University of Essex
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MA981: DISSERTATION

Generating RANDOM 'VALID' TRIALS for an AI- based game for the prevention of the development of Dyscalculia in children.

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Abstract

A particular learning disorder called Dyscalculia hinders a person's capacity to give mathematical and numerical concepts a quantitative meaning. In reality, dyscalculic individuals struggle to understand how calculations and tallying work. Finding methods to teach dyscalculic kids will help them become better at arithmetic calculations and will lessen the challenges that may arise in many areas of their daily life, according to a portion of scientific research that focuses on the study of this disease. In preschool or the very early years of primary school which is when almost 90 percent of us humans begin to develop our logical and arithmetical skills, it can be made possible to instruct children who have trouble understanding arithmetic and numerical ideas to help them understand and develop the concept of numerics in an effort to improve their mathematical abilities.

1 Introduction

Arithmetic skills are highly valued in today's society in a multitude of situations, including daily life, the school, and the workplace. Early development of children reveals that they have a concept of numbers and mathematical ability. According to studies, a baby's capacity to count is assumed to be based on two distinct systems: one that only analyses little numbers and tracks their visual characteristics, and another that estimates the numerals to reflect the largest numerosities.[1]

Human beings, along with several other species of animals, are evolved with what is known as Number Sense, as articulated by Kesler et al.[7] " Humans and Animal's numeric sense can be termed as a nonverbal skill that involves spatial relationships among numbers together with a mental arithmetic that facilitates number comparison and analysis" It is critical to understand and define the concept of number sense because persons who struggle with mathematics are thought to have a basic deficit in number sense. Scientists can identify whether or not a person has mathematics issues by testing their Number Sense in some way. Researchers can evaluate numerical and arithmetic sense in a number of ways, including:

1. Physical distance tests: this is used to determine the distance between two separate integers.
2. Computation and Counting test: Examples of computation and counting tests include addition and subtraction of numbers.
3. Comparison test: Test for comparison of two numbers

It is possible to develop a training program to handle such obstacles and improve mathematical abilities following those examinations, as well as many others, and then after determining whether or not there are arithmetic limits.

A specific disorder that handicaps us arithmetically is called dyscalculia and treating which is the core obective of this proposed effort. "Dyscalculia, commonly referred to as developmental dyscalculia, is one of the primary SLDs (Specific Learning Disorders) that impacts the capacity to learn school-level mathematical skills, similar to the more well-known Dyslexia which affects language and linguistic skills. [16] Limiting those who have this disorder from developing their arithmetic skills as other people, likely to result in a progressive decline in skill acquisition and, of course, a deficit in the process of learning"[3]. Young children can only be regarded as "at risk of a Dyscalculia diagnosis" until they begin primary school since dyscalculia cannot be diagnosed until then. Children's brains may be able to be taught, ideally in preschool or early primary school, to help prevent or mitigate the impacts of dyscalculia. This dissertation's idea might be utilised as an intervention tool for children who struggle with math difficulties. Its purpose is to assist children in developing their mathematical talents to the maximum extent possible so that they may approach the world of numbers with a more positive attitude and drive. In order to effectively train children and support them in improving their learning capacity of arithmetics and quantities, the main goal of this dissertation has been to thoroughly research Dyscalculia and develop a computer game consisting of a client and a server that is intended to be played by children in their final years of kindergarten, who are essentially children around 3 to 5 years old. It is crucial to note that the decision to implement a game-like assessment is not a case because there are numerous studies that report progress with his or her "training" games because it is a way for children to have fun while also being proven that they improve their arithmetic abilities or competence in gene-related tasks. The next chapter will examine all of the research to date as well as some potential remedies for dyscalculia. The following chapters will examine "Rescue Calcularis" [8] and "The Number Race" [19][18], two instances that are especially intriguing. The last one is the best known because it was the first game ever to be recorded with actual utility and because it was made accessible in many languages on both Windows and Mac systems. The works of Manuela Piazza, Vito De Feo, Stefano Panzeri, and Stanislas Dehaene served as inspiration for the idea. A dyscalculic person has trouble concentrating only on numerical information while responding to questions involving it, allowing oneself or herself to be influenced by potential non-numerical features such as size, shape, or colour, according to the research Learning to focus on number [14]. The experiment, which will be the subject of its own chapter in this dissertation, provides a training technique that should reduce the interference effect by teaching the dyscalculic child to focus solely on the number while filtering out any potentially distracting information. The game will be discussed in full in the next chapter, as well as how it was created, the technology employed, and how far the project has progressed. The last chapter will also discuss the efforts made to ensure that the entire project is compatible with the literature and the input variables which are often mentioned in it, as well as that it works with the Unity environment, which is used to construct the client-side of the game.

The report's conclusion will explain how the project and game could change in the future and what are the developments that can be made to use it for a greater good.

2 The Disability of Dyscalculia

Math abilities are highly appreciated in today's culture in a variety of contexts, including not only the classroom and industry, but also in everyday life. Children have a knowledge of numbers and fundamental arithmetic skills from an early age. An infant's sense of number is said to be made up of two separate systems: one that only evaluates and approximates extremely small numbers, and another that only loosely mimics the number system to represent the biggest numerosities. As these systems mature, they widen the linear mapping of the numerical idea of multiple numbers as well as the sensory experience of number. They develop and strengthen these brain functions through study and education by expanding their knowledge and mental representations of numbers. For certain people, however, who suffer from the disease dyscalculia, all of this is delayed or difficult.

2.1 Our Brain and how it understands numbers and numerical variables

Humans and other animals like fish and non-human primates have the ability to comprehend and compare the numerical magnitudes of groups of objects as natural talents from birth [20], suggesting that humans possess what is known as "number sense." The notion that this ability depends on the approximation number system, or ANS for short, an archaic cognitive system, has been furthered by certain studies, including that by Khanum et al. [20]. The capacity to approximate numerals and their magnitude is known as the automatic number system (ANS), but humans differ from many other species in that we also get formal education with the purpose of improving and upgrading the ANS to a far more exact and comprehensive image. The development of the ANS is thought to be advantageous for teaching fundamental symbolic arithmetic to both adults and children, whether in a laboratory context or outside, such as in general education classrooms or settings. The implication that humans are born with ANS gives rise to the idea that there is an inherent quality of the mathematical and arithmetic concept, amongst quantitative magnitude and comparison, which is simply known as "number sense," according to a study by Honorè and Noel [6]. The ANS is therefore viewed as the foundation of numerical and arithmetic abilities, the way it is shared in between several animal species, and how it relates to the notion of "number sense." It is defined as a nonverbal talent that "incorporates spatial relationships among a mental number line, allowing for arithmetic and quantitative comparison and processing" (Kesler et al., [7]). In other terms, it refers to a psychological representation of the concept of an amount that exists in people's minds and assists actions such as comparison and arithmetic processing. Given that number sense is regarded to be an intrinsic trait, it is feasible to assert that ANS ideas and number sense are intimately connected and vital in deciding whether or not a person suffers with numbers in any manner. Based on these ideas, youngsters may be assigned a range of training activities to determine if they have an arithmetic handicap, the degree of that handicap, and whether or not that arithmetic handicap is dyscalculia. Several research provide various tactics and approaches for developing the ANS and number sense. It could be a good idea to begin with a game-like application or online software that includes practise activities such as numeric comparison, intellectual calculations, and sorting responsibilities as seen in [7].

A software programme called IDR, or Integrated Dynamic Representation, aims to evaluate people's formal and informal understanding of mathematics and numerology. The fascinating study "Influence of early mathematical competencies on the efficacy of a classroom-based intervention" [2] makes this claim. "While it is possible to learn anything formally, such as through the use of a game or by intuition or even through everyday experience, formal education and particular exercises and activities, like practising addition and subtraction, are required to build knowledge in the latter scenario," says the author.

