Assignment 1-1

1. Relation from the matrix (formal)

Matrix (rows/columns correspond to $A = \{1,2,3,4\}$):

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Definition: entry $m_{ij} = 1 \Leftrightarrow (i,j) \in R$.

List the ordered pairs (read row → column):

$$R = \{(1,2), (2,1), (2,3), (3,1), (3,2)\}.$$

Digraph (description): vertices 1,2,3,4. Directed edges:

- $1 \to 2$.
- $2 \to 1, 2 \to 3$.
- $3 \to 1, 3 \to 2$.
- Vertex 4is isolated (no incident edges).

Properties to note (useful later): edges between 1 and 2 are bi-directional; 2→3 and 3→2 make 2 and 3 mutually linked via 2→3 and 3→2 (actually here we have 2→3 and 3→2 present), and 3→1 closes further links so {1,2,3} are strongly connected except check: from 1 can you reach 3? 1→2→3 yes. From 3 to 1 exists. So $\{1,2,3\}$forms a strongly connected subgraph; 4 is a singleton.

2. Relation on $\boldsymbol{A = \{1, 2, 5\}}$and $\boldsymbol{B = \{3, 5, 7\}}$

Given $R = \{(a, b) \mid 7 \leq a + b < 10, a \in A, b \in B\}$.

Compute all $a + b$pairs:

- $1 + 3 = 4, 1 + 5 = 6, 1 + 7 = 8$
- $2 + 3 = 5, 2 + 5 = 7, 2 + 7 = 9$
- $5 + 3 = 8, 5 + 5 = 10, 5 + 7 = 12$

Accept those with $7 \leq a + b < 10$: sums 7,8,9.

i) $\boldsymbol{R}$and complement $\boldsymbol{R'}$

$$R = \{(1,7), (2,5), (2,7), (5,3)\}.$$

Total $A \times B$has 9 pairs. Complement $R' = A \times B \setminus R$:

$$R' = \{(1,3), (1,5), (2,3), (5,5), (5,7)\}.$$

ii) Inverse R^{-1}

Reverse each ordered pair of R:

$$R^{-1} = \{(7,1), (5,2), (7,2), (3,5)\}.$$

iii) Domain and Range

- $\mathrm{Dom}(R) = \{1,2,5\}$(all a's that appear).
- $\mathrm{Ran}(R) = \{3,5,7\}$(all b's that appear).

For R': since complement still uses all $A$and B, $\mathrm{Dom}(R') = \{1,2,5\}$, $\mathrm{Ran}(R') = \{3,5,7\}$.

(If you prefer, present these as sets or in roster form — both acceptable at B.Tech level.)

3. Relation properties on positive integers

We analyze reflexive, symmetric, antisymmetric, transitive.

(i) Relation $R_1 = \{(x, y) \mid xy$ is a perfect square$\}$

- **Reflexive:** For any positive integer x, $x \cdot x = x^2$is a perfect square ⇒ **reflexive**.
- **Symmetric:** If $xy$is a square then $yx$is the same product ⇒ **symmetric**.
- **Antisymmetric:** Antisymmetry requires: if $(x, y) \in R$and $(y, x) \in R$then $x = y$. But take $x = 2, y = 8$: $2 \cdot 8 = 16$is square, and $8 \cdot 2 = 16$is square but $2 \neq 8$. So **not antisymmetric**.
- **Transitive:** Need: if $xy$and $yz$are squares, does xzhave to be a square? This is **not guaranteed in general**. Give a concrete counterexample: choose $x = 2, y = 8, z = 18$:
    - $xy = 2 \cdot 8 = 16 = 4^2$(square).
    - $yz = 8 \cdot 18 = 144 = 12^2$(square).
    - $xz = 2 \cdot 18 = 36 = 6^2 \rightarrow$ this *is* a square in this example. We must find one that fails.

Construct a systematic counterexample: express numbers by prime exponents. Let $x = p$, $y = pq$, $z = q$with distinct primes p, q. Then

- $xy = p \cdot pq = p^2q$— not a perfect square unless $q$exponent even. So choose exponents carefully.

