

Arithmetic operation.

Arithmetic operation in a computer are done using binary number. and not decimal number, and these take place in its arithmetic unit.

Binary Arithmetic.

The arithmetic rules for Addition Subtraction, multiplication and Division of Binary numbers is

Addition

- 1) $0+0=0$
- 2) $0+1=1$
- 3) $1+0=1$
- 4) $1+1=10$, carry = 1

Subtraction.

- $0-0=0$
- ~~0~~ $1-0=1$
- $1-1=0$
- $10-1=1$

1. multiplication.

- 1 $0 \times 0 = 0$
- 2 $0 \times 1 = 0$
- 3 $1 \times 0 = 0$
- 4 $1 \times 1 = 1$

Division.

0/

Binary Addition :- Two binary number can be added in the same way a two decimal numbers are added. The addition

Example — 1010 and 1111

MSB $c=1$ $c=1$

LSB

1 1 1 1 \rightarrow 15

1 0 1 0 10

Step 1 → The least significant bits are added
i.e. $0+1=1$ with a carry 0

Step-2 - The carry in the previous step is added to the next higher significant bits.
i.e. $= 1+1=0$ with a carry 1

Step-3 The carry in the above step is added to the next higher significant bit
i.e. $0+1+1=0$ with a carry 1

Step-4 - The preceding carry is added to the most significant bits. i.e. $1+1+1=1$
with carry = 1

Binary Subtraction ÷ Binary Subtraction is also carried out in the same way as decimal number are subtracted. ~~The subtraction is carried out from the least significant bits and proceeds to the~~

Case-1

MSB

LSB

$$\begin{array}{r} 1101 \\ - 1001 \\ \hline 0000 \end{array} \quad \begin{array}{r} 13 \\ - 9 \\ \hline 4 \end{array}$$

Step- the LSB in the first column are 1 and 1
Hence, the difference is $1-1=0$

Step-2 - In the second column, the subtraction is performed as $0-0=0$

Step-3 In the third column, the difference is given by $1-0=1$

Step 1 \Rightarrow In the 4th column (MSB),
the difference is given by $1-1=0$

case - 2

$$\begin{array}{r} 1 \\ 1001 \\ 0111 \\ \hline 0010 \end{array} \quad \begin{array}{r} \text{Decimal} \\ 9 \\ 7 \\ \hline 2 \end{array}$$

multiply the following binary number system

(a) 1011 and 1101

(b) 100110 and 1001

(c) 1.01 and 10.1

$$\begin{array}{r} 1011 \\ \times 1101 \\ \hline c=1 \quad 1011 \\ c=1 \quad 0000x \\ c=1 \quad 1011xx \\ 1011xxxx \end{array}$$

carry \rightarrow 1 0 0 0 1 1 1 1 ans

Sign
→ Addition in 2's complement system:-

Addition can be explained with four possible cases. (i) when both the numbers are positive. (ii) when augend is a positive and addend is negative. (iii) when augend is negative and addend is positive. (iv) when both the numbers are negative.

Case - 1 Two positive numbers

	16	8	4	2	1	
+ 29 →	0	0	1	1	1	0
+ 19 →	0	0	0	1	0	0
<u>48</u>	0	0	1	1	0	0
	0	0	1	1	0	0

res.

sign → 32 16

Case-3 positive addend number and negative augend number.

Consider the addition of -47 and $+29$

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & 32 & 16 & 8 & 4 & 2 & 1 \\
 -47 \rightarrow & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & \text{one's complement} \\
 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & \\
 & & & & & & & & +1 \\
 \hline
 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & -2\text{'s complement}
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 +29 \\
 \hline
 -18
 \end{array}$$

$$\begin{array}{r}
 00011101 \\
 \text{sign bit} \rightarrow \textcircled{1} 1101110 \\
 \hline
 0010001 \\
 +1 \\
 \hline
 0010010
 \end{array}$$

$$\textcircled{-} \rightarrow 0010010 \quad \textcircled{-18} \text{ Ans}$$

Case-4 Two negative numbers.

