

Arithmetic Operation.

Arithmetic operation in a computer are done using binary number and not decimal number, and these take place in its arithmetic unit.

Binary Arithmetic.

The arithmetic rules for Addition, Subtraction, multiplication and Division of Binary numbers is

Addition

- 1) $0+0=0$
- 2) $0+1=1$
- 3) $1+0=1$
- 4) $1+1=10, \text{ carry } = 1$

Subtraction.

- 1) $0-0=0$
- 2) ~~$1-0=1$~~
- 3) $1-1=0$
- 4) $10-1=1$

multiplication.

- 1) $0 \times 0=0$
- 2) $0 \times 1=0$
- 3) $1 \times 0=0$
- 4) $1 \times 1=1$

Division.

- 1) $0 \div 0=0$

Binary Addition :- Two binary numbers can be added in the same way as two decimal numbers are added. The addition

Example — 1010 and 1111

$$\begin{array}{r}
 \text{MSB} \ C=1 \ C=1 \quad \text{LSB} \\
 \underline{\underline{C=1}} \quad 1 \ 1 \ 1 \ 1 \rightarrow 15 \\
 1 \ 0 \ 1 \ 0 \quad \underline{\underline{10}}
 \end{array}$$

Date: / /

Step 1 → The least significant bits are added
i.e. $0+1=1$ with a carry 0

Step-2 - The carry in the previous step is added to the next higher significant bits.
i.e. $=1+1=0$ with a carry 1

Step-3 The carry in the above step is added to the next higher significant bit
i.e. $0+1+1=0$ with a carry 1

Step-4 - The preceding carry is added to the most significant bits. i.e. $1+1+1=1$ with carry = 1

Binary Subtraction : Binary Subtraction is also carried out in the same way as decimal numbers are subtracted. The subtraction is carried out from the least significant bits and proceeds to the most significant bits.

Case-1

MSB LSB

$$\begin{array}{r} 11101 \\ - 13 \\ \hline 1001 \\ \hline 1000 \end{array}$$

Step-1 the LSB in the first column are 1 and 1
Hence, the difference is $1-1=0$

Step-2 - In the second column, the subtraction is performed as $0-0=0$

Step-3 In the Third column, the difference is given by $1-0=1$

Step 2 In the 4th column (MSB),
the difference is given by $1-1=0$

case - 2

$$\begin{array}{r} & 1 \\ & 1 \\ 1001 & \xrightarrow{\text{Decimal}} \\ 0111 & \\ \hline 0010 & \end{array}$$

$\frac{7}{2}$

Multiply the following binary number system

(a) 1011 and 1101

(b) 100110 and 1001

(c) 1.01 and 10.1

$$1011$$

$$1101$$

$$\begin{array}{r} C=1 \cdot 1011 \\ C=1 0 \cdot 060X \\ C=1 C=1 1011XX \end{array}$$

$$11011XX$$

carry \rightarrow 1 0001111 ans

$$10001111$$

Sign → Addition in 2's complement system!

Addition can be explained with four possible cases:

- (I) when both the numbers are positive.
- (II) when augend is a positive and addend is negative.
- (III) when augend is negative and addend is positive.
- (IV) when both the numbers are negative.

Case - I Two positive numbers

$$\begin{array}{r} \text{16} \ 8 \ 4 \ 2 \ 1 \\ + 29 \rightarrow 00011101 \\ + 19 \rightarrow \underline{\underline{0001001}} \\ \hline 48 \qquad 00110000 \text{ rem.} \\ \text{sign} \swarrow \qquad \downarrow \\ \qquad \qquad 32 = 16 \end{array}$$

Case-3 positive addend number and negative augend number.

Consider the addition of -47 and $+29$

$$\begin{array}{r} \text{32 } 16 \ 8 \ 4 \ 2 \\ -47 \rightarrow 00101111 \text{ one's complement} \\ 11010000 \\ +1 \\ \hline 11010001 \text{ - 2's complement} \end{array}$$

$$\begin{array}{r} +29 \\ -18 \text{ (circled)} \\ \hline \text{Symbol} \rightarrow 00011101 \\ 0010001 \\ +1 \\ \hline 0010010 \text{ (circled) } -18 \text{ Ans} \end{array}$$

Case-4 Two negative numbers.

