Shri Vile Parle Kelavani Mandal's

DWARKADAS J. SANGHVI COLLEGE OF ENGINEERING

(Autonomous College Affiliated to the University of Mumbai)

NAAC Accredited with "A" Grade (CGPA : 3.18)

RUBRICS INDEX

Academic Year 2022-23

Name: Prerna Jadhav **SAP ID:** 60004220127

Department /Branch: Computer Engineering **Division:** B

Course / Subject Name: Discrete Structure **Course Code:** DJ19CET303

Perfomance Indicators	T1	T2	T3	T4	T5	T6	T6	T7	T8	Avg.
Course Outcome										
1.Knowledge (4) (Factual/Conceptual)	4	4	4	4	4	4		4	4	4
2.Describe (2) (Methods/Procedure) (Procedural)	2	2	2	2	2	2		2	2	2
3.Demonstration (2) (Correct solutions) (Procedural/Metacognitive)	2	2	2	2	2	2		2	2	2
4. Attitude towards learning (2) (receiving, attending, responding, valuing, organizing, characterization by value)	2	2	2	2	1	1		2	2	2 2
Total (10)	10	10	10	10	9	9		10	10	10
Signature	K	K	K	K	K	K		K	K	K

Exceed Expectations (100%), Meet Expectations (75%), Below Expectations (40%)

Signature of the Teacher Head of the Department Principal

Name of the Teacher: Mrs. Vishakha Shelke Date: 12.1.23

15 $\frac{10}{10}$

NAME: PRERNA SUNIL JADHAV
SAPID: 60004220127
BRANCH: COMPUTER ENGINEERING

COURSE ID: DJ19CET303
COURSE: DISCRETE STRUCTURE TUTORIAL
TUTORIAL I

Q1 Determine the number of integers between 1 & 250 that are divisible by 2 or 3 or 5 or 7.

Soln A is a set of all integers from 1 to 250 divisible by 2.
B is a set of all integers from 1 to 250 divisible by 3
C is a set of all integers from 1 to 250 divisible by 5.
D is a set of all integers from 1 to 250 divisible by 7.

Now,

$|A| = \frac{250}{2} = 125$ → cardinality of set A

$|B| = \frac{250}{3} = 83$ → cardinality of set B

$|C| = \frac{250}{5} = 50$ → cardinality of set C

$|D| = \frac{250}{7} = 35$ → cardinality of set D

$|A \cap B| = \frac{250}{2 \times 3} = \frac{250}{6} = 41$

$|\cancel{A} C \cap D| = \frac{250}{5 \times 7} = \frac{250}{35} = 7$

we know, $|A \cup B| = |A| + |B| - |A \cap B|$
By addition principle

$|A \cup B| = |A| + |B| - |A \cap B|$
$= 125 + 83 - 41$
$= 167$

Similarly,
$|C \cup B| = |C| + |D| - |C \cap D|$
$= 50 + 35 - 7$
$= 78$

$\big| |A \cup B| \cup |C \cup D| \big| = |A \cup B| + |C \cup D| - \big| |A \cup B| \cap |C \cup D| \big|$

$\big| |A \cup B| \cap |C \cup D| \big| = \frac{250}{3 \times 2 \times 5 \times 7} = 1$

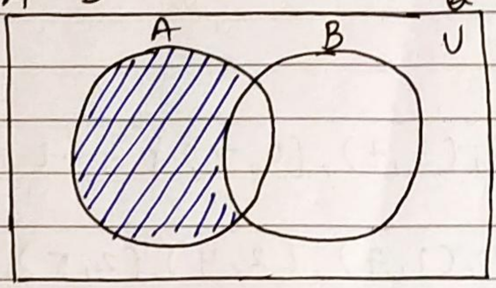
$\therefore \big| |A \cup B| \cup |C \cup D| \big| = 167 + 78 - 1$
$= 245 - 1 = 244$

Q2 Prove the following
$(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

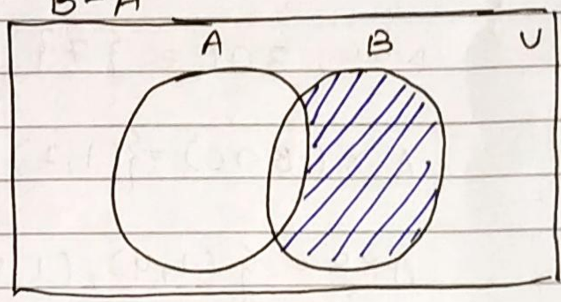
Soln:

