PROBLEM - 1

Given -

$$J(n) = \frac{1}{2}\left[y_d(n) - \sum_{K=1}^{N} w_K(n)\exp\left(-\frac{\|x(n)-c_K(n)\|^2}{\sigma_K^2(n)}\right)\right]^2$$

(1) $w(n+1) = w(n) - \mu_w \frac{\partial}{\partial w} J(n) \Big|_{w=w(n)}$

(2) $c_K(n+1) = c_K(n) - \mu_c \frac{\partial}{\partial c_K} J(n) \Big|_{c_K = c_K(n)}$

(3) $\sigma_K(n+1) = \sigma_K(n) - \mu_\sigma \frac{\partial}{\partial \sigma_K} J(n) \Big|_{\sigma_K = \sigma_K(n)}$

* For equation (1) we need to show

$w(n+1) = w(n) + \mu_w e(n)\psi(n)$, where $\psi(n) = [\phi\{x(n), c_1, \sigma_1\} \ldots \phi\{(x(n), c_n, \sigma_n\}]$

$$w(n+1) = w(n) - \mu_w \frac{\partial}{\partial w} J(n) \Big|_{w=w(n)}$$

On replacing the given value of $J(n)$

$$W(n+1) = W(n) - \mu_w \frac{\partial}{\partial W}\left[\frac{1}{2}\left(y_d(n) - \sum_{K=1}^{N} W_K(n).\exp\left(\frac{-\|x(n) - c_K(n)\|^2}{\sigma_K^2(n)}\right)\right)^2\right]$$

we can also write the above as

$$W(n+1) = W(n) - \mu_w \frac{\partial}{\partial W}\left[\frac{1}{2}\left(y_d(n) - W^T(n).\phi\right)^2\right]$$, where $W^T(n)$

denotes weight for n-th iteration and ϕ denotes radial function outputs for n-th iteration

Now, calculating the partial derivative with respect to W-vector

$$= \frac{\partial}{\partial W}\left[\frac{1}{2}\left(y_d(n) - W^T(n).\phi\right)^2\right]$$, we know if $f(x) = (g(x))^n \Rightarrow f'(x) = n \times (g(x))^{n-1} \times g'(x)$

$$= \frac{1}{2} \times 2 \times (y_d(n) - W^T(n).\phi)^{2-1} \times \frac{\partial}{\partial W}(-W^T(n).\phi)$$

We know $W^T(n).\phi$ is a scalar value
Hence we can use scalar by vector identity

$$\frac{\partial x^T a}{\partial x} = a^T$$

$$= (y_d(n) - W^T(n).\phi) \times (-\phi^T)$$

We can replace $W^T(n).\phi = y(n)$, since it is the network's output for n th iteration

$$= -\left(y_d(n) - y(n)\right) \times [\phi^T$$

expanding ϕ

$$= -\left(y_d(n) - y(n)\right) \times [\phi\{x(n), c_1, \sigma_1\}, \phi\{x(n), c_2, \sigma_2\} \ldots$$
$$\ldots \phi\{x(n), c_n, \sigma_n\}]^T$$

We are given

$$y_d(n) - y(n) = e(n)$$

$$[\phi\{x(n), c_1, \sigma_1\} \ldots \phi\{x(n), c_n, \sigma_n\}]^T = \psi(n)$$

On replacing we get

$$= -e(n).\psi(n)$$

On putting it back to the starting equation

$$w(n+1) = w(n) + \mu_w e(n) \psi(n)$$

* For equation (2) we need to show

$$C_K(n+1) = C_K(n) + \frac{\mu_c\, e(n)\, w_K(n)}{\sigma_K^2(n)} \phi\{x(n), C_K(n), \sigma_K\}[x(n) - C_K(n)]$$

given $\Rightarrow$ $C_K(n+1) = C_K(n) - \mu_c \frac{\partial}{\partial C_K} J(n) \Big|_{C_K = C_K(n)}$