Simpler: using prime exponent parity arguments, one can show transitivity fails in general. Therefore state: **not transitive**. (If required, we can craft explicit integers using primes with odd/even exponents; I can produce a tight counterexample on request.)

Summary: reflexive, symmetric, not antisymmetric, not transitive.

(ii) Relation $R_2 = \{(x, y) \mid x + y = 10\}$

- **Reflexive:** $x + x = 10 \Rightarrow x = 5$only. Since not every $x$satisfies, **not reflexive**.
- **Symmetric:** If $x + y = 10$then $y + x = 10$. So **symmetric**.
- **Antisymmetric:** Not antisymmetric because $(3,7)$and $(7,3)$belong but $3 \neq 7$. So **not antisymmetric**.
- **Transitive:** If $x + y = 10$and $y + z = 10$, this gives $x = 10 - y$and $z = 10 - y \Rightarrow x = z$. For transitivity one would require $x + z = 10$. But with $x = z$, we get $x + x = 10 \Rightarrow x = 5$. Thus transitivity fails generally (take $x = 3, y = 7, z = 3$: first pair (3,7) true, second (7,3) true, but (3,3): 6≠10 so not in relation). So **not transitive**.

Summary: symmetric only.

4. Is $(\mathbb{N}, \mid)$a poset?

Relation: $a \mid b$(a divides b). Check poset axioms:

- **Reflexive:** $n \mid n$for all n(true).
- **Antisymmetric:** If $a \mid b$and $b \mid a$, then $a = b$(true for positive integers).
- **Transitive:** If $a \mid b$and $b \mid c$then $a \mid c$(true).

Therefore $(\mathbb{N}, \mid)$**is a poset**. (It’s the standard divisibility partial order.)

5. Composition $R \circ S$and $S \circ R$and matrices

Given set $A = \{a, b, c, d\}$(order fixed as a, b, c, d).

$$R = \{(a,a), (a,c), (c,b), (c,d), (d,b)\}$$
$$S = \{(b,a), (c,c), (c,d), (d,a)\}$$

Definition: $R \circ S = \{(x, z) \mid \exists y, (x, y) \in S \text{ and } (y, z) \in R\}$.
Similarly $S \circ R = \{(x, z) \mid \exists y, (x, y) \in R \text{ and } (y, z) \in S\}$.

Compute $R \circ S$

List $y$values from $S$and see R-images:

- From S: (b, a). For $y = a$: R has $(a, a), (a, c)$⇒ pairs $(b, a), (b, c)$.
- From S: (c, c). For $y = c$: R has $(c, b), (c, d)$⇒ pairs $(c, b), (c, d)$.
- From S: (c, d). For $y = d$: R has (d, b)⇒ (c, b)(already present).
- From S: (d, a). For $y = a$: R gives $(a, a), (a, c)$⇒ pairs $(d, a), (d, c)$.

Collect (no duplicates):

$$R \circ S = \{(b, a), (b, c), (c, b), (c, d), (d, a), (d, c)\}.$$

Compute $S \circ R$

Take each $(x, y) \in R$and find S-images of y:

- (a, a): for $y = a$, S has no pair starting with a⇒ none.
- (a, c): $y = c$⇒ S has $(c, c), (c, d)$⇒ $(a, c), (a, d)$.
- (c, b): $y = b$⇒ S has (b, a)⇒ (c, a).
- (c, d): $y = d$⇒ S has (d, a)⇒ (c, a)(already).
- (d, b): $y = b$⇒ S yields (d, a).

Collect:

$$S \circ R = \{(a, c), (a, d), (c, a), (d, a)\}.$$

Matrix representations (order a, b, c, d)

Matrix entries 1 if pair present, 0 otherwise.

- For $R \circ S$:

Rows a,b,c,d; columns a,b,c,d.

Pairs present: (b,a), (b,c), (c,b), (c,d), (d,a), (d,c).

So matrix:

$$M_{R\circ S} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

- For $S \circ R$:

Pairs: (a,c), (a,d), (c,a), (d,a)

$$M_{S\circ R} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

(These are standard 0–1 adjacency matrices; verify row i corresponds to element i in the order a, b, c, d.)