L.H.S $\Rightarrow$ $(A-B) \cup (B-A)$

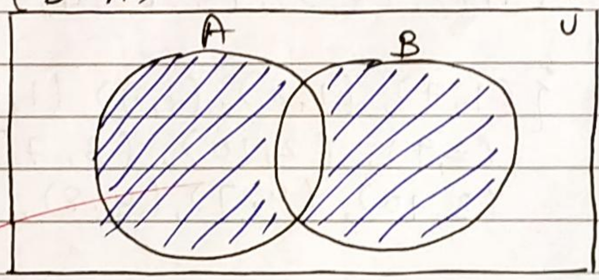
$A-B$



$B-A$

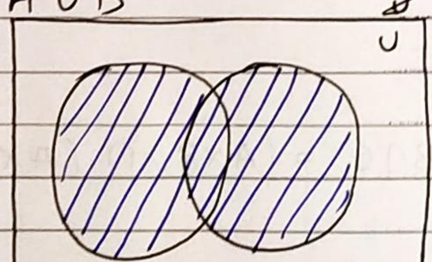


So, $(A-B) \cup (B-A)$

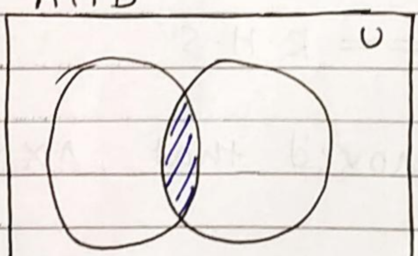


R.H.S $\Rightarrow$ $(A \cup B) - (A \cap B)$

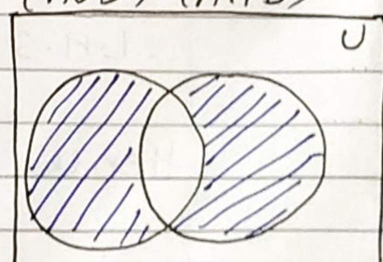
$A \cup B$



$A \cap B$



$(A \cup B) - (A \cap B)$



Hence proved. Since L.H.S == R.H.S
$(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

Q3 Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Soln: Let the sets be as follows:-

$A = \{1, 2, 3, 4\}$

$B = \{4, 5, 6, 7\}$

$C = \{7, 8, 9, 10\}$

Now, $B \cap C = \{7\}$

$A \times (B \cap C) = \{(1,7), (2,7), (3,7), (4,7)\}$ .... L.H.S

$A \times B = \{(1,4), (1,5), (1,6), (1,7), (2,4), (2,5), (2,6), (2,7), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6) (4,7)\}$

$A \times C = \{(1,7), (1,8), (1,9), (1,10), (2,7), (2,8), (2,9), (2,10), (3,7), (3,8), (3,9), (3,10), (4,7), (4,8), (4,9), (4,10)\}$

then

$(A \times B) \cap (A \times C) = \{(1,7), (2,7), (3,7), (4,7)\}$ .... R.H.S

$\therefore$ L.H.S == R.H.S

Hence proved that, $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Q4 Prove using Laws of Logic

$[(p \vee q) \wedge (p \vee \sim q)] \vee q \leftrightarrow p \vee q$

Soln $L.H.S = [(p \vee q) \wedge (p \vee \sim q)] \vee q$

$= [p \vee (q \wedge \sim q)] \vee q$ ... By Distribution property

$= [p \vee F] \vee q$ .... ($\because q \wedge \sim q$ is always F)

$= p \vee q$ ... ($\because p \vee F = p$)

$= R.H.S$

Hence proved, $[(p \vee q) \wedge (p \vee \sim q)]\, q \leftrightarrow p \vee q$

Q5 Verify that the preposition $p \vee \sim(p \wedge q)$ is a tautology.

Soln Truthtable:

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T
				Tautology

Since the truth table for $p \vee \sim(p \wedge q)$ is (T T T T). We say $p \vee \sim(p \wedge q)$ is a tautology.

Q6 Construct truth table to determine whether each of the following is tautology, contradiction, or contingency.

i) $(q \wedge p) \vee (q \wedge \sim p)$

ii) $q \rightarrow (q \rightarrow p)$

iii) $p \rightarrow (q \wedge p)$

Soln: i) $(q \wedge p) \vee (q \wedge \sim p)$

p	q	$q \wedge p$	$\sim p$	$q \wedge \sim p$	$(q \wedge p) \vee (q \wedge \sim p)$
T	T	T	F	F	T
T	F	F	F	F	F
F	T	F	T	T	T
F	F	F	T	F	F

contingency

Since the truth table for $(q \wedge p) \vee (q \wedge \sim p)$ is (T F T F) its contingency.

ii) $q \rightarrow (q \rightarrow p)$

p	q	$q \rightarrow p$	$q \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	T	T

contingency

Since the truth table for $(q \rightarrow (q \rightarrow p))$ is (T T F T) its contingency.

iii) $p \rightarrow (q \wedge p)$

p	q	$q \wedge p$	$p \rightarrow (q \wedge p)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T
			Contingency

- Since the truth table for $p \rightarrow (q \wedge p)$ is (TFTT) its contingency.

Q7 Write the english sentences for the following where, $P(x)$: x is even
$Q(x)$: x is prime
$R(x, y)$: $x + y$

i> $\exists x \, \forall y \; R(x, y)$
ii> $\sim (\exists x \; P(x))$
iii> $\sim (\cancel{\exists x} \, \forall x \; Q(x))$
iv) $\forall x \, (\sim Q(x))$

- Soln: i) $\exists x \; \forall y \; R(x, y)$
→ There exist a value of x for all y such that relation $R(x, y)$ where relation $(x + y)$ exists.

ii) $\sim (\exists x \; P(x))$
→ There doesnt exist any value for x for which x is even

iii) $\sim (\forall x \; Q(x))$
→ There doesnt exist any value of x which is prime

iv) $\forall x \; (\sim Q(x))$
→ for all x, x is not a prime

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TUTORIAL II

Q1 Given that the truth values of x, y and z as T, and those of u & v as F, find the truth values of

$(X \wedge (Y \vee Z)) \wedge \sim((X \vee Z) \wedge (U \vee V) \wedge Z)$

Soln:

$X \rightarrow T$ $U \rightarrow F$

$Y \rightarrow T$ $V \rightarrow F$

$Z \rightarrow T$

$(X \wedge (Y \vee Z)) \wedge \sim((X \vee Z) \wedge (U \vee V) \wedge Z)$

$(T \wedge (T \vee T)) \wedge \sim((T \vee T) \wedge (F \vee F) \wedge T)$

$(T \wedge T) \wedge \sim(T \wedge F \wedge T)$

$(T) \wedge \sim(F)$

$T \wedge T$

$\boxed{T}$

Q2 Prove using laws of logic

a) $a \rightarrow (p \vee c) \Leftrightarrow (a \wedge \sim p) \rightarrow c$

b) $\sim(p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q)) \leftrightarrow (\sim p \vee q)$

Soln a) $a \rightarrow (p \vee c) \Leftrightarrow (a \wedge \sim p) \rightarrow c$

L.H.S : $a \rightarrow (p \vee c)$

$\sim a \vee (p \vee c)$ ..... Implicative law.

R.H.S : $\sim(a \wedge \sim p) \vee c$

$(\sim a \vee p) \vee c$

$\sim a \vee (p \vee c)$ ....($\because a \vee (b \vee c) = (a \vee b) \vee c$)

L.H.S == R.H.S, Hence proved

b) $\sim(p \wedge q) \to (\sim p \vee (\sim p \vee q)) \leftrightarrow (\sim p \vee q)$

L.H.S : $(p \wedge q) \vee (\sim p \vee (\sim p \vee q))$

$(p \wedge q) \vee ((\sim p \vee \sim p) \vee (\sim p \vee q))$

$(p \wedge q) \vee (\sim p \vee q)$

$[(p \vee \sim p) \vee (p \vee q)$

$[(p \vee \sim p) \vee (p \vee q)] \wedge [(q \vee \sim p) \vee (q)]$

$T \wedge [(\sim p \vee q) \vee q]$

$T \wedge [\sim p \vee q]$

$\sim p \vee q$

$=$ R.H.S

$\therefore$ L.H.S == R.H.S, Hence proved

Q3 Write English sentence for the following where

$P(x)$: x is even

$Q(x)$: x is prime

$R(x, y)$: $x \cdot y$ is even

Soln: i) $\exists x \ \forall y \ R(x,y)$

There exist a value of x for all values of y for which the relationship $x \cdot y$ is even holds true.

ii) $\forall x \ \exists y \ R(x,y)$

For all values of x there exists values of y for which the relationship $x \cdot y$ is even holds true.

iii) $\sim(\exists x \ P(x))$

It is not true that there exist a value of x for which x is even

iv) $\sim(\forall x \ Q(x))$

It is not true for all values x for ~~with~~ which x is prime

v) $\exists y\ (\sim P(y))$
There exists a value of y for which y is not even

vi) $\forall x\ (\sim Q(x))$
For all values of x, x is not prime.

Q4 Its known that at the university
60% of professors play tennis
50% of professors play bridge
70% of professors jog.
20% Tennis and Bridge
30% Tennis and jog
40% Bridge and jog
If someone claimed that 20% of the professors jog and play bridge & play tennis. Would you believe the claim? why?

soln: Let the professors playing tennis be represented by set T.
Let the professors playing Bridge be represented by set B
Let the professors who jog be represented by set J
Universal set $|U| = 100 = |T \cup B \cup J|$

$\therefore |T| = 60$ & $|T \cup J| = 30$

$|J| = 70$ $\quad |T \cup B| = 20$

$|B| = 50$ $\quad |B \cup J| = 40$

$|T \cup B \cup J| = |T| + |B| + |J| - |T \cup B| - |B \cup J|$
$- |T \cup J| + |T \cap B \cap J|$

$100 = 60 + 50 + 70 - 20 - 30 - 40 + |T \cap B \cap J|$

$\therefore 100 = 180 - 90 + |T \cap B \cap J|$

$\therefore 100 - 180 + 90 = |T \cap B \cap J|$

$\therefore |T \cap B \cap J| = 10$

We can say, professors playing all three Tennis, jog and bridge are 10%.
So, it cannot be claimed that 20% professors jog, play ~~pri~~ ~~p~~ bridge, play tennis

Q5 Let $A = \{a, b, c, d, e, f, g, h\}$. Consider the following subsets of A.

$A_1 = \{a, b, c, d\}$
$A_2 = \{a, c, e, f, g, h\}$
$A_3 = \{a, c, e, g\}$
$A_4 = \{b, d\}$
$A_5 = \{f, h\}$

Determine whether each of the following is partition of A or not?

Soln: i) $\{A_1, A_2\}$

$\{A_1, A_2\} = \{(a, b, c, d), (a, c, e, f, g, h)\}$
It is not a partition because its not mutually disjoint.

ii) $\{A_1, A_3\}$

$\{A_1, A_3\} = \{(a, b, c, d), (a, c, e, g)\}$
Since $A_1 \cup A_3 \neq A$,
its not a partition

iii> $\{A_3, A_4, A_5\}$

$\{A_3, A_4, A_5\} = \{(a, c, eg), (b, d), (f, h)\}$

Since $(A_3 \cup A_4 \cup A_5) = A$

and $A_3 \cap A_4 \cap A_5 = \phi$

Sets A_3, A_4, A_5 are mutually disjoint and their union is A, hence its a partition.

