

Binary addition

How to add two decimal no.:

$$\begin{array}{r}
 ^0\ 1 \\
 547 - | 5 \times 10^2 + 4 \times 10^2 + 7 \times 10^0) 12 \times 10^0 \\
 25 | 2 \times 10^1 + 5 \times 10^0 \\
 \hline
 572 \text{ rms.} \quad 5 \quad 7 \quad 2
 \end{array}$$

is nota
 decimal
 num.

	$\text{base}(r) = 10$
$7 < 9$	$12 > 9 \quad 0 \rightarrow 9$
$7 = 0 + 7$	$(10 + 2) \times 10^0$
$0 \times 10^2 + 7 \times 10^1$	$10 \times 10^1 + 2 \times 10^0$

$$\begin{array}{r} \text{Ans} \\ \begin{array}{c} 110 \rightarrow 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \quad \text{base}(2)=2 \\ 101 \rightarrow 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \quad 0 \text{ and } 1 \\ \hline 1011 \end{array} \end{array}$$

Sum	1	0	1	$1 \times 2^1 + 1 \times 2^0$	$2 \times 2^2 > 1$
				$2 = 2 + 0$	
	S	C		$2 + 0 \times 2^2$	
	$0+0$	0	0	$1 \times 2^3 + 0 \times 2^2$	
	$0+1$	1	0		
	$1+0$	1	0		
	$1+1$	0	1		

Binary Subtraction

$$\begin{array}{r}
 \text{Ques} \\
 \text{17) } \begin{array}{c} 16 & 8 & 4 & 2 & 1 \\ \hline 1 & 1 & 0 & 1 & 1 \end{array} \quad -27 \quad 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\
 \begin{array}{r} 22 \\ \hline -1 & 0 & 1 & 1 & 0 \end{array} \quad 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\
 \hline \begin{array}{r} 0 & 0 & 1 & 0 & 1 \\ \hline \text{Ans} \end{array} \quad 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0
 \end{array}$$

(11)

The diagram illustrates a binary search tree and its in-order traversal. The tree has the following structure:

```

    graph TD
        Root(( )) --> L1_1(( ))
        Root --> L1_2(( ))
        L1_1 --> L2_1(( ))
        L1_1 --> L2_2(( ))
        L1_2 --> L2_3(( ))
        L1_2 --> L2_4(( ))
        L2_1 --> L3_1(( ))
        L2_1 --> L3_2(( ))
        L2_2 --> L3_3(( ))
        L2_3 --> L3_4(( ))
        L2_4 --> L3_5(( ))
        L3_1 --> L4_1(( ))
        L3_1 --> L4_2(( ))
        L3_2 --> L4_3(( ))
        L3_3 --> L4_4(( ))
        L3_4 --> L4_5(( ))
        L3_5 --> L4_6(( ))
        L4_1 --> L5_1(( ))
        L4_1 --> L5_2(( ))
        L4_2 --> L5_3(( ))
        L4_3 --> L5_4(( ))
        L4_4 --> L5_5(( ))
        L4_5 --> L5_6(( ))
        L5_1 --> L6_1(( ))
        L5_1 --> L6_2(( ))
        L5_2 --> L6_3(( ))
        L5_3 --> L6_4(( ))
        L5_4 --> L6_5(( ))
        L5_5 --> L6_6(( ))
        L6_1 --> L7_1(( ))
        L6_1 --> L7_2(( ))
        L6_2 --> L7_3(( ))
        L6_3 --> L7_4(( ))
        L6_4 --> L7_5(( ))
        L6_5 --> L7_6(( ))
        L7_1 --> L8_1(( ))
        L7_1 --> L8_2(( ))
        L7_2 --> L8_3(( ))
        L7_3 --> L8_4(( ))
        L7_4 --> L8_5(( ))
        L7_5 --> L8_6(( ))
        L8_1 --> L9_1(( ))
        L8_1 --> L9_2(( ))
        L8_2 --> L9_3(( ))
        L8_3 --> L9_4(( ))
        L8_4 --> L9_5(( ))
        L8_5 --> L9_6(( ))
        L9_1 --> L10_1(( ))
        L9_1 --> L10_2(( ))
        L9_2 --> L10_3(( ))
        L9_3 --> L10_4(( ))
        L9_4 --> L10_5(( ))
        L9_5 --> L10_6(( ))
        L10_1 --> L11_1(( ))
        L10_1 --> L11_2(( ))
        L10_2 --> L11_3(( ))
        L10_3 --> L11_4(( ))
        L10_4 --> L11_5(( ))
        L10_5 --> L11_6(( ))
        L11_1 --> L12_1(( ))
        L11_1 --> L12_2(( ))
        L11_2 --> L12_3(( ))
        L11_3 --> L12_4(( ))
        L11_4 --> L12_5(( ))
        L11_5 --> L12_6(( ))
        L12_1 --> L13_1(( ))
        L12_1 --> L13_2(( ))
        L12_2 --> L13_3(( ))
        L12_3 --> L13_4(( ))
        L12_4 --> L13_5(( ))
        L12_5 --> L13_6(( ))
        L13_1 --> L14_1(( ))
        L13_1 --> L14_2(( ))
    
```

The tree is traversed in-order as follows: 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14.

H. W.

$$10 - \overbrace{2 + 2^3}^{2^4 + 1 \times 2^3}$$

$$\begin{array}{r} \text{2} \\ \text{0} \\ \text{1} \\ \hline \text{1} \end{array} \quad \begin{array}{r} \text{0} \\ \text{1} \\ \text{0} \\ \hline \text{1} \end{array} \quad \begin{array}{r} \text{0} \\ \text{1} \\ \text{0} \\ \hline \text{1} \end{array} \quad \begin{array}{r} \text{0} \\ \text{1} \\ \text{0} \\ \hline \text{1} \end{array} \quad \begin{array}{r} \text{0} \\ \text{1} \\ \text{0} \\ \hline \text{1} \end{array}$$

$$\begin{array}{r} \underline{-} \quad \underline{l} \quad \underline{l} \quad \underline{l} \quad \underline{l} \\ \hline \underline{\underline{c}} \end{array} \quad \begin{array}{r} 15 \\ - \\ 6 \\ \hline 9 \end{array}$$

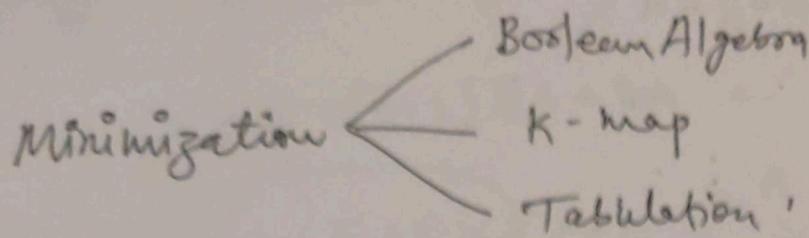
$$\begin{array}{r} 2^1 \alpha^0 \\ \underline{-} 2^1 \\ \hline 10 \rightarrow 2 \end{array}$$

1

10

0 1 1 1 1
0 1 0 1 0

Minimization



Boolean algebra is used when no. of variables are less and o/p may be either zero or one.

K-map is used when no. of variables are 2, 3, 4, 5 and o/p may be 0, 1 or X.

If no. of variables are very high then Tabulation Method is used.

Boolean Algebra

Invented in 1854 by George Boole.

Not operation :-
 $A \rightarrow \bar{A}$ (or) A'

$$\bar{\bar{A}} = A$$

And operation :-

$$0 \cdot 0 = 0$$

$$0 \cdot 1 = 0$$

$$1 \cdot 0 = 0$$

$$1 \cdot 1 = 1$$

$$A \cdot A = A$$

$$A \cdot 0 = 0 \checkmark$$

$$A \cdot 1 = A$$

$$A \cdot \bar{A} = 0 \checkmark$$

Or operation :-

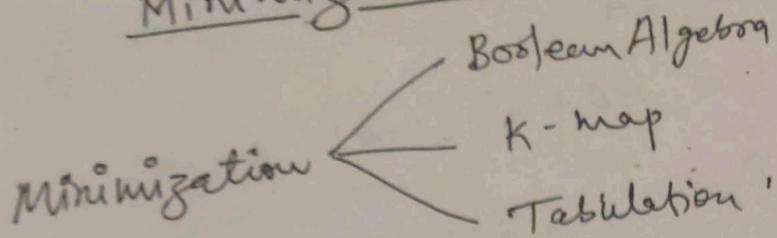
$$A + A = A$$

$$A + 0 = A$$

$$A + 1 = 1 \checkmark$$

$$1 + 1 = 1$$

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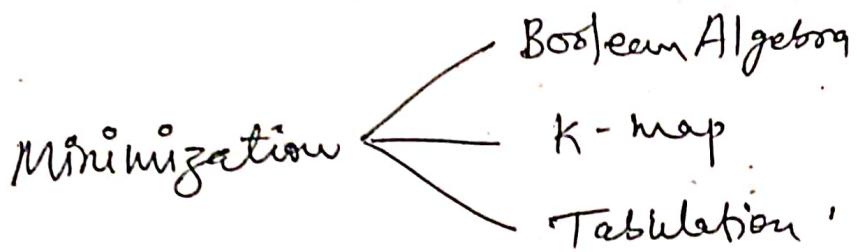
$$\begin{aligned} A \cdot A &= A \\ A \cdot 0 &= 0 \\ A \cdot 1 &= A \\ A \cdot \bar{A} &= 0 \end{aligned}$$