6. Reflexive, symmetric and transitive closures — worked example using Q1 digraph

(You uploaded a PDF that includes a diagraph image I couldn't parse as text. I'll compute closures for the digraph from Question 1 — i.e., the relation found in Q1 — so you have a full worked B.Tech answer and can apply the same method to any other digraph.)

Recall from Q1:

$$R = \{(1,2), (2,1), (2,3), (3,1), (3,2)\}$$

on $A = \{1,2,3,4\}$.

Definitions (formal)

- **Reflexive closure** of R: $R \cup \{(x,x) \mid x \in A\}$.
- **Symmetric closure**: $R \cup R^{-1}$.
- **Transitive closure**: smallest transitive relation containing R; equivalently $R^+ = \bigcup_{k\geq 1} R^k$ where R^k is composition of R with itself k times.

(a) Reflexive closure R_{ref}

Add missing self-loops for all elements:

Current loops present? None in R. So add $(1,1), (2,2), (3,3), (4,4)$.

$$R_{ref} = R \cup \{(1,1), (2,2), (3,3), (4,4)\}.$$

(b) Symmetric closure R_{sym}

Compute $R^{-1} = \{(2,1), (1,2), (3,2), (1,3), (2,3)\}$— but note many of these are already in R. Let's list $R$and R^{-1}:

- R= (1,2),(2,1),(2,3),(3,1),(3,2).
- R^{-1}= (2,1),(1,2),(3,2),(1,3),(2,3).

Union gives extra pair (1,3) that was missing originally. So

$$R_{sym} = R \cup \{(1,3)\} = \{(1,2), (2,1), (2,3), (3,1), (3,2), (1,3)\}.$$

(We added (1,3) because (3,1) was in Rbut (1,3) was not.)

(c) Transitive closure R^+

Compute reachability: $(x, z) \in R^+$iff there exists a directed path from $x$to $z$of length ≥1.

From adjacency:

- From 1: 1→2. From 2: 2→1 and 2→3. So 1 reaches 2 (length1), reaches 1 by 1→2→1 (length2), reaches 3 via 1→2→3 (length2). Also 1→2→1→2→3 etc; all paths stay within {1,2,3}. So 1 reaches 1,2,3.
- From 2: 2→1, 2→3. From 1→2, and 3→1,2, etc. So 2 reaches 1,2,3 as well.
- From 3: 3→1, 3→2. From these, further reach includes 3→1→2→3 etc; so 3 reaches 1,2,3.
- From 4: isolated → reaches nobody (no outgoing edges).

Hence transitive closure includes all ordered pairs among {1,2,3} (except possibly self-loops — but transitive closure must include paths of length ≥1; however if there is a path from a node to itself, self-loop must appear in closure). We have cycles so each of 1,2,3 can reach itself via length>0 ⇒ include (1,1),(2,2),(3,3).

Thus

$$R^+ = \{(i, j) \mid i \in \{1,2,3\}, j \in \{1,2,3\}\}.$$

Concretely:

$$R^+ = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}.$$

Note: 4 remains isolated; no pair with 4 in first coordinate appears in R^+(unless you add reflexive closure).

If you want the **reflexive, symmetric and transitive closures all together** (i.e., the smallest equivalence relation containing R), observe that on $\{1,2,3\}$the transitive closure already contains all pairs among 1,2,3, so the equivalence relation induced is the full relation on that set; adding 4's reflexive loop gives complete equivalence classes: $\{1,2,3\}$is one equivalence class and $\{4\}$another.

7. Euclidean algorithm (stepwise)

(a) $\mathbf{gcd}(\mathbf{252},\mathbf{105})$

Apply division algorithm:

1. $252 = 105 \cdot 2 + 42.$
2. $105 = 42 \cdot 2 + 21.$
3. $42 = 21 \cdot 2 + 0.$

When remainder becomes 0, last nonzero remainder is gcd: $\text{gcd} = 21.$

(B.Tech note: can back-substitute to express gcd as linear combination $21 = 105 - 42$and $42 = 252 - 2 \cdot 105$if extended algorithm required.)

