



Q1. Which of these sentences are propositions ? What are the truth values of those that are propositions ?

- a) Boston is the capital of Massachusetts. b) Miami is the capital of Florida.
c) $2 + 3 = 5$. d) $5 + 7 = 10$.
e) $x + 2 = 11$. f) Answer this question.

Q2. Answer the following questions:

1. What is the negation of each of these propositions ?

- (a) Jennifer and Teja are friends. (b) There are 13 items in a baker's dozen.
(c) 121 is a perfect square. (d) Abby sent more than 100 text messages everyday

2. Which of following sentences is a proposition ? [Winter 2018 – 19]

- (a) What is a group? (b) $2n > 100$.
(c) Wish you all the best (d) A simple graph has a loop

3. Which of the following is proposition ? [Winter 2023 – 24]

- (a) Get me a glass of milkshake (b) God bless you!
(c) What is the time now? (d) The only odd prime number is 2.

4. Which of the following is knowing a proposition ? [Summer 2023 – 24]

- (a) Four is even number. (b) $x + y = 10$
(c) Sun rises in west. (d) (a) and (c)

5. Which of the following is not proposition ? [Winter 2022 - 23]

- (a) My name is Kamal. (b) Four is even number.
(c) May God bless you. (d) Nine is prime number.

6. Which of the following is not proposition ? [Winter 2023 – 24]

- (a) $\sqrt{3}$ is a prime number. (b) $\sqrt{2}$ is rational number.
(c) Mathematics is interesting. (d) 5 is even integer.

Q3. Suppose that,

Smartphone A has 256 MB RAM and 32GB ROM, and the resolution of its camera is 8 MP;
Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and
Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP.
Determine the truth value of each of these propositions.

- a) Smartphone B has the most RAM of these three smartphones.
b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

- Q4.** Let p and q be the propositions “The election is decided” and “The votes have been counted,” respectively. Express each of these compound propositions as an English sentence.

$$\begin{array}{llll} \text{a) } \neg p & \text{b) } p \vee q & \text{c) } \neg p \wedge q & \text{d) } q \rightarrow p \\ \text{e) } \neg q \vee (\neg p \wedge q) & \text{f) } \neg p \rightarrow \neg q & \text{g) } p \leftrightarrow q & \text{h) } \neg q \rightarrow \neg p \end{array}$$

- Q5.** Construct a truth table for each of these compound propositions.

$$\text{a) } ((p \rightarrow q) \rightarrow r) \rightarrow s \quad \text{b) } (p \wedge q) \rightarrow (p \vee q)$$

- Q6.** Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of each of these pairs of bit strings.

$$\begin{array}{ll} \text{a) } 00\ 0111\ 0001, 10\ 0100\ 1000 \\ \text{b) } 11\ 1111\ 1111, 00\ 0000\ 0000 \end{array}$$

- Q7.** a) Show that $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ are logically equivalent.

b) Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

- Q8.** Determine whether each of these compound propositions is satisfiable.

$$\begin{array}{ll} \text{a) } (p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q) \\ \text{b) } (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r) \end{array}$$

- Q9.** Let $C(x)$ be the statement “ x has a cat,” let $D(x)$ be the statement “ x has a dog,” and let $F(x)$ be the statement “ x has a ferret.”

Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

- a) A student in your class has a cat, a dog, and a ferret.
- b) All students in your class have a cat, a dog, or a ferret.
- c) Some student in your class has a cat and a ferret, but not a dog.
- d) No student in your class has a cat, a dog, and a ferret.
- e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

- Q10.** Translate the logical equivalence $(T \wedge T) \vee \neg F \equiv T$ into an identity in Boolean algebra. [Winter 2019 – 20]

- Q11.** Give a direct proof that if m and n are both perfect squares, then nm is also a perfect square.

- Q12.** Prove that if n is an integer and n^2 is odd, then n is odd.

- Q13.** Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction. [Winter 2021 – 22]

OR

By contradiction method prove that $\sqrt{6}$ is an irrational number. [Winter 2023 – 24]

- Q14.** Use a direct proof to show that the sum of two odd integers is even.

- Q15.** Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

- a) a proof by contraposition.
- b) a proof by contradiction.

- Q16.** Prove that $n^2 + 1 \geq 2^n$ when n is a positive integer with $1 \leq n \leq 4$. [Winter 2019 – 20]