Consider the addition of -32 and -44

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & 32 & 16 & 8 & 4 & 2 & 1 \\
 -32 \rightarrow & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1\text{'s comp} \\
 & & & & & & & & +1 \\
 \hline
 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 2\text{'s complement}
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 -44 \rightarrow \begin{array}{ccccccc} 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{array} \\
 \begin{array}{ccccccc} 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \end{array} \rightarrow 1\text{'s complement} \\
 \hline
 \begin{array}{ccccccc} & & & & & & & +1 \end{array}
 \end{array}$$

$$\begin{array}{r}
 -44 \rightarrow 11010100
 \end{array}$$

$$\begin{array}{r}
 -32 \rightarrow 11100000
 \end{array}$$

$$\begin{array}{r}
 -76 \rightarrow 11010100
 \end{array}$$

$$\begin{array}{r}
 11001011 \\
 +1 \\
 \hline
 11001100
 \end{array}$$

$$64 + 8 + 4$$

$$\textcircled{-76}$$

9's complement

The 9's complement of a decimal number can be found by subtracting each digit in the number from 9's complement of decimal digits 0 to 9 is shown.

Decimal digit	9's complement
---------------	----------------

0	9
---	---

1	8
---	---

2	7
---	---

3	6
---	---

4	5
---	---

5	4
---	---

6	3
---	---

7	2
---	---

8	1
---	---

9	0
---	---

Example. Find the 9's complement of each of the following number.

(a) 19 (b) 146 (c) 469 (d) 4397

(a)

99

-19

80

9's complement of 19

b)

999

146

853

→ 9's complement of 146

c)

999

469

530 → 9's complement of 469

9999

4397

5602 \rightarrow 9's complement of 4397

Subtraction of 9's complement

Example. 18 - 06

$$\begin{array}{r} 18 \\ - 6 \\ \hline 12 \end{array}$$

(a) 18

$$\begin{array}{r} 18 \\ + 99 \\ \hline 111 \\ + 1 \\ \hline 12 \rightarrow \text{Ans} \end{array}$$

(b) 39

$$\begin{array}{r} 39 \\ - 23 \\ \hline 16 \end{array}$$

$$\begin{array}{r} 39 \\ + 76 \\ \hline 115 \\ + 1 \\ \hline 116 \rightarrow \text{Ans} \end{array}$$

(c) 34

$$\begin{array}{r} 34 \\ - 49 \\ \hline -15 \end{array}$$

34

$$\begin{array}{r} 34 \\ + 50 \text{ --- 9's complement} \\ \hline 84 \\ 99 \\ \hline -15 \text{ --- 9's complement of 84} \end{array}$$

$$\begin{array}{r} 49 \\ - 84 \\ \hline -35 \end{array}$$

49

$$\begin{array}{r} 49 \\ + 15 \text{ --- 9's complement of 84} \\ \hline 64 \\ - 35 \text{ --- 9's complement of 64} \\ \hline \end{array}$$

② 10's complement - The 10's complement of decimal number is equal to 9's complement + 1

Example (a) 9, (b) 46, (c) 739

(a) 9

$$\begin{array}{r} 9 \\ - 9 \\ \hline 0 \\ + 1 \\ \hline 1 \end{array}$$

9's complement of 9

10's complement of 9

$$\begin{array}{r} (b) \quad 46 \\ 99 \\ \hline \end{array}$$

53 — 9's complement of 46

$$+ 1$$

54 — 10's complement of 46

$$\begin{array}{r} 739 \\ - 999 \\ \hline \end{array}$$

88

260 — 9's complement of 739

$$+ 1$$

261 — 10's complement of 739

Subtract the following decimal number.

Ex — (a) 9-4 (b) 20-09, (c) 69-32, (d) 347-265

Regular Subtraction.