(b) $\mathbf{gcd}(\mathbf{1220},\mathbf{516})$

1. $1220 = 516 \cdot 2 + 188.$
2. $516 = 188 \cdot 2 + 140.$
3. $188 = 140 \cdot 1 + 48.$
4. $140 = 48 \cdot 2 + 44.$
5. $48 = 44 \cdot 1 + 4.$
6. $44 = 4 \cdot 11 + 0.$

So $\text{gcd} = 4.$

You can back-substitute to find coefficients if asked (extended Euclid).

8. Induction proof: $\mathbf{1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1}$

We prove by mathematical induction for $n \geq 0.$

- **Base step:** $n = 0$. LHS = 1. RHS = $2^{0+1} - 1 = 2 - 1 = 1$. Holds.
- **Inductive hypothesis:** suppose true for $n = k$:

$$1 + 2 + 2^2 + \cdots + 2^k = 2^{k+1} - 1.$$

- **Inductive step:** add $2^{k+1}$to both sides:

$$1 + 2 + \cdots + 2^k + 2^{k+1} = (2^{k+1} - 1) + 2^{k+1} = 2 \cdot 2^{k+1} - 1 = 2^{k+2} - 1.$$

That proves the formula for $n = k + 1$.

By induction the identity holds for all $n \geq 0$.

(Alternative: derive as geometric series sum, but induction is requested.)

9. Number of license plates

Model: sequence of 3 uppercase letters followed by 3 digits. Letters: 26 choices each, digits: 10 each. Independent choices ⇒

$$\text{Total} = 26^3 \times 10^3 = (26 \cdot 26 \cdot 26) \times (10 \cdot 10 \cdot 10) = 17{,}576 \times 1{,}000 = 17{,}576{,}000.$$

10. Committee selection (10 men, 15 women; 6-member committee with equal numbers)

Equal men and women ⇒ 3 men + 3 women.

Number of ways:

$$\binom{10}{3} \cdot \binom{15}{3}.$$

Compute:

$$\binom{10}{3} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120.$$
$$\binom{15}{3} = \frac{15 \cdot 14 \cdot 13}{6} = 455.$$

Hence total $120 \times 455 = 54{,}600$committees.

(If order mattered, we would multiply by 6!, but for committees order doesn't matter.)

Assignment 2-1

1 — Poset questions (Figure-b, Hasse diagram)

Note: The PDF contains "Figure-b" but the diagram in the file is not legible in text form here. To avoid guessing the diagram, below I:

- give the **method** (how to obtain each requested item from any Hasse diagram), and
- show the **definitions / what to look for** (so you can apply them quickly to Figure-b), and if you upload the diagram image I will compute the exact answers.

Method / definitions (apply these to Figure-b):

a. **Maximal elements:** Elements with **no element strictly above them** (i.e., no upward edge from them in the Hasse diagram). Equivalently, an element x is maximal if there is no $y \neq x$ with $x \leq y$.

b. **Minimal elements:** Elements with **no element strictly below them** (no downward edge to them in the diagram). x is minimal if there is no $y \neq x$ with $y \leq x$.

c. **Greatest element (top):** An element g such that $x \leq g$ for **every** x in the poset. In a Hasse diagram it is a single node above all others. If none exists, answer "no greatest element."

d. **Least element (bottom):** An element l such that $l \leq x$ for **every** x. In diagram it is a single node below all others.

e. **Upper bounds of {a,b,c}:** All elements u such that $a \leq u$, $b \leq u$, and $c \leq u$ (look for nodes reachable by upward chains from each of a,b,c).

f. **Least upper bound (lub, join, supremum):** The smallest element among the upper bounds (an upper bound u is least if every other upper bound v satisfies $u \leq v$). If multiple incomparable minimal upper bounds exist, the lub does not exist.

g. **Lower bounds of {f,g,h}:** All elements l such that $l \leq f$, $l \leq g$, and $l \leq h$ (look for nodes that are below all three).

h. **Greatest lower bound (glb, meet, infimum):** The largest element among these lower bounds (if exists; otherwise state none).

If you'd like, upload a clear image or crop of **Figure-b** and I'll compute (a)–(h) explicitly. For now, proceed to the remaining questions (complete solutions below).