Q6 Let the universal set be

$U = \{1, 2, 3, \ldots 10\}$

Let $A = \{2, 4, 7, 9\}$, $B = \{1, 4, 6, 7, 10\}$

$C = \{3, 5, 9, 7\}$

Find:

Soln: i> $A \cup B$

$A \cup B = \{1, 2, 4, 6, 7, 9, 10\}$

ii> $A \cap C$

$A \cap C = \{7, 9\}$

iii> $B \cap \bar{C}$

$\bar{C} = \{1, 2, 4, 6, 8, 10\}$

$B \cap \bar{C} = \{1, 4, 6, 10\}$

iv> $(A \cap \bar{B}) \cup C$

$\bar{B} = \{2, 3, 5, 8, 9\}$

$(A \cap \bar{B}) = \{2, 9\}$

$(A \cap \bar{B}) \cup C = \{2, 3, 5, 7, 9\}$

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TUTORIAL III

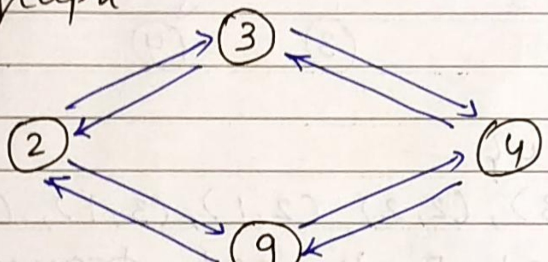
Q1 Given,

$A = \{2, 3, 4, 6, 9\}$

Relation = 'x is relatively prime to y'

Soln: $R = \{(2,3), (2,9), (3,4), (4,9), (9,4), (4,3), (9,2), (3,2)\}$

Directed graph

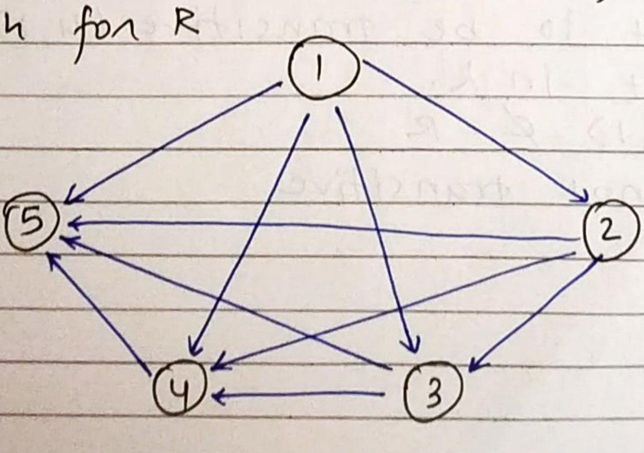


Q2 Given, $A = \{1, 2, 3, 4, 5\}$

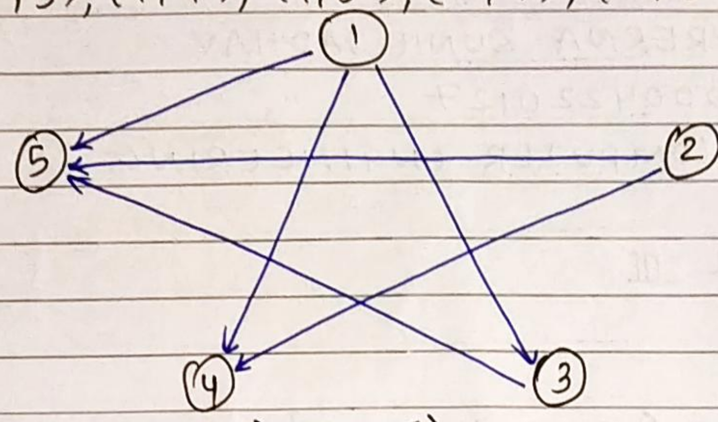
To find, R, R^2, R^3

Soln: $R = \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\}$

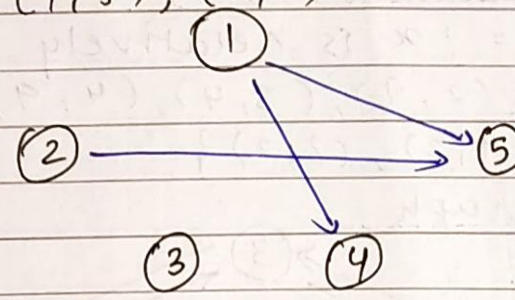
Diagraph for R



$R^2 = \{(1,3), (1,4), (1,5), (2,4), (2,5), (3,5)\}$



$R^3 = \{(1,4), (1,5), (2,5)\}$



Q3 $S = \{1, 2, 3, 4\}$

$R = \{(4,3), (2,2), (2,1), (3,1), (1,2)\}$

(a) Show that R is not transitive.

Soln: For R to be transitive, it must satify the condition, That is aRb & bRc then aRc.

Here, $(4,1) \subset R$ should be satisfied.

$\because (4,3), (3,1) \subset R$

For it to be transitive $(4,1)$ should be present in R.

$\because (4,1) \not\subset R$

R is not transitive

Q3

(b) Find a Relation $R_1 \subseteq R$ such that R is transitive.