This particular study is really noteworthy since it demonstrates how IDR might enhance problem-solving abilities in both verbal and mathematical domains, not only in mathematics. There are three ways to practise math and train yourself for arithmetic problem solving:

1. Iconic Interpretation is a type of training that only uses visual methods of information delivery.

2. Symbolic Interpretation, or instruction using information communicated through language.

3. Mixed Interpretation which is training through both Iconic and Symbolic interpretations.

The challenges cover number facts, amount comparability, informal/formal calculations, informal/formal ideas, and informal/formal calculations. Computations that are both informal and formal are arithmetic calculations that are stated twice, once informally and once formally. Because the IDR is a flexible instrument, individuals who experienced the most difficulty reported more consistent development than those who experienced less difficulty yet improved after training. The term "MLD," which stands for Mathematical Learning Disorder, is also investigated in the current study. The mathematical and quantitative difficulty levels are classified as low, medium, or high. This research focuses on an important concept: persons with ADHD (Attention Deficit/Hyperactivity Disorder) have the most difficulty grasping mathematical ideas. Furthermore, someone with ADHD is more likely to struggle with arithmetic, or that having ADHD would exacerbate their problems (ADHD-MLD comorbidity). This highly exciting discovery has already been noted by Gonzales-Castro et al. [5]. It not only validates MLD kids' inequity and the fact that these students continue to struggle with mathematics in upper-level courses like high school, but it also tells cities and counties that these issues are compounded by the existence of ADHD. In actuality, children with MLD and ADHD have a number of psychological deficiencies that must be documented, resulting in a range of memory and focus issues. Additionally, it asserts that whereas only 3to 13 percentage of people with MLD have significant specific arithmetic and quantitative challenges, this number substantially increases to over 11 to 26 percentahe in cases of MLD-ADHD condition.

The concept of working memory will be the final topic to be explored. Working memory is described as "a cognitive workspace that allows a person to maintain knowledge in memory while concurrently undertaking other complex cognitive operations" by Passolunghi et al. [13]. To put it another way, working memory may be thought of as a box in which information and data are held for as long as it is necessary for the successful completion of a more sophisticated job, which could be a mathematical or numeric action as well as one of a different sort. This may alternatively be conceived of as a major domain-general determinant of arithmetic or a numeric competence. Furthermore, it is bound by both childhood and maturity arithmetic abilities, necessitating the employment of working memory for any arithmetic or numeric tasks, even the most basic ones. The assumption that working memory impacts math or numerical accomplishments and that weak working memory leads to bad math or numerical performances emphasises the significance of working memory. The key point to remember is that there is no specific training that increases WM capacity exclusively; rather, because working memory is a precursor to early arithmetic and mathematical accomplishments, training it has a "transferring influence" on those skills. In addition, a kid who receives working memory training improves their early arithmetic and numeracy skills as a result of the transferring effect of knowledge on information. They are also noted in another research [9] that uses the same working memory technique and ideas. This research claims that working memory is divided into a variety of subsystems, in addition to a basic system known as the central executive. A visuo-spatial sketch pad for information visualisation, two information storage units, and a unit for linguistic and dialect data are among the components. The two parts of working memory that are thought to best predict mathematical problem solving abilities are central executive and visuo-spatial. The purpose of the research was to see if two different types of training improved participants' mathematical and numerical abilities and, if so, which training had a greater influence. The findings corroborate the idea of the "transferring effect" stated in the preceding section by demonstrating that number sense or working memory activities can improve arithmetic and numerical abilities. However, the study sums up that the effects of spatial working memory exercises were not as substantial as those of number sense activities.

Although it is necessary to have a separate discussion given the research community's involvement in this disorder as well as the claim that far too many people are afflicted with it, dyscalculia is the most researched among all the disorders in the study by Gonzales-Castro et al., [5] 'given that it is one of the most common arithmetic and numerical disorders in the world' as stated in the study.

2.2 An Intro to Dyscalculia and its effects on an individual

Developmental dyscalculia affects a person's ability to grasp digits, recognise mathematical objects like numbers and their patterns, and carry out basic arithmetic and quantative operations. When "performance in mathematics, as evaluated by self-report standardised tests, is significantly below average when compared to the expected score given the person's biological age, evaluated intellect, and age-appropriate schooling," the "Diagnostic and Statistical Manual of Mental Disorders" advises creating a treatment plan for dyscalculia.[16]

According to research, there is also a distinction between the two kinds of developmental dyscalculia known as "primary dyscalculia" and "secondary dyscalculia." The secondary references to a math and numeric deficit that is caused by outside factors such as insufficient education, behavioural and attention problems, and low socio-economic condition. "The first one has to do with the part of the brain that processes numerical data not developing as it should normally be".[1]

This impairment, albeit less widely researched by scientists than dyslexia, is continually developing. This condition can thus be explained in a variety of ways, including physiological and social factors. Several studies have indicated that familial and genetic variables can impact the development of dyscalculia. The above-mentioned learning problems, or "attention deficit hyperactivity disorder," may be associated to developmental dyscalculia. Inherited dyscalculia can also occur as a result of brain damage, for example. The former is associated to a brain injury or disorder, but it is not classified as an SLD because it did not evolve.

2.3 Common symptoms for an individual with Dyscalculia

Patients with dyscalculia have some or all of the symptoms listed below: "They struggle with working with numbers, they struggle with understanding and remembering mathematical, arithmetic, and quantative symbols, they struggle with basic arithmetic operations like addition, subtraction, multiplication, and division, they frequently reverse or rearrange numbers in their heads, for example, (45: 54), they struggle with mental arithmetic, they struggle with telling time and directions, and they struggle with mental arithmetic, they also struggle with tasks that requires strategic planning, They struggle with predicting and estimating activities, seeing patterns, distinct components of a math issue, or finding crucial info needed in finding solutions. They rely on concrete aids such counting using fingers and tallying marks. They also have record of underachievement in academics, which is known to contribute to the formation of defeatism in arithmetics", as studied from the works...[10][3].

Furthermore, dyscalculia significantly affects the child's numeracy skills. The youngster might not be able to analyse the meaning of quantity and the interactions between the regular and non- numeric dimensions. The so far discussed "sense of number," or the capacity to measure and quantitatively assess things in commonplace imagery or sceneries, is greatly hampered in dyscalculics. This has a significant impact on arithmetic and logical thinking as well as other activities that require handling symbolic quantities.[15]

2.4 Training and Larning practices to improve this condition

A component of scientific study into this condition is the development of strategies for teaching dyscalculics to improve their arithmetic computation abilities and reduce the challenges that may emerge in many aspects of their daily life. Children with dyscalculia require more resources to acquire and grasp mathematical concepts such as numbers and arithmetic than their classmates. As a result, learning may be facilitated and accelerated by the use of tablets or laptops loaded with programmes geared solely at enhancing and integrating these abilities. Electronic instruments are also quite useful since they can track children's growth in real time, which is critical for selecting the most efficient rehabilitation strategy to minimise this condition.

2.5 Using game application with kids in order to train them

A growing number of people begin playing video games as young children. On the other hand, as technology develops, children are exposed to more electronic media from both inside and beyond the home. This promotes continuous research and the creation of enhanced instructional software for children. Therefore, educational games provide the advantage of applying pertinent and efficient methods for improvement in the area of cohesion; in reality, gamification-based applications reward users who push themselves to overcome extremely tough degrees of difficulty and so intellectually develop and learn. Mobile phones and tablets are the most suggested electronic teaching tools for young children since they are portable, compact, and do not require specialised interfaces like keyboards and mouse.

According to a recent research from England [12] the usage of tablets with built-in mathematical apps has greatly enhanced instruction. Children who used the app in addition to their regular math lessons, those who just used it during school hours, and those who didn't were all evaluated. The first group of students had much greater learning gains than those who only received standard instruction.

These findings are incredibly positive for the creation of preventive and therapeutic interventions for people with learning difficulties, particularly because they accelerate the process of creating ad hoc pathways by analysing answers one at a time while the youngster is having a great time while also learning.

3 Theory: Learning To Focus on Numbers

The study is based on research published in 2018 by "M. Piazza, V. De Feo, S. Panzerid, and S. Dehaene." The researchers' study, dubbed the "filtering hypothesis," demonstrates that the capacity to concentrate on numbers while filtering out possibly contradictory information on non-numerical aspects increases with age and education. [14] This study also revealed that dyscalculic people, who developed at the same rate as healthy people, have trouble distinguishing between numerical and non-numerical aspects. This may be used to identify problems in children as young as preschool age.

3.1 Looking into the experiment

The study involved 156 participants, including 44 Italian elementary school students, 29 young Italian students, 20 Italian educated adults, 25 Italian children with the effects of dyscalculia, and 38 Munduruc, children and adults with poor computational skills. It provided detailed information on the potential causes of this disorder.

Individuals were shown many combinations of numeric values, each represented by a dot, and directed to identify the group with the highest proportion without tallying the dots. In reality, they were given a short period of time to make a decision, therefore they were unable to tally. The number of dots and other non-numerical features, such as the size of the dots, were changed as the test progressed. As a result, the investigators claim that the trials were "congruent and incongruent." The largest group in congruent trials also had the highest value of non-numerical attributes, resulting in a higher rating. In contrast, un incongruent trials, the smallest group would always include numerous non-numerical variables with a higher significance. The following image depicts these numerous types of testing.

