Simulating Chaotic Weather Patterns: Analyzing Weather Forecasting Insights Using Differential Equations

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***Abstract*—This research work outlines the chaotic weather patterns with respect to the lorenz equations and their relationship with fluid dynamics and atmospheric predictabilities. The approach is a three-dimensional computational model that solves reduced lorenz equations with a finite difference method. By observing different development patterns of randomized initial velocity fields via advection, diffusion and pressure gradient terms, we aim to study extension of microscale perturbations toward macroscale chaotic behavior. Simulation captures basic constraint of the weather prediction termed as butterfly effect which means that small differences in initial conditions will have large variations in outcome related to time. With most advanced numerical schemes, our results show that causal long-term weather forecasts decrease exponentially into time matching with signature results of Lorenz.**

In addition, our results show how temperature gradients expressed in terms of velocity field magnitudes create the sort of complicated flow patterns that have been observed to occur in the circulation characteristics of the atmosphere. The findings of this work will be of interest to meteorologists and atmospheric scientists to the extent that they want to improve forecast models by better describing chaotic ingredients in the weather. Also includes upgrading from Lorenz 96 to Lorenz equations. Index Terms: Lorenz Equations, Lorenz96 System, LSTM model, Butterfly effect

This work also consists the upgradation of Lorenz equation to Lorenz 96 system.

***Index Terms*—Lorenz Equations, Lorenz96 System, LSTM model, Butterfly effect**

I. Introduction

Our project is chiefly concerned with modeling chaotic weather patterns: Analysis of meteorological forecasting data using differential equations concerned with the various complex nonlinear dynamical short-term predictors.

Weather prediction is one of the toughest problems of computational science because the atmosphere is itself a chaotic medium. The main purpose of this project was to model unpredicted change by means of differential equations. Here we particularly focused upon the equations of Lorenz- and involving his Lorenz-96.

These are trivializable-reduced-threesome simple mathematical equations showing some chaotic features of atmospheric convection, and these equations have been formulated in 1963 by Edward Lorenz. This three-dimensional system is probably best known for showing the butterfly effect: how small changes in initial conditions can lead to wildly different results. Therefore, one of these major barriers concerned with long-term forecasts is well represented in Lorenz attractor, which has a butterfly-like trajectory in phase space.

This original Lorenz model is very good at chaos theory; however, very poor with regard to the actual weather prediction. Hence we now shift our attention to the Lorenz-96 system that allows amore spatially complex interpretation of atmospheric dynamics. Such a model simulates the weather along a latitude circle with variables corresponding to the key elements of temperature, pressure, and wind speed at different locations under investigation.

That comparison helps us define the limits of predictability; then, at what point prediction results become meaningless due to growth of measurement errors. We'll numerically solve those differential equations and visualize the behavior resulting from it while assessing the prediction capability in light of measurement and forecast timescales.

So this would feed back the results of this work into ensemble forecasting-the new-fangled way of weather forecasting that involves running heterogeneous simulations with slightly different initial conditions to improve reliability in predictions.

The Lorenz Equations were not meant for the long-term predictions of the weather, so we upgraded Lorenz equations to Lorenz96 system using LSTM model.

II. LITERATURE REVIEW

1) **Lorenz Equations: The Definitive Model for Meteorological Prediction**

Authors: Robert Ferro

In this paper, the author mainly described about the numerical weather prediction using the lorenz equations for its chaotic nature. These chaotic equations are numerically solved by 4th order RK method. The results in this paper mainly describes the different trajectories diverge rapidly even with slight change in the initial conditions In this they have taken only 2 initial conditions, when compared to our paper there are many initial conditions either we used data set. They finally concluded that the system is unpredictable in long term predictions.

2) Direct statistical simulation of the Lorenz96 system in model reduction approaches

Authors: Kuan Li, Steven M

This paper approaches a Direct Statistical simulation(DSS) to analyze the lorenz96 system. This paper states that instead of using the traditional methods, DSS directly solve the statistical properties by evolving the statistical cumulants. This paper used Cumulation methods either CE2, CE2.5. The results of this paper gives the chaotic refimes and periodic regimes. This paper helps our project, how actually lorenz96 system evolve periodic and statistically.

3) Visualising chaos with RK4

Authors: Aarana Vitarana

This paper breifly describes about the lorenz equations and the chaotic graphs that come when we use lorenz equations. This paper clearly explains the RK4 method, gives the clear equations invovled in this method and solvation of the equations and also different regimes. This paper clearly explains the 3 constants that are used in the lorenz equations. There is no specific method involved but the main method is the RK4 method with lorenz equations. This paper helps our project easier making the clear understanding of the chaos of lorenz equations and the different graphs involved in that and also the the 3 main constant in the lorenz equations.