Soln: For a transitive set if aRb & bRc then aRc should be present

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & & & & \\ 2 & & & & \\ 3 & & & & \\ 4 & & & & \end{array}$$

$$M_R = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Subset $M_{R_1} = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \end{array}$ is transitive.

since $(2,1), (1,2) \subset R$

& $(2,2) \subset R$

i.e., its transitive

$R_1 = \{(2,1), (1,2), (2,2)\}$

Q3

(C) Transitive closure by warshalls, $S = \{1,2,3,4\}$

$R = \{(4,3), (2,2), (2,1), (3,1), (1,2)\}$

Soln: Let $W_0 = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$

Selecting Row 1 and column 1,

$C_1 = \cancel{\{(2,1), (3,1)\}}\ \{2, 3\}$

$R_1 = \cancel{\{(1,2)\}}\ \{2\}$

$\therefore C_1 \times R_1 = \{(2,2), (3,2)\}$

Adding the pairs which dont exists in W_0.

$$\therefore W_1 = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Here, selecting Row 2 and Column 2,

$C_2 = \{1, 2, 3\}$

$R_2 = \{1, 2\}$

$C_2 \times R_2 = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)\}$

Adding these pairs which dont exist in W_1

$$\therefore W_2 = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Selecting Row 3 and Column 3,

$C_3 = \{4\}$

$R_3 = \{1, 2\}$

$C_3 \times R_3 = \{(4,1), (4,2)\}$

Adding pairs which are not present in W_2

$$\therefore W_3 = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Selecting Row 4 and Column 4

$C_4 = \phi$

$R_4 = \{1, 2, 3\}$

$C_4 \times R_4 = \phi$ ... so no new addition.

$\therefore R_T' = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,1), (4,2), (4,3)\}$

Q4 $A = \{11, 12, 13, 14\}$
$R = \{(11,12), (12,13), (13,14), (12,11)\}$
Find the transitive closure of R using Warshall's algorithm.

Sol'n:

$$W_0 = \begin{array}{c|c} & \begin{array}{cccc} 11 & 12 & 13 & 14 \end{array} \\ \begin{array}{c} 11 \\ 12 \\ 13 \\ 14 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Selecting Row 1 and column 1

$C_1 = \{12\}$
$R_1 = \{12\}$
$\therefore C_1 \times R_1 = \{(12,12)\}$

$$\therefore W_1 = \begin{array}{c|c} & \begin{array}{cccc} 11 & 12 & 13 & 14 \end{array} \\ \begin{array}{c} 11 \\ 12 \\ 13 \\ 14 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Selecting Row 2 and column 2

$C_2 = \{11, 12\}$
$R_2 = \{11, 12, 13\}$
$\therefore C_2 \times R_2 = \{(11,11), (11,12), (11,13), (12,11), (12,12), (12,13)\}$

$$\therefore W_2 = \begin{array}{c|c} & \begin{array}{cccc} 11 & 12 & 13 & 14 \end{array} \\ \begin{array}{c} 11 \\ 12 \\ 13 \\ 14 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Selecting Row 3 and ~~P~~ column 3

$C_3 = \{11, 12\}$

$R_3 = \{14\}$

$C_3 \times R_3 = \{(11, 14), (12, 14)\}$

$$\therefore W_3 = \begin{array}{c c} & \begin{array}{cccc} 11 & 12 & 13 & 14 \end{array} \\ \begin{array}{c} 11 \\ 12 \\ 13 \\ 14 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Selecting Row 4 and column 4

$C_{\cancel{3}4} = \{11, 12, 13\}$

$R_4 = \phi$

$\therefore R_4 \times C_4 = \phi$ .... No new addition.

$\therefore R_T = \{(11,11), (11,12), (11,13), (11,14), (12,11),$
$(12,12), (12,13), (12,14), (13,14)\}$

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NAME: PRERNA SUNIL JADHAV
SAP ID: 60004220127
BRANCH: COMPUTER ENGINEERING

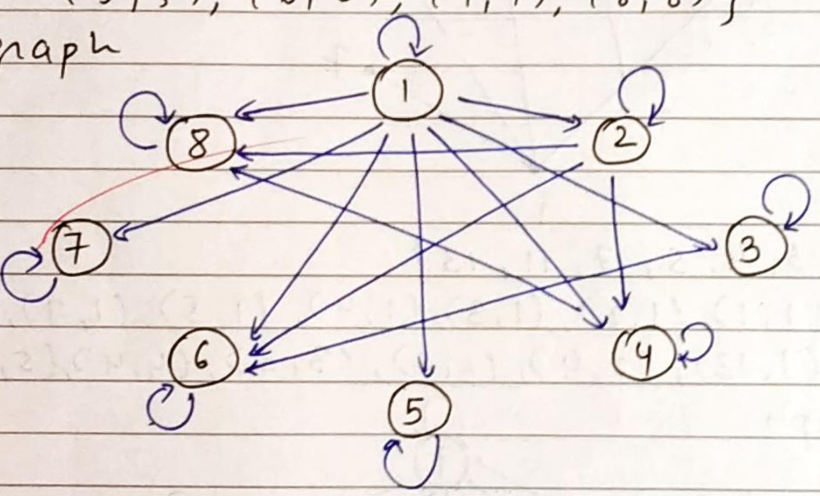
TUTORIAL IV

Q1 Draw the Hasse Diagram for Divisibility set on the set.