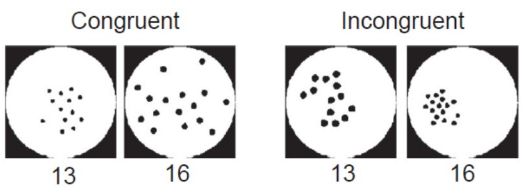


Figure 1: Visualization of congruent and Incongruent[14]

The investigators defined the stimulus space using four non-numerical variables expressed in terms of the number of pixels, which explicitly indicate quantity, and one numerical variable representing the number of dots:

1. Item Surface Area (ISA), which is the area of each dots in terms of number of pixels.
2. Total Surface Area (TSA) which is the product of ISA and the number of items.
3. Field Area, (FA) or "convex hull", which is the area of space in pixels where the dots are displayed.

4. Sparsity, (SPARS), which shows how sparse the dots are displayed inside their area, it's the quotient of FA and the number of items.

Using the principal component analysis approach, these four non-numerical features were integrated into a single summary measure known as the "Non-Numerical Dimension (NND) of the stimulus space storing the sets" for research purposes.[14]

3.2 The Analytics

The researchers examined the findings in light of the fact that an individual's decision-making mechanism often advances with age, becoming capable of differentiating minor numerical changes of 15 to 20 percent by maturity. Every dot in the next image stands for a single attempt, which is made up of two integers and their respective non-numerical features. The vertical scale indicates the log ratio of the non-numeric variables and the vertical line, whereas the horizontal axis displays the logarithmic ratio of the numbers. The "Optimal Decision Boundary" is displayed on the graph showing the "Optimal Decision Boundary" [14]. It symbolises a boundary in a two-dimensional area where, as seen in the image, the dots that belong to two different classes are positioned. If indeed the line is crossed, the classification will shift.

Each dot in this plot and the subsequent ones indicates a trial that was conducted as part of the study. All of the locations in which the greatest numerical variable is available in the second sector, n2¿n1, and all of the places in which the main numerical variable is represented in the first sector, n2¡n1, are included in the first category. When the decision boundary properly divides all instances of the group on its left from the instances of the group on its right, that boundary is said to be optimum. The numerosities of the dots in the two-dimensional space vary very little while they are near the line; nevertheless, as they travel away from the line, both their distances grow, the right and to the left.

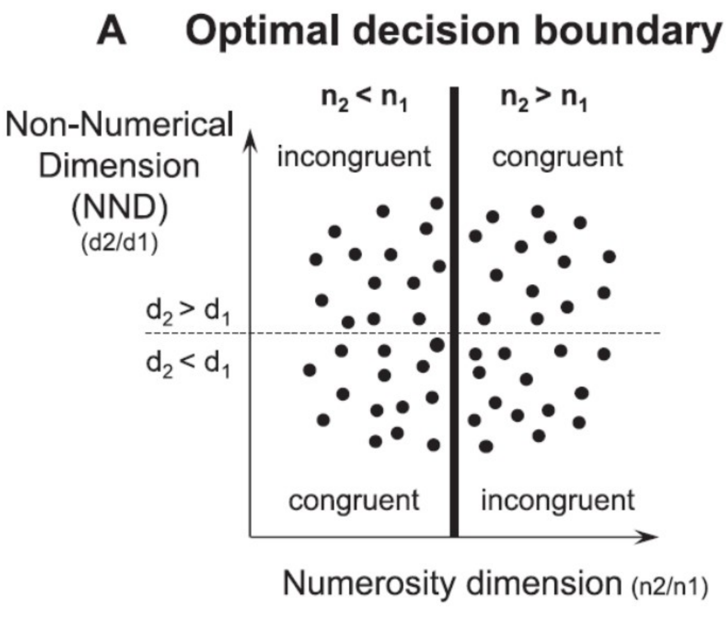


Figure 2: Visualization of "Optimal decision boundary"[14]

The research talks about and makes use of two hypotheses:the sharpening and the filtering hypothesis.

1. According to the sharpening idea, individuals improve their mental representation of numbers and mathematical quantities amounts with age and education. Actually, as youngsters get older, they lose the ability to detect numerical disparities clearly and precisely.

2. Contrarily, the filtering hypothesis postulates that youngsters take into consideration both numerical and non-numerical factors from a young age, but as they get older, their decision-making mechanism learns to prioritise numbers and numeric symbols and ignore interference from non-numerical variables.

The graphs linked with the two ideas are depicted in the figure below. The ambiguity of mental representations of numbers and another non-numerical dimension are represented by the size of the dots in these panels (NND). According to the sharpening model, representations are chaotic at first but grow sharper as people age and gain more education. According to the filtering process, what is altering is the capacity to progressively learn to disregard NND components and focus just on numbers, rather than how numbers are numerically represented.[14]

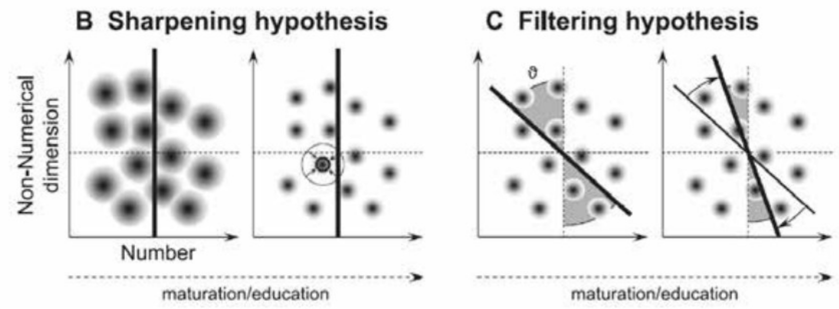


Figure 3: Visualization of "Sharpenning and Filtering Hypothesis"[14]

The filtering model displays a steady tilting of the decision border in the direction of the ideal line, resulting in a constant decrease in the angle between the choice and the ideal slope, enhancing the accuracy of the decision-making process. According to the sharpening model, the "Optimal Decision Boundary" remains constant throughout life.[14]

While studies demonstrate that it improves considerably more in incongruent trials, if the sharpening idea were true and distinct by enhancing one's visual acuity, there would be an improvement in both congruent and incongruent trials, with no difference. The filtering hypothesis must be considered, as it best depicts the developing brain, because dyscalculia is a handicap that makes it difficult to focus just on the important size, which is the number.

3.3 The Results from the Analytics

At the completion of the testing, the researchers separately evaluated the results of the experiment where there was congruence between the non-numerical and numerical dimensions and the incongruent ones. Greater overall magnitudes from congruent trials were seen across all experimental groups.

Following that, the data was analysed using two methods: Shannon data analysis and logistic regression analysis. The first, based on a model, demonstrates how the decision between right and left, that is, which set is correct, varies depending on all of the variables involved; the second, done without a model, employs mutual information [17] by devising a broader method to assess the impact of numerical and non-numerical factors. All of this study was done to determine the relative relevance of each variable. The findings distinguish the influence of education from that of age, with education found to be a major determinant.

In actuality, education frequently has the effect of lessening the impact non-numerical features have on decision-making. The results of the data analysis then provide evidence in favour of the filtering hypothesis by showing that a successful number analogy results from a better ability to focus on the number rather than on the other variables.

3.4 Implementing the gained knowledge

The research will be based on an experiment that applies the study's filtering hypothesis using a gaming platform. It is specifically designed to be used to educate children how to identify variations between sets of numbers while ignoring non-numerical aspects. These trails train the young brain to grow better at blocking out irrelevant outside factors and concentrating just on the numerical value. The game will also include AI technology that, after exposing the pupils to a range of randomly generated tasks of increasing degrees of complexity, will evaluate their progress.

By evaluating the responses, the AI agent will create new and more targeted tasks that resemble the region of the child's brain that is responsible for counting numbers. By figuring out how mathematically intelligent a pupil is, this modelling aims to improve filtering. The game seeks to position the decision-making border as ideally as possible and, in doing so, aids in avoiding the onset of dyscalculia by providing the child with specialised trials.