OR operation :-

$$A + A = A$$

$$A + 0 = A$$

Minimization



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$$\text{Or operation :-}$$

$$A + A = A$$

$$A + 0 = A$$

$$A + 1 = 1 \checkmark$$

$$A + \bar{A} = 1 \checkmark$$

$$\textcircled{1} \quad A B + A \bar{B} \rightarrow A [B + \bar{B}] = A$$

$$\textcircled{2} \quad A B + A \bar{B} C + A \bar{B} \bar{C} \rightarrow A B + A \bar{B} [C + \bar{C}]$$
$$\equiv A [B + \bar{B}] = A //$$

Advantages of minimization :-

- \textcircled{1} No. of logic gates decreases
- \textcircled{2} Speed is increased
- \textcircled{3} Power dissipation is decreased
- \textcircled{4} Fan in may be reduced.
- \textcircled{5} Complexity will decrease.

I) Simplify $(A+B)(A+C)$.

Sol:

$$\begin{aligned} & A + \underbrace{ACT+AB+BC}_{A[1+C+B]+BC} \\ &= A[1+C+B] + BC \\ &= A + BC \end{aligned}$$

\textcircled{1} $(\underline{\underline{A+B}})(\underline{\underline{A+C}}) = A + BC$ Transposition Theorem.

\textcircled{2} $(\underline{\underline{X+Y}})(\underline{\underline{\bar{X}+Z}}) = \bar{X} + YZ$

\textcircled{3} $(\underline{\underline{A+B+\bar{C}}})(\underline{\underline{A+B+C}})(A+\bar{B}+\bar{C})$

$$\begin{aligned} &= (\underline{\underline{A+B}})(\underline{\underline{A+\bar{B}+\bar{C}}}) \\ &= A + B(\bar{B}+\bar{C}) \\ &= A + B\bar{C} \end{aligned}$$

$$\textcircled{4} \quad (\underline{\underline{A}} + \underline{\underline{B}})(\underline{\underline{A}} + \overline{\overline{B}})(\overline{\overline{A}} + B)(\overline{\overline{A}} + \overline{\overline{B}})$$

$$= (A) \cdot \overline{A} = 0$$

$$\textcircled{5} \quad (\underline{\underline{x}} + \underline{\underline{y}})(\overline{\overline{x}} + \overline{y})(\overline{x} + \underline{\underline{y}}) = y(\overline{x} + \overline{y}) = \overline{x}y$$

$$\boxed{(A+B)(A+C) = A+BC} \quad \text{Transposition theorem}$$

$$A+BC = (A+B)(A+C)$$

$$\textcircled{1} + \textcircled{2}\textcircled{3} = (\textcircled{1} + \textcircled{2})(\textcircled{1} + \textcircled{3})$$

$$A + \overline{A}B = (A + \overline{A})(A + B)$$

$$A + \overline{A}\overline{B} = (A + \overline{A})(A + \overline{B})$$

$$\overline{A} + AB = \overline{A} + B$$

$$\overline{A} + A\overline{B} = \overline{A} + \overline{B}$$

$$\underbrace{A\overline{B} + \cancel{AC} + \overline{A}B}_{A[\overline{B} + \overline{A}]} + B[A + \overline{A}]$$

$$A + \overline{A}B \Rightarrow A + B.$$

$$\overline{AB} + AB + \overline{A}\overline{B}$$

$$B[\overline{A} + \overline{A}] + \cancel{A}\overline{B} \cdot \overline{A}[B + \overline{B}] \\ = \overline{A} + B$$

$$AB + BC + AC = \overline{B}\overline{C} + \overline{A}C$$

$$AB + \overline{B}C + AC = AB + \overline{C}C$$

$$(\overline{A} + B)(\overline{A} + C)(B + C) = (\overline{A} + B)(\overline{A} + C)$$

$$(\overline{A} + \overline{B})(A + C)(B + C) = (\overline{A} + \overline{B})(B + C)$$

$$\overline{A} \cdot \overline{B} + A \overline{C} + \overline{B} \cdot \overline{C} = \overline{A} \overline{B} + A \overline{C}$$

$$(\overline{A} + \overline{B})(\overline{B} + C)(\overline{A} + \overline{C}) = (\overline{B} + C)(A + \overline{C})$$

Demorgan's Theorems :-

$$A \overline{B} \overline{C} = \overline{A} + \overline{B} + \overline{C}$$

$$\overline{A + B + C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$$

Boolean algebra

- ① Minimization
- ② SOP
 - minimal
 - canonical
- ③ POS
 - minimal
 - canonical
- ④ Truth Tables
- ⑤ Duality
- ⑥ Complements
- ⑦ Karnaugh diagrams
- ⑧ Switching Circuits
- ⑨ L-S statement.

- SOP form will occur when O/P of logic ckt is one.
- SOP form each product terms is known as minterm (or) compliment.

$$5 \rightarrow 101 \\ A \bar{B}^C$$

$$9 \rightarrow 1001 \\ A \bar{B} \bar{C} D$$

Minterms of

for given truth table minimize that

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

$\therefore \bar{A}\bar{B} + A\bar{B}$
 $= \bar{B} [A + \bar{A}]$
 $= \bar{B}$

$$Y(A, B) = \sum m(0, 2)$$

$$\begin{aligned}
 & 00, 10 \\
 & \bar{A}\bar{B} + A\bar{B} \\
 & \bar{B} [A + \bar{A}] \\
 & = \bar{B}
 \end{aligned}$$

for given expression.

$$Y(A, B) = \sum m(0, 2, 3)$$

Minimize the expression.

$$\begin{aligned} &= \overline{AB} + \overline{A}\overline{B} + AB \\ &= A[B + \overline{B}] + \overline{A}\overline{B} \\ &= A + \overline{A}\overline{B} \\ &= A + B \quad // \text{ Minimal} \end{aligned}$$

$$A + \overline{AB} = \text{minimal} \rightarrow A + B$$

$$\begin{aligned} &\text{Canonical} \rightarrow A \cdot 1 + \overline{A} \cdot \overline{B} \\ &\text{(Standard form)} \rightarrow A[B + \overline{B}] + \overline{A}\overline{B} \\ &\Rightarrow AB + A\overline{B} + \overline{A}\overline{B} \end{aligned}$$

No. of minterms in Canonical form

$$\begin{aligned} &A + \overline{BC} \cdot 1 && \textcircled{1} 4 \\ &= A[B + \overline{B}][C + \overline{C}] && \textcircled{2} 5 \\ &\quad + \overline{BC}[A + \overline{A}] && \textcircled{3} 6 \\ &= A[B(C + BC + \overline{B}C + \overline{B}\overline{C})] + \cancel{A}\overline{B}C + (\overline{A}\overline{B}C) && \textcircled{4} 7 \\ &= ABC + A\overline{B}C + A\overline{B}C + A\overline{B}\overline{C} + \overline{A}\overline{B}C \end{aligned}$$

POS

→ POS form is used when opp of logic clk is zero.

Eg: $(A+B+C)(\bar{A}+B+C)(A+\bar{B}+C)$

Max. form

→ In pos form each term is known as Max terms.

$$S \rightarrow 101$$

$$\bar{A}+B+\bar{C}$$

→ For given truth table minimize pos form.

A	B	Y
0	0	1
0	1	0
1	1	0

$$(A+\bar{B})(\bar{A}+\bar{B}) \\ = \pi M(1,3)$$

$$Y = \bar{A}\bar{B} + A\bar{B}$$

$$\Rightarrow \bar{B}(A+\bar{A})$$

$$\Rightarrow \sum m(0,2)$$

$$\sum m(0,2) = \pi M(1,3)$$

$$\sum m(0, 1, 5, 7) = \prod M(2, 3, 4, 6)$$

for given truth table this is dual pos
expression & pos expression same.

Dual

→ Dual Expression is used to convert +ve logic to negative logic or negative logic to +ve logic.

+ve logic AND

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

-ve logic (OR)

1	↑	1
1	0	1
0	1	1
0	0	0

$$+ve \text{ logic AND} = -ve \text{ logic OR}$$

$$+ve \text{ logic OR} = -ve \text{ logic AND}$$

Dual

① AND \leftrightarrow OR

• \leftrightarrow +

1 \leftrightarrow 0

④ keep variable as it is.

$$A\bar{B}C + \bar{A}BC + ABC \quad \text{Dual.}$$

$$(A + \bar{B} + C) (\bar{A} + B + C) \cdot (A + B + \bar{C})$$

→ for any logical expression if two times dual is used resulting same expression.