4) Using the probabilistic machione learning to better model temporal patterns in parameterizations: a case study with lorenz96 model

Authors: Raghul Parthipan, Hannah M. , J. Scott hosking, DamonJ.

This paper addresses the challenge of accurately modelling the small-scale processes in climate models. This paper says that in the traditional methods they capture the different noises. Here, they used the special model called L96-RNN which is designed to evaluate the unsolved processes of loren96. This consist of the two gated recurrent layers. The result of the paper shows the models of various metrics, including lower Kullback–Leibler divergence scores for probability density functions arising from long-range simulations.

5) The Dual Nature of Chaos and Order in the Atmosphere

Authors: Bo-Wen Shen, Roger Pielke, Xubin Zeng, Robert Atlas, Amit Kesarkar, Xiping Zeng, Sara Faghih Naini, Jialin Cai.

This paper reveals the coexistence of the Lorenz models that differentiate the chaotic and non-chaotic behavior in the atmosphere. The model used in this paper are L63, L69, GLM respectively. The findings of the paper shows the existence of the attractors, time-varying forces etc. The authors says that the atmosphere show chaotic nature with these models. The authors says that the chaotic solutions have limited predictability where as non- chaos solutions predictable life time.

6) Performance Analysis of LSTM Vs GRU in Predicting Weather Patterns For Climate Change Models

Authors: Pavithra E, Mayuri Reddy, Suresh Bhukya

This paper mainly shows the differences of working of LSTM model and Gated Recurrent unit either GRU model, which model is most effective for predicting weather patterns. The parameters used here are temperature, humidity and wind speed. They conclude by saying that the LSTM model performs better than GRU by getting better RMSE and R$^2 score$.

7) Performance Comparison Analysis on Weather Prediction using LSTM and TKAN

Authors: Ajie Kusuma Wardhana, Yudha Riwanto, Budi Wijaya Rauf

The goal of this paper to show the comparision of TKAN(Temporal Kolmogorov Arnold Network) and LSTM for weather predicting using mean loss and MAE. They state that the upgradation of LSTM and GRU either for getting more accuracy the new model KAN have come in. Again KAN is modelled to TKAN. They stated the result as the model for both LSTM and TKAN achieved 0.09 and 0.11 for model loss and 0.08 and 0.96 for MAE.

8) Modelling Weather Conditions Using Encoder-Decoder and Attention Based on LSTM Deep Regression Model

Authors: Amr Badr, Khder Alakkari, Mostafa Abotaleb, El sayed

This paper states that the weather report has been taken from the Narmadapuram District. This project includes the models encoder and decoder LSTM and Attention LSTM. The result shows that Encoder and Decoder LSTM model performed better than the Attention LSTM.

9) Comparison of RNN-LSTM, TFDF and stacking model approach for weather forecasting in Bangladesh using historical data from 1963 to 2022

Authors: Md. Mahamudul Hasan, Md. Jahid Hasan, Parisha Binte Rahman

This paper reveals that the weather is data is 60 years meterological data of Bangladesh from 1933 to 2022. The parameters taken are temperature, rainfall, humidity and pressure. The main algoithms used here are RNN- LSTM and TFDF(

tensorFlow decision forest) and stacking ensemble models. They conclude that the Stacking ensemble model acheived best performance with RMSLE 1.3002 and 10 percent more accurate than LSTM.

Taking all the papers into consideration we first used Lorenz equations using RK4 method to get clearly clarify about the chaotic nature of the weather conditions and how the graphs vary, than we upgraded it to Lorenz96 system using LSTM model which is more complex than the earlier lorenz equations.

The above papers when compared to our project, considered only either using lorenz equations or only lorenz96 system but our project first used the lorenz equations then upgraded it to lorenz96 system using the LSTM models. But the above papers helped by getting the clear information about the chaotic nature and non-chaotic nature and how the graphs vary with the different parameters and also how different models work differently.

III. METHODOLOGY

A. lORENZ EQUATIONS

These are the three famous non- linear differential equations which are mostly used for predicted chaotic behaviour of weather patterns. These are mainly used for predicted short-term weather patterns.