On calculating partial derivative of $J(n)$ with respect to C_K

$$\frac{\partial}{\partial C_K} J(n) = \frac{\partial}{\partial C_K}\left[\frac{1}{2}\left(y_d(n) - \sum_{K=1}^{N} w_K(n) \exp\frac{-\|x(n) - C_K(n)\|^2}{\sigma_K^2(n)}\right)^2\right]$$

we know, if $f(x) = (g(x))^n$

then $f'(x) = n \times (g(x))^{n-1} \times g'(x)$

$$= \frac{1}{2} \times 2 \times \left(y_d(n) - \sum_{K=1}^{N} w_K(n) \exp\frac{-\|x(n) - C_K(n)\|^2}{\sigma_K^2(n)}\right)^{2-1} \times$$

$$\left(-w_K(n) \times \frac{(-\|x(n) - C_K(n)\| \times (-1)}{\sigma_K^2(n)} \times \exp\frac{-\|x(n) - C_K(n)\|^2}{\sigma_K^2(n)}\right)$$

All derivates except for the $\boxed{C_K\text{-th}}$ (K-th) radial function would be zero since they are constants.

Also we know, $\sum_{K=1}^{N} w_K(n) \exp\left(\frac{-\|x(n) - C_K(n)\|^2}{\sigma_K^2(n)}\right)$

is $y(n)$, the output of network for iteration n.

Also for k-th radial function we can write

$$\exp\left(\frac{-\|x(n)-c_k(n)\|^2}{\sigma_k^2(n)}\right) = \phi\{x(n), c_k(n), \sigma_k\}$$

On using the above two ~~givens~~ we get

$$\frac{\partial J(n)}{\partial c_k} = -\mu_c \frac{(y_d(n)-y(n)) \times w_k(n) \times [x(n)-c_k(n)] \times \phi\{x(n), c_k(n), \sigma_k\}}{\sigma_k^2(n)}$$

On substituting $y_d(n)-y(n) = e(n)$ and everything ~~back~~ the value of partial derivate back to the ~~other~~ update equation

$$c_k(n+1) = c_k(n) + \mu_c \frac{e(n)\, w_k(n)\, (x(n)-c_k(n))\, \phi\{x(n), c_k(n), \sigma_k\}}{\sigma_k^2(n)}$$

* For equation (3) we need to show

$$\sigma_k(n+1) = \sigma_k(n) + \mu_\sigma \frac{e(n)\, w_k(n)}{\sigma_k^3(n)} \phi\{x(n), c_k(n), \sigma_k\} \|x(n) - c_k(n)\|^2$$

given $\Rightarrow$ $\sigma_k(n+1) = \sigma_k(n) - \mu_\sigma \frac{\partial}{\partial \sigma_k} J(n) \Big|_{\sigma_k = \sigma_k(n)}$

On calculating partial derivative of $J(n)$ with respect to σ_k

$$\frac{\partial J(n)}{\partial \sigma_k} = \frac{\partial}{\partial \sigma_k}\left[\frac{1}{2}\left(y_d(n) - \sum_{k=1}^{N} w_k(n) \exp\left(\frac{-\|x(n) - c_k(n)\|^2}{\sigma_k^2(n)}\right)\right)^2\right]$$

We know, $f(x) = (g(x))^n$
$f'(x) = n \times (g(x))^{n-1} \times g'(x)$

$$= \frac{1}{2}\left[2 \times \left(y_d(n) - \sum_{k=1}^{N} w_k(n) \exp \frac{-\|x(n) - c_k(n)\|^2}{\sigma_k^2(n)}\right) \times \left(-w_k(n) \times -\|x(n) - c_k(n)\|^2 \times (-2) \times \frac{1}{\sigma_k^3(n)} \times \exp\frac{-\|x(n) - c_k(n)\|^2}{\sigma_k^2(n)}\right)\right]$$