Unit 2 Minimization Techniques

↳ There are 3 types of Algebra :-

1. Boolean Algebra
2. K-map, K-map with Don't Care
3. Quine MC-Cluskey and Quine MC-Cluskey with don't care

⟹ Boolean Algebra :-

↳ Boolean Addition :-

$0+0=0 \Rightarrow x+\bar{x}=1$

$0+1=1 \quad x+x=x$

$1+0=1 \quad x+1=1$

$1+1=1$

↳ Boolean Multiplication :-

$0.0=0 \longrightarrow x.x=x$

$0.1=0 \quad x.\bar{x}=1$

$1.0=0 \quad \bar{x}.\bar{x}=\bar{x}$

$1.1=1$

⟶ Properties of Boolean Algebra :-

1. Commutative Property :-

$$A+B = B+A$$
$$A.B = B.A$$

2. Associative Property :-

$$A+(B+C) = (A+B)+C$$
$$A.(B.C) = (A.B)C$$

3. Distributive Property :-

$$A + BC = (A+B)(A+C)$$

Proof :- $A + BC$

$A.1 + BC$

$A(1+B) + BC$ ($\because 1+B = 1$)

$A + AB + BC$

$A(1+C) + AB + BC$ ($\because 1+C = 1$)

$A.A + AC + AB + BC$ ($\because A.A = A$)

$A(A+C) + B(A+C)$

$(A+B).(A+C)$ Hence Proved

4. Absorption Property / Law :-

$$A + AB = A$$

Proof :- $A + AB$

$A + AB$

$A(1+B)$ ($1+B = 1$)

$A.1$

A Hence Proved

Proof :- $A(A+B)$

$A.A + AB$

$A + AB$

$A(1+B)$ ($1+B = 1$)

$A(1)$

A Hence Proved

Ex :- $A + \bar{A}B = (A+B)$

Ans Proof :- $A + \bar{A}B = (A+B)$

$(A+\bar{A})(A+B)$ $(\because A+\bar{A} = 1)$

$1(A+B)$

$(A+B)$ Hence Proof / Proved

Ex :- $A(\bar{A}+B) = AB$

Ans Proof :- $A(\bar{A}+B) = AB$

$(A+\bar{A})+AB$ $(\because A+\bar{A})$

$1+AB$

AB Hence Proved

5. Consensus law :-

1. $AB + \bar{A}C + BC = AB + \bar{A}C$

Proof :- $AB + \bar{A}C + BC = AB + \bar{A}C$

$AB + \bar{A}C + BC \cdot 1$

$AB + \bar{A}C + BC(A+\bar{A})$

$AB + \bar{A}C + BCA + BC\bar{A}$

$AB + \bar{A}C + ABC + \bar{A}BC$

$AB(1+C) + \bar{A}C(1+B)$

$AB \cdot 1 + \bar{A}C \cdot 1$

$AB + \bar{A}C$

Hence Proved

2. $(A+B)(\bar{A}+C)(B+C) = (A+B).(\bar{A}+C)$

Proof:- $(A+B)(\bar{A}+C)(B+C) = (A+B).(\bar{A}+C)$

$(A+B)(\bar{A}+C)(B+C+A.\bar{A})$ → Distributive Property

$(A+B)(\bar{A}+C)(B+C+A)(B+C+\bar{A})$

$(A+B)(A+B+C)(\bar{A}+C)(\bar{A}+B+C)$ Absorption law

$(A+B).(\bar{A}+C)$ $(\because A(A+B) = A)$

Hence Proved

3. $AB + BC + \bar{B}C = AB + C$

Proof:- $AB + BC + \bar{B}C = AB + C$

$AB + C(B+\bar{B})$ $\because B+\bar{B} = 1$

$AB + C(1)$

$AB + C$

Hence Proved

6. Quality Theorem :-

↳ To get dual of any Boolean Expression is called duality theorem.

