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NAME : PRERNA SUNIL JADHAV
SAP ID: 60004220127

COURSE NAME: ENGINEERING MATHEMATICS - III
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TERM TEST - I ASSIGNMENT

Q1 Find $L\left\{\frac{1-\cos(at)}{t}\right\}$

Soln:

$$f(t) = 1 - \cos(at)$$

$$L\{f(t)\} = L\{1 - \cos(at)\}$$

$$= L\{1\} - L\{\cos(at)\}$$

$$= \frac{1}{s} - \frac{s}{s^2+a^2}$$

$$= f(s)$$

By using division by 't' property.

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty f(s)\, ds$$

$$= \int_s^\infty \frac{1}{s} - \frac{s}{s^2+a^2}\, ds$$

$$= \int_s^\infty \frac{1}{s}\, ds - \int_s^\infty \frac{s}{s^2+a^2}\, ds$$

$$= \left[\frac{2}{2}\log s\right]_s^\infty - \frac{1}{2}\int_s^\infty \frac{2s}{s^2+a^2}\,ds$$

$$= \frac{1}{2}\left[\log s^2 - \log(s^2+a^2)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2}{s^2+a^2}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[-\log\left(\frac{s^2}{s^2+a^2}\right)\right]$$

$$= \frac{1}{2}\log\frac{s^2+a^2}{s}$$

$$\boxed{\therefore L\left\{\frac{1-\cos(at)}{t}\right\} = \frac{1}{2}\log\frac{s^2+a^2}{s}}$$

Q2 Using convolution theorem find
$$L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$$

Soln:

$$\frac{1}{s^2(s+1)^2} = \frac{1}{s^2} \times \frac{1}{(s+1)^2}$$

$$\therefore f_1(s) = \frac{1}{s^2}$$

$$\therefore f_2(s) = \frac{1}{(s+1)^2}$$

Now, taking Laplace Inverse Transform of $f_1(s)$ and $f_2(s)$,

$$\therefore L^{-1}\{f_1(s)\} = L^{-1}\left\{\frac{1}{s^2}\right\} = \frac{t^{(2-1)}}{(2-1)!} = \underline{\underline{t}}$$

$$\therefore L^{-1}\{f_2(s)\} = L^{-1}\left\{\frac{1}{(s+1)^2}\right\} = e^{-t} \times \frac{t^1}{1!} = \underline{\underline{e^{-t}\cdot t}}$$

By first shifting property

By convolution theorem,

$$L^{-1}\{f_1(s)\cdot f_2(s)\} = \int_0^t f_1(u)\cdot f_2(t-u)\,du$$

$$= \int_0^t (t-u)\cdot ue^{-u}\,du$$

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$$= \int_0^t (tu - u^2)\, e^{-u}\, du$$

$$= \left\{ \left[+(tu-u^2) \cdot \frac{e^{-u}}{-1} \right] - \left[(t-2u) \cdot e^{-u} \right] + \left[-2 \cdot \frac{e^{-u}}{-1} \right] \right\}_0^t$$

$$= \left[-(tu-u^2)\, e^{-u} - (t-2u)e^{-u} + 2e^{-u} \right]_0^t$$

$$= \left\{ \left[-\overbrace{(t^2-t^2)}^{0} e^{-t} - (t-2t)e^{-t} + 2e^{-t} \right] - \right.$$

$$\left. \left[-0e^{0} - (t-0)e^{0} + 2e^{0} \right] \right\}$$

$$= \left[+te^{-t} + 2e^{-t} \right] - \left[-t + 2 \right]$$

$$= te^{-t} + 2e^{-t} + t - 2$$

Q3 Solving using Laplace transform

$$(D^2 - 3D + 2)y = 4e^{2t}, \quad y(0) = -3, \quad y'(0) = 5$$

Soln:

$$(D^2 - 3D + 2)y = 4e^{2t}$$

$$\therefore D^2y + 3Dy + 2y = 4e^{2t}$$

$$\therefore y'' - 3y' + 2y = 4e^{2t}$$

$$y(0) = -3$$

$$y'(0) = 5$$

Now, Taking Laplace Transform on both sides

$$L\{y''\} - 3L\{y'\} + 2L\{y\} = 4L\{e^{2t}\}$$

$$\therefore s^2Y(s) - sy(0) - y'(0) - 3[sY(s) - y(0)] + 2Y(s) = \frac{4}{s-2}$$

$$\therefore s^2Y(s) - s(-3) - 5 - 3[sY(s) - (-3)] + 2Y(s) = \frac{4}{s-2}$$

$$\therefore s^2Y(s) + 3s - 5 - 3sY(s) - 9 + 2Y(s) = \frac{4}{s-2}$$

$$\therefore s^2Y(s) + 3s - 14 - 3sY(s) + 2Y(s) = \frac{4}{s-2}$$

$$\therefore s^2Y(s) - 3sY(s) + 2Y(s) = \frac{4}{s-2} - 3s + 14$$

$$\therefore \left(s^2 - 3s + 2\right) Y(s) = \frac{4 - 3s(s-2) + 14(s-2)}{s-2}$$