4 Inspirations:

Today's kids are more drawn to and absorbed in technology; in fact, they start using it at an early age. The researchers made the assumption that employing video game apps would be a more effective way to engage dyscalculic children than using more traditional methods like books and simple activities. The purpose of these programmes is to advance and deepen the kids' conceptual knowledge of arithmetic and numbers. Applications will need to be updated with any new results in this sector because current study is continually changing. This chapter will focus particularly on three gaming applications because, despite certain limitations, they are based on published research and scientific studies, as opposed to other tools on the market that lack this foundation in tried-and-true testing. So, to help the intervention and prevention of Dyscalculia in Children, Researchers have created apps like,

1. 'The Number Race'[18]
2. 'Rescue Calcularis'[8]

The "Number Race" was designed by the INSERM-CEA Cognitive Neuroimaging Unit, a French research organisation that is a pioneer in the study of mathematical cognition. [18] The first software concentrates on smaller single-digit numbers, while the "Rescue Calcularis" app focuses on bigger two-digit numerals. Swiss academics developed the most current app under scrutiny. [8]

4.1 'The Number Race' by INSERM-CEA Cognitive Neuroimaging Unit[18]

The target audience for this game is children aged 4 to 8 who are just starting to learn about arithmetic and numbers. Older students will practise arithmetic and be able to measure their grasp of numbers through the game, while smaller children will learn the fundamentals of math and number ideas. The game focuses on three primary theme macrogroups: number formats, counting from 1 to 40, and addition and subtraction activities from 1 to 10. Because dyscalculia is a significant weakness that affects the links between mathematical representations of numbers and symbols, the structure of the game has been explored to see whether it can be used to develop and benefit both children without specific issues and dyscalculic youngsters.

To give the best training possible, an adaptive algorithm that adapts the game's difficulty in response to player performance is used. Furthermore, the complexity can be raised by lowering the reaction time for numerical comparison, displaying the numbers mostly in symbolic form, or including fundamental arithmetic operations. All of this is regularly assessed in light of the child's growth and input.

The game has two major screens where students must complete various tasks. On the first screen, they must compare two sets and determine which is the major one, as well as perform some math. They must move their characters and the characters of their opponents on the secondary display according on how many coins they have. They will receive a gift if they arrive first.

Researchers A. Wilson and S. Dehaene [11] developed and tested the programme with the intent of testing children's numerical sense, or their capacity to explain quantities of numbers without using words, as well as their ability to represent actual numbers. Thus, the game aims to accomplish both objectives through stages that use estimate the number and their spatial depiction as well as through comparison of non-symbolic and symbolic numerical figures. Nine kids around the ages of 7 and 9 participated in the game's testing by the researchers.

The results demonstrated an improvement in fundamental number cognition as well as a partial consolidation of the relationships between numeric concepts. There was only a better knowledge of the rules, no significant advancement in mathematical concepts. Nonetheless, there were several limitations to this study, such as the limited sample size and the lack of a control group.

4.2 A launch into 'Rescue Calcularis'[8]

The programme favours the link between the numerical amount and the space in order to improve the representation of numbers. It also seeks to comprehend estimating, mathematics, and quantitative ordinariness. To keep the child's attention, the instruction is integrated into a tale. The player's home planet, "Calcularis," which has limited energy resources, must be saved. Because the planet "Heureka" is far away, the young astronaut must refuel on 10 other planets before proceeding to the planet with his or her spaceship to retrieve the Rocket fuel. The 30 light-years separating the two planets corresponds to 30 more difficult stages. For each level, the child must steer the spaceship to the right position on the number line. To engage learners, whenever a player makes a mistake, they must replay the level until they figure it out.[8]

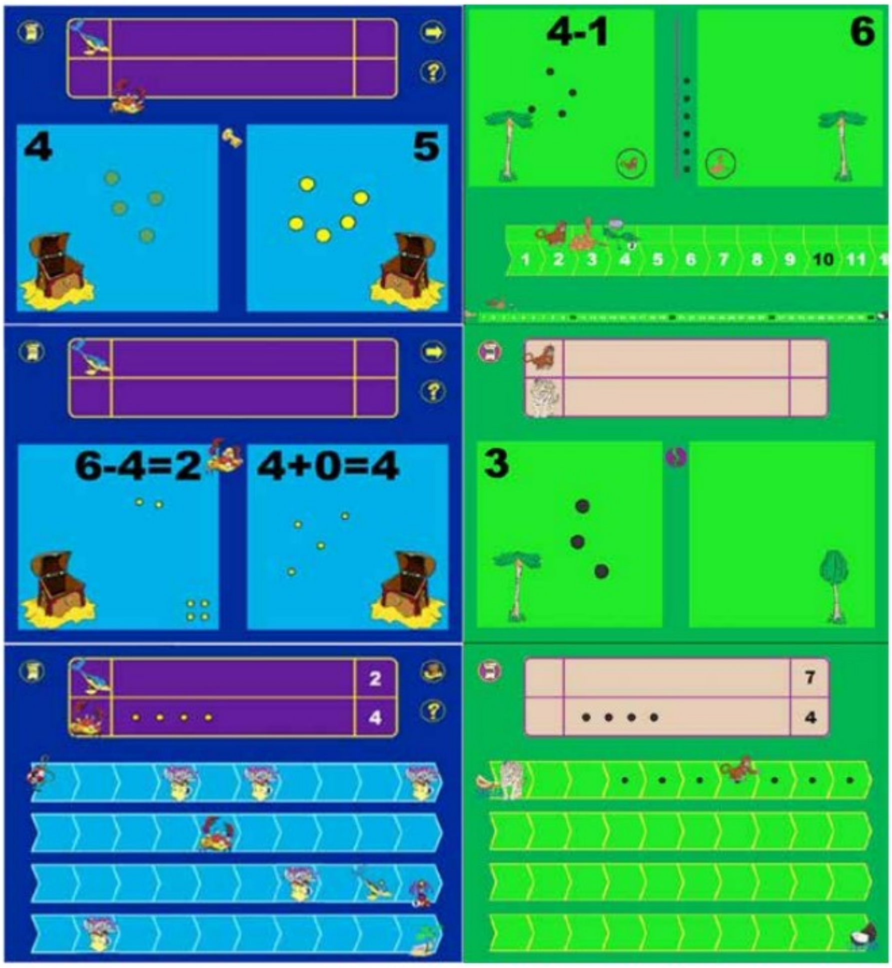


Figure 4: 'The Number Race'[18]

"Following the game, both the intervention and control groups improved in determining the right location of a number on a number line, according to the evaluations. Dyscalculic youngsters performed considerably better on tests, which bodes well for the future of people with disabilities. The game also helped players visualise the number more vividly in their minds. However, all of these discoveries and advancements might be the consequence of training and greater mathematical knowledge."[8]

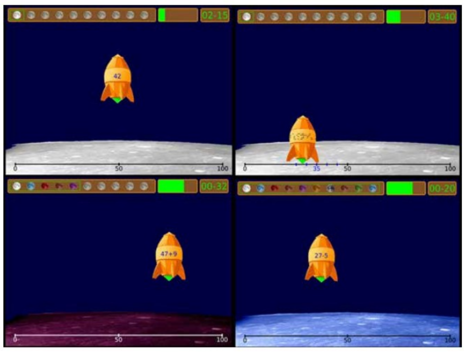


Figure 5: 'Rescue Calcularis'[8]

4.3 How different is our game from these these APPLICATIONS

According to studies, the games described above are good instruments for enhancing and minimising the diseases caused by dyscalculia. They do not, however, discuss more recent study on this handicap, which results in some discrepancies. In actuality, the potential interferences that non-numerical elements may have on the awarding of the number itself are not improved and analysed during the playing process of these apps.

They are not meant to prevent dyscalculia in primary school pupils, who show early signs of the disease, but rather to teach dyscalculic children of school age in mathematics and arithmetic. They cannot aid in the diagnosis by evaluating the relationship between the child's behaviour and them because they are not even based on AI technology. The last distinction is that the results obtained are not entirely satisfactory due to the tiny sample size employed for testing.

To sum up, the primary differences between these games for dyscalculic youngsters and the project's objectives are as follows:

1. not ignoring or failing to examine the impact of non-numerical factors on the perception of numbers;
2. Because these applications are for schoolkids with dyscalculia rather than for preventing its emergence in early childhood, the target audience is distinct;
3. They do not implement AI technology

5 The Game: "The Number Farm"

The game I chose to work on for my dissertation is developed by Ivana Orefice and Brugo Gaia [11] [4] from the Italian University Politecnico di Torino.

