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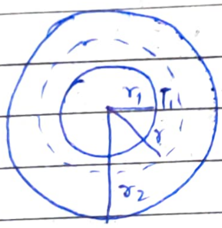
Assignment - 3

Q-1 Derive critical insulation thickness for sphere using conduction for spherical co-ordinates. State the necessary assumptions along with derivation.

Soln **Assumptions:**

- Steady state
- Neglegible heat transfer by radiation
- $e_{gen} = 0$ (No heat generation)
- Uniform thermal conductivity



Cond. eqn for spherical co-ordinates

$$\cancel{\nabla^2 T}_{\,s.s}^{\,0} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(K r^2 \frac{\partial T}{\partial r}\right) + \cancel{\left(\frac{1}{r^2 \sin\theta}\right)\frac{\partial}{\partial \theta}\left(K \sin\theta \frac{\partial T}{\partial \theta}\right)}^{\,0} + \cancel{\left(\frac{K}{r^2 \sin^2\theta}\right)\left(\frac{\partial^2 T}{\partial \phi^2}\right)}_{\,0} + \cancel{e_{gen}}^{\,0}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(K r^2 \frac{\partial T}{\partial r}\right) = 0$$

where, r is radial dist from centre of sphere & T is temp.

$$T = A + B/r$$

Boundary condn.

$r = r_1 \Rightarrow T = T_1$ $\qquad$ $A + B/r_1 = T_1$

$r = r_2 \Rightarrow T = T_2$ $\qquad$ $A + B/r_2 = T_2$

$$B\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = T_1 - T_2, \quad A = T_1 - \frac{B}{r_1}$$

$$T = \frac{T_2 r_2 - T_1 r_1}{r_2 - r_1} + \frac{(T_1 - T_2)\, r_1 r_2}{(r_2 - r_1)\, r}$$

$$Q = -KA\frac{dT}{dr}, \quad Q_{cond.} = -K(4\pi r^2)\frac{dT}{dr}$$

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$$\frac{dT}{dr} = \frac{-(T_1-T_2)r_1 r_2}{(r_2-r_1)r^2}$$

$$Q_{cond.} = \frac{4\pi k r^2 (T_1-T_2) r_1 r_2}{(r_2-r_1) r^2} = M_1 (T_1-T_2)$$

$$Q_{convection} = (T_1-T_2) h_o A = h_o 4\pi r_2^2 (T_1-T_2)$$
$$= M_2 (T_1-T_2)$$

$$Q_{net} = \frac{T_1-T_2}{\left(\frac{1}{M_1}+\frac{1}{M_2}\right)} = \frac{T_1 \; T_2}{\underbrace{\frac{R_2-R_1}{4\pi k R_1 R_2} + \frac{1}{4\pi h_o R_2^2}}_{\text{min}}}$$

$$\therefore \frac{\partial}{\partial r_2}\left(\frac{r_2-r_1}{4\pi k r_1 r_2} + \frac{1}{4\pi h_o r_2^2}\right) = 0$$

$$\frac{\partial}{\partial r_2}\left(\frac{1}{4\pi k r_1} - \frac{1}{4\pi k r_2} + \frac{1}{4\pi h_o r_2^2}\right) = 0$$

$$0 + \frac{1}{4\pi k r_2^2} - \frac{2}{4\pi h_o r_2^3} = 0$$

$$\frac{1}{k} = \frac{2}{r_2 h_o} \Rightarrow r_2 = \frac{2k}{h_o}$$

$$\boxed{t = \frac{2k}{h_o} - r_1}$$