1.a

$$K(x, z) = x^T z + (x^T z)^2 + 4$$

$$= (x^T z + 2)^2 - x^T z$$

$$\left[\begin{array}{l} \{(x^T z)^2 + 2.x^T z.2 + 2^2\} - x^T z \\ ((x^T z)^2 + 4x^T z + 2^2) - x^T z \end{array}\right.$$

we hav,

$$(x^T z)^2 \rightarrow (x_1 z_1 + x_2 z_2 + x_3 z_3)^2$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j z_i z_j$$

$$+ 4x^T z \rightarrow 2.2(x_1 z_1 + x_2 z_2 + x_3 z_3)$$

$$= \sum_{i=1}^{n} 2x_i \, 2z_i$$

$$+ 2^2$$

$$-x^T z \rightarrow (-x_1 z_1 - x_2 z_2 - x_3 z_3)$$

$$= \sum_{i=1}^{n} -x_i \cdot z_i$$

So,
$$\phi(x) = \begin{bmatrix} x_1 x_1 \\ x_1 x_2 \\ x_1 x_3 \\ x_2 x_1 \\ x_2 x_2 \\ x_2 x_3 \\ x_3 x_1 \\ x_3 x_2 \\ x_3 x_2 \\ -x_1 \\ -x_2 \\ -x_3 \end{bmatrix}$$ //

Scanned with CamScanner

Teacher's formula:

$$\min_{\omega, b} \frac{1}{2}||\omega||^2 + \frac{c}{n} \sum_{i=1}^{n} \max\{0, 1 - y_i(\omega^T x_i + b)\}$$

---

1.b) → Here,

$K_1(x, z) \Rightarrow \phi_1(x)$ and

$K_2(x, z) = \phi_2(x)$

$K(x, z) = ?$

we know, $K(x,z) = K_1(x,z) + K_2(x,z)$

$$= \phi_1(x)^T \phi_1(z) + \phi_2(x)^T \phi_2(z)$$

$$= \begin{bmatrix} \phi_1(x) & \phi_2(x) \end{bmatrix} \begin{bmatrix} \phi_1(z) \\ \phi_2(z) \end{bmatrix}$$

$$= \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \end{bmatrix}^T \begin{bmatrix} \phi_1(z) \\ \phi_2(z) \end{bmatrix}$$

so, $\phi(x) = \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \end{bmatrix}$

so, the kernel $K(x, z)$ maps to $\phi(x) = \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \end{bmatrix}$.