→ with n variable max. possible no. terms as
max. term $\rightarrow 2^n$.

→ with n variable max. possible logical expression
is 2^{2^n}

$$n \Rightarrow 2 \Rightarrow 2^{2^2} \Rightarrow 2^4 = 16$$

$$n=3 \Rightarrow 2^{2^3} = 2^8 = 256$$

$$n \Rightarrow 4 = 2^{2^4} = 2^{16} = 2^6 \times 2^{10} = 64 \times$$

$$64 \times 1024 = 65536$$

Self Dual $\Rightarrow AB + BC + AC$

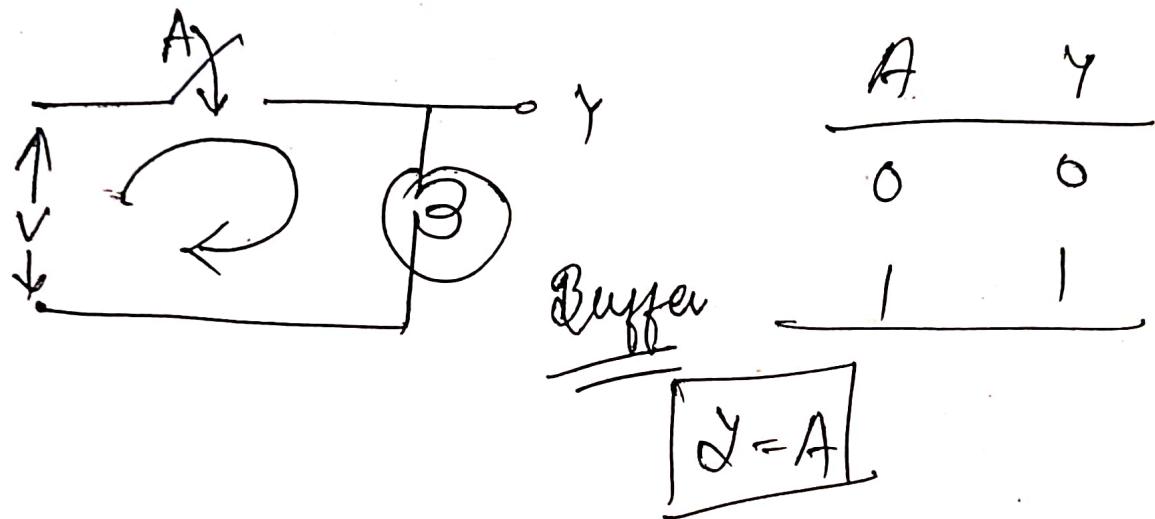
$$(A + B) (B + C) (A + C)$$

$$\Rightarrow \{B + (AC)\} (A + C)$$

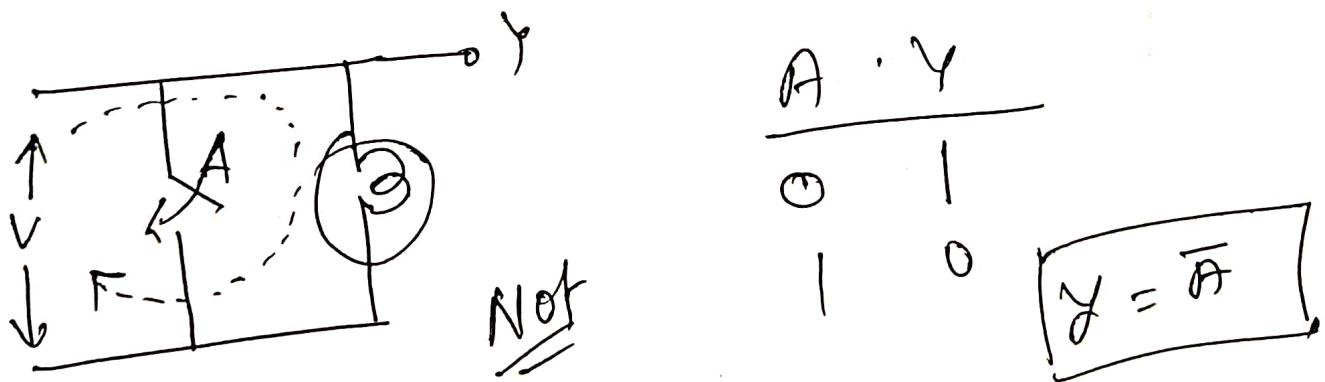
$$\Rightarrow AB + BC + AC$$

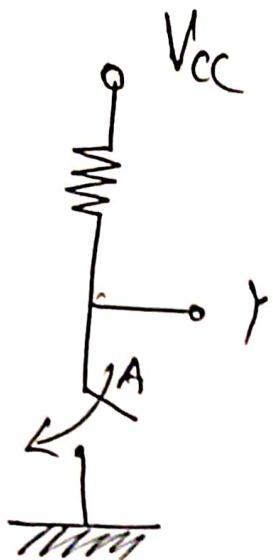
Switching Ckts

① For given switching ckt o/p Y is.