The three equations are:

$$dx/dt = \sigma(y - x) \tag{1}$$

$$dy/dt = x(\rho - z) - y \tag{2}$$

$$dz/dt = xy - \beta(z) \tag{3}$$

Here the 3 constants represent:

- σ(Prandtl number) shows how quickly temperature variations dissipate relative to velocity variations.
- ρ(Rayleigh number) shows temperature difference driving convection.
- β(Geometric ratio parameter) shows how pressure patterns (z) evolve.

The 3 state variables represent:

· x represent temperature

· y represent wind velocity

· z represent pressure

Here, we are taking only 3 state variables but when we consider the weather there are many parameters other than temperature, wind velocity, pressure like humidity etc.

So we upgraded the Lorenz equations to Lorenz96 System where this system has multiple state variables.

B. lORENZ 96 SYSTEM

This consist of single equation, also invented by Edward Lorenz which is the upgradation of Lorenz equations.

$$\mathbf{dx_i/dt=(x_{i+1}-x_{i-2})*x_{i-1}-x_i+F}$$

Here,

· x_i represents the state variable

· i represent the grid points i.e i= 1,2,. . . ..N

Here N typically range between 20 to 40

· F represents external influences like heating from the Sun or other atmospheric sources (controls energy input).

C. RK4 Method

This is the numerical method used for solving the first order differential equations in the form dx/dt=f(x,y). It estimates the next state of the system using the following step:

$$\mathbf{X^{(n+1)}=x^{(n)}+\ t/6(k_1+2k_2+2k_3+k_4)}$$

where:

k_1 is the derivative at the current step.

k_2 is the derivative at a half-step forward.

k_3 is another estimate at a half-step forward using k2

k_4 is the derivative at the full step forward.

D. DATA STRUCTURE : Data structure is an important thing here because when we use a set of values at a particular time, it must be stored somewhere so that model can easily acces that values at the particular time, for this purpose we used TREE data structure.

Fig. 1. TREE data structures where values are stored

This consists of particular time , what are the predicted values of the 3 variables i.e the temperature, pressure, wind speed. Graph data structure is more better than trees for more values etc. but when we use graph, it is used for storing sequential time series data.

IV. MODEL

For the Lorenz96 we are using the LSTM (Long short- term memory) model.

This the type of deep learning model that are mostly used in the project where time series data is used because they can remember long term dependencies that is ideal for predicting the chaotic patterns.

This network is mainly used because:

1) Learning temporary dependencies in Lorenz 96 system.

2) Capturing the long-term relationships between the variables.

3) Making strong predictions in chaotic environments.

Structure of LSTM:

LSTM network consists of LSTM cells which basically contains 4 gates which are used for flow of information.

1) Forget gate:

This is determined by f_t.

This gate mainly determines which past information should be discarded either mainly focus on past weather states.

This is shown by formula:

$\mathbf{f_t = \sigma(W_f.[h_{t-1},x_t]+b_f}$

Here,

- X_t represents current state variable.
- W_f is the weight matrix
- H_{t-1} is the previous hidden state
- b_f is the bias

σ is the sigmoid activation function

2) Input Gate

This is determined by i_t .

This gate determines what new information should be added to the memory.

This is shown by the formula:

$\mathbf{i_t = (W_i.[h_{t-1},x_t]+b_i)}$

3) Cell state:

This is determined by C_t .

After the input gate, a new cell state is created.

This is created by the formula:

$\mathbf{C_t = tanh(W_c.[h_{t-1},x_t]+b_c}$

Here,

- Tanh is the hyperbolic tangent activation function.

This is the main memory unit that store the long term dependencies.

This is represented by the formula:

$\mathbf{C_t = f_t .C_{t-1} + i_t . C_t}$

4) Output gate:

This is determined by O_t .

This gate determines which part of the memory should be used to predict the next state of the Lorenz96 system.

This is shown by the formula:

$\mathbf{O_t = \sigma(w_o . [h_{t-1},x_t t]+ b_o)}$

The final hidden state wither which represent the model prediction is given by:

$\mathbf{h_t = O_t . tanh(C_t)}$

Working of LSTM:

1) Input data:

- Collect the previous data of the system.
- Uses previous X_i values as the input features.
- Normalize the data to improve the training efficiency.

2) Model training:

- Train the model using the past time steps predicting the future values.
- The model learns the chaotic patterns.

