1) <u>Complement of A</u>

The complement of crisp set A is set of all elements in universe X that do not belong to A

$$\chi_{A'} = 1 - \chi_A$$

Given $\chi_A = 1$ $(x \in A)$
$= 0$ (otherwise)

<u>Case I</u> →

a) If $\chi_A(x) = 1$ $(x \in A)$
Thus $\chi_{A'} = 1 - \chi_A = 1 - 1 = 0$
Implying (x does not belong to A)

Case II →

b) If $\chi_{A'}(x) = 0$ $(x \notin A)$
Thus $\chi_{A'} = 1 - \chi_A = 1 - 0 = 0$
Implying (x belong to A)

<u>Implication</u> →

The complement ~~of~~ for crisp sets is similar to <u>NOT</u> operation in binary logic.

It allows to find elements that do not belong to set A.

2) <u>Union of A and B</u> ($A \cup B$)

Union of 2 crisp sets A and B includes all elements that belong to either A or B or both.

$$\chi_{A \cup B} = \max(\chi_A, \chi_B)$$

<u>Case I</u>

1) If <u>either</u> χ_A or $\chi_B = 1$ then maximum is 1, means $x \in A$ or B (or both)

Case II

1) If <u>both</u> χ_A or $\chi_B = 0$ then maximum is 0, means $x \notin A$ or B.

<u>Implication</u>

→ The union operation for crisp sets is similar to OR operation in binary logic.

→ It allows to combine both sets A and B and help in finding elements belonging to either A or B (or both)

3) <u>Intersection of A and B</u> ($A \cap B$)

Intersection of 2 crisp sets A and B include all elements that belong to both A and B

$$\chi_{A \cap B} = \min(\chi_A, \chi_B)$$

→ <u>Case I</u>

1) If <u>both</u> χ_A and χ_B are 1, then minimum is 1 representing $x \in$ to both A and B.

<u>Case II</u>

2) If <u>either</u> χ_A or χ_B is 0, then minimum is 0, meaning $x \notin$ to both A and B.

<u>Implication</u>

→ The intersection for crisp sets is similar to the AND operation in binary logic.

→ It allows to find elements that belong to both A and B sets.

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4) Implication b/w A and B ($A \to B$)

The implication of 2 crisp sets A and B represent fuzzy relationship where we want to infer the membership of B based on the membership of A.

$$\chi_{A\to B} = \min(1, \{1 - \chi_{A(x)} + \chi_{B(y)}\})$$

Here x & y can be 2 diff universe X and Y

Case I

1) If $\chi_{A(x)} = 0$ (A does not contain x)

then $\chi_{A\to B} = \min(1, \{1 - 0 + \chi_{B(y)}\})$

$= \min[1, \{1 + \chi_{B(y)}\}]$

Means that if A does not contain x. B's membership in y does not change and remain unaffected

2) If $\chi_A(x) = 1$ (A contains x)

then $\chi_{A\to B} = \min(1, \{1 - 1 + \chi_{B(y)}\})$

$= \min[1, \{\chi_B(y)\}]$

Means that if A contain x, then B's membership in y is affected by membership of x in A.

~~III~~

Implication

The implication for crisp sets is similar to fuzzy relation b/w A & B.
It helps us to infer membership of B based on membership of A.

Thus all these results demonstrate relation b/w crisp sets & binary logic.
They allow to apply principle from binary to crisp set operation.
These help us in reasoning, calculation & solving problems involving crisp sets.