Assignment - 2

1) Given, $k = 2, d = 2$

$Pr\{Y=1\} = Pr\{Y=2\} = 0.5$

$X|Y=1 \sim N\{\mu_1, \Sigma\}$

$X|Y=2 \sim N\{\mu_2, \Sigma\}$

$\mu_1 (\text{mean}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $\mu_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$Pr\{X = x | Y = i\} = (2\pi)^{-d/2} \det(\Sigma)^{-1/2} . e^{-1/2 (x-\mu_i)^T \Sigma^{-1} (x-\mu_i)} \quad \text{—(a)}$$

Now,

$$P(X|Y=1) \cdot P(Y=1) \underset{y=2}{\overset{y=1}{\gtrless}} P(X|Y=2) \cdot P(Y=2)$$

or, From the relation (a)

$$(2\pi)^{-d/2} . \det(\Sigma)^{-1/2} . e^{-1/2 (x-\mu_1)^T \Sigma^{-1} (x-\mu_1)} \underset{y=2}{\overset{y=1}{\gtrless}}$$

$$(2\pi)^{-d/2} . \det(\Sigma)^{-1/2} . e^{-1/2 (x-\mu_2)^T \Sigma^{-1} (x-\mu_2)}$$

Taking $\log_e$ on both sides, we get

$$\log_e[(2\pi)^{-d/2} . \det(\Sigma)^{-1/2} . e^{-1/2 (x-\mu_1)^T \Sigma^{-1} (x-\mu_1)}] \underset{y=2}{\overset{y=1}{\gtrless}}$$

$$\log_e[(2\pi)^{-d/2} . \det(\Sigma)^{-1/2} . e^{-1/2 (x-\mu_2)^T \Sigma^{-1} (x-\mu_2)}]$$

or, $$\cancel{-\frac{d}{2} \ln(2\pi) + \ln(\det(\Sigma)^{-1/2})} - \frac{1}{2} (x-\mu_1)^T \Sigma^{-1} (x-\mu_1)$$

$$\underset{y=2}{\overset{y=1}{\gtrless}} \cancel{-\frac{d}{2} \ln(2\pi) + \ln(\det(\Sigma)^{-1/2})} - \frac{1}{2} (x-\mu_2)^T \Sigma^{-1} (x-\mu_2)$$

or, $$-\frac{1}{2} (x-\mu_1)^T \Sigma^{-1} (x-\mu_1) \underset{y=2}{\overset{y=1}{\gtrless}} -\frac{1}{2} (x-\mu_2)^T \Sigma^{-1} (x-\mu_2)$$

or, $$(x-\mu_1)^T \Sigma^{-1} (x-\mu_1) \underset{y=1}{\overset{y=2}{\gtrless}} (x-\mu_2)^T \Sigma^{-1} (x-\mu_2) \quad \text{—(b)}$$

We have, $\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ & we need the inverse of Σ

So, determinant of (Σ) :

$$\det(\Sigma) = 2\times2-1 = 4-3 = 3$$

and, $\Sigma^{-1} = \frac{1}{\det(\Sigma)} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$$= \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \quad \text{—— (c)}$$

Since, we have $d = 2$

let's assume

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ & we have $\mu_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $\mu_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

Then, we have the previous rel^n

Eq^n(b) :

$$\Sigma^{-1}(x-\mu_1)^T(x-\mu_1) \underset{y=1}{\overset{y=2}{\gtrless}} \Sigma^{-1}(x-\mu_2)^T(x-\mu_2)$$

Substituting the value of Σ^{-1} from (c),

$$\frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1-1 \\ x_2-1 \end{bmatrix}^T\begin{bmatrix} x_1-1 \\ x_2-1 \end{bmatrix} \underset{y=1}{\overset{y=2}{\gtrless}} \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1+1 \\ x_2+1 \end{bmatrix}^T\begin{bmatrix} x_1+1 \\ x_2+1 \end{bmatrix}$$

or, $$\frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1-1 & x_2-1 \end{bmatrix}\begin{bmatrix} x_1-1 \\ x_2-1 \end{bmatrix} \underset{y=1}{\overset{y=2}{\gtrless}} \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1+1 & x_2+1 \end{bmatrix}\begin{bmatrix} x_1+ \\ x_2+ \end{bmatrix}$$

or, $$\frac{1}{3}\underbrace{\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}}_{2\times2}\underbrace{\begin{bmatrix} x_1-1 \\ x_2-1 \end{bmatrix}}_{2\times1}\underbrace{\begin{bmatrix} x_1-1 & x_2-1 \end{bmatrix}}_{1\times2} \underset{y=1}{\overset{y=2}{\gtrless}} \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1+1 \\ x_2+1 \end{bmatrix}\begin{bmatrix} x_1+1 & x \end{bmatrix}$$

or $$\frac{1}{3}\begin{bmatrix} 2(x_1-1)+(-1)(x_2-1) \\ (-1)(x_1-1)+2(x_2-1) \end{bmatrix}\begin{bmatrix} x_1-1 & x_2-1 \end{bmatrix} \underset{y=1}{\overset{y=2}{\gtrless}} \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1+1 \\ x_2+1 \end{bmatrix}\begin{bmatrix} x_1+1 & x_2+1 \end{bmatrix}$$

$$\frac{1}{3}\begin{bmatrix} 2x_1-2+(-x_2+1) \\ -x_1+1+2x_2-2 \end{bmatrix}\begin{bmatrix} x_1-1 & x_2-1 \end{bmatrix} \underset{y=1}{\overset{y=2}{\gtrless}} \frac{1}{3}\begin{bmatrix} 2(x_1+1)-(x_2+1) \\ -(x_1+1)+2(x_2+1) \end{bmatrix}\begin{bmatrix} x_1+1 & x_2+ \end{bmatrix}$$