5.1 AIMS

Mathematics has an impact on many aspects of people's daily lives. Many businesses and activities require the insight and flexibility that mathematical operations provide. All of this can cause discomfort and annoyance, as well as a lack of interest in recruiting people with dyscalculia. As a disorder that has just lately been the subject of different studies and debates, no definitive method for mitigating the effects of this impairment on the individual has been identified. The primary purpose of this proposal is to prevent the emergence of any form of dyscalculia in preschoolers. Actually, kids between the ages of 4 and 5 who might or might not have trouble counting make up the majority of the application's target demographic. The most current studies in this area served as the foundation for the project. The idea is that this problem appears in a young child who is learning to count and recognise numerical amounts as a result of interference from non-numerical information. In actuality, when dyscalculic youngsters aged 7 or 8 are compared to those aged 3 or 4, the results are the same, and both groups are recognised to have the same difficulties in understanding the same numerical concepts. The game's AI is designed to offer each player with a personalised and unique experience. It provides a continuous and continuing real-time study of the child's replies; as a consequence, if it sees that the player is experiencing difficulties on some trials due to interruptions caused by non-numerical factors to the disadvantage of numerical variables, it decreases the level of difficulty. As a result, the filtering system may start improving.

To summarise, the fundamental concern motivating this project is that dyscalculic children may never totally recover and will only have a greater comprehension of quantity and number concepts, restricting their capacity to fully integrate into all aspects of daily life. The goal is to avoid the onset of any symptom linked with this condition by training children as early as kindergarten to observe and pay attention to the amount and numerical dimension rather than any other characteristic.

5.2 Insight of the game

The idea behind the game is very simple, There is a farmer who has two fences where, once the game is started the chickens of different sizes will enter. The task of the player is to help the farmer choose out of the two fences which one has the highest number of chickens.

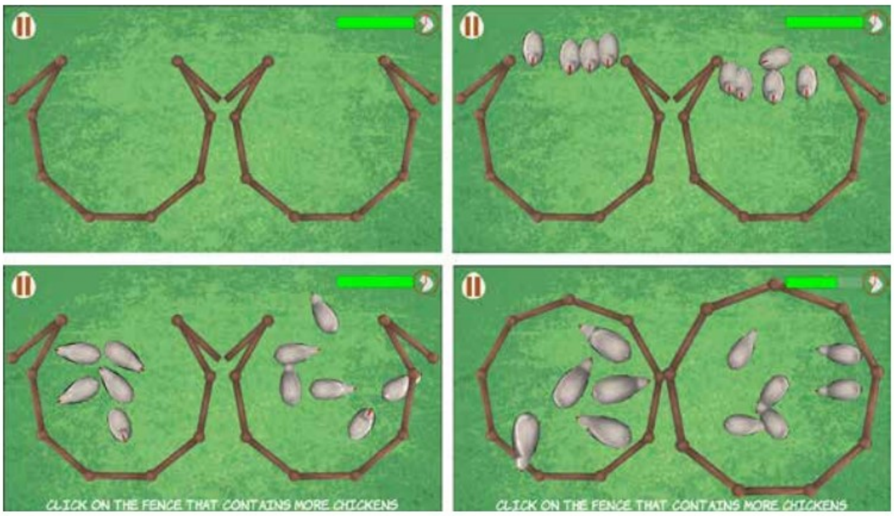


Figure 6: 'Showing how the game works'[11]

If the user clicks on the fence with the highest number of chickens, the user passes the level a the game displays a victory message and moves on to the next trial else if the user click on the wrong fence, the game displays a failure message. There is also a timer on top within which the user must click on the fence and there is also a sub-timer only within which the chickens will be displayed.

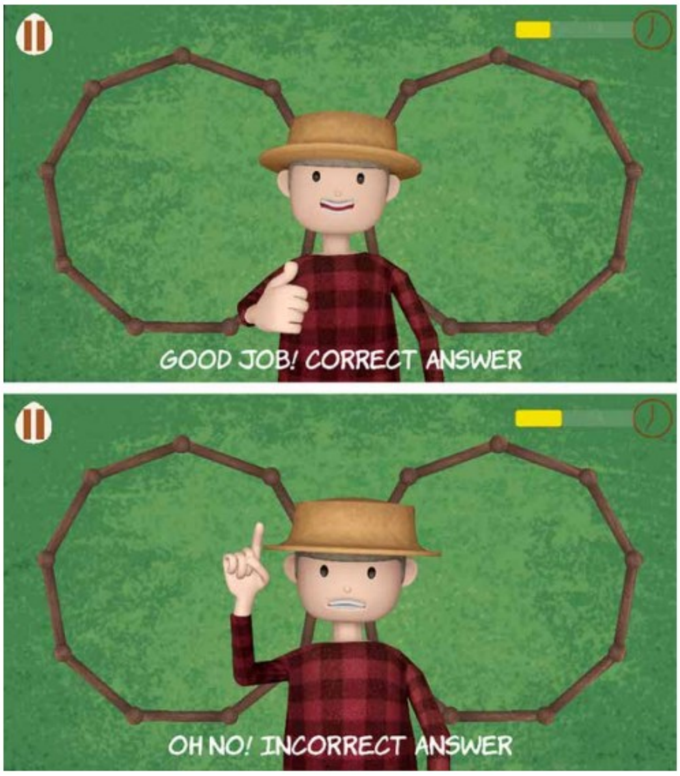


Figure 7: 'Victory or Failure message after each trial'[11]

The software was designed to be used nearly totally independently by elementary students, without the assistance of other adults or their parents. This will provide complete training from the perspective of number sense and filtering mechanism. The research detailed in the article "Learning to focus on number" [14]is recreated in the game.

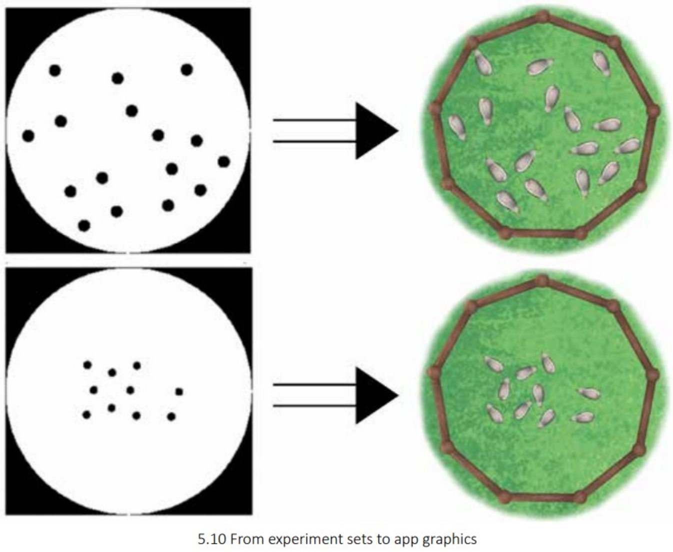


Figure 8: Comparing the research with the Game trials

To create a backdrop plot that would capture the player's attention, the two sets that hold the game's dots have been updated to simulate fences, and the points have been tweaked to symbolise animals. The variables and data from the research are acknowledged throughout the program[14]. The number of chickens, the size of the fence, the size of the chicken, and their proximity to one another characterise the game's distinction between numerical and non-numerical variables, which in the research were represented by the number of dots and the size of the fence, the points, and their mutual distance. As seen in theory, a kid that suffers from Dyscalculia might suffer from perform simple counting task by being distracted by other non numerical dimensions such as Chicken size and Field areas.

The target for this game, being young kids the game is designed in a manner a child would enjoy but at the same time learn and train themselves with the task of counting. This game also serves as a training kit for those suffering from the disorder.

5.3 The Backend

Figure 9: [11][4]Client to server communication

The game connects a server that creates trials using an algorithm that makes references to the filtering theory and the study that served as the foundation for the original research[14] and a client that manipulates this data to create a real game. In other words, the server generates test data and sends it to the client, or Unity engine, and the client then sends the results back to the server once the player has finished the several trials. My task was to work on the mathematical side of the game that is the trial generation so that the server can generate and send valid trials over to the client.

6 Generating Random 'Valid' trials:

When Ivana and Gaia shared their works with me, it was a running game but it ran using a predefined trial matrix.

