Q - Evaluate

① $\log_3 243$

② $\log_{5\sqrt{2}} 2500$

③ $(1024)^{\log_2 m}$

④ $(729)^{\log_3 m}$

⑤ $\log_\alpha\left(\frac{1}{\alpha}\right)$

⑥ $\log 81 \div \log 27$

② $\log_{5\sqrt{2}} (2500)$

$$= \frac{\log 2500}{\log 5\sqrt{2}}$$

$$= \frac{\log (2500)}{\log (\sqrt{50})}$$

$$= \frac{\log (50)^2}{\log (50)^{1/2}}$$

$$= \frac{2 \times \cancel{\log 50}}{\frac{1}{2} \times \cancel{\log 50}}$$

$$= \frac{2}{1/2} = 2 \times 2$$

$$= 4.$$

$5\sqrt{2}$

$= \sqrt{25} \times \sqrt{2}$

$= \sqrt{25 \times 2}$

$= \sqrt{50}$

④ $(729)^{\log_3 m}$

$= (3^6)^{\log_3 m}$

$= (3)^{6 \times \log_3 m}$

$= (3)^{\log_3 (m)^6}$

$= (m)^6$

⑤ $\log_\alpha\left(\frac{1}{\alpha}\right)$

$= \log_\alpha (\alpha)^{-1}$

$= (-1) \times \log_\alpha \alpha$

$= (-1) \times 1$

$= -1$

⑥ $\log 81 \div \log 27$

$$= \frac{\log 81}{\log 27}$$

$$= \frac{\log (3)^4}{\log (3)^3}$$

$$= \frac{4 \times \log 3}{3 \times \log 3}$$

$$= 4/3$$

① $\log_3 (243)$

$= \log_3 (3)^5$

$= 5 \times \log_3 3$

$= 5 \times 1$

$= 5$

③ $(1024)^{\log_2 m}$

$= (2^{10})^{\log_2 m}$

$= (2)^{10 \times \log_2 m}$

$= (2)^{\log_2 (m)^{10}}$

$= (m)^{10}$

Q:→ Prove that

① $L.H.S. = \log\left(\frac{9}{14}\right) - \log\left(\frac{15}{16}\right) + \log\left(\frac{35}{24}\right)$

$= \log\left(\frac{9}{14}\right) + \log\left(\frac{16}{15}\right) + \log\left(\frac{35}{24}\right)$

$= \log\left[\frac{\cancel{9}^{\,3}}{\cancel{14}_{\,2}} \times \frac{\cancel{16}^{\,8}}{\cancel{15}_{\,5}} \times \frac{\cancel{35}^{\,7}}{\cancel{24}_{\,3}}\right]$

$= \log[1]$

$= 0$

$= RHS$

② $L.H.S = \frac{1}{\log_{24} 12} + \frac{1}{\log_{8} 12} + \frac{1}{\log_{9} 12}$

$= \frac{1}{\frac{\log 12}{\log 24}} + \frac{1}{\frac{\log 12}{\log 8}} + \frac{1}{\frac{\log 12}{\log 9}}$

$= \frac{\log 24}{\log 12} + \frac{\log 8}{\log 12} + \frac{\log 9}{\log 12}$

$= \frac{\log 24 + \log 8 + \log 9}{\log 12}$

$= \frac{\log(24 \times 8 \times 9)}{\log 12}$

$= \frac{\log(1728)}{\log 12} = \frac{\log(12)^3}{\log 12} = \frac{3 \times \cancel{\log 12}}{\cancel{\log 12}}$

$= 3$

$= RHS$

Q :-> Prove that

(3) $$L.H.S = \frac{1}{\log_2 64} + \frac{1}{\log_{\sqrt{2}} 64} + \frac{1}{\log_{2\sqrt{2}} 64}$$

$$= \frac{1}{\frac{\log 64}{\log 2}} + \frac{1}{\frac{\log 64}{\log \sqrt{2}}} + \frac{1}{\frac{\log 64}{\log 2\sqrt{2}}}$$

$$= \frac{\log 2}{\log 64} + \frac{\log \sqrt{2}}{\log 64} + \frac{\log 2\sqrt{2}}{\log 64}$$

$$= \frac{\log 2 + \log \sqrt{2} + \log 2\sqrt{2}}{\log 64}$$

$$= \frac{\log (2 \times \sqrt{2} \times 2\sqrt{2})}{\log 64}$$

$$= \frac{\log 8}{\log 64} = \frac{\log 8}{\log (8)^2} = \frac{\cancel{\log 8}}{2 \times \cancel{\log 8}}$$

$$= \frac{1}{2} = RHS$$

(4) $$L.H.S = \log\left(\sqrt{x^2+1} + x\right) + \log\left(\sqrt{x^2+1} - x\right)$$