$\odot \rightarrow \oplus$

$\oplus \rightarrow \odot$

OR → AND

AND → OR

NAND → NOR

NOR → NAND

EX-OR → EX-NOR

EX-NOR → EX-OR

0 → 1

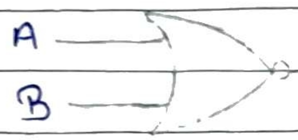
1 → 0

7. Demorgans Theorem :-

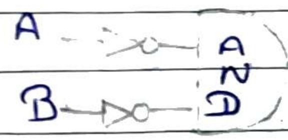
Gate
↓

↳ Whole complement of two or more ORed Variable is equal to AND of the equivalement and vice versa also True.

1. $\overline{A+B} = \bar{A} \cdot \bar{B}$



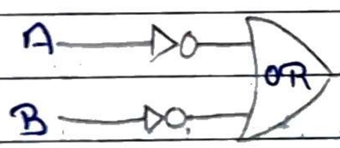
$x = \overline{A+B}$
$= \bar{A} \cdot \bar{B}$



$x = \bar{A} \cdot \bar{B}$

2. $\overline{A \cdot B} = \bar{A} + \bar{B}$

$x = \overline{A \cdot B}$
$= \bar{A} + \bar{B}$



$x = \bar{A} + \bar{B}$

A	B	$\bar{A}$	$\bar{B}$	$A+B$	$A \cdot B$	$\overline{A+B}$	$\overline{A \cdot B}$	$\bar{A} \cdot \bar{B}$	$\bar{A}+\bar{B}$
0	1	1	1	0	0	1	1	1	1
0	0	1	0	1	0	0	1	0	1
1	1	0	1	1	0	0	1	0	1
1	0	0	0	1	1	0	0	0	0

Proof $\overline{A \cdot B} = \bar{A} + \bar{B}$ HP
$\overline{A+B} = \bar{A} \cdot \bar{B}$ HP

Minimization of Boolean Expression using Boolean Algebra

Ex:- Proof that $AB + BC + \bar{B}C = AB + C$

Ans

$AB + C(B + \bar{B})$ $(\because B + \bar{B} = 1)$

$AB + C(1)$

$AB + C$ Hence Proved

Ex:- Simplify the expression $\bar{A}B + AB + \bar{A}\bar{B}$

Ans

$B(\bar{A} + A) + \bar{A}\cdot\bar{B}$

$B + \bar{A}\bar{B}$ $(\because A + \bar{A}B = A + B)$

$\bar{A} + B$ Hence Proved

Ex:- Simplify the given expression $A + A\bar{B} + \bar{A}B$

Ans

$A + A\bar{B} + \bar{A}B$

$A(1 + \bar{B}) + \bar{A}B$ $(\because 1 + B = 1)$

$A\cdot 1 + \bar{A}B$ $(\because A + \bar{A}B = A + B)$

$A + \bar{A}B$

$A + B$ Hence Proved

Ex:- Simplify the following expression $Y = (\bar{A} + B)(A + B)$

Ans

$\bar{A}\cdot A + \bar{A}\cdot B + AB + BB$

$0 + \bar{A}\cdot B + AB + B$ $(\because BB = B)$ $(\because A\cdot\bar{A} = 0)$

$\bar{A}B + B(A + 1)$ $(\because 1 + A = 1)$

$B + B\bar{A}$

$B(1 + \bar{A})$

B Hence Proved

⇒ Literals :-

↳ In througout the Boolean Expression and solving K-map we have some what like literal.

↳ Literal is nothing, it is a variable and its complement

$$f(A,B) = A + \bar{A}B$$

$$A, \bar{A}, B, \bar{B}, C, \bar{C}, \ldots$$

↳ In n variable function we have 2nd literals

⇒ Product Term :-

The AND Function is reffered to as a product, it may be complemented or uncomplemented

Example :- $AB\bar{C}$

⇒ Sum Term :-

The OR Function is reffered to as a addition, it may be complement or uncomplemented.

Example :- $A + \bar{B} + C$

⇒ SOP (Sum of Product) :-

↳ SOP is a group of product term sum together

→ Sum

↳ $AB + ABC + BC$

→ Product

⇒ POS (Product of Sum) :-

↳ Product of Sum is a group of sum term multiply together.