Consider the addition of -32 and -44

$$\begin{array}{r} \text{32 } 16 \ 8 \ 4 \ 2 \\ -32 \rightarrow 00100000 \\ 11010111 \text{ 1's comp} \\ +1 \\ \hline 11100000 \text{ 2's complement} \end{array}$$

$$\begin{array}{r} -44 \rightarrow 00101100 \\ 11010111 \rightarrow 1's complement \\ +1 \\ \hline 11010100 \\ -32 \quad 11100000 \\ -76 \quad 11010100 \\ \hline 11001100 \text{ (circled) } -76 \text{ Ans} \end{array}$$

q's complement

The q's complement of a decimal number can be found by subtracting each digit in the number from q's complement of decimal digits 0 to q is shown.

Decimal digit	q's complement
0	9
1	8
2	7
3	6
4	5
5	4
6	3
7	2
8	1
9	0

Example. find the q's complement of each of the following numbers.

- (a) 19 (b) 1461 (c) 469 (d) 4397

a)
$$\begin{array}{r} 99 \\ - 19 \\ \hline 80 \end{array}$$
 q's complement of 19

b)
$$\begin{array}{r} 999 \\ - 146 \\ \hline 853 \end{array}$$
 \rightarrow q's complement of 146

c)
$$\begin{array}{r} 999 \\ - 469 \\ \hline 530 \end{array}$$
 \rightarrow q's complement of 469

9999

4397

5602

 \rightarrow q's complement of 4397

Subtraction of q's complement

Example. $18 - 06$

$$\begin{array}{r} 18 \\ - 6 \\ \hline 12 \end{array}$$

(a)

$$\begin{array}{r} 18 \\ + 98 \\ \hline 111 \\ \boxed{+1} \\ \hline 12 \rightarrow \underline{\text{Ans}} \end{array}$$

(b) . 39

$$\begin{array}{r} 99 \\ - 23 \\ \hline 76 \\ + 96 \\ \hline \boxed{+1} \end{array}$$

99

- 06 \rightarrow

93

99

- 23

76

$$\begin{array}{r} 34 \\ - 49 \\ \hline - 15 \end{array}$$

$$\begin{array}{r} 34 \\ + 50 - \text{q's complement} \\ \hline 84 \\ 99 \\ \hline - 15 \end{array}$$

 $- 15 \rightarrow \underline{\text{Ans}}$ - q's complement at 84

$$\begin{array}{r} 49 \\ - 84 \\ \hline - 35 \end{array}$$

$$\begin{array}{r} 49 \\ + 15 - \text{q's complement of 84} \\ \hline 64 \\ \boxed{-1} \\ \hline - 35 \end{array}$$

$$\begin{array}{r} 99 \\ - 84 \\ \hline 15 \\ \boxed{-1} \\ \hline - 35 \end{array}$$

: q's complement of 64.

(2) 10's complement - The 10's complement of Decimal number is equal to q's complement +1

Example

(a) 9, (b) 46, (c) 739

$$\begin{array}{r} 9 \\ - 9 \\ \hline 0 \end{array}$$

- q's complement of 9

$$\begin{array}{r} +1 \\ \hline 1 \end{array}$$

1 10's complement of 9

$$(b) \quad \begin{array}{r} 46 \\ - 99 \\ \hline \end{array}$$

$53 - 9^{\text{'}}\text{s complement of } 46$

$$\underline{+ 1}$$

$54 - 10^{\text{'}}\text{s complement of } 46$

$$\begin{array}{r} 739 \\ - 999 \\ \hline \end{array}$$

$\underline{88}$

$260 - 9^{\text{'}}\text{s complement of } 739$

$$\begin{array}{r} + 1 \\ \hline 1261 \end{array}$$

$10^{\text{'}}\text{s complement of } 739$

Subtract the following decimal number.

Ex - (a) $9 - 4$ (b) $20 - 09$, (c) $69 - 32$, (d) $347 - 265$

Regular Subtraction.