$$= \log\left[\left(\sqrt{x^2+1} + x\right)\left(\sqrt{x^2+1} - x\right)\right]$$

$$\left[\begin{array}{l} (a+b)\ (a-b) = a^2 - b^2 \\ a = \sqrt{x^2+1}\ ;\ b = x \end{array}\right]$$

$$= \log\left[\left(\sqrt{x^2+1}\right)^2 - (x)^2\right]$$

$$= \log\left[\cancel{x^2} + 1 - \cancel{x^2}\right]$$

$$= \log[1]$$

$$= 0$$

$$= RHS$$

Q :→ Prove that

(5) $L.H.S = \log\left(\frac{32}{25}\right) + \log\left(\frac{64}{225}\right) + \log\left(\frac{25}{128}\right) + \log\left(\frac{450}{32}\right)$

$= \log\left[\frac{\cancel{32}}{\cancel{25}} \times \frac{\cancel{64}}{\cancel{225}} \times \frac{\cancel{25}}{\cancel{128}_{2}} \times \frac{\cancel{450}^{\cancel{225}}}{\cancel{32}}\right]$

$= \log[1]$

$= 0$

$=$ RHS.

Q :- Solve the following

(1) $\log_x 243 = 5$

$\Rightarrow 243 = (x)^5$

$\Rightarrow (3)^{\cancel{5}} = (x)^{\cancel{5}}$

$\Rightarrow 3 = x$

$\Rightarrow x = 3$

(2) $\log_{32} x = \frac{2}{5}$

$\Rightarrow x = (32)^{2/5}$

$\Rightarrow x = (2^5)^{2/5}$

$\Rightarrow x = (2)^{5 \times \frac{2}{5}}$

$\Rightarrow x = (2)^2$

$\Rightarrow x = 4$

(3) $\frac{\log x \times \log 16}{\log 32} = \log 256$

$\therefore \log x = \frac{\log 256 \times \log 32}{\log 16}$

$= \frac{\log (2)^8 \times \log (2)^5}{\log (2)^4}$

$= \frac{\overset{2}{\cancel{8}} \times \cancel{\log 2} \times 5 \times \log 2}{\cancel{4} \times \cancel{\log 2}}$

$= 2 \times 5 \times \log 2$

$\therefore \log x = 10 \times \log 2$

$\therefore \log x = \log (2)^{10}$

By Taking Antilog

$\therefore x = (2)^{10}$

$\therefore x = 1024$

Q :→ Solve the following

(4) $\log_x 4 + \log_x 8 + \log_x 16 = 9$

$\therefore \log_x (4 \times 8 \times 16) = 9$

$\therefore \log_x (512) = 9$

$\therefore 512 = (x)^9$

$\therefore (2)^{\cancel{9}} = (x)^{\cancel{9}}$

$\therefore 2 = x$

$\therefore x = 2$

(5) $\log_2 (\log_3 (2x+1)) = 1$

$$\boxed{\begin{array}{l} P = \log_3 (2x+1) \\ \log_2 (P) = 1 \;\therefore P = (2)^1 \end{array}}$$

$\therefore \log_3 (2x+1) = (2)^1$

$\therefore \log_3 (2x+1) = 2$

$\therefore (2x+1) = (3)^2$

$\therefore 2x + 1 = 9$

$\therefore 2x = 9 - 1$

$\therefore 2x = 8$

$\therefore x = 8/2$

$\therefore x = 4$

(6) $\dfrac{1}{\log_8 x} + \dfrac{1}{\log_9 x} + \dfrac{1}{\log_{16} x} = 3$

$\therefore \dfrac{1}{\frac{\log x}{\log 8}} + \dfrac{1}{\frac{\log x}{\log 9}} + \dfrac{1}{\frac{\log x}{\log 16}} = 3$

$\therefore \dfrac{\log 8}{\log x} + \dfrac{\log 9}{\log x} + \dfrac{\log 16}{\log x} = 3$

$\therefore \dfrac{\log 8 + \log 9 + \log 16}{\log x} = 3$

$\therefore \dfrac{\log (8 \times 9 \times 16)}{\log x} = 3$

$\therefore \log (1152) = 3 \times \log x$

$\therefore \log (1152) = \log (x)^3$

By Taking Antilog

$1152 = x^3$

$\therefore x^3 = 1152$

$\therefore x^3 = (2)^3 \times (3)^2 \times (2)^4$

$= (2)^3 \times 9 \times (2)^3 \times 2$

$\therefore x^3 = (2)^3 \times (2)^3 \times 18$

$\therefore x = 2 \times 2 \times \sqrt[3]{18}$

$\therefore x = 4\sqrt[3]{18}$