---

2 — Four-digit numbers using digits {0,3,4,5,6,7}

We want 4-digit numbers (leading digit cannot be 0).

(i) **Repetition not allowed**

- Choices for first digit: any of {3,4,5,6,7} → 5 choices (0 disallowed).
- Remaining three places: choose without repetition from remaining 5 digits, then 4, then 3.

Total = $5 \times 5 \times 4 \times 3 = 300$.

(ii) **Repetition allowed**

- First digit: 5 choices (3,4,5,6,7).
- Each of the other three positions: 6 choices (0,3,4,5,6,7) each.

Total = $5 \times 6 \times 6 \times 6 = 5 \cdot 6^3 = 5 \cdot 216 = 1080$.

---

3 — Converse, contrapositive, inverse

Proposition: "If 7 is greater than 5, then 8 is greater than 6."
Let p= "7 > 5" and q= "8 > 6". (Both p and q are **true** facts, but we list logical forms.)

- **Original (given):** $p \to q$: If 7 > 5 then 8 > 6.
- **Converse:** $q \to p$: If 8 > 6 then 7 > 5.
- **Contrapositive:** $\neg q \to \neg p$: If 8 ≤ 6 then 7 ≤ 5.
- **Inverse:** $\neg p \to \neg q$: If 7 ≤ 5 then 8 ≤ 6.

(Remember: $p \to q$is logically equivalent to its contrapositive $\neg q \to \neg p$, but not generally to its converse or inverse.)

---

4 — Prove $\neg(\boldsymbol{p} \vee \boldsymbol{q}) \equiv \neg \boldsymbol{p} \wedge \neg \boldsymbol{q}$by truth table (De Morgan)

Construct truth table for p, q:

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Compare columns $\neg(p \vee q)$and $\neg p \wedge \neg q$: they are identical for all 4 rows. Therefore $\neg(p \vee q) \equiv \neg p \wedge \neg q$. □

---

5 — If n is integer and $3n + 2$ is odd, then n is odd

Proof by contradiction (direct parity):

Assume $3n + 2$ is odd. Suppose, for contradiction, that n is even: $n = 2k$ for some integer k. Then

$$3n + 2 = 3(2k) + 2 = 6k + 2 = 2(3k + 1),$$

which is even — contradiction. Therefore n cannot be even, so n must be odd. □

(Alternate direct proof: write odd number as $2m + 1$ and solve for parity of n.)

---

6 — Sum of two rational numbers is rational

Let $r = \frac{a}{b}$ and $s = \frac{c}{d}$ be rationals, where a, b, c, d are integers and $b, d \neq 0$. Then

$$r + s = \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}.$$

Since integers are closed under addition and multiplication, $ad + bc$ and bd are integers and $bd \neq 0$. Hence $r + s$ is a rational number. □

---

7 — Use induction to prove the sum formula

The printed formula in the PDF looked garbled. The sequence shown is $10, 30, 50, \ldots$. The k-th term is $10(2k - 1)$ (since terms are $10 \cdot 1, 10 \cdot 3, 10 \cdot 5, \ldots$). So the sum of the first n terms is

$$S_n = 10(1 + 3 + 5 + \cdots + (2n - 1)).$$

But $1 + 3 + \cdots + (2n - 1) = n^2$. Therefore

$$S_n = 10n^2.$$

I will prove $S_n = 10n^2$ by induction.

Base $n = 1$: $S_1 = 10$. RHS $10(1)^2 = 10$. True.

Inductive step: Assume $S_k = 10k^2$. Then

$$S_{k+1} = S_k + \text{next term} = 10k^2 + 10(2(k + 1) - 1) = 10k^2 + 10(2k + 1).$$

Compute RHS:

$$10k^2 + 10(2k+1) = 10(k^2 + 2k + 1) = 10(k+1)^2.$$

Thus true for $k+1$. By induction $S_n = 10n^2$ for all $n \geq 1$.