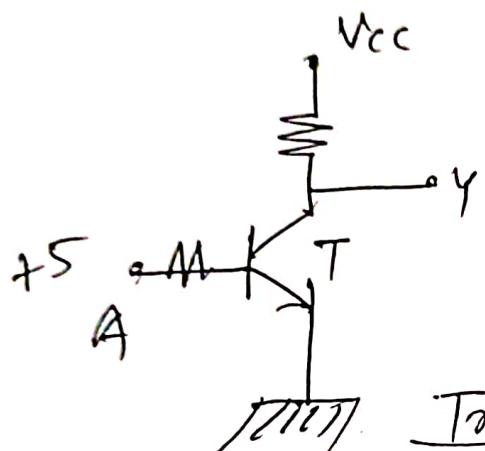


②



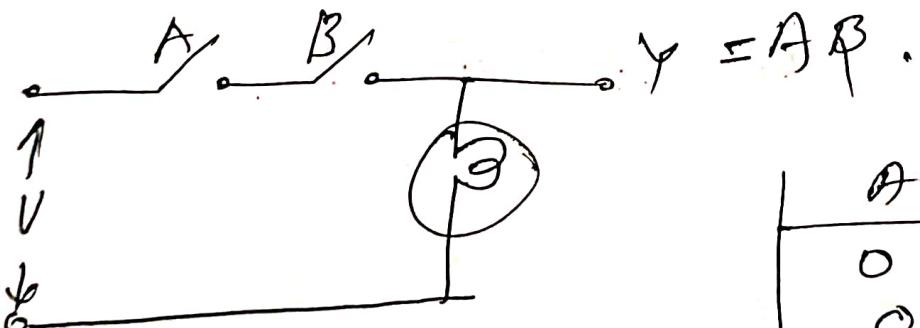


A	Y
0	1
1	0
$Y = \bar{A}$	

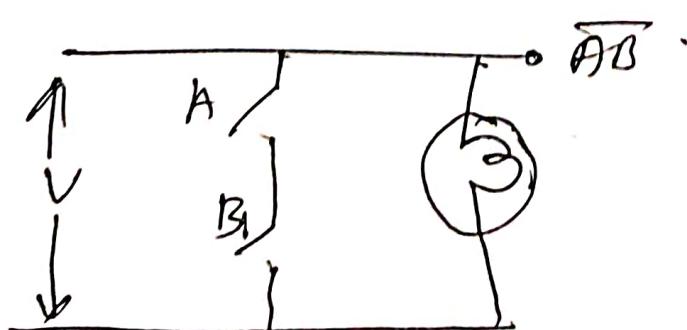


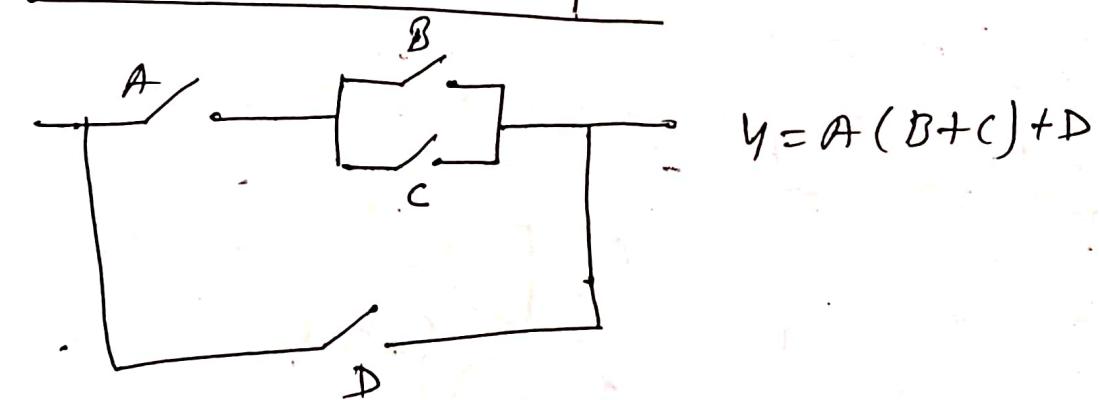
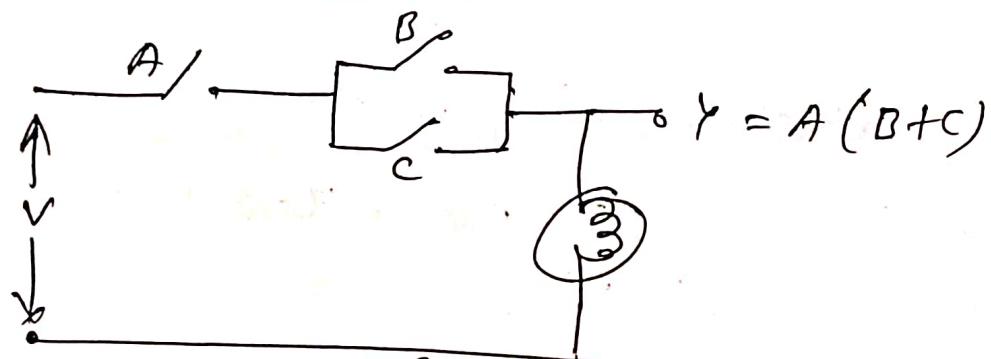
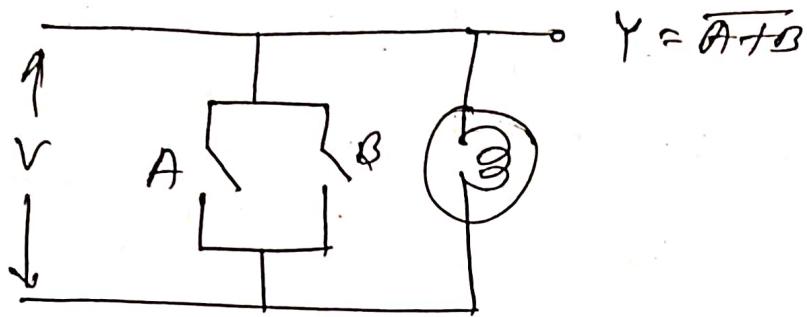
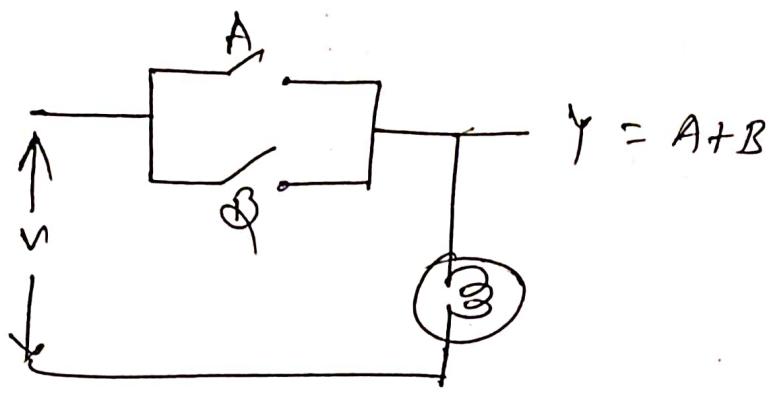
A	T	Y
0	off	1
1	on	0

Transistor
Not gate



A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1





→ A logic ckt have 3-i/p A & B & C & o/p is
 $Y, Y=1$, for the following combination.

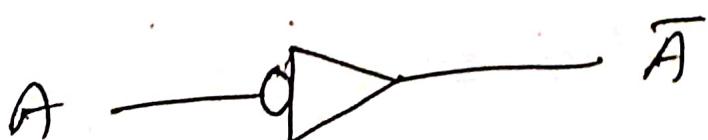
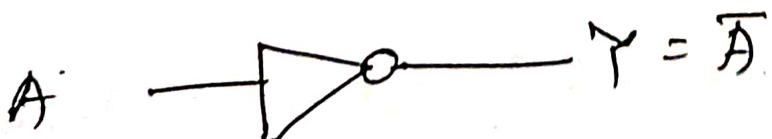
- ① B and C are false $\rightarrow \overline{BC}$
- ② A and C are true. $\rightarrow AC$

Logic Gates

Basic building blocks of digital Ckt.

- ① NOT
 - ② AND
 - ③ OR
 - ④ NAND
 - ⑤ NOR
 - ⑥ EXOR
 - ⑦ EXNOR
- } Basic Gates
- } universal Gates
- } Arithmetic.
Comparison
Parity Generation/Checking.
Code Converter.

NOT



A	Y
0	1
1	0

AND Gate

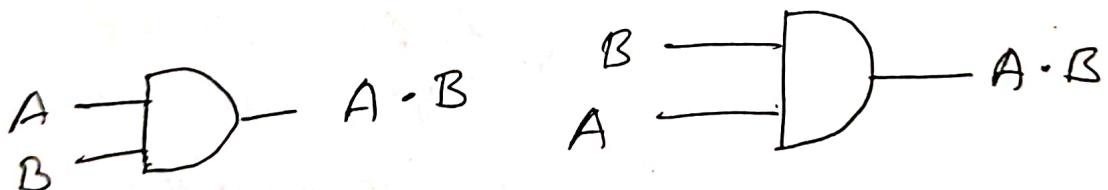


A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Series switch

Property

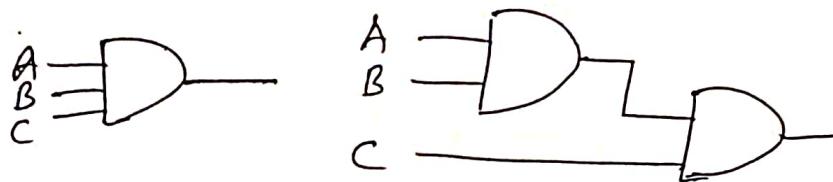
① Commutative



$$AB = BA$$

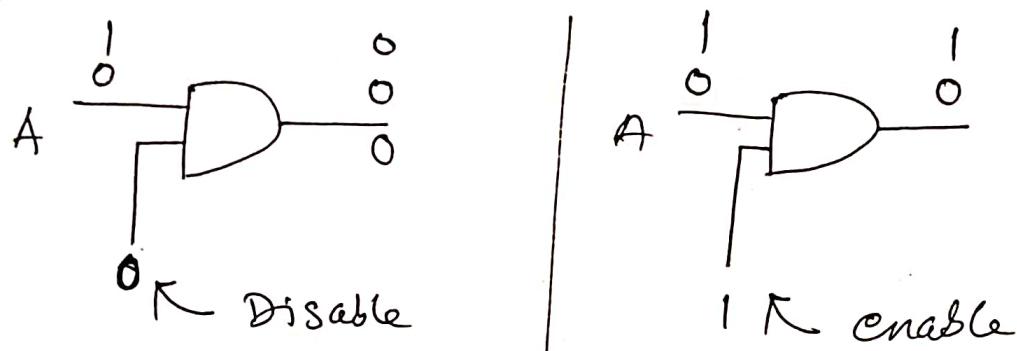
⑨

Associative

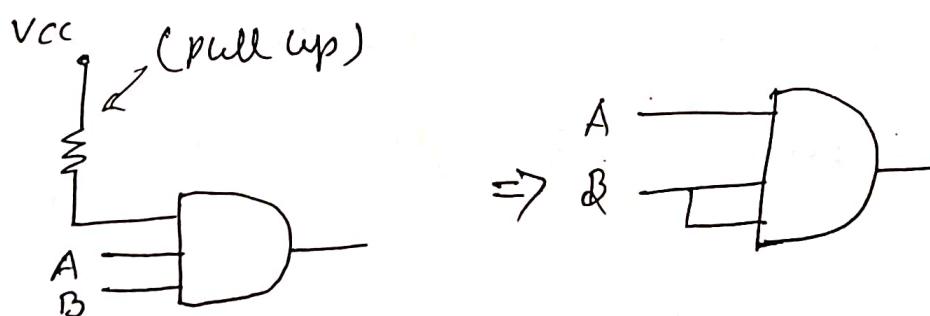


$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

⇒ And Gate follows both commutative & associative law.



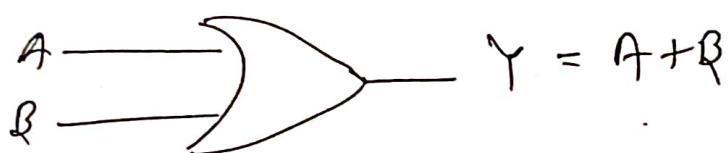
And $\rightarrow 0 \rightarrow$ Disable \rightarrow O/p does not change
 And $\rightarrow 1 \rightarrow$ Enable \rightarrow O/p does not change



- ⇒ In TTL logic family any i/p is open or floating then it will act as logic one (1).
- ⇒ whereas in ECL logic family floating i/p is acts as logic zero (0).