10's Complement Subtraction

(a)

$$\begin{array}{r} 9 \\ - 4 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 9 \\ + 5 \\ \hline 14 \end{array} \text{ — 10's complement of 4}$$

drop carry

(b)

$$\begin{array}{r} 20 \\ - 9 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 20 \\ + 90 \\ \hline 110 \end{array} \text{ — 10's complement}$$

drop carry

$$\begin{array}{r} 11 \\ \hline 11 \end{array}$$

(c)

$$\begin{array}{r} 69 \\ - 32 \\ \hline 37 \end{array}$$

$$\begin{array}{r} 69 \\ + 69 \\ \hline 138 \end{array} \text{ — 10's complement}$$

$$\begin{array}{r} 137 \\ \hline \end{array} \text{ — ans.}$$

drop carry

(d)

$$\begin{array}{r} 347 \\ - 265 \\ \hline 82 \end{array}$$

$$\begin{array}{r} 347 \\ + 735 \\ \hline 1082 \end{array}$$

$$\begin{array}{r} 082 \\ \hline \end{array} \text{ — ans.}$$

$$\begin{array}{r} 99 \\ - 32 \\ \hline 67 \\ + 1 \\ \hline 68 \end{array}$$

$$\begin{array}{r} 999 \\ - 265 \\ \hline 734 \\ + 1 \\ \hline 735 \end{array}$$

B_{CD} Binary coded decimal (BCD)

→ Binary coded decimal (BCD) number is a combination of four binary digits that represent decimal number. and it is also called as 8421 code:

→ The 8421 is a type of binary coded decimal. It has 4 bits and represent the decimal digits 0 to 9.

→ To express any decimal number in BCD each decimal digit should be replaced by the appropriate four bit code.

Decimal	Binary	BCD code
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	0101
6	0110	0110
7	0111	0111
8	1000	1000
9	1001	1001
10	1010	0000, 0000
11	1011	0001 0001
12	1100	0001 0010
13	1101	0001 0011
14	1110	0001 0100
15	1111	0001 0101

BCD Arithmetic.

BCD is a numerical code. Many applications require arithmetic operations. Addition is the most important of these because the other 3 operations, which are subtraction, multiplication and division, can be done using addition.

→ The rules for addition of two BCD numbers

→ • Add the two numbers using the rules for binary addition.

→ • If four-bit sum is equal to or less than 9, it is a valid BCD number.

→ If a four-bit sum is greater than 9, or if a carry-out of group is generated it is an invalid result.

→ Add $6 (0110)_2$ to the four-bit sum in order to skip the six invalid states and return to BCD.

If a carry results when 6 is added, add the carry to the next four-bit group.

Ex. Add the following BCD numbers

(a) 1001 and 0100

(b) 00011001 and 00010100

Solution

1001

$$\begin{array}{r}
 1001 \\
 0100 \\
 \hline
 1101 \rightarrow \text{invalid BCD number} \\
 + 0110 \rightarrow \text{Add 6} \\
 \hline
 00010011 \\
 \hline
 1 \quad 3 \quad \quad (13)_{10} \text{ Ans}
 \end{array}$$

(b)

$$\begin{array}{r}
 00011001 \\
 + 00010100 \\
 \hline
 00101101 \rightarrow \text{Right group is invalid} \\
 \hline
 000100 + 0110 \text{ Add 6} \\
 \hline
 00110011 - \text{Valid BCD number} \\
 \hline
 3 \quad 3 \quad \quad (33)_{10} \text{ Ans}
 \end{array}$$

⇒ BCD Subtraction ⇒

$$206 - 147$$

$$\begin{array}{r}
 206 \rightarrow 0010 \quad 0000 \quad 0110 \\
 - 147 \rightarrow 0001 \quad 0100 \quad 0111 \\
 \hline
 0000 \quad 1011 \quad 1111 \\
 \downarrow \quad \quad \quad \text{invalid 11} \quad \quad \text{invalid 15} \\
 (0000, 0101, 1001)^0 \text{ BCD} \\
 \hline
 0110 \quad - 0110 \\
 0101 \quad 1001 \\
 \hline
 5 \quad 9 \\
 (59) \text{ Ans}
 \end{array}$$