Layer (Type)	Output Shape	Param #
input-layer-1 (InputLayer)	(None, 72, 6)	0
lstm-1 (LSTM)	(None, 32)	4,992
dense-1 (Dense)	(None, 1)	33

TABLE I
LSTM ARCHITECTURE

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V. RESULTS

1) Using Lorenz Equations:

First we collected the data from the real time dataset and then the initial conditions are given manually and the the lorenz equations using RK4 method normalised the values in the data set as it is hard to present the real values on the graph to compare it with the values after solving Lorenz equations. We took the 3 state variables either x,y,z as temperature, wind speed and pressure.

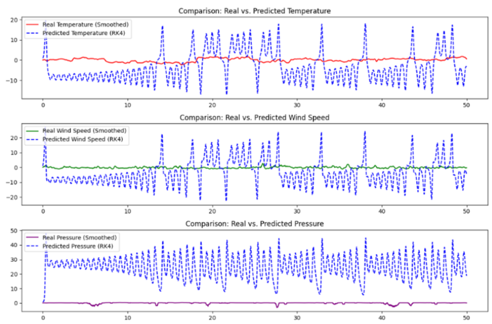

Fig. 2. Comparision of temperature, wind speed and pressure

In Fig:2 the graphs shows the comparison between the real normalised values and the predicted normalised values of Temperature, Wind speed, Pressure.

We can see that the values ranges are less because the 3 parameter values are normalised because when we take the original values without normalising it will be very difficult to show the comparison of the real values and the predicted values.

We can clearly observe that the short term real values of the Temperature, Wind speed and Pressure are almost constantly going but the same values when predicted using the Lorenz equations shows a lot of deviations where it can be predicted that there may be a temperature hike or there may be a tornado expected.

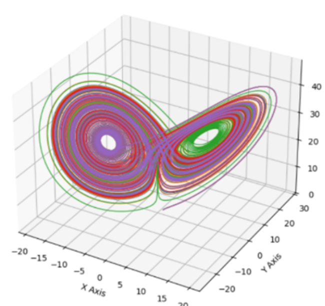

Fig. 3. Chaotic weather pattern of temperature and pressure

In Fig:3

X-axis represent Temperature

Y-axis represent the Pressure

In fig:3 the chaotic graph shown, the shape is so practical that this is called butterfly effect. This means that when we take only limited values like 5 to 10 values of the parameters the graph will be a single irregular curved which is complicated to predict, but when we use many values like at some range of time up to 6 hours or 12 hours, the all values that we get, when plotted on a graph will be in the form of butterfly wings hence this is called butterfly effect.

There are 2 lobes in the above graph.

Upper lobe attract the curves which are more than the real values taken, for example when we take temperature parameter, when the real normalised value is 10 and the predicted normalised value is 20, as the predicted value is more than the real value the upper lobe attract the curve. Lower lobe attract the curves which are less than the real values taken, for example when we take the same temperature parameter, when the real normalised value is 10 and the predicted normalised value is -10 then the lower lobe attract the curves.

2) Using Lorenz96 system:

The upgraded version of Lorenz equations are used to make the asolution or the chaotic graphs more practical. Below are the outputs of the temperature, Wind speed and velocity graphs solved by Lorenz96 system and LSTM model.

The below 2 graphs represent the comparison and the variations of the Temperature of the real values, Real LSTM values and the values after solving with Lorenz96 system with LSTM.

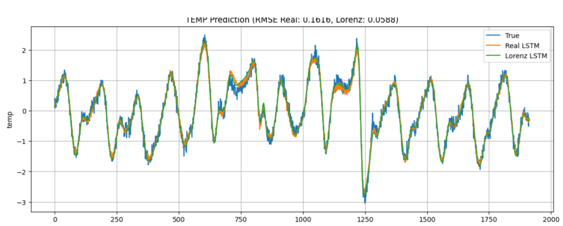

Fig. 4. Comparison of temperature with true values, Real LSTM values and Lorenz LSTM values

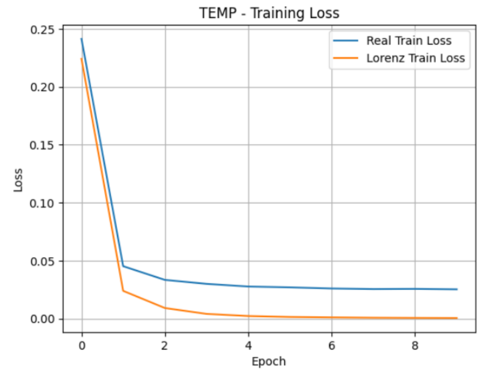

Fig. 5. Temperature Training loss graph of Real and Lorenz

The below graphs fig 6 and fig 7 shows the variations of Pressure. The first graph shows the comparision of the pressure values in real, real LSTM values and predicted Lorenz96 LSTM values. The next graph shows the training loss of the pressure.