A trial matrix is a matrix that contains the input to the game. A default trial matrix looks like this:

```
import socket
from db.db_connector import DBConnector
from server import Create_Game
from mapping_matrix import dummy_matrix_generator
from plot_trials import PlotTrials
from transform_matrix import TransformMatrix
from ai import SelectTrial

ServerSocket = socket.socket()
DB = DBConnector()
host = '127.0.0.1'
port = 65432
ThreadCount = 0


simulation_on = 1


nnd_number = 5

alpha = 20

nnd_general = 0

# REAL GAME
if simulation_on == 0:

    trials_matrix_original = [[8, 7, 40000, 40000, 130.04, 79.95, 4, 8]
                             [5, 6, 27777.78, 37777.78, 273.13, 173.13, 4, 8],
                                  [7, 6, 27777.78, 27777.78, 173.13, 173.13, 4, 8],
                                 [2, 7, 27777.78, 27777.78, 173.13, 173.13, 4, 8]]

    # To transform our parameters into the ones accepted by the real game, it is
    # mandatory to call the TransformMatrix function to obtain the right matrix
    trials_matrix = TransformMatrix(trials_matrix_original)
```

Figure 10: [11][4] Trial Matrix with predefined trials

[Number of chickens(Left), [Number of chickens(right), Field Area (Left), Field Area(Right), Item Surface Area(left), Item Surface Area(Right), Chicken display timer, Trial display timer] where, Field Area is the size of the fence in pixels, Item surface are is the size of each chicken in pixels, Chicken display timer is the time till which the chickens are displayed in seconds, and Trial display timer is the time till which each level of the game is displayed in seconds.

As we can see in Figure 10 trial matrix original is predefined with a set of 4 predefined trial matrices. So, the game would display only those four trials. My task is to find a way to generate Random valid trials that the game can use.

6.1 What we have so far and Ideology

I was given a lookup table by Ivana and Gaia with a set of valid combinations that they have collected by mannually inputing them in the game.

NumChickens	FieldArea	temSurfaceArea
9	5625	27.7
13	40000	222.38
13	40000	249.31
13	40000	277.78
21	40000	150.82
21	40000	173.13
21	40000	196.98
25	40000	130.04
37	40000	79.95
37	40000	93.11
37	40000	110.8
45	40000	62.33
57	40000	49.25
69	40000	37.7
89	40000	27.7
112	40000	19.24
9	5625	37.7
13	36736.11	222.38
13	36736.11	249.31
13	36736.11	277.78
21	36736.11	130.04
21	36736.11	150.82
21	36736.11	173.13
21	36736.11	196.98
29	36736.11	93.11
29	36736.11	110.8
37	36736.11	79.95
45	36736.11	62.33
49	36736.11	49.25
69	36736.11	37.7
81	36736.11	27.7
101	36736.11	19.24
9	5625	49.25
9	5625	62.33
9	6944.44	37.7
13	33611.11	196.98
13	33611.11	222.38
21	33611.11	130.04
21	33611.11	150.82
21	33611.11	173.13
25	33611.11	93.11
25	33611.11	110.8
37	33611.11	79.95
45	33611.11	49.25
45	33611.11	62.33
61	33611.11	37.7
69	33611.11	27.7
97	33611.11	19.24

Figure 11: Dataset of Valid trials

Now, we know that every single combination from the lookup table is a valid combination which is manually verified by Ivana and Gaia but it is very limited. It can only generate chickens with numerosity [1, 5, 9, 13, 21, 25, 28, 29, 37, 45, 48, 49, 57, 61, 69, 81, 89, 97, 101, 112] on either sides. We need to generate every possible combinations within this range valid combinations.

To do so, we need an equation that can generate those combinations. To approximate an equation wee need to approximate the plane of possivle combinations from Ivana and Gaia. Thus I put the dataset into a mesh to generate a plane in 3d.

	40000	36736.1	33611.1	30625	27777.8	25069.4	22500	20069.4	17777.8	15625	13611.1	11736.1	10000	8402.78	6944.44	5625
19.24	112	101	97	89	69	69	69	61	48	45	37	37	27	21	21	21
27.7	89	81	69	69	61	57	45	45	37	37	29	25	21	21	13	9
37.7	69	69	61	49	45	45	37	37	29	25	21	21	21	13	9	9
49.25	57	49	45	45	37	37	37	25	21	21	21	21	13	9	9	9
62.33	45	45	37	37	37	29	25	21	21	21	21	13	9	9	9	9
76.95	37	37	37	29	25	21	21	21	21	21	13	9	9	9	9	5
93.11	37	29	28	25	21	21	21	21	13	13	9	9	9	9	5	5
110.8	29	25	21	21	21	21	21	13	13	9	9	9	9	9	5	1
130.04	25	21	21	21	21	21	13	13	9	9	9	9	9	5	5	1
150.82	21	21	21	21	13	13	9	9	9	9	9	9	5	5	5	1
173.13	21	21	21	13	13	9	9	9	9	9	9	5	5	5	5	1
196.98	21	21	13	13	9	9	9	9	9	9	5	5	5	5	1	1
222.38	21	13	13	9	9	9	9	9	9	5	5	5	5	1	1	1
249.31	13	13	9	9	9	9	9	9	9	5	5	5	1	1	1	1
277.78	13	9	9	9	9	9	9	9	5	5	5	5	1	1	1	1
307.79	9	9	9	9	9	9	9	5	5	5	5	1	1	1	1	1

Figure 12: Mesh to plot the data in 3d

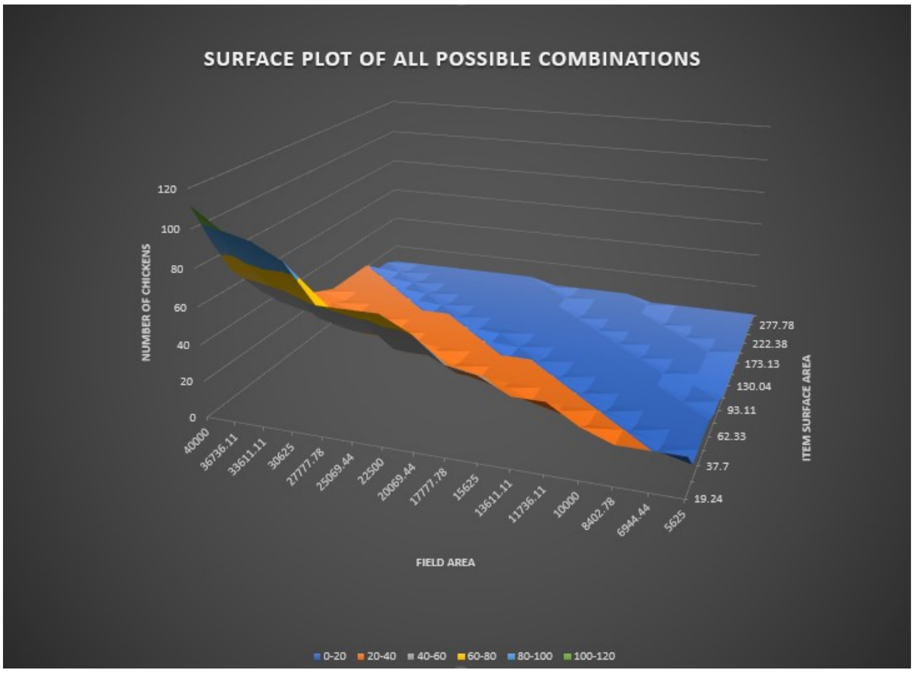


Figure 13: Plane of possible valid combinations in 3d

6.2 Equation

The plot below shows the plane of possible valid combinations from a different perspective, so that it would be easy for us to understand. So, from what we knew before, we can say that the plane we see below is the surface plot of possible valid combinations and the space below the plane or let's just say that the space that falls under the area of the plane is the space of all possible valid combinations. Which means in theory, if the plot of future combinations falls within or under the plane, it is considered a valid trial or if the plot of the future combinations fall above the plane, the combination is considered invalid.

To do this, we need to approximate a plane with respective to the plane of possible valid combinations. The trick is to find 3 extreme points on the plane and when we join those 3 points we would have the approximated plane for the plane of possible valid combinations and the equation of that plane would be the approximate equation for the plane of possible valid combinations.

