41}{\sqrt{2257}} \quad (17)$$

3) Finding the value of angle  $C$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (18)$$

and

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} \quad (19)$$

also calculating the values of norms

$$\|\mathbf{A} - \mathbf{C}\| = \sqrt{32} \quad (20)$$

$$\|\mathbf{B} - \mathbf{C}\| = \sqrt{122} \quad (21)$$

and by doing matrix multiplication we get,

$$\begin{aligned} (\mathbf{A} - \mathbf{C})^T(\mathbf{B} - \mathbf{C}) &= \begin{pmatrix} 4 & 4 \end{pmatrix} \begin{pmatrix} -1 \\ 11 \end{pmatrix} \\ &= 40 \end{aligned} \quad (22)$$

so

$$\cos C = \frac{40}{\sqrt{32} \sqrt{122}} \quad (23)$$

$$= \frac{5}{\sqrt{61}} \quad (24)$$

$$\Rightarrow C = \cos^{-1} \frac{5}{\sqrt{61}} \quad (25)$$