$10^{\text{'}}\text{s complement subtraction}$

$$\textcircled{(a)} \quad \begin{array}{r} 9 \\ - 4 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 9 \\ + 5 \\ \hline 14 \end{array} \rightarrow 10^{\text{'}}\text{s complement of } 4$$

$$\textcircled{(b)} \quad \begin{array}{r} 20 \\ - 9 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 20 \\ + 9 \\ \hline 29 \end{array} \rightarrow 10^{\text{'}}\text{s complement}$$

$$\begin{array}{r} 99 \\ - 32 \\ \hline 67 \end{array}$$

$$\textcircled{(c)} \quad \begin{array}{r} 69 \\ - 32 \\ \hline 37 \end{array}$$

$$\begin{array}{r} 69 \\ + 32 \\ \hline 101 \end{array} \rightarrow 10^{\text{'}}\text{s complement}$$

drop carry

$$\textcircled{1} 37 - \text{ans.}$$

$$\textcircled{(d)} \quad \begin{array}{r} 347 \\ - 265 \\ \hline 82 \end{array}$$

$$\begin{array}{r} 347 \\ + 65 \\ \hline 412 \end{array} \rightarrow 10^{\text{'}}\text{s complement}$$

BCD Binary Coded decimal (BCD)

- Binary coded decimal (BCD) number is a combination of four binary digits that represent decimal number and it is also called as 8421 code.
- The 8421 is a type of binary coded decimal. It has 4 bits and represent the decimal digits 0 to 9.
- To express any decimal number in BCD each decimal digit should be replaced by the appropriate four bit code.

Decimal	Binary	BCD code
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	0101
6	0110	0110
7	0111	0111
8	1000	1000
9	1001	1001
10	1010	1010
11	1011	1011
12	1100	1100
13	1101	1101
14	1110	1110
15	1111	1111

BCD Arithmetic.

BCD is a numerical code. Many application require arithmetic operations. Addition is the most important of these because the other 3 operations, which is subtraction, multiplication and division, can be done using addition.

→ The rules for addition of two BCD numbers

- Add the two numbers using the rules for binary addition.
- If four-bit sum is equal to or less than 9, it is a valid BCD number.
- If a four-bit sum is greater than 9, or if a carry-out of group 5 is generated it is an invalid result.

→ Add 6 (0110_2) to the four bit sum in order to skip the six invalid states and return to BCD.
if a carry results when 6 is added, add the carry to the next four bit group.

(a.) Add the following BCD number

(a) 1001 and 0100

(b) 00011001 and
00010100

Solution

1001

$$\begin{array}{r}
 1001 \\
 0100 \\
 \hline
 1101 \rightarrow \text{invalid BCD number. } (13)_{10}
 \end{array}$$

$$\begin{array}{r}
 + 0110 \rightarrow \text{Add 6} \\
 \hline
 \underbrace{000}_{1} \underbrace{10011}_{3} \quad (13)_{10} \underline{\text{Ans}}
 \end{array}$$

(b)

$$\begin{array}{r}
 00011001 \\
 + 00010100 \\
 \hline
 00101101 \rightarrow \text{Right group is invalid.}
 \end{array}$$

$$\begin{array}{r}
 \cancel{0010} + 0110 \text{ Add 6} \\
 \hline
 \underbrace{001}_{3} \underbrace{1001}_{3} \rightarrow \text{Valid BCD number. } (33)_{10} \underline{\text{Ans}}
 \end{array}$$

\Rightarrow BCD Subtraction \Rightarrow

$$206 - 147$$

$$\begin{array}{r}
 206 \rightarrow \overbrace{0010}^1 \rightarrow \overbrace{0000}^1 \rightarrow \overbrace{0110}^0 \\
 - 147 \rightarrow \overbrace{0001}^1 \quad \overbrace{0100}^1 \quad \overbrace{0111}^1 \\
 \hline
 \underbrace{0000}_{11} \quad \underbrace{1011}_{\text{invalid.}} \quad \underbrace{1111}_{15 \text{ invalid}}
 \end{array}$$

(0000, 0101, 1001)
BCD-

59 ms.

$$\begin{array}{r}
 0110 - 0110 \\
 \hline
 0101 \quad 1001
 \end{array}$$

5

9