$$\therefore \left(s^2 - s - 2s + 2\right) Y(s) = \frac{4 - 3s^2 + 6s + 14s - 28}{s-2}$$

$$\therefore \left(s(s-1) - 2(s-1)\right) Y(s) = \frac{-3s^2 + 20s - 24}{s-2}$$

$$\therefore Y(s) = \frac{-3s^2 + 20s - 24}{(s-1)(s-2)(s-2)}$$

$$\therefore Y(s) = \frac{-3s^2 + 20s - 24}{(s-1)(s-2)^2}$$

Partial fractions:

$$\therefore \frac{-3s^2 + 20s - 24}{(s-1)(s-2)^2} = \frac{a}{(s-1)} + \frac{b}{(s-2)} + \frac{c}{(s-2)^2}$$

$$\therefore -3s^2 + 20s - 24 = a(s-2)^2 + b(s-1)(s-2)^2 + c(s-1)$$

$$\therefore -3s^2 + 20s - 24 = a(s^2 - 4s + 4) + b(s^2 - 3s + 2) + cs - c$$

$$\therefore -3s^2 + 20s - 24 = as^2 - 4as + 4a + bs^2 - 3bs + 2b + cs - c$$

$$\therefore -3s^2 + 20s - 24 = (a+b)s^2 + (-4a-3b+c)s + (4a+2b-c)$$

~~$\therefore 3s^2$~~ Comparing coefficients:

$$\therefore a + b = -3 \quad \ldots\ldots (i)$$
$$-4a - 3b + c = 20 \quad \ldots\ldots (ii)$$
$$4a + 2b - c = -24 \quad \ldots\ldots (iii)$$

Adding equations (ii) and (iii)

$$-b = -4$$
$$\boxed{\therefore b = 4}$$

put $b=4$ in eq. i, we get

$$a + b = -3$$
$$a = -3 - 4$$
$$\boxed{\therefore a = -7}$$

put $a = -7$ in eq. ii, we get

$$-4a - 3b + c = 20$$
$$-4(-7) - 3(4) + c = 20$$
$$28 - 12 + c = 20$$
$$c = 20 - 16$$

$$\boxed{\therefore c = 4}$$

$$\frac{-3s^2+20s-24}{(s-1)(s-2)^2} = \frac{-7}{s-1} + \frac{4}{s-2} + \frac{4}{(s-2)^2}$$

$$\therefore Y(s) = \frac{-7}{s-1} + \frac{4}{s-2} + \frac{4}{(s-2)^2}$$

Taking inverse laplace transform,

$$y(t) = -7\, L^{-1}\left\{\frac{1}{s-1}\right\} + 4\, L^{-1}\left\{\frac{1}{s-2}\right\} + 4\, L^{-1}\left\{\frac{1}{(s-2)^2}\right\}$$

$$y(t) = -7e^{t} + 4e^{2t} + 4te^{2t}$$

Q4a Evaluate the following integral by using Laplace Transform

$$\int_0^\infty \underbrace{e^{-3t}}_{\text{definition}} \underbrace{t}_{\substack{\text{multiplication} \\ \text{by } t \\ \text{property}}} \underbrace{\sin t}_{f(t)} \, dt$$

Soln:

$f(t) = \sin t$

$\therefore L\{f(t)\} = L\{\sin t\} = \dfrac{1}{s^2+1} = f(s)$

... where $a = 1$

By multiplication by t property,

$$L\{t \times f(t)\} = (-1)^n \frac{d^n}{ds^n} f(s)$$

$$= (-1)^1 \frac{d}{ds}\left(\frac{1}{s^2+1}\right)$$

$$= (-1) \frac{-2s}{(s^2+1)^2}$$

$$= \frac{2s}{(s^2+1)^2} \quad \ldots\ldots (i)$$

To find Particular value,

$e^{-st} = e^{-3t}$

$\boxed{\therefore s = 3}$

Substituting $s = 3$ in eq.(i)

$$\therefore \frac{2s}{(s^2+1)^2} = \frac{2(3)}{(3^2+1)^2} = \frac{6}{100} = \frac{3}{50}$$

Q4b Find $L\{f(t)\}$ for $f(t) = k\frac{t}{T}$, $0 < t < T$
and $f(t+T) = f(t)$

Soln:

Function is periodic with period 'T'
Laplace Transform of periodic function is given as

$$L\{f(t)\} = \frac{1}{1-e^{-Ts}} \int_0^T e^{-st} \cdot f(t)\, dt$$

$$= \frac{1}{1-e^{-Ts}} \int_0^T e^{-st} \cdot k\frac{t}{T}\, dt$$

$$= \frac{K}{T} \cdot \frac{1}{1-e^{-Ts}} \int_0^T e^{-st} \cdot t\, dt$$

$$= \frac{K}{T(1-e^{-Ts})} \left[\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2} \right]_0^T$$

$$= \frac{K}{T(1-e^{-Ts})} \left[\frac{-Te^{-sT}}{s} - \frac{e^{-sT}}{s^2} - \left(0 - \frac{1}{s^2}\right) \right]$$

$$= \frac{K}{T(1-e^{-Ts})} \left[\frac{-Te^{-sT}}{s} - \frac{e^{-sT}}{s^2} + \frac{1}{s^2} \right]$$

$$= \frac{1}{1-e^{-Ts}} \cdot \frac{K}{T} \left[\frac{1}{s^2}(1-e^{-sT}) - \frac{Te^{-sT}}{s} \right]$$

$$= K \left[\frac{1}{Ts^2} - \frac{e^{-sT}}{s(1-e^{-Ts})} \right]$$

Q5 a find $L^{-1}\left\{\frac{3s+1}{(s+1)(s^2+2)}\right\}$

Soln:

$$f(s) = \frac{3s+1}{(s+1)(s^2+2)}$$

Partial Fraction

$$\frac{3s+1}{(s+1)(s^2+2)} = \frac{A}{(s+1)} + \frac{Bs+C}{(s^2+2)} \quad \ldots \text{I}$$

$$\frac{3s+1}{\cancel{(s+1)(s^2+2)}} = \frac{A(s^2+2)+(Bs+C)(s+1)}{\cancel{(s+1)(s^2+2)}}$$

$$\therefore 3s+1 = As^2 + 2A + Bs^2 + Bs + Cs + C$$

$$\therefore 3s+1 = s^2(A+B) + s(B+C) + (2A+C)$$

Equating,

$$A+B = 0 \quad \ldots (i)$$

$$B+C = 3 \quad \ldots (ii)$$

$$2A+C = 1 \quad \ldots (iii)$$

$\therefore$ Adding eq. i & eq ii & eq iii

$$3A + 2(B+C) = 4$$

$$3A + 2(3) = 4$$

$$3A + 6 = 4$$

$$3A = -2$$

$$\boxed{\therefore A = -\frac{2}{3}}$$

Substituting $A = \frac{-2}{3}$ in eq i

$$\frac{-2}{3} + B = 0$$

$$\boxed{\therefore B = \frac{2}{3}}$$

Substituting $A = -\frac{2}{3}$, $B = \frac{2}{3}$ in eq. iii

$$2\left(\frac{-2}{3}\right) + C = 1$$

$$\frac{-4}{3} + C = 1$$

$$C = 1 + \frac{4}{3}$$

$$\boxed{\therefore C = \frac{7}{3}}$$

From I,

$$\frac{3s+1}{(s+1)(s^2+2)} = \frac{-2}{3(s+1)} + \cancel{\frac{2s}{3(s^2}} \frac{(2/3)s + (7/3)}{(s^2+2)}$$

$$= -\frac{2}{3}\left(\frac{1}{s+1}\right) + \frac{2}{3}\left(\frac{s}{s^2+2}\right) + \frac{7}{3}\left(\frac{1}{s^2+2}\right)$$

Taking Laplace Inverse Transform,

$$\therefore L^{-1}\left\{\frac{3s+1}{(s+1)(s^2+2)}\right\} = \frac{-2}{3} L^{-1}\left\{\frac{1}{s+1}\right\} + \frac{2}{3} L^{-1}\left\{\frac{s}{s^2+2}\right\}$$

$$+ \frac{7}{3} L^{-1}\left\{\frac{1}{s^2+2}\right\}$$

$$= \frac{-2}{3} e^{-t} + \frac{2}{3} \cos\sqrt{2}t + \frac{7}{3} \cdot \frac{1}{\sqrt{2}} \sin\sqrt{2}t$$

$$= \frac{1}{3}\left[-2e^{-t} + 2\cos\sqrt{2}t + \frac{7}{\sqrt{2}} \sin\sqrt{2}t\right]$$

Q5 b Find $L\{\sin t\ H(t-\pi) + t^2\delta(t-2)\}$

Soln: $L\{\sin t\ H(t-\pi)\}$

$\therefore f(t) = \sin t$

$f(t+\pi) = \sin(t+\pi)$

$= -\sin t$

$\therefore L\{f(t+\pi)\} = \dfrac{-1}{s^2+1}$

$\therefore L\{\sin t\ H(t-\pi)\} = -e^{-\pi s}\dfrac{1}{s^2+1}$ .... (i)

$L\{t^2\delta(t-2)\}$

Here, $f(t) = t^2$

$L\{\delta(t-2)\} = e^{-2s}$

$\therefore L\{t^2\delta(t-2)\} = 4e^{-2s}$ .... (ii)

$\therefore L\{\sin\ H(t-\pi) + t^2\delta(t-2)\}$

$= e^{-\pi s}\dfrac{1}{s^2+1} + 4e^{-2s}$ ... from (i) & (ii)