$$\frac{1}{3}\begin{bmatrix} 2x_1-2-x_2+1 \\ -x_1+1+2x_2-2 \end{bmatrix}\begin{bmatrix} x_1-1 & x_2-1 \end{bmatrix} \underset{y=1}{\overset{y=2}{\gtrless}} \frac{1}{3}\begin{bmatrix} 2x_1+2-x_2-1 \\ -x_1-1+2x_2+2 \end{bmatrix}\begin{bmatrix} x_1+1 & x_2+1 \end{bmatrix}$$

$$\frac{1}{3}\underbrace{\begin{bmatrix} 2x_1-x_2-1 \\ 2x_2-x_1-1 \end{bmatrix}}_{2\times1}\underbrace{\begin{bmatrix} x_1-1 & x_2-1 \end{bmatrix}}_{1\times2} \underset{y=1}{\overset{y=2}{\gtrless}} \frac{1}{3}\underbrace{\begin{bmatrix} 2x_1-x_2+1 \\ 2x_2-x_1+1 \end{bmatrix}}_{2\times1}\underbrace{\begin{bmatrix} x_1+1 & x \end{bmatrix}}_{1\times2}$$

$$\frac{1}{3}\begin{bmatrix} x_1-1 & x_2-1 \end{bmatrix}\begin{bmatrix} 2x_1-x_2-1 \\ 2x_2-x_1-1 \end{bmatrix} \underset{y=1}{\overset{y=2}{\gtrless}} \frac{1}{3}\begin{bmatrix} x_1+1 & x_2+1 \end{bmatrix}\begin{bmatrix} 2x_1-x_2 \\ 2x_2-x_1 \end{bmatrix}$$

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or, $\frac{1}{3}\left[(x_1-1)(2x_1-x_2-1)+(x_2-1)(2x_2-x_1-1)\right] \underset{y=1}{\overset{y=2}{\gtrless}} \frac{1}{3}\left[(x_1+1)(2x_1-x_2+1)+(x_2+1)(2x_2-x_1+1)\right]$

or, $\frac{1}{3}\left[2x_1^2-2x_1x_2-x_1-2x_1+x_2+1+2x_2^2-x_1x_2-x_2-2x_2+x_1+1\right]$
$\underset{y=1}{\overset{y=2}{\gtrless}} \frac{1}{3}\left[2x_1^2+2x_2^2-2x_1x_2+2x_1+2x_2+2\right]$

or $\cancel{6x_1^2-6}$ $\frac{1}{3}\left[2x_1^2+2x_2^2-2x_1x_2-2x_1-2x_2+2\right. \underset{y=1}{\overset{y=2}{\gtrless}} \frac{1}{3}\left[2x_1^2+2x_2^2-2x_1x_2+2x_1+2x_2+2\right.$

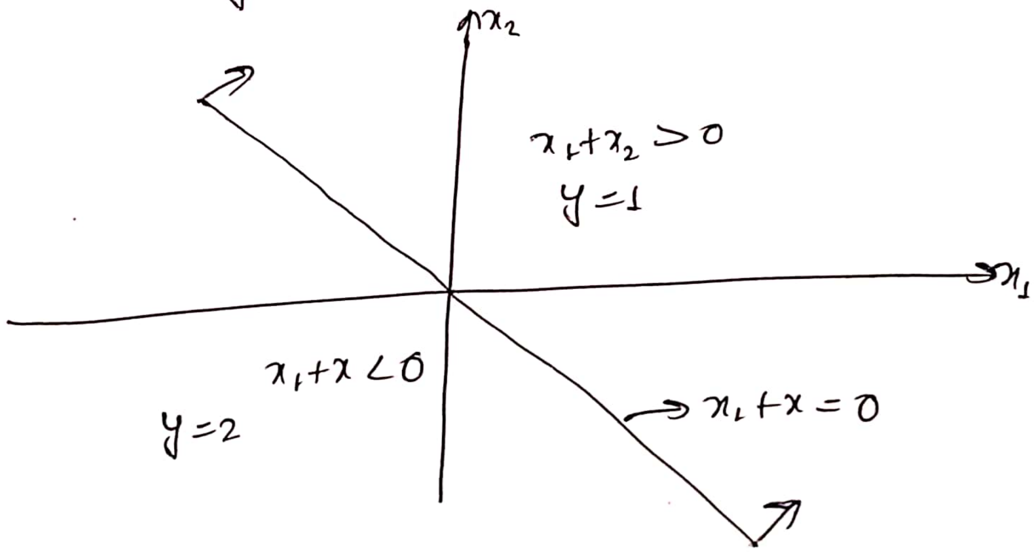
or $\cancel{6x_1^2}+\cancel{6x_2^2}-\cancel{6x_1x_2}-6x_1-6x_2+\cancel{6} \underset{y=1}{\overset{y=2}{\gtrless}} \cancel{6x_1^2}+\cancel{6x_2^2}-\cancel{6x_1x_2}+6x_1+6x_2+\cancel{6}$.

or $-6x_1-6x_2 \underset{y=1}{\overset{y=2}{\gtrless}} 6x_1+6x_2$

or $0 \underset{y=1}{\overset{y=2}{\gtrless}} 12x_1+12x_2$

or $x_1+x_2 \underset{y=1}{\overset{y=2}{\lessgtr}} 0$

The boundary condition is : $x_1+x_2=0$



Fig(1) Bayes classifier with boundary condition.