↳ $\underbrace{\overbrace{(A+B)\cdot(B+C)\cdot(A+\bar{C})}^{\text{Sum}}}_{\text{Product}}$

⇒ Minterms :-

↳ Minterm each individual term in SSOP (Standard SOP) is called Minterms.

⇒ Maxterms :-

↳ Each individual term is SPOS (Standard POS) is called Maxterms.

⇒ SSOP (Standard Sum of Product) :-

↳ Each product terms containt all the variable of the function.

Ex :- $\bar{A}BC + AB\bar{C} + B\bar{C}\bar{A}$

⇒ SPOS (Standard Product of Sum) :-

↳ Each sum terms containt all the variable of the function.

Ex :- $(A+B+C)(B+C+A)(A+C+B)$

Minterms → m_i & Maxterms → M_i

Ques For Two variable SSOP and SPOS :- $A=0 \Rightarrow \bar{A}$
$A=1 \Rightarrow A$

Ans

A	B	Minterms SSOP	Maxterm SPOS
0	1	$\bar{A}\cdot\bar{B} = m_0$	$A+B = \cancel{m_0}\ M_0$
0	0	$\bar{A}\cdot B = m_1$	$A+\bar{B} = \cancel{m_1}\ M_1$
1	1	$A\cdot\bar{B} = m_2$	$\bar{A}+B = \cancel{m_2}\ M_2$
1	0	$A\cdot B = m_3$	$\bar{A}+\bar{B} = \cancel{m_4}\ M_3$

Minterms → m_i
Maxterms → M_i
$A=0 \Rightarrow \bar{A}$ $B=0 \Rightarrow \bar{B}$
$A=1 \Rightarrow A$ $B=1 \Rightarrow B$

Imp :- This is the SOP form Not an canonical form because here B + A is Missing Ex:- A + B

⟹ SOP → SSOP Conversion :-

1. Identify the missing variable in product term.
2. Multiply (variable + complement of variable)
3. Neglect the repeated term.

↳ SSOP is denoted by $\Sigma m(\dots)$

→

Ex :- $f(A, B, C) = AB + AB\bar{C} + BC$

Ans $AB + AB\bar{C} + BC$ → A (Missing)

C (Missing)

		A	B	C
7 =	m_7 =	1	1	1
6 =	m_6 =	1	1	0
3 =	m_3 =	0	1	1

$AB(C + \bar{C}) + AB\bar{C} + BC(A + \bar{A})$

$\underbrace{ABC} + AB\bar{C} + AB\bar{C} + \underbrace{ABC} + \bar{A}BC$

$ABC + \underbrace{AB\bar{C}} + \underbrace{AB\bar{C}} + \bar{A}BC$

$ABC + AB\bar{C} + \bar{A}BC$

$f(A,B,C) = \Sigma m\,(m_7, m_6, m_3)$

$= \Sigma m\,(3, 6, 7)$

– Ans

→ POS → SPOS Conversion :-

1. Identify the missing variable in product term
2. Add that variable and its complement
3. Neglect the repeated term.

↳ SPOS is denoted by $\Sigma M(\ldots\ldots)$

→

Ex :- $f(A,B,C) = A(B+C)$

<u>Ans</u>

$A(B+C)$ → A (Missing)

↗ B, C (Missing)

$= (A+B+\bar{B}+C+\bar{C})\cdot(B+C+\bar{A}+A)$

$= A(B+C+A+\bar{A}) + B(B+C+A+\bar{A}) + \bar{B}(B+C+A+\bar{A}) + C(B+C+A+\bar{A}) + \bar{C}(B+C+A+\bar{A})$

$= AB + AC + A + \overset{0}{A\cdot\bar{A}} + B + BC + AB + \bar{A}B + \overset{0}{\bar{B}\cdot B} + BC + A\bar{B} + \bar{A}\bar{B} + BC + C + AC + \bar{A}C + B\bar{C} + \overset{0}{\bar{C}\cdot C} + A\bar{C} + \bar{A}\bar{C}$