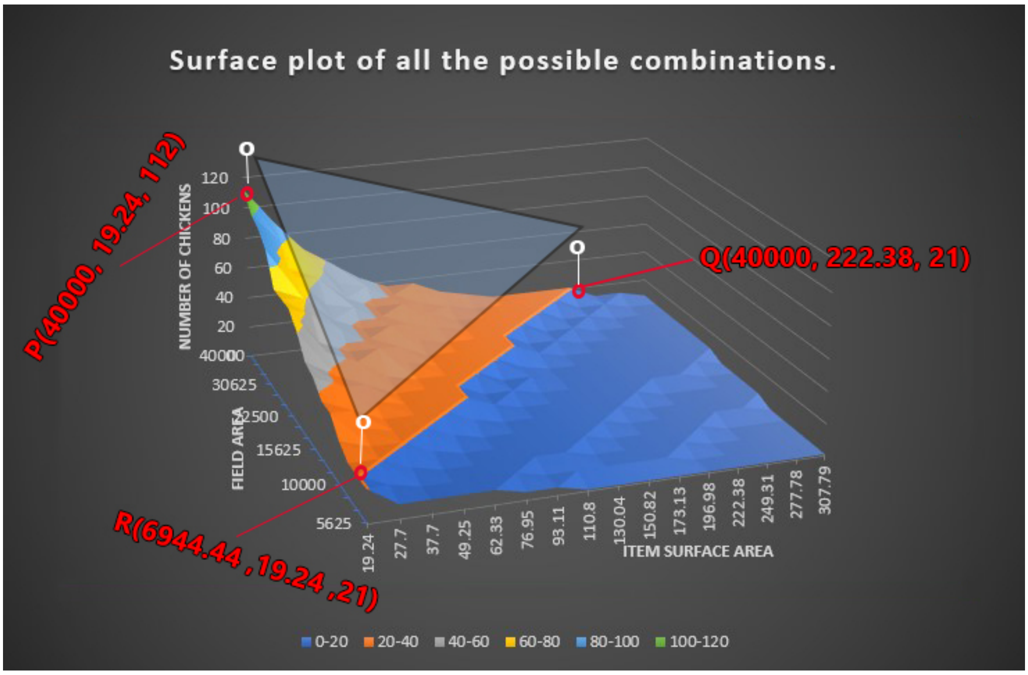


Figure 14: Representation of the approximated plane for the plane of possible valid combinations

To find the equation of the approximated plane, Let's take three extreme points on the plane of possible combinations:

P(40000, 19.24, 112)
Q(40000, 222.38, 21)
R(6944.44, 19.24, 21)

Let's find vector PQ and PR

$\overrightarrow{PQ} = Q - P = (0, 203.14, -91)$

$\overrightarrow{PR} = R - P = (-33055.56, 0, -91)$

Now to find a normal vector find cross product of vector a and b:

$$\overrightarrow{n} = \overrightarrow{a} * \overrightarrow{b} = \begin{bmatrix} i & j & k \\ 0 & 203.14 & -91 \\ -33055.56 & 0 & -91 \end{bmatrix}$$

$$= \begin{bmatrix} 203.14 & -91 \\ 0 & -91 \end{bmatrix} \text{ i - } \begin{bmatrix} 0 & -91 \\ -33055.56 & -91 \end{bmatrix} \text{ j + } \begin{bmatrix} 0 & 203.14 \\ -33055.56 & 0 \end{bmatrix} \text{ k}$$

=-18485.74 i + 3008055.96 j + 6714906.4584 k

Therefore, $\overrightarrow{n} = (-18485.74, 3008055.96, 6714906.4584)$

Now, lets define the equation of the plane:

This is the equation of the plane:
$a(X - X_o) + b(Y - Y_o) + c(Z - Z_o) = 0$

Where, (a, b, c) = $\overrightarrow{n}$ and $(X_o, Y_o, Z_o) = P$

Substituting the known values,

-18485.74 (X - 40000) + 3008055.96 (Y - 19.24) + 6714906.4584 (Z - 112) = 0

-18485.74X + 739429600 + 3008055.96Y - 57874996.6704 + 6714906.4584Z - 752069523.34 = 0

Thus, we have found the equation of a plane that approximates the plane of possible combinations

[-18485.74 X + 3008055.96 Y + 6714906.4584 = 70514920.0112] ———-Equation 1

where, X = Field Area
Y = Item Surface Area
Z = Number of Chickens

Thus from the above equation, we can conclude that, if we substitute the values of the future combinations (X, Y, Z) in the equation and the value of the respective solution is less than 70514920.0112, the respective combination falls below or under the area of the approximated plane for the plane of possible valid combinations. Thus, the respective combination is considered valid. Else, if the value of the respective solution is higher than 70514920.0112, the respective combination falls above the area of the approximated plane for the plane of possible valid combinations and the respective combination is considered invalid.

6.3 Implementing the equation to generate a Random Valid trial

Now let us try to implement the above found equation into a function for a python code so that we could use it to generate a random valid trial. Below is the code where I randomised the ISA and FA available in Unity and described by Ivana and Gaia's Look up table based on the Number of chickens using a simple for loop.

```python
if 21> i <=25:
    trial.append(random.choice(S_25))
elif 1> i <=9:
    trial.append(random.choice(S_5_9))
elif 9> i <=13:
    trial.append(random.choice(S_13))
elif 13> i <=21:
    trial.append(random.choice(S_21))
elif 25> i <=27:
    trial.append(random.choice(S_27))
elif  i ==28:
    trial.append(random.choice(S_28))
elif  i ==29:
    trial.append(random.choice(S_29))
elif 29> i <=37:
    trial.append(random.choice(S_37))
elif 37> i <=45:
    trial.append(random.choice(S_45))
elif 45> i <=48:
    trial.append(random.choice(S_48))
elif i ==49:
    trial.append(random.choice(S_49))
elif 49> i <=57:
    trial.append(random.choice(S_57))
elif 57> i <=61:
    trial.append(random.choice(S_61))
elif 61> i <=69:
    trial.append(random.choice(S_69))
elif 69> i <=81:
    trial.append(random.choice(S_81))
elif 81> i <=89:
    trial.append(random.choice(S_89))
elif 89> i <=97:
    trial.append(random.choice(S_97))
elif 97> i <=112:
    trial.append(random.choice(S_101112))
else:
    trial.append(random.choice(S_1))
```

Figure 15: Randomising ISA to Number of chickens

```python
for i in range(1,113):
    trial = []
    #######

    if 21> i <=25:
        trial.append(random.choice(FS_25))
    elif 1> i <=9:
        trial.append(random.choice(FS_5_9))
    elif 9> i <=13:
        trial.append(random.choice(FS_13))
    elif 13> i <=21:
        trial.append(random.choice(FS_21))
    elif 25> i <=27:
        trial.append(random.choice(FS_27))
    elif  i ==28:
        trial.append(random.choice(FS_28))
    elif  i ==29:
        trial.append(random.choice(FS_29))
    elif 29> i <=37:
        trial.append(random.choice(FS_37))
    elif 37> i <=45:
        trial.append(random.choice(FS_45))
    elif 45> i <=48:
        trial.append(random.choice(FS_48))
    elif i ==49:
        trial.append(random.choice(FS_49))
    elif 49> i <=57:
        trial.append(random.choice(FS_57))
    elif 57> i <=61:
        trial.append(random.choice(FS_61))
    elif 61> i <=69:
        trial.append(random.choice(FS_69))
    elif 69> i <=81:
        trial.append(random.choice(FS_81))
    elif 81> i <=89:
        trial.append(random.choice(FS_89))
    elif 89> i <=97:
        trial.append(random.choice(FS_97))
    elif 97> i <=112:
        trial.append(random.choice(FS_101112))
    else:
        trial.append(random.choice(FS_1))
```

Figure 16: Randomising FA to Number of chickens

After randomizing, I send them into my equation and the equation produces all the possible combinations.

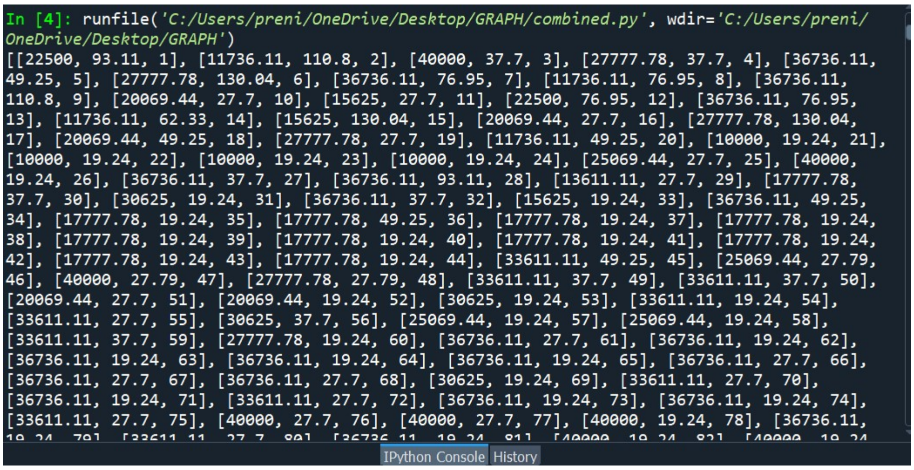
```python
for i in range(len(a)):

    c = -18485.74 * a[i][0] + 3008055.96 * a[i][1] + 6714906.4584 * a[i][2]
    if c <= 70514920.0112:
        X.append(a[i][0])
        Y.append(a[i][1])
        Z.append(a[i][2])
        ax.scatter(X, Y, Z, c='g', marker='o')
        print("trial is valid = ",i)
    else:
        X1.append(a[i][0])
        Y1.append(a[i][1])
        Z1.append(a[i][2])
        ax.scatter(X1, Y1, Z1, c='r', marker='o')
        print("trial is invalid = ",i)
```