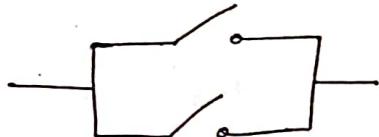
OR Gate

Symbol :-



$$Y = A + B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1



Property -

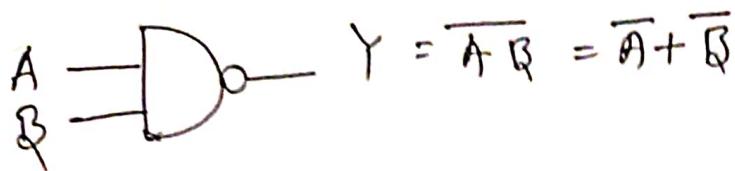
- ① It follows commutative law.

$$A + B = B + A$$

- ② It follows associative law.

$$A + (B + C) = (A + B) + C$$

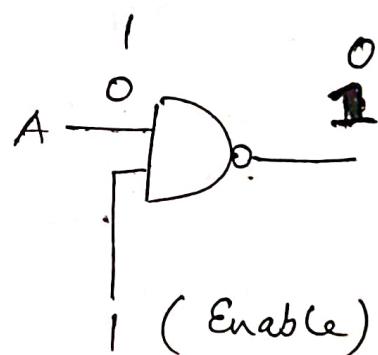
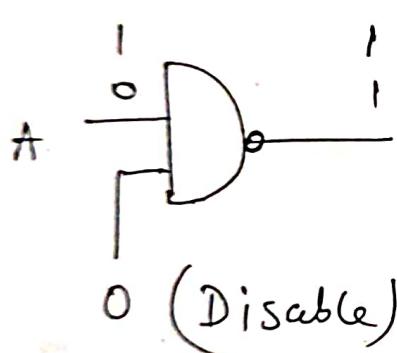
NAND GATE



Bubbled OR Gate

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

→ If Any i/p is zero o/p is one. and both the i/p is one o/p is zero.



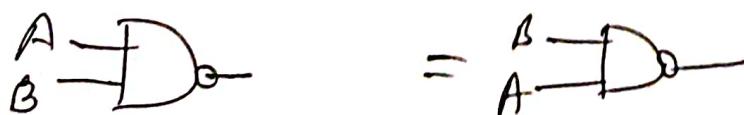
NAND → enable → 1 → o/p changes

NAND → disable → 0 → o/p does not change

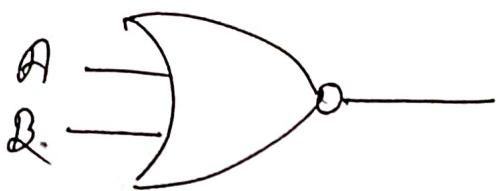
Property

① It follows commutative law.

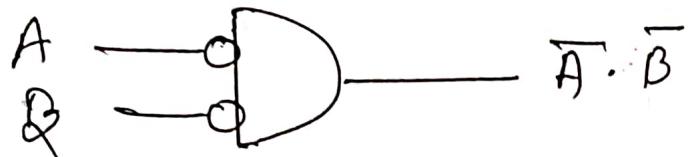
$$\overline{AB} = \overline{BA}$$



NOR Gate



$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

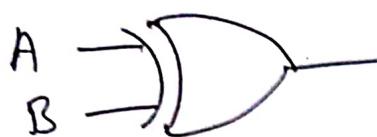


A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Bubbled AND gate

⇒ Any i/p if one out o/p is zero & if both i/p is zero o/p is one.

EXOR (or) XOR (Exclusive OR)



$$Y = A \oplus B$$

$$= \overline{A}B + A\overline{B}$$

Note :

~~OR gate = Inclusive OR~~

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

~~* * *~~

$$Y = (A+B)(\overline{A}+\overline{B})$$

$$Y = \overline{AB} + A\overline{B}$$

$$A = B \Rightarrow Y = 0$$

$$A \neq B \Rightarrow Y = 1$$

3 Variable XOR gate. (Truth Table)

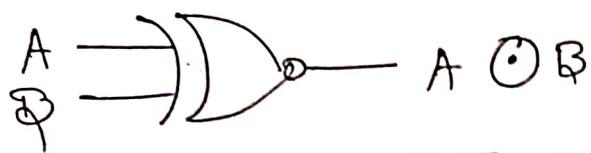
A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Logical Expression :-

$$Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

* * * → XOR gate is available with only two inputs. If we use multiple XOR gate we will go for associative law. This cause delay in O/P.

EXNOR Gate



$$\begin{aligned}
 &= \overline{A} \overline{B} + A B \\
 &= (A + \overline{B})(\overline{A} + B)
 \end{aligned}$$

A	B	P
0	0	1
0	1	0
1	0	0
1	1	1

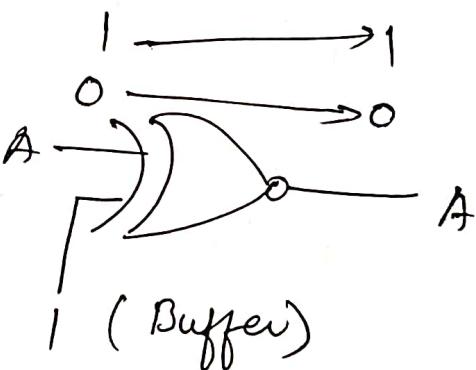
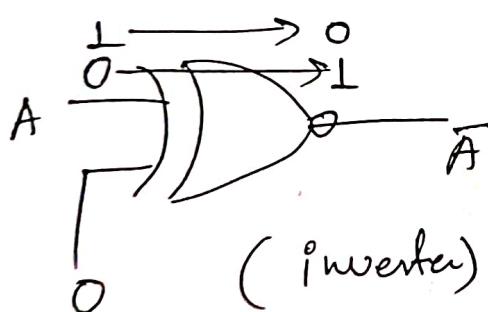
Synonyms -

- ① Equivalence Gate
- ② Coincidence logic gate.

Property -

$$A = B \Rightarrow 1$$

$$A \neq B \Rightarrow 0$$



Conclusion :-

$$\begin{aligned}
 A \odot A &\stackrel{?}{=} 1 \\
 A \odot \overline{A} &= 0 \\
 A \odot 1 &\stackrel{?}{=} A \\
 A \odot 0 &= \overline{A}
 \end{aligned}$$

Relations

$$A \oplus B = \overline{AOB}$$

$$A \oplus B \oplus C \oplus D = \overline{AOB \odot COD}$$

★

$$A \oplus B \oplus C = A \odot B \odot C$$

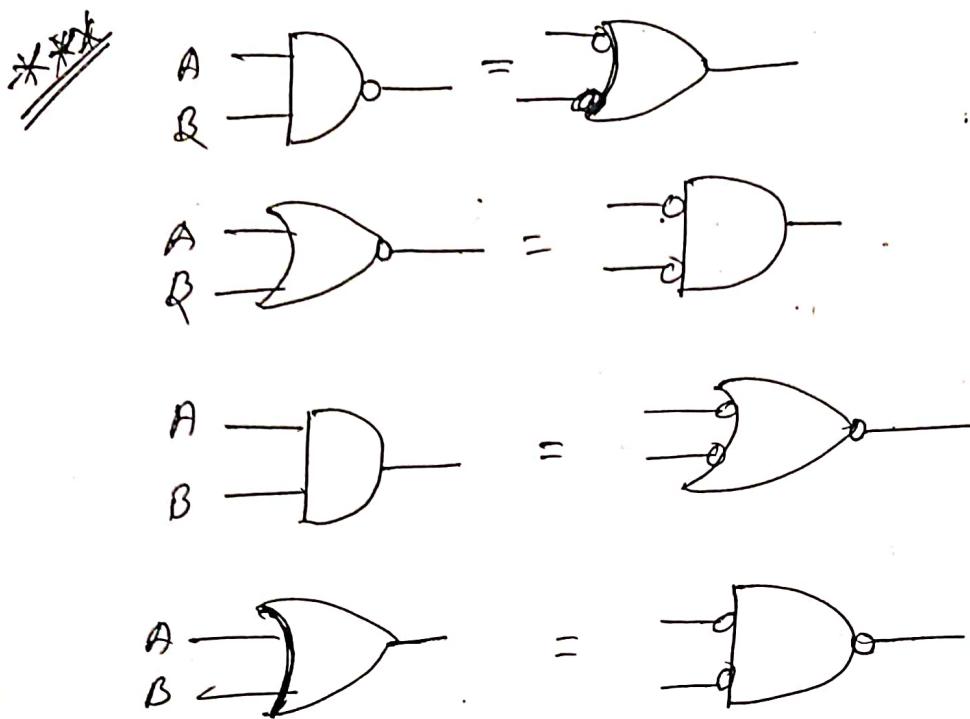
Let LHS : $\bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + ABC$