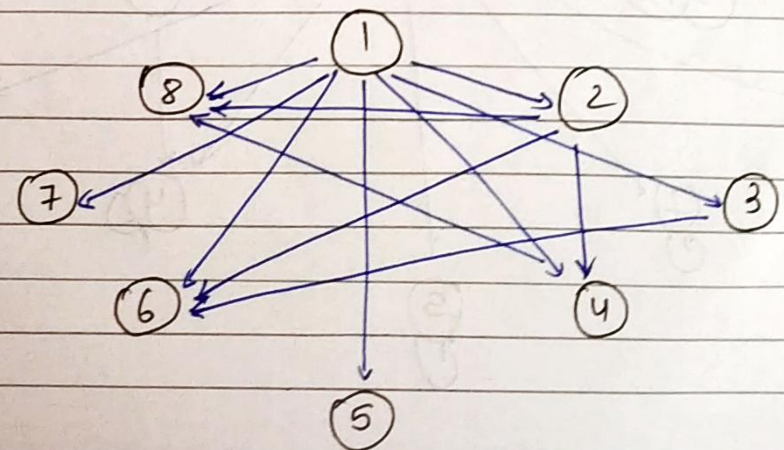
i) {1, 2, 3, 4, 5, 6, 7, 8}

Soln: R = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (2,2), (2,4), (2,6), (2,8), (3,3), (3,6), (4,4), (4,8), (5,5), (6,6), (7,7), (8,8)}

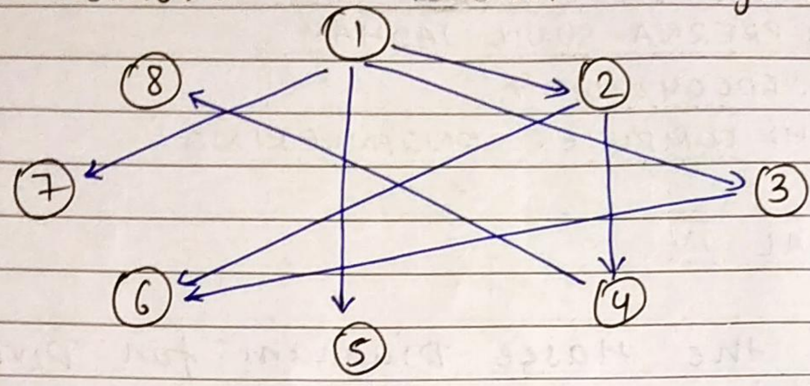
Diagraph



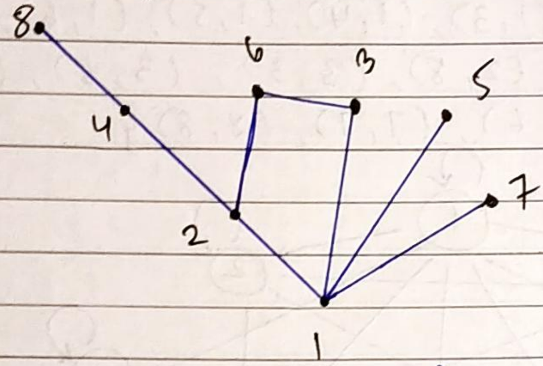
Step 1: Remove the loops



Step 2: Remove the transitive edges.



