

Complex No

$$\textcircled{1} Z_1 = 3 - 2i$$

$$Z_2 = 2 + 5i$$

$$\begin{aligned} Z_1 + Z_2 &= (3 - 2i) + (2 + 5i) \\ &= 3 - 2i + 2 + 5i \\ &= 5 + 3i \end{aligned}$$

$$\begin{aligned} \rightarrow |Z_1 + Z_2| &= \sqrt{(x)^2 + (y)^2} \\ &= \sqrt{(5)^2 + (3)^2} \\ &= \sqrt{25 + 9} \\ &= \sqrt{34} \end{aligned}$$

$$\textcircled{2} Z_1 = -3 + 5i$$

$$Z_2 = 5 - i$$

$$\begin{aligned} Z_1 - Z_2 &= (-3 + 5i) - (5 - i) \\ &= -3 + 5i - 5 + i \\ &= -8 + 6i \end{aligned}$$

$$\begin{aligned} |Z_1 - Z_2| &= \sqrt{(x)^2 + (y)^2} \\ &= \sqrt{(-8)^2 + (6)^2} \\ &= \sqrt{64 + 36} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

$$\textcircled{3} Z = 4 - 3i$$

$$\bar{Z} = 4 + 3i$$

$$\begin{aligned} \textcircled{1} Z + \bar{Z} &= (4 - 3i) + (4 + 3i) \\ &= 4 - 3i + 4 + 3i \\ &= 4 + 4 \\ &= 8 \end{aligned}$$

$$\begin{aligned} \textcircled{2} Z - \bar{Z} &= (4 - 3i) - (4 + 3i) \\ &= 4 - 3i - 4 - 3i \\ &= -3i - 3i \\ &= -6i \end{aligned}$$

$$\begin{aligned} \textcircled{3} Z \cdot \bar{Z} &= (4 - 3i)(4 + 3i) \\ &= 16 + 12i - 12i - 9i^2 \\ &= 16 - 9(-1) \\ &= 16 + 9 \\ &= 25 \end{aligned}$$

Complex No

$$\begin{aligned} \textcircled{4} \quad z_1 &= 5 - 2i \\ z_2 &= 2 + i \\ z_1 \cdot z_2 &= (5 - 2i) \cdot (2 + i) \\ &= 10 + 5i - 4i - 2i^2 \\ &= 10 + i - 2(-1) \\ &= 10 + i + 2 \\ &= 12 + i \\ |z_1 \cdot z_2| &= \sqrt{(x)^2 + (y)^2} \\ &= \sqrt{(12)^2 + (1)^2} \\ &= \sqrt{144 + 1} \\ &= \sqrt{145} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad \left| \frac{5 + 12i}{4 + 3i} \right| &= \frac{|5 + 12i|}{|4 + 3i|} \\ &= \frac{\sqrt{(5)^2 + (12)^2}}{\sqrt{(4)^2 + (3)^2}} \\ &= \frac{\sqrt{25 + 144}}{\sqrt{16 + 9}} \\ &= \frac{\sqrt{169}}{\sqrt{25}} \\ &= \frac{13}{5} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad \text{Find value} & \\ \textcircled{1} \quad i^{19} &= i^3 = -i \\ \textcircled{2} \quad i^{108} &= (i^4)^{27} = 1 \\ \textcircled{3} \quad i^{97} &= (i^4)^{24} \cdot i^1 = i \\ \textcircled{4} \quad i^{76} &= (i^4)^{19} = 1 \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad z &= \frac{3}{5} - \frac{4}{5}i \\ \operatorname{Re}(z) &= \frac{3}{5} \\ \operatorname{Im}(z) &= -\frac{4}{5} \\ \textcircled{8} \quad \frac{\cos(2\theta) + i \sin(2\theta)}{(\cos \theta + i \sin \theta)^2} &= \frac{\cos(2\theta) + i \sin(2\theta)}{(\cos \theta + i \sin \theta)^{2-(-1)}} \\ &= (\cos \theta + i \sin \theta)^3 \\ &= \cos 3\theta + i \sin 3\theta \end{aligned}$$

Complex No

$$\begin{aligned}
 \textcircled{9} \quad z &= \frac{2+3i}{3+2i} \\
 &= \frac{2+3i}{3+2i} \times \frac{3-2i}{3-2i} \\
 &= \frac{(2+3i)(3-2i)}{(3+2i)(3-2i)} \\
 &= \frac{6-4i+9i-6i^2}{9-6i+6i-4i^2} \\
 &= \frac{6+5i-6(-1)}{9-4(-1)} \\
 &= \frac{6+5i+6}{9+4} = \frac{12+5i}{13} \\
 \therefore z &= \frac{12}{13} + \frac{5}{13}i \\
 \therefore \bar{z} &= \frac{12}{13} - \frac{5}{13}i
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{10} \quad z &= 2+3i \\
 &= x+iy \\
 \therefore x &= 2, \quad y=3 \\
 \bar{z} &= \frac{x-iy}{x^2+y^2} \\
 &= \frac{2-i(3)}{(2)^2+(3)^2} \\
 &= \frac{2-3i}{4+9} \\
 &= \frac{2-3i}{13} \\
 &= \frac{2}{13} - \frac{3}{13}i
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{11} \quad z &= \frac{1}{4+i} \\
 &= \frac{1}{4+i} \times \frac{4-i}{4-i} \\
 &= \frac{4-i}{(4+i)(4-i)} \\
 &= \frac{4-i}{16+4i-4i-i^2} \\
 &= \frac{4-i}{16-(-1)} \\
 &= \frac{4-i}{17} \\
 &= \frac{4}{17} - \frac{1}{17}i
 \end{aligned}$$