$= AB + AC + A + B + C + BC + \bar{A}B + \bar{B}C + A\bar{B} + \bar{A}\bar{B} + \bar{A}C + B\bar{C} + A\bar{C} + \bar{A}\bar{C}$

Ex :- $AB + A\bar{C} + BC$

<u>Ans</u> $AB(C+\bar{C}) + A\bar{C}(B+\bar{B}) + BC(A+\bar{A})$

$ABC + AB\bar{C} + A\bar{C}B + A\bar{C}\bar{B} + BCA + BC\bar{A}$

$ABC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC$

Canonical SOP form

Ex :- $f(A, B, C) = A(A+C)$

B (Missing)

Ans $A(A+C)$

B, C (Missing)

$(A+B+C)(A+\bar{B}+\bar{C})(A+B+C)(A+\bar{B}C)$

$(A+B+C)(A+\bar{B}+\bar{C})(A+\bar{B}C)$ POS

$\Sigma m(M4, M5, M7)$

$\Sigma m(4, 5, 7)$

Ans

Ex :- $(A+BC)(B+\bar{C}A)$

Ans $(A+B)(A+C)(B+\bar{C})(B+A)$

$(A+B)(A+C)(B+\bar{C})$

$\uparrow C*$ $\uparrow B*$ $\uparrow A*$

$(A+B+0)(A+C+0)(B+C+0)$

$(A+B+C.\bar{C})(A+B.\bar{B}+C)(A.\bar{A}+B+\bar{C})$

$\underbrace{(A+B+C)}\underbrace{(A+B+\bar{C})}\underbrace{(A+B+C)}(A+\bar{B}+C)$

$\underbrace{(A+B+\bar{C})}(\bar{A}+B+\bar{C})$

$(A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+\bar{C})$

-Ans

⇒ K - Map :-

↳ Karnaugh Map is also known as K-Map.

↳ This is develop by Scientist Karnaugh by his name.

↳ K-Map is developed in 1953.

↳ K-Map is used to simplifier boolean expression without using boolean theorem.

↳ K-Map follows gray code.

↳ 2 Variable K-Map :-

$2^2 = 4$

(MSB) A \ B (LSB)	0	1
0	0	1
1	2	3

Table :-

A	B	Binary
0	0	0
0	1	1
1	0	2
1	1	3

↳ 3 Variable K-Map :-

$f(A, B, C) = 2^3 = 8$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	BC 10
$\bar{A}$ 0	0	1	3	2
A 1	4	5	7	6

Table :-

A	B	C	Binary
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

↳ 4 Variable K-Map :-

$2^4 = 16$

AB \ CD	CD 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	3	2
$\bar{A}B$ 01	4	5	7	6
AB 11	12	13	15	14
$A\bar{B}$ 10	8	9	11	10

Rules for Simplifying K-Map :-

- It is Build on Gray Code
- Usually used simplify SOP
- Group may not contain "0".
- We can make a group 1, 2, 4, 8
- Group containing 1 must be group
- Each group should be as large as possible.
- Group may be overlap.
- Opposite grouping and corner grouping allowed.

Ques Representation of K-Map with truth table

$2^3 = 8$

Ans

$A = 0 = \bar{A}$
$A = 1 = A$

	A	B	C	f
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

$$f = \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

AB \ C		$\bar{C}$ 0	C 1
$\bar{A}\bar{B}$	00	0 [0]	1 [1]
$\bar{A}B$	01	0 [2]	0 [3]
AB	11	1 [6]	1 [7]
$A\bar{B}$	10	1 [4]	0 [5]

