Programming Assignment 4: Integration and ODE: Initial Value Problems

1. Write a computer program for Romberg integration starting from the trapezoidal rule,and Gauss-Legendre Quadrature to evaluate one-dimensional integrals.The program should have the following features:

Input: The program should read - (i) the function to be integrated, $f(x)$ (ii) lower limit, a, and upper limit, b, of integration domain; and (iii) the maximum allowable approximate relative error (%).

Options: The user should have the option of selecting one or more of the following methods–

a. Romberg Integration: Start with $h=b-a$, and keep halving the step size till the approximate error is within allowable limit. Apply the Romberg algorithm up to the maximum possible accuracy before halving the step size.
b. Gauss-Legendre quadrature: Start with 1 Gauss point and keep increasing till the approximate error is within allowable limit

Output: The output from the program should be in the form of

(a) the value of integral, I ;
(b) number of intervals or the number of Gauss points, needed to achieve the desired accuracy;
(c) approximate relative error in the estimated value of the integral; and
(d) a figure showing the location of points where the function was evaluated and the corresponding function value.

NOTE: To compute the approximate error in the Romberg method, there could be several ways in which the error at any step is computed, since we have several estimates with different order of accuracy and different step-size. It is possible that a Romberg estimate of $O(h^4)$ is less accurate than a trapezoidal estimate with a smaller h. One option for computing the error is thatwhenever we refine the interval, we compute the Romberg estimate up to the highest possible order and compare it with the best Romberg estimate of the next lower order. The other possibility is to compare the current estimate (whether it is trapezoidal or Romberg of a lower order) with the highest order Romberg estimate with the smallest h available till that time. This is what was followed in the sample problem (it allows you to stop before reaching the highest possible Romberg estimate. However, since most of the computational effort is required in obtaining the trapezoidal estimate, it does not really mean much saving). You are free to choose any method, the answer **may** vary accordingly.

2. Write a computer program for solving Initial Value Problems. The program should have the following features:

Input: (i) Ordinary differential equation to be solved $\frac{dy}{dt} = f(t, y)$; (ii) initial values t_0 and y_0; (iii) final value t_f and (iv) interval size h.

<u>Options:</u> The user should have the option of selecting one or more of the following –

a. Forward Euler method
b. 2^{nd} order RK method (Midpoint method)
c. 4^{th} order RK method

<u>Output:</u> The output from the program should be:

(a) A text file containing the values of t_i and corresponding y_i;
(b) A figure showing y vs t.

Test Data, Problem 1:

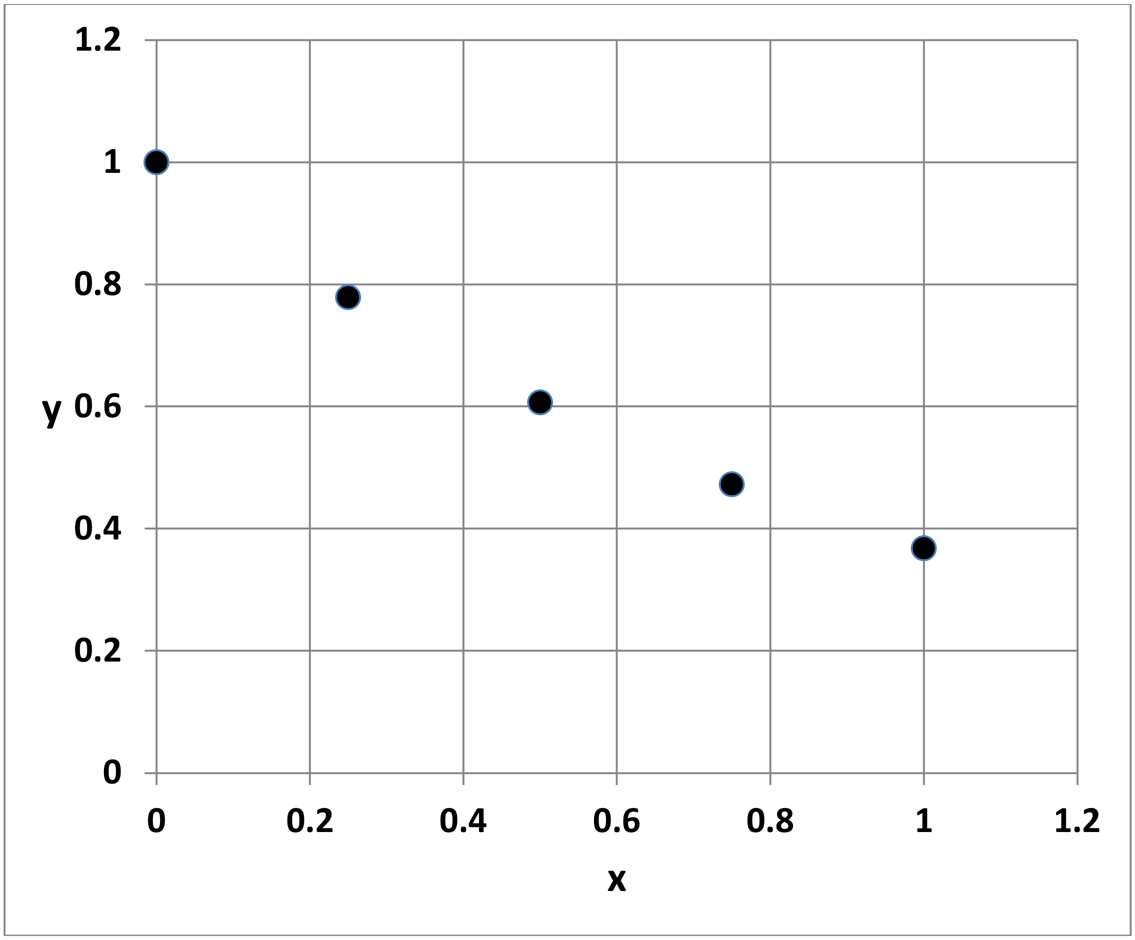
Sample input file

$f(x)=\exp(-x)$! Function
0, 1 ! a, b
0.01 ! Allowable error (%)
1 ! Romberg

Sample output files

I=0.63212088
Number of intervals = 4
Approximate relative error (%) = -0.0021

Sample Figure



Test Data, Problem 2:

Sample input file

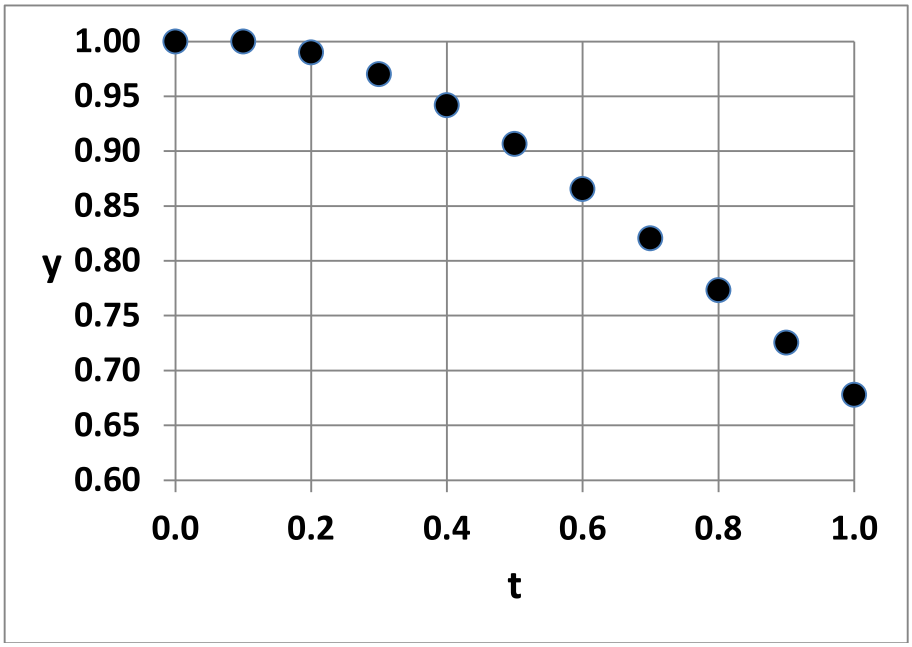
$f(t,y) = -y^2 t$! Functiondy/dt
0, 1 ! t_0, y_0
1 ! t_f
0.1 ! h
1 ! Euler Forward

Sample output files

t, y
0.0 1.00000000
0.1 1.00000000
0.2 0.99000000
0.3 0.97039800
0.4 0.94214783
0.5 0.90664213
0.6 0.86554213

0.7 0.82059234
0.8 0.77345632
0.9 0.72559754
1.0 0.67821328

Sample Figure



<u>Due date</u>: Friday, November 10, 2022, 11:59 pm. No assignments will be accepted over email.

Submit a **single zip folder** in the Brihaspati server under Assignment 4. The name of the zip-folder should be your roll-number (e.g., If your roll no.is 123456, the folder name should be '123456.zip'). The folder shouldinclude -

(i) All the computer program file(s)
(ii) A PDF file of the solution and the required figures for the test cases given in the assignment.