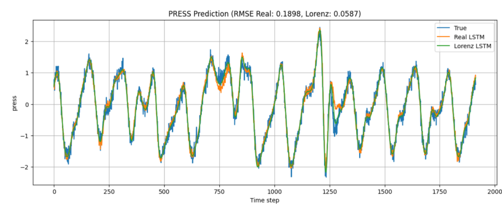

Fig. 6. Variations of pressure

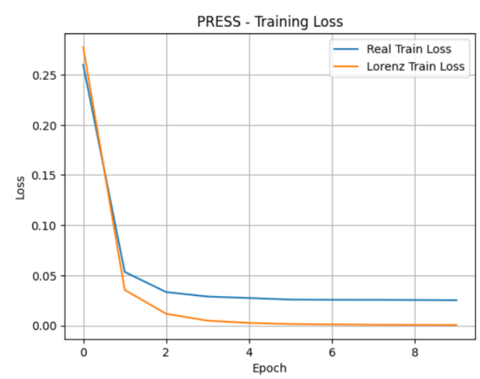

Fig. 7. Pressure training Loss

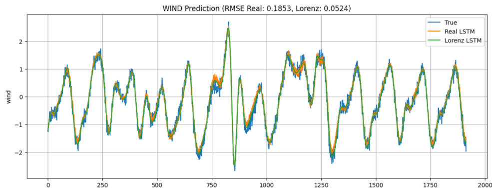

Fig. 8. Variations of wind speed

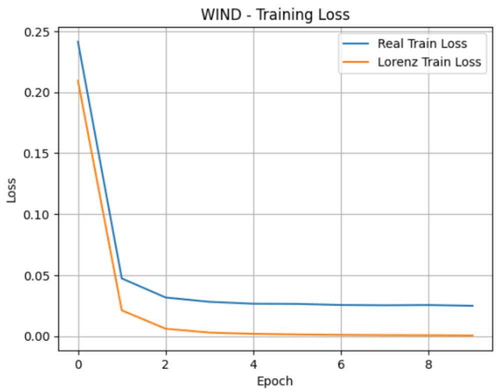

Fig. 9. Training loss of wind speed

The above graphs in fig 8 and fig 9 shows the variations of Wind speed. The first graph shows the comparison of the pressure values in real, real LSTM values and predicted Lorenz96 LSTM values. The next graph shows the training loss of the pressure with real values and Lorenz values. The below graph show the 3D representation when the temperature, pressure and the wind speed values are plotted.

In fig 10:
The black line in the graph represent the true values. The green line represent the real LSTM values. The red line represent the predicted Lorenz96 LSTM values.

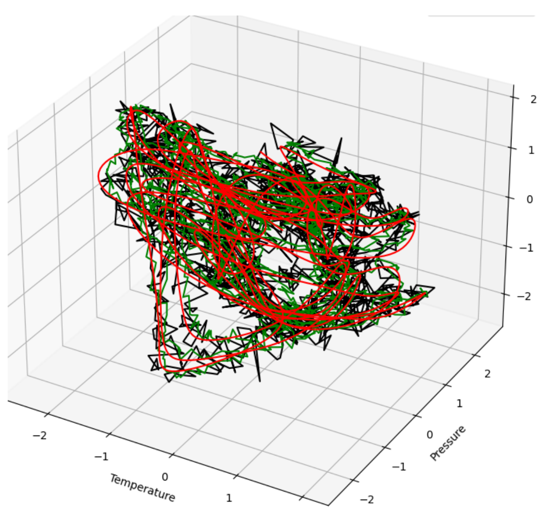

Fig. 10. 3D representation of temperature, pressure and wind speed

We mentioned there are multiple state varibales but we have taken only 3 parameters because in the previous lorenz equations we included only 3 parameters so to be similar we used the same 3 parameters for the Lorenz96 system also.

VI. CONCLUSION

In this project we first used Lorenz equation to predict the short term weather predictions using temperature, wind speed and pressure as the 3 variables. The result that we acquired in the form of graphs are stated above. Then to be more complex we upgraded Lorenz equations to Lorenz96 system using LSTM. These have multiple state variables but we have taken the same parameters what we have taken for Lorenz equations also to be more accurate. The results for the LSTM model is also pasted in the above results section.

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