Figure 17: implementing equation in code

7 Results discussion

Thus the equation produces all the possible combinations and the values are plotted.



```
In [4]: runfile('C:/Users/preni/OneDrive/Desktop/GRAPH/combined.py', wdir='C:/Users/preni/
OneDrive/Desktop/GRAPH')
[[22500, 93.11, 1], [11736.11, 110.8, 2], [40000, 37.7, 3], [27777.78, 37.7, 4], [36736.11,
49.25, 5], [27777.78, 130.04, 6], [36736.11, 76.95, 7], [11736.11, 76.95, 8], [36736.11,
110.8, 9], [20069.44, 27.7, 10], [15625, 27.7, 11], [22500, 76.95, 12], [36736.11, 76.95,
13], [11736.11, 62.33, 14], [15625, 130.04, 15], [20069.44, 27.7, 16], [27777.78, 130.04,
17], [20069.44, 49.25, 18], [27777.78, 27.7, 19], [11736.11, 49.25, 20], [10000, 19.24, 21],
[10000, 19.24, 22], [10000, 19.24, 23], [10000, 19.24, 24], [25069.44, 27.7, 25], [40000,
19.24, 26], [36736.11, 37.7, 27], [36736.11, 93.11, 28], [13611.11, 27.7, 29], [17777.78,
37.7, 30], [30625, 19.24, 31], [36736.11, 37.7, 32], [15625, 19.24, 33], [36736.11, 49.25,
34], [17777.78, 19.24, 35], [17777.78, 49.25, 36], [17777.78, 19.24, 37], [17777.78, 19.24,
38], [17777.78, 19.24, 39], [17777.78, 19.24, 40], [17777.78, 19.24, 41], [17777.78, 19.24,
42], [17777.78, 19.24, 43], [17777.78, 19.24, 44], [33611.11, 49.25, 45], [25069.44, 27.79,
46], [40000, 27.79, 47], [27777.78, 27.79, 48], [33611.11, 37.7, 49], [33611.11, 37.7, 50],
[20069.44, 27.7, 51], [20069.44, 19.24, 52], [30625, 19.24, 53], [33611.11, 19.24, 54],
[33611.11, 27.7, 55], [30625, 37.7, 56], [25069.44, 19.24, 57], [25069.44, 19.24, 58],
[33611.11, 37.7, 59], [27777.78, 19.24, 60], [36736.11, 27.7, 61], [36736.11, 19.24, 62],
[36736.11, 19.24, 63], [36736.11, 19.24, 64], [36736.11, 19.24, 65], [36736.11, 27.7, 66],
[36736.11, 27.7, 67], [36736.11, 27.7, 68], [30625, 19.24, 69], [33611.11, 27.7, 70],
[36736.11, 19.24, 71], [33611.11, 27.7, 72], [36736.11, 19.24, 73], [36736.11, 19.24, 74],
[33611.11, 27.7, 75], [40000, 27.7, 76], [40000, 27.7, 77], [40000, 19.24, 78], [36736.11,
```

IPython Console History

Figure 18: List of all possible combinations generated from the equation.

The code in Fig 17 also says if the combinations are valid or not when compared to the game and the look up table.

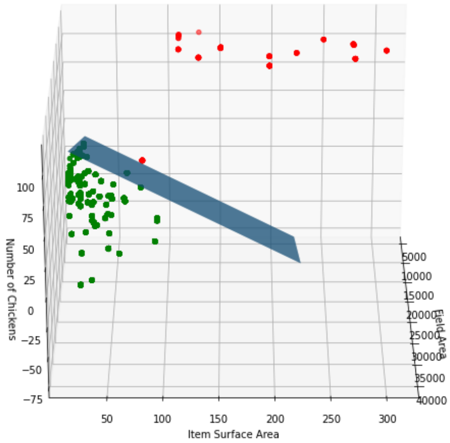


Figure 19: Visualisation of the equated plane all possible combinations generated from the equation.

The above plot represents the approximated plane of possible combinations that is generated from the equation and scatter points of all the possible combinations if they are valid they are represented in green and if they are invalid, they are represented in red. As theorised, we can say that the plane we see below is the surface plot of possible valid combinations and the space below the plane or let's just say that the space that falls under the area of the plane is the space of all possible valid combinations. Which means in theory, if the plot of future combinations falls within or under the plane, it is considered a valid trial or if the plot of the future combinations fall above the plane, the combination is considered invalid.

To prove that the equation is working, we combine the generated valid combinations into trials and sent into the trial matrix. Now, eventhough the trials are not from the lookup table they are displayed by the game proving that the equation is working.

Here are a few screenshots from the game with trials that are not from the look up table proving that the equation is working.

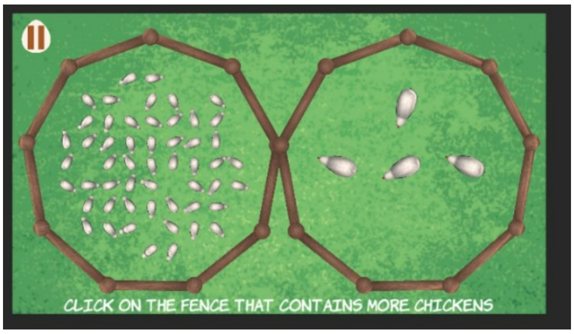


Figure 20: Trial: [53, 4, 30625, 27777.78, 19.24, 93.11, 4, 8]

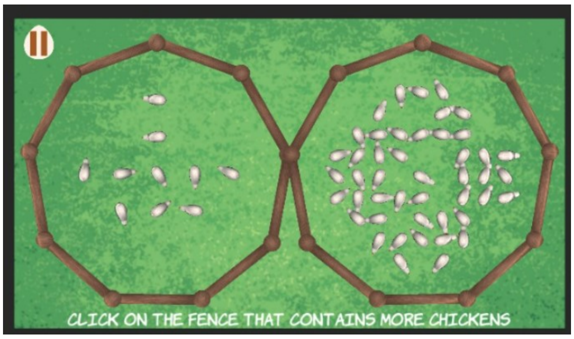


Figure 21: Trial: [10, 46, 20069.44, 25069.44 , 27.7, 27.79, 4, 8]

Figure 22: Trial:[84, 85, 40000, 36736.11, 19.24, 19.24, 4, 8]

8 Conclusions

Thus from the above results, we can conclude that the equation can generate Random 'Valid' trials for the game to use. Further, we can apply filtering hypothesis on this trial and obtain their difficulty coefficient.

```python
for i in combinations:
    left = i[0]
    right = i[1]
    Ratio= left/right
    #print(Ratio)
    log_Ratio= math.log10(Ratio)
    #print(log_Ratio)
    lr.append(log_Ratio)
#print(lr)
nr = []
lr_max = max(lr)
dc= []
#print(lr_max)
for i in lr:
    nor_Ratio = abs(i/lr_max)
    nr.append(nor_Ratio)
    difficulty_coeff = round(1- nor_Ratio,2)
    dc.append(difficulty_coeff)

index = []
while(True):
    Diff_coeff = float(input('Enter a Difficulty coefficient: '))
    # print(Diff_coeff)
    if Diff_coeff in dc:
        for i in range(len(dc)):
            if dc[i] == Diff_coeff:
                index.append(i)
        break
    else:
        print("Difficulty coeffient is not available , Enter a new number")
```

Figure 23: Implementing Filtering Hypothesis

```
In [6]: runfile('C:/Users/preni/OneDrive/Desktop/trial.py', wdir='C:/Users/preni/OneDrive/
Desktop')
Enter a Difficulty coefficient: 0.99
[22, 21, 40000, 40000, 173.13, 222.38, 4, 8]
```

Figure 24: Generating valid trial using filtering Hypothesis

This code takes in the difficulty coefficient from 0 to 1. If the difficulty coefficient is close to zero, it generates a easy difficulty trial, else if the difficulty coefficient is close to 1 it generates a hard trial. In the future, this can be used by the AI in the server side to change the difficulty of trials based on the players input for the purpose of training. This concludes my dissertation and I hope this project can help to shine some light for the individuals that suffers from dyscalculia. I'd like to thank Ivan and Gaia for letting me take part in their project and my supervisor Dr. Vito De Feo for supervising me and The University of Essex for this wonderful oppurtunity.

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