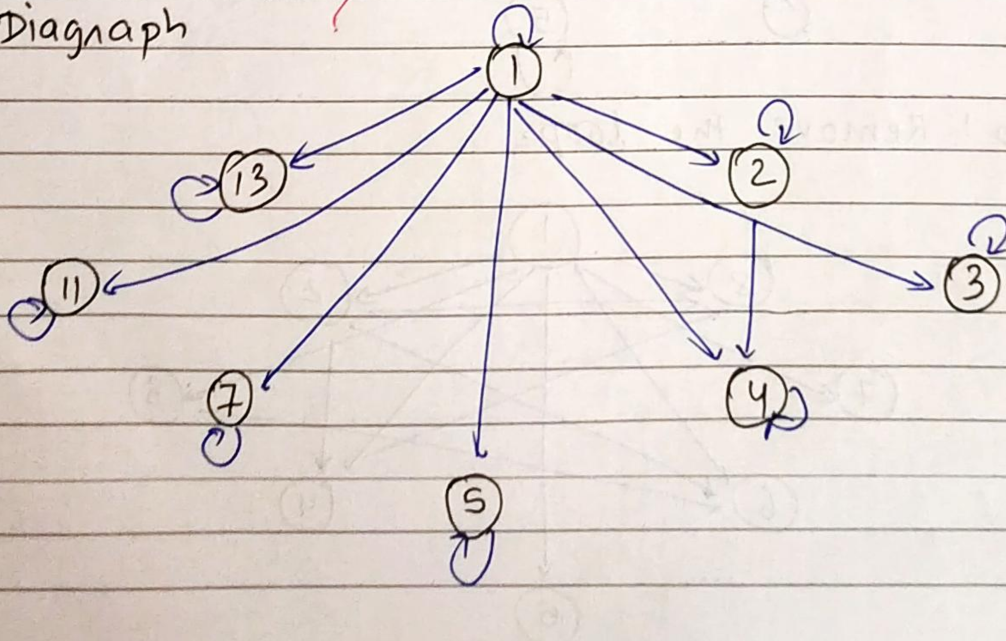
Hasse Diagram



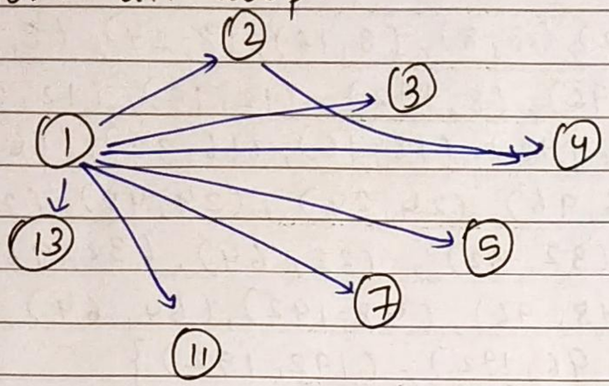
ii> {1, 2, 3, 4, 5, 7, 11, 13}

R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 7), (1, 11), (1, 13), (2, 4), (2, 2), (3, 3), (4, 4), (5, 5), (7, 7), (11, 11), (13, 13)}

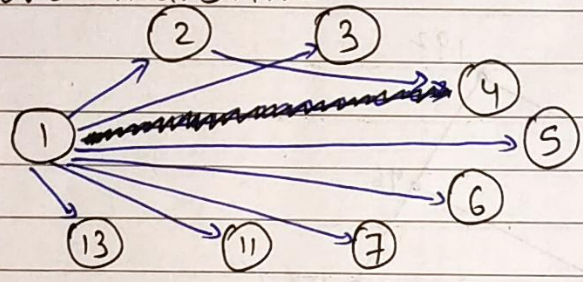
Diagraph



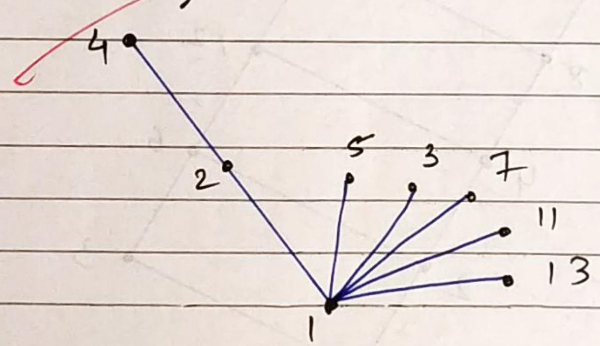
Remove all loops



Remove Transitive



Remove circles & arrows
Hasse diagram.



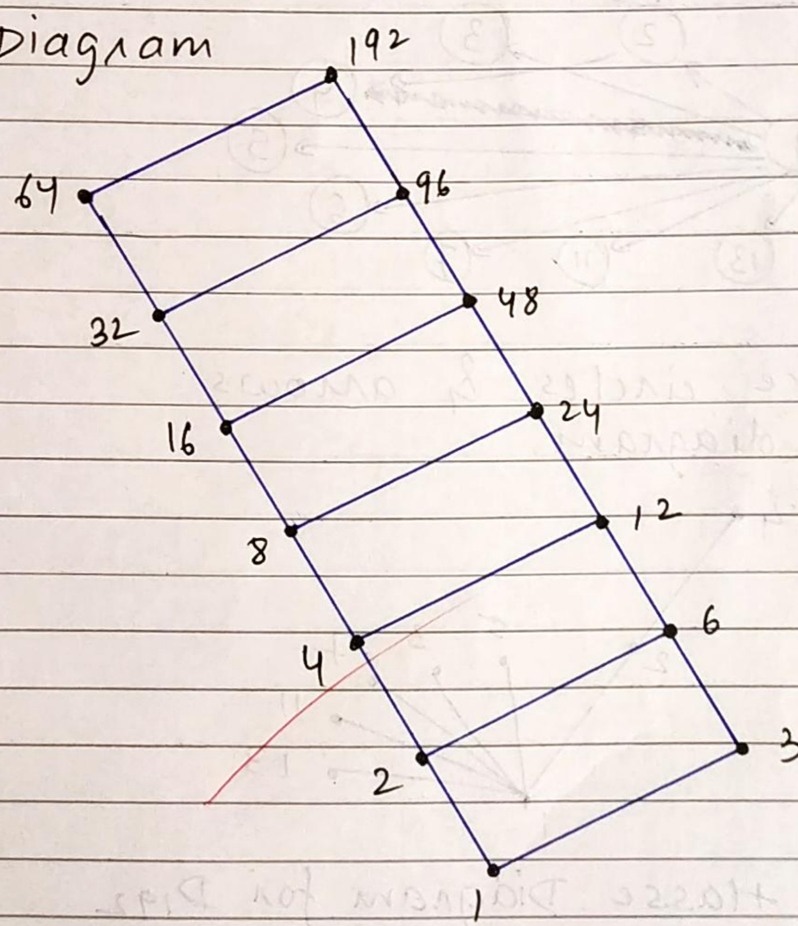
Q2 Draw Hasse Diagram for D_{192}

Soln: $D_{192} = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, 96, 192\}$