RHS

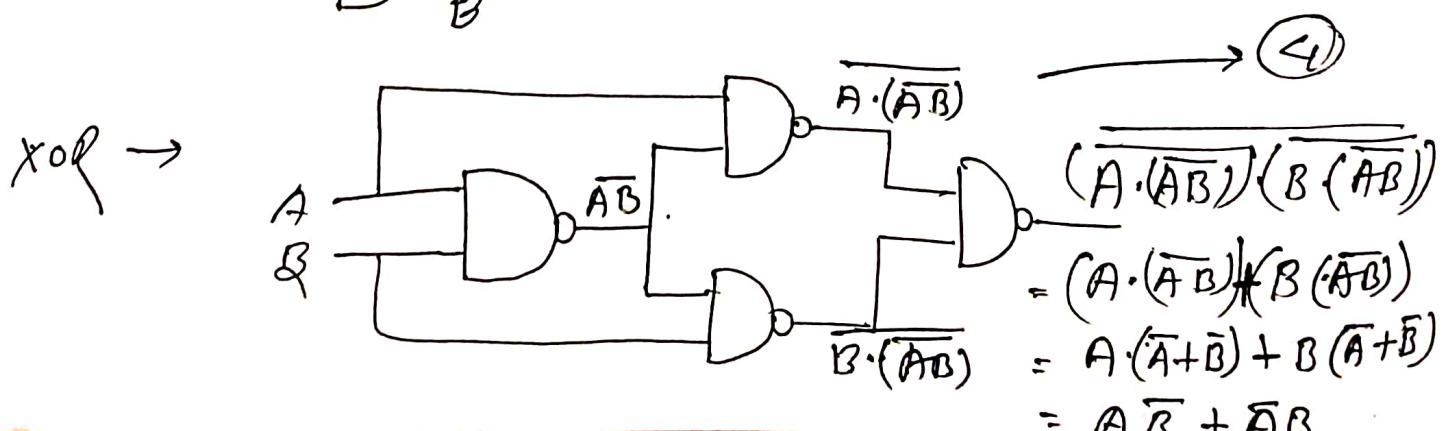
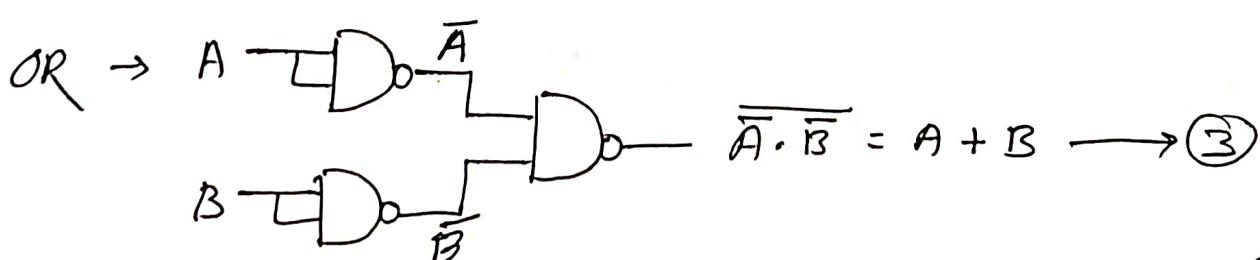
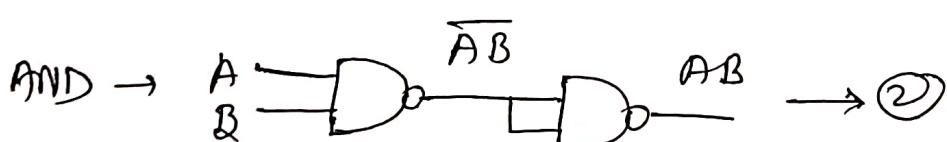
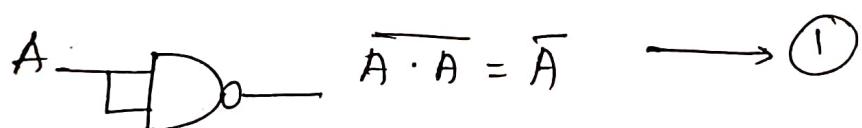
$$\begin{aligned}
 A \odot B \odot C &= (AB + \bar{A}\bar{B}) \odot C \\
 &= X \odot C = XC + \bar{X}\bar{C} \\
 &= (AB + \bar{A}\bar{B})C + (\bar{A}B + A\bar{B})\bar{C} \\
 &= ABC + \bar{A}\bar{B}C + (\bar{A}B + A\bar{B})\bar{C} \\
 &= ABC + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} \\
 &= LHS
 \end{aligned}$$

Hence, $A \oplus B \oplus C = A \odot B \odot C$ (Proved)