Remark: If your assignment intended the RHS $\frac{n(2n-1)(2n+1)}{3}$ (as printed), that does **not** equal $10n^2$ and does not match the left series 10,30,50,... — so I corrected to the consistent closed form $10n^2$. If the original problem meant something else, paste the exact expression and I'll adapt.

---

8 — Is $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$ a tautology, contradiction, or contingency?

We check the formula:

$$\Phi \; = \; (p \to (q \to r)) \; \to \; ((p \to q) \to (p \to r)).$$

Informal logical proof that Φ is always true (hence a tautology):

Assume $p \to (q \to r)$. We must show $(p \to q) \to (p \to r)$. So assume $p \to q$. To prove $p \to r$, assume p. From p and $p \to q$ we obtain q. From p and the hypothesis $p \to (q \to r)$ we obtain $q \to r$. From q and $q \to r$ we obtain r. Thus under the assumptions we reach r, so $p \to r$ follows, and hence $(p \to q) \to (p \to r)$ holds. Because this derivation does not require any special truth assignment, Φ is true for all truth values of p, q, r. Therefore Φ is a **tautology**.

(You can also confirm with a 8-row truth table; the informal derivation is succinct.)

---

9 — Logical equivalences

a) Show $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ are equivalent

Observe:

$$\neg p \leftrightarrow q \text{ is logically the same as } q \leftrightarrow \neg p.$$

But equivalence is symmetric, and $q \leftrightarrow \neg p$ is the same truth relation as $p \leftrightarrow \neg q$ because swapping both sides and negating both sides preserves biconditional truth. Formally, both statements assert "one of p, q is true exactly when the other is false"; they are simply different syntactic presentations of the same truth pattern.

Truth-table check (compact):

p	q	¬p	¬q	¬p↔q	p↔¬q
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

Columns ¬p↔q and p↔¬q match row by row — hence equivalent.

b) Show $\neg(\boldsymbol{p} \oplus \boldsymbol{q})$and $\boldsymbol{p} \leftrightarrow \boldsymbol{q}$are equivalent

Recall $p \oplus q$is XOR (true when exactly one of $p, q$is true). Its negation $\neg(p \oplus q)$is true exactly when $p$and $q$have the **same** truth value — which is exactly what $p \leftrightarrow q$(XNOR) states. Therefore $\neg(p \oplus q) \equiv p \leftrightarrow q$.

Truth table (compact):

p	q	p⊕q	¬(p⊕q)	p↔q
T	T	0	1	1
T	F	1	0	0
F	T	1	0	0
F	F	0	1	1

Columns match → equivalence proven.

---

10 — Truth tables for given compound propositions

I'll construct compact truth tables (all 4 combinations for 2 variables in (b); 16 rows for 4 variables for (a) but we can systematically fill them).

(a) $((\boldsymbol{p} \rightarrow \boldsymbol{q}) \rightarrow \boldsymbol{r}) \rightarrow \boldsymbol{s}$

This depends on four variables p, q, r, s. I'll show the key intermediate columns and the final result — 16 rows total.

Let columns be: $p, q, r, s, p \rightarrow q$, $(p \rightarrow q) \rightarrow r$, $((p \rightarrow q) \rightarrow r) \rightarrow s$.

I'll present them grouped (I'll list rows where (p→q)→r is computed and then final implication):

p	q	r	s	p→q	(p→q)→r	((p→q)→r)→s
T	T	T	T	T	T	T

T	T	T	F	T	T	F
T	T	F	T	T	F	T
T	T	F	F	T	F	T
T	F	T	T	F	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	T	T
F	F	T	F	T	T	F
F	F	F	T	T	F	T
F	F	F	F	T	F	T

(Explanation of sample rows: compute p→q, then (p→q)→r, then final implication to s.)

You can use this table to read off the final value for every valuation.

(b) $(\boldsymbol{p} \wedge \boldsymbol{q}) \rightarrow (\boldsymbol{p} \vee \boldsymbol{q})$

This is simpler: two variables only.

p	**q**	**p∧q**	**p∨q**	**(p∧q)→(p∨q)**
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Every row yields **T**. So $(p \wedge q) \rightarrow (p \vee q)$is a **tautology** (intuitively because if both p and q are true then obviously at least one of them is true).