LINEAR SIMULTANEOUS EQUATIONS

Input Test Case

$$4x_1 + 2x_2 = 10$$

$$2x_1 + 4x_2 + x_3 = 11.5$$

$$x_2 + 5x_3 = 4.5$$

1) Gauss Elimination (Without Pivoting)

INPUT :

3

4 2 0 10

2 4 1 11.5

0 1 5 4.5

OUTPUT :

X

1.500000

2.000000

0.500000

Gauss Elimination Method is good for finding roots of simultaneous equations but it diverges if matrix is not pivoted and completely different answers with high error can be obtained.

2) Gauss Elimination (With Partial Pivoting)

INPUT :

3

4 2 0 10

2 4 1 11.5

0 1 5 4.5

OUTPUT :

X

1.500000

2.000000

0.500000

The partial pivoting technique is used to avoid round-off errors that could be caused when dividing every entry of a row by a pivot value that is relatively small in comparison to its remaining row entries.

3) LU Decomposition (Doolittle Method – Without Pivoting)

INPUT :

3

4 2 0 10

2 4 1 11.5

0 1 5 4.5

OUTPUT :

X

1.500000

2.000000

0.500000

L

1.000000 0.000000 0.000000

0.500000 1.000000 0.000000

0.000000 0.333333 1.000000

U

4.000000 2.000000 0.000000

0.000000 3.000000 1.000000

0.000000 0.000000 4.666667

n^2 equations and n^2 unknowns

Equations are solved simultaneously in two steps:

1. UX = Y
2. LY = B

To solve AX = B where A=LU

4) LU Decomposition (Crout Method – Without Pivoting)

OUTPUT :

X

1.500000

2.000000

0.500000

L

4.000000 0.000000 0.000000

2.000000 3.000000 0.000000

0.000000 1.000000 4.666667

U

1.000000 0.500000 0.000000

0.000000 1.000000 0.333333

0.000000 0.000000 1.000000

INPUT :

3

4 2 0 10

2 4 1 11.5

0 1 5 4.5

Complexity of the Crout Method is similar to Doolittle Method, involves the same procedure. It differ slightly in formulation of L and U .

5) LU Decomposition (Cholesky Method – Without Pivoting)

OUTPUT :

X

1.500000

2.000000

0.500000

L

2.000000 0.000000 0.000000

1.000000 1.732051 0.000000

0.000000 0.577350 2.160247

INPUT :

3

4 2 0 10

2 4 1 11.5

0 1 5 4.5

n (n+1)/2 equations and n(n+1)/2 unknowns

Equations are solved simultaneously in two steps:

1. UX = Y
2. LY = B

To solve AX = B where A=LU

Effective for Symmetric Matrices

EIGENVALUES

Input Test Case

$$A = \begin{bmatrix} 8 & -1 & -1 \\ -1 & 4 & -2 \\ -1 & -2 & 10 \end{bmatrix}$$

Maximum iterations: 50

Maximum relative approximate error: 0.001%

Find Eigenvalue closest to: 8

1) Power Method

OUTPUT :

Eigenvalue

10.7788

Eigenvector

-0.267587

-0.255597

1.000000

Iterations

32

INPUT :

3

8 -1 -1

-1 4 -2

-1 -2 10

100

0.001

Power Method works best when there is dominant eigenvalue.

No. of Eigenvalues = Dimension of Symmetric Matrix

But, with power method we are limited to finding only the maximum eigenvalue.

2) Inverse Power Method

INPUT :

3

8 -1 -1

-1 4 -2

-1 -2 10

100

0.001

OUTPUT :

Eigenvalue

3.0749

Eigenvector

0.269593

1.000000

0.327737

Iterations

12

Inverse Power Method is for finding the smallest eigenvalues, works best when there is dominant eigenvalue.

Similar complexity as that of Power Method

3) Inverse Power Method with Shift

INPUT :

3

8 -1 -1

-1 4 -2

-1 -2 10

100

0.001

OUTPUT :

Eigenvalue

8.1461

Eigenvector

1.000000

-0.329774

0.183644

Iterations

6

Inverse Power Method with Shift is for finding the eigenvalues that is close to given value, but still we can find one at a time

Similar complexity as that of Power Method

4) QR Method

INPUT :

3

8 -1 -1

-1 4 -2

-1 -2 10

100

0.001

OUTPUT :

Eigenvalues

10.7789

8.1462

3.0749

Iterations

25