NAND As Universal

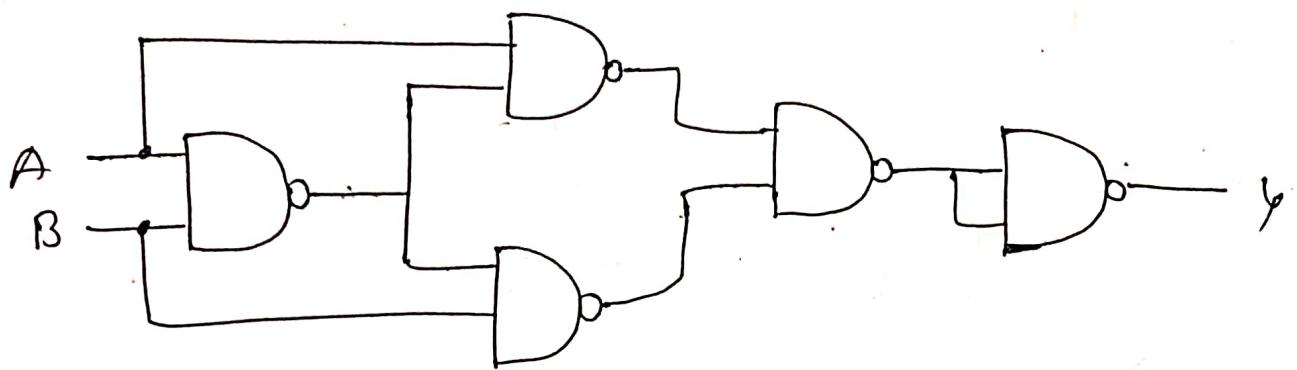


As NOT gate :-



Ex NOR →

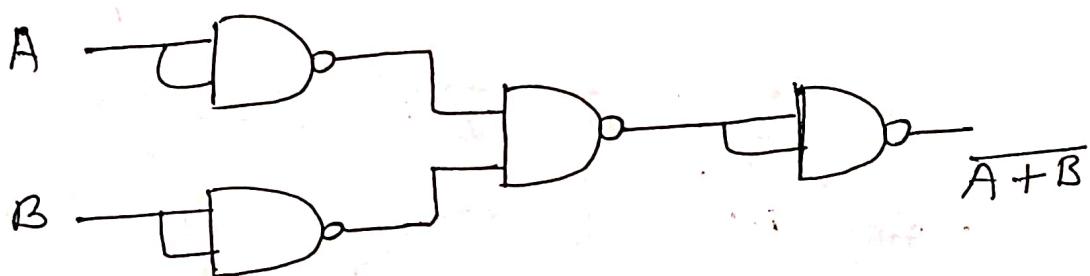
→ ⑤ gate



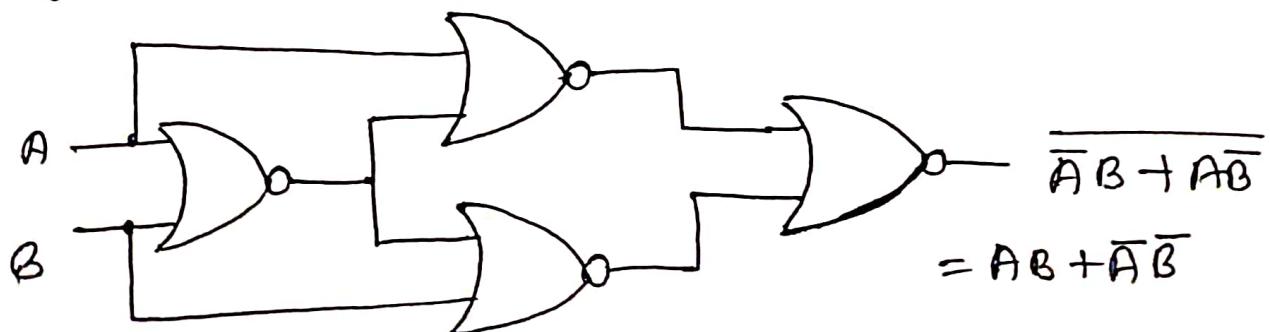
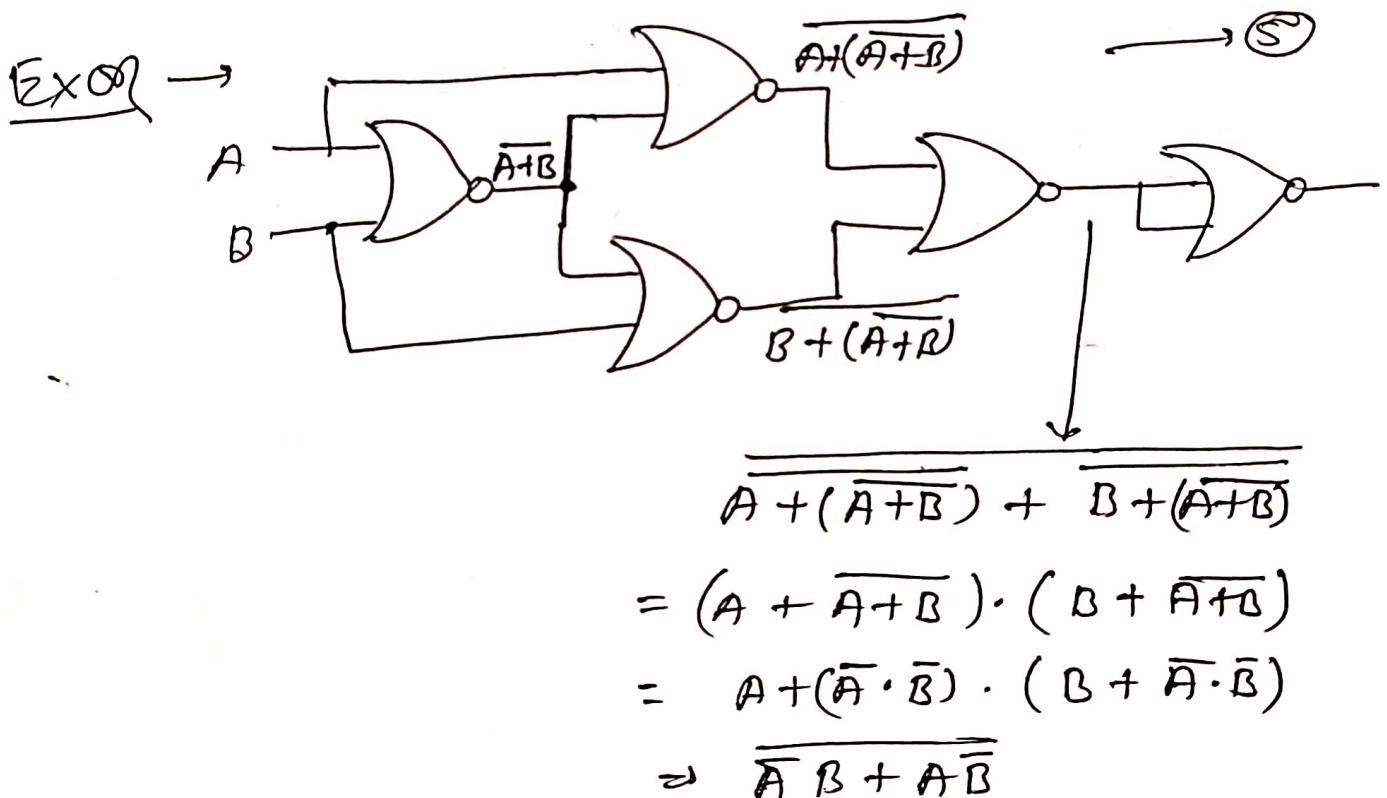
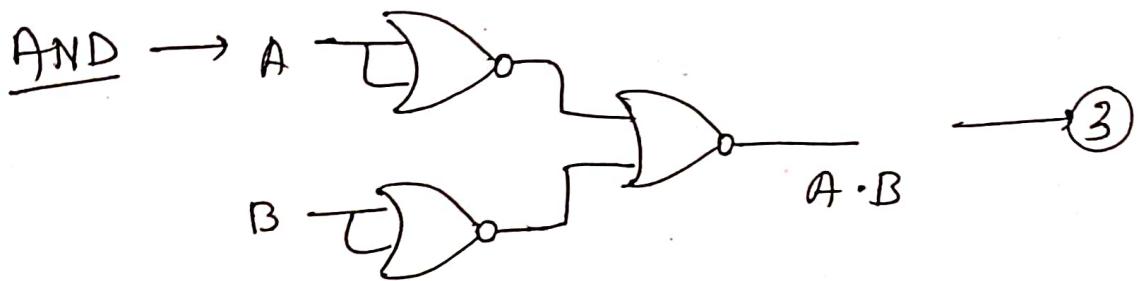
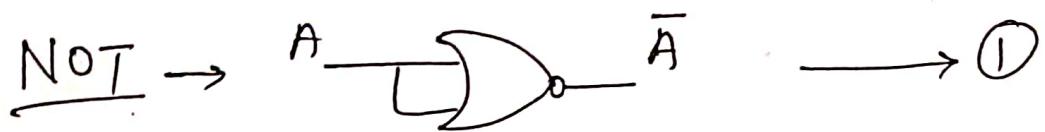
~~Ex NOR~~

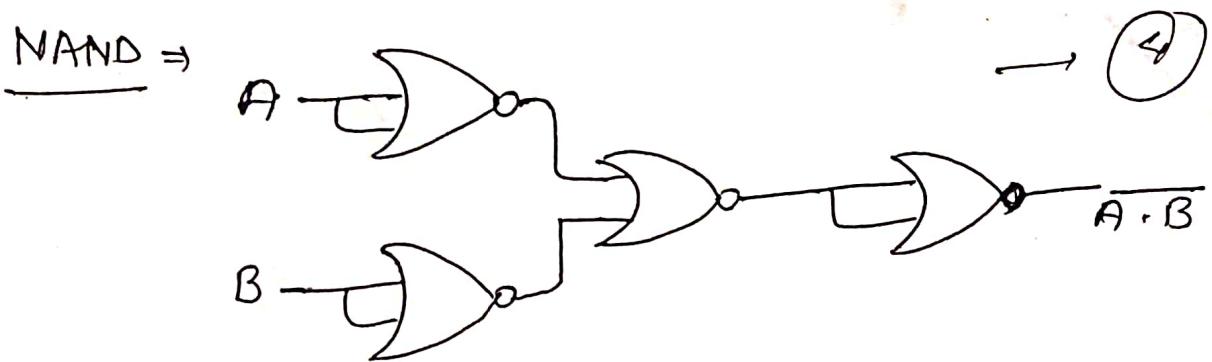
NOR:

→ ④



Not as Universal





Summary :-

logic gate	No. of NAND	No. of NOR
NOT	1	1
AND	2	3
OR	3	2
EXOR	4	5
EXNOR	5	4
NOR	4	1
NAND	1	4

Minterm and Maxterm:

Minterms: Each individual term in SSOP is called as minterm.

Maxterm: Each individual SPOS form terms in SPOS is called as Maxterm.

For two variable function \rightarrow

F ₁	Variable	Minterm		Maxterm	
		A	B	SSOP	SPOS
1	0 0	$\bar{A}\bar{B} \rightarrow m_0$		$\bar{A}+\bar{B}-M_0$	
0	0 1	$\bar{A}B \rightarrow m_1$		$A+\bar{B}-M_1$	
1	1 0	$A\bar{B} \rightarrow m_2$		$\bar{A}+B-M_2$	
1	1 1	$AB \rightarrow m_3$		$\bar{A}+\bar{B}-M_3$	

Ex: $f(A, B) = AB + \bar{A}B$ - this is in SSOP form.

pos $(A+B)(\bar{A}+B)$ $\sum_m(0, 2, 3)$ Anywhere you see like this nine its minterm in SSOP form.

Σ denotes minterm in SSOP form.
what is maximum value is there; two variable.

$$f = \bar{A}\bar{B} + A\bar{B} + AB$$

$$f = \sum_m(0, 2, 3) \sum_m(0, 2, 3)$$

Maxterm is complement to each other.

How can directly we can identify by Max term
and minterms each other.

Example:

$$F_{ABC} = \sum_m (0, 2, 3, 5) \rightarrow F(A\bar{B}\bar{C}) \text{ M}(1, 4, 6, 7)$$

variable \rightarrow These are three variables
have many minterms and maxterm.

Total minterms and maxterm. $2^n = 2^3 = 8$
(0, 1, 2, 3, 4, 5, 6, 7, 8)

* SOP to SSOP *

* steps *

- 1: Identify the missing variable in product term
- 2: multiply (variable + its complements)
- 3: Neglect the repeated term.

$$\text{Example } f(A, B, C) = AB + A\bar{B}\bar{C} + \bar{B}C$$

This is a three variable function.

$$= AB(C + \bar{C}) + A\bar{B}\bar{C} + BC(A + \bar{A})$$

$$= ABC + A\bar{B}\bar{C} + A\bar{B}\bar{C} + BA\bar{B}C + \bar{A}BC$$

$$= ABC + A\bar{B}\bar{C} + \bar{A}BC, \sum_m (3, 5, 7)$$

This is resultant of SSOP form, m₆ m₇

Now we need conversion this SOP in SSOP
For example in your exams they asking by giving
above expression what are the minterm.

of this expression, then we can't say what
are the minterm for in your expression

We can't say by this SOP expression so definitely
we need to SSOP expression to defined the
minterm.