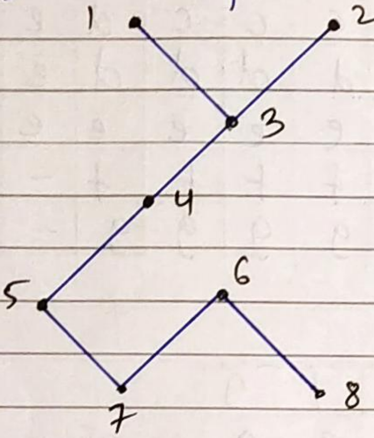
$\therefore R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,8), (1,12), (1,16), (1,24),$
$(1,32), (1,48), (1,64), (1,96), (1,192), (2,2), (2,4), (2,6),$
$(2,8), (2,12), (2,16), (2,24), (2,32), (2,48), (2,64), (2,96),$
$(2,192), (3,6), (3,12), (3,24), (3,48), (3,96), (3,192),$
$(4,4), (3,3), (4,8), (4,12), (4,16), (4,24), (4,32), (4,48),$
$(4,64), (4,96), (4,192), (6,6), (6,12), (6,24), (6,48),$

(6,96), (6,192), (8,8), (8,16), (8,24), (8,32), (8,48),
(8,64), (8,96), (8,192), (12,12), (12,24), (12,48),
(12,96), (12,192), (16,16), (16,32), (16,48),
(16,64), (16,96), (24,24), (24,48), (24,96),
(24,192), (32,32), (32,64), (32,96), (32,192),
(48,48), (48,96), (48,192), (64,64), (64,192),
(96,96), (96,192), (192,192)}

Hasse Diagram



Q3 Determine whether the following Hasse diagram represent a lattice or not.

a) 

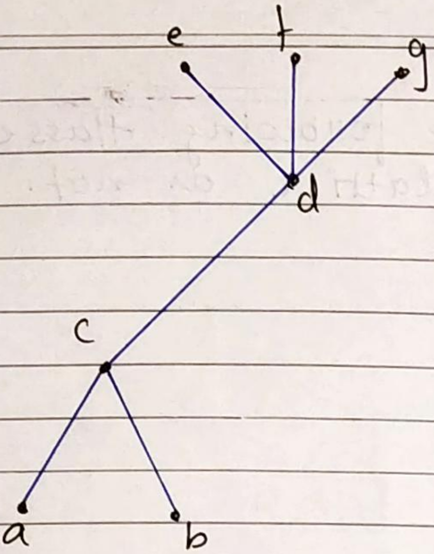
LUB : Least upper bound

∨	8	7	6	5	4	3	2	1
8	8	6	6	5	4	3	2	1
7	6	7	6	5	4	3	2	1
6	6	6	6	–	4	3	2	1
5	5	5	–	5	4	3	2	1
4	4	4	4	4	4	3	2	1
3	3	3	3	3	3	3	2	1
2	2	2	2	2	2	2	2	–
1	1	1	1	1	1	1	–	1

∧ GLB	8	7	6	5	4	3	2	1
8	8	–	8	8	8	8	8	8
7	–	7	7	7	7	7	7	7
6	8	7	6	7	7	7	7	7
5	8	7	–	5	5	5	5	5
4	8	7	6	5	4	4	4	4
3	8	7	3	5	4	3	3	3
2	8	7	2	5	4	3	2	-
1	8	7	1	5	4	3	-	1

(2,1), (2,2) have no LUB. So it is not a lattice.

Q3) b)



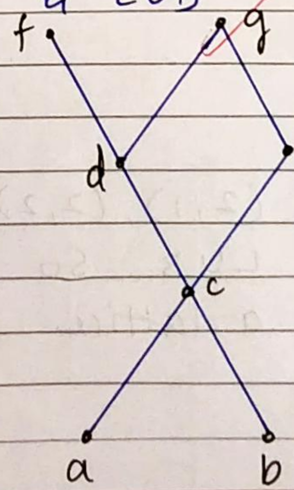
LUB ∨	a	b	c	d	e	f	g
a	a		c	d	e	f	g
b		b	c	d	e	f	g
c	c	c	c	d	e	f	g
d	d	d	d	d	e	f	g
e	e	e	e	e	e	-	-
f	f	f	f	f	-	f	-
g	g	g	g	g	-	-	g

GLB ∧	a	b	c	d	e	f	g
a	a		a	a	a	a	a
b	-	b	b	b	b	b	b
c	a	b	c	c	c	c	c
d	a	b	c	d	d	d	d
e	a	b	c	d	e	-	-
f	a	b	c	d	-	f	-
g	a	b	c	d	-	-	g

There are NULL values, hence not a lattice.

Q4 Find the upper bounds, lower bound, GLB & LUB

a)



⇒ {a, b, c}

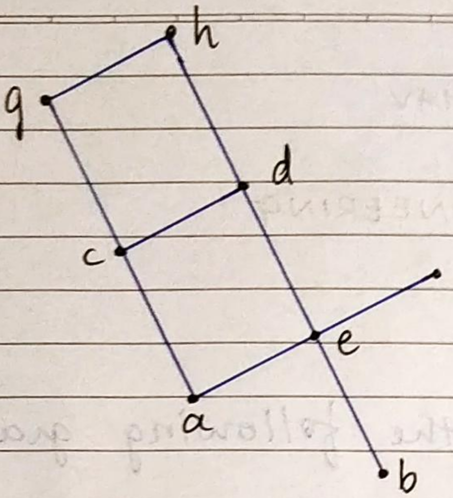
upper bound = {d, e, f, g, c}

Lower bound = { }

LUB = {C}

GLB = { }

b)



$\Rightarrow \{d, e, f\}$

Upper bound $= \{\ \}$

Lower bound $= \{a, e, b\}$

LUB $= \phi$

GLB $= \{e\}$