$$A\bar{C} + AB + \bar{A}\bar{B}C$$

$$f = \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

① $\bar{A}\bar{B}C$

② $A\bar{C} = A\bar{B}\bar{C} + AB\bar{C}$

$= A\bar{C}(\bar{B} \cdot B)$ $\quad \because \bar{B} \cdot B = 1$

$= A\bar{C}(1)$

$= A\bar{C}$

③ $AB = AB\bar{C} + ABC$

$= AB(\bar{C} \cdot C)$ $\quad \because \bar{C} \cdot C = 1$

$= AB(1)$

$= AB$

$$\bar{A}\bar{B}C + A\bar{C} + AB$$

Hence Proved

Ques Representation of K-Map with truth table

$$2^4 = 16$$

Ans

A	B	C	D	F
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

AB \ CD	$\bar C\bar D$ 00	$\bar C D$ 01	CD 11	$C\bar D$ 10
$\bar A\bar B$ 00	1 (0)	0 (1)	1 (3)	0 (2)
$\bar A B$ 01	0 (4)	0 (5)	1 (7)	0 (6)
AB 11	0 (12)	0 (13)	0 (15)	0 (14)
$A\bar B$ 10	0 (8)	1 (9)	0 (11)	0 (10)

$$\bar A\bar B\bar C\bar D + \bar A C D + A\bar B\bar C D$$

– Ans

⇒ K-Map Grouping & Implecant :-

1. Implecant :- Group of 1's is called implecant
i.e, 1, 2, 3, 4, 6, 16

2. Prime Implecant :- Largest possible group of 1's is called prime implecant.

3. Essential Implecant :- At least there is single 1's is in the group which can not be combined in any other way.

↳ Example :- $f(A,B,C,D) = \Sigma m\,(1, 5, 7, 12, 13)$
$= m_1 + m_5 + m_7 + m_{12} + m_{13}$

AB \ CD		$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	00	0	1 1	2	3
$\bar{A}B$	01	4	1 5	1 7	6
AB	11	1 12	1 13	15	14
$A\bar{B}$	10	8	9	11	10

No. of Implecant = 5
No. of Prime Implecant = 2
No. of Essential Implecant = 3

– Ans

$\Rightarrow$ K-Map Example with don't care :-

Ques Simplify the Boolean Expression using K-Map

Ans $f(A, B, C) = \Sigma m(1, 3, 4, 5)$ SOP form

$2^n = 2^3 = 8$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1 1	1 3	2
A 1	1 4	1 5	7	6

I   II

$f = \mathrm{I} + \mathrm{II}$

$f = A\bar{B} + \bar{A}C$

-Ans

Ques Simplify the Boolean expression using K-Map

$f = \bar{A}\bar{B}C + A\bar{B}C + AB\bar{C} + ABC$

Ans $f = \underbrace{\underbrace{\bar{A}\bar{B}C}_{001}}_{1} + \underbrace{\underbrace{A\bar{B}C}_{101}}_{5} + \underbrace{\underbrace{AB\bar{C}}_{110}}_{6} + \underbrace{\underbrace{ABC}_{111}}_{7}$

$f = (A, B, C) = \Sigma m(1, 5, 6, 7)$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1 1	3	2
A 1	4	1 5	1 7	1 6

$f = \mathrm{I} + \mathrm{II} + \mathrm{III}$

$f = AC + AB + \bar{B}C$

- Ans

Ques Minimize the logic function :-

Ans $f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 5, 7, 8, 12, 14)$

$2^n = 2^4 = 16$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1 0	1 1	1 3	1 2
$\bar{A}B$ 01	4	1 5	1 7	6
AB 11	1 12	13	15	1 14
$A\bar{B}$ 10	1 8	9	11	10

$f = \mathrm{I} + \mathrm{II} + \mathrm{III} + \mathrm{IV}$

$f = \bar{A}\bar{B} + \bar{A}D + AB\bar{D} + A\bar{C}\bar{D}$

- Ans

Simple Structure :-

1 1 1 1 → I

1 1 → II

1 → III

$\boxed{\text{Fold} \rightarrow}$

1
1
→ IV

Ques Simplify the Boolean Expression with don't care

$$f(A,B,C,D) = \Sigma_m (1, 3, 7, 10, 11, 14, 15) + d(0,2,5)$$

don't care stands in symbol ⊗

Ans $2^n = 2^4 = 16$

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	x 0	1 1	1 3	x 2
$\bar{A}B$ 01	4	x 5	1 7	6
AB 11	12	13	1 15	1 14
$A\bar{B}$ 10	8	9	1 11	1 10

$f = I + II + III$

$f = ACD + \bar{A}\bar{B}D + AC$

-Ans

Simple Structure :-