POS to SPOS

Steps

* Identify the missing variable.

* Add with variable & its complement separately.

* Neglect the repeated terms.

Example: $f(A, B, C) = \underset{\text{POS}}{A(A+C)}$

$\downarrow \quad \downarrow$

B, C are missed.
B is missed,
missed

$$= (A + \bar{B} + \bar{C}) (A + \bar{B} + C) (A + B + \bar{C}) (A + B + C)$$

$$(A + C + B) (A + C + \bar{B})$$

$$= \underset{0\ 0\ 0}{(A+B+C)} \underset{0\ 1\ 0}{(A+\bar{B}+C)} \underset{0\ 1\ 0}{(A+\bar{B}+\bar{C})} \underset{0\ 0\ 1}{(A+B+\bar{C})} \underset{\text{SPOS}}{\underline{(A+C+B)}}$$

$$\prod_m [M_0, M_2, M_3, M_1]$$

$$\prod_m [0, 1, 2, 3] \text{ mms}$$

Converting SSOP to SPOS

$$f(A, B, C) \doteq \sum_m (0, 1, 3, 4, 7,)$$

This is three variable.

minterms and maxterms $2^3 = 8$

what are the Associated Maxterm: Just take the complement. So complement whatever. Min-terms are not present that number will come Maxterm.

$$f(A, B, C), \prod_m [2, 5, 6]$$

$$010, 101, 110, \frac{A\bar{B}C + (A+\bar{B}+C)(\bar{A}+\bar{B}C)}{(A+\bar{B}C)}$$

Ex Express the function $y = A + \bar{B}C$ in (a) canonical SOP and (b) canonical POS form.

Sol

$$y = A + \bar{B}C$$

$$= \bar{A}(B + \bar{B}) + \bar{B}C(A + \bar{A})$$

$$= AB + A\bar{B} \neq A\bar{B}C + \bar{A}\bar{B}C$$

$$A(B + \bar{B})(C + \bar{C}) + \bar{B}C(A + \bar{A})$$

$$\cancel{ABC} + \cancel{A\bar{B}C} +$$

$$(AB + A\bar{B})(C + \bar{C}) + A\bar{B}C + \bar{A}\bar{B}C$$

$$\underline{ABC + ABC\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + A\bar{B}C + A\bar{B}\bar{C}}$$

$$\begin{array}{ccccccccc} ABC & + & ABC\bar{C} & + & A\bar{B}C & + & A\bar{B}\bar{C} & + & A\bar{B}C \\ \text{m}_7 & & \text{m}_6 & & \text{m}_5 & & \text{m}_4 & & \text{m}_3 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{array}$$

~~Therefore~~

$$Y = \sum_m (m_1, m_4, m_5, m_6, m_7)$$

(b)

canonical POS

$$Y = A + \bar{B}C$$

$$= (A + \bar{B})(A + C) +$$

$$[\because A + BC = (A + B)(A + C)]$$

$$= (A + \bar{B} + C)(A + \bar{B} + \bar{C})(A + B + C)(A + B + \bar{C})$$

$$= (\cancel{A + \bar{B} + C})(A + \bar{B} + \bar{C})(A + B + \bar{C})(A + B + C)$$

$$\therefore Y = \prod [m_1, m_2, m_3]^{m_1, m_2, m_3}$$

SPOS \rightarrow POS SOP

Date _____ Page No. _____

$$f(A, B) = A(A+B)$$

$$= (A+B)(A+\bar{B})(A+B)$$

$$= (A+B)(A+\bar{B})$$

$$\therefore \Sigma = \Sigma_{m0} \cup \Sigma_m \subseteq \Sigma_m[0,1] \Rightarrow \overline{F}$$

$$\Sigma \subseteq \Sigma_m[2,3]$$

$$\text{SOP: } A\bar{B} + AB$$

* KARNAUGH MAP [K-map] *

Karnaugh map is also called as K-map
this is developed by scientist Karnaugh by his
name so this is developed in 1953

- * Developed by Karnaugh in 1953
- * used to simply simplify boolean algebraic expressions
without using boolean theorems.

So when we are solving a boolean expression, we
use to consider boolean laws and theorems.
so that may be some what tricky typical for us.
So that may whenever you forget that laws and
theorems.

* whenever you forget that laws and theorems
you can make use of this K-map to simplify that
boolean expression

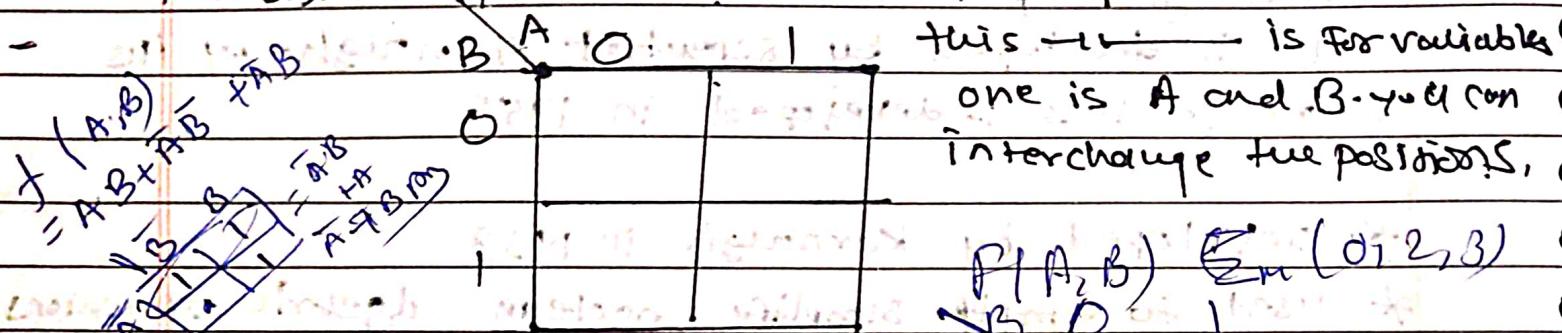
* Now we will say how the K-map will be
what is the structure of K-map and how
we are having variables in that and that
will see now.

* K-map may be for 2 variable. we may have:
 $2^2 = 4$ cells in K-map.

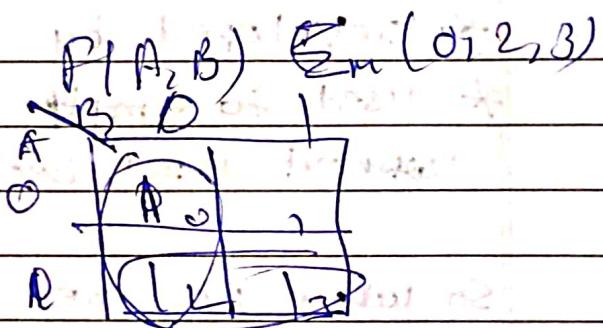
* K-map is nothing its a pictorial representation representation or pictorial solving method of Boolean expression. without using boolean laws and theorems.

Example for 2 variable K-map.

this box always contained with the separation. and



* number of cells $\geq 2^n$
 $= 2^2$



... n = number of variables

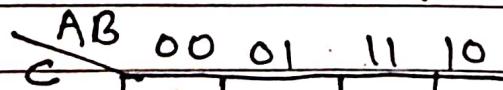
... 2^n = number of cells

* Three variable K-map. more

How to structure of 3 variable K-map

* we can take vertically or horizontally

* what is the cell means the possibility of your variable combination.



	00	01	11	10
C	0			
	1			
		1		
			1	

$$n = 3$$

$$2^n = 2^3 = 8$$

* the reason behind this is for any state change for every state change you need change one bit

BESTER - Dual of Boolean EXPRESSION.

* Dual is nothing but Interchanging OR with AND and AND with OR

Replace .

* OR with AND ($+ \rightarrow \cdot$)

* AND with OR ($\cdot \rightarrow +$)

* 1 with zero

* 0 with 1

Ex : For the identity

$$AB + \overline{AC} + BC = AB + \overline{AC}$$

The Dual form is

$$(A+B) \cdot (\overline{A}+C) \cdot (B+C) =$$

$$\Rightarrow A\overline{A} + AC + \overline{A}B + BC (B+C)$$

$$= 0 + ABC + \overline{A}B + BC + AC + \overline{A}BC + BC$$

$$= ABC + \overline{A}B + \overline{B}C + AC + \overline{A}BC$$

$$= BC + \overline{A}B + \overline{B}C + AC = BC + \overline{A}B + AC$$

$$= B(\overline{A}+C) + C(A+\overline{B}) = B(\overline{A}+C) + AC$$