Problem 2:

$$\mu_A(x) = e^{-\lambda(x-a)^n} \qquad \lambda = 2, n = 2, a = 3$$
$$S = [0,6]$$

Substituting the values:-

$$\mu_A(x) = e^{-2(x-3)^2}$$

When $x = 0$,

$$\mu_A(0) = e^{-2(0-3)^2} = e^{-18}$$
$$= 1.52 \times 10^{-8} \simeq 0$$

When $x = 1$;

$$\mu_A(1) = e^{-2(1-3)^2} = 0.00033 \simeq 0$$

When $x = 2$;

$$\mu_A(2) = e^{-2(2-3)^2} = 0.135$$

When $x = 3$:

$$\mu_A(3) = e^{-2(3-3)^2} = e^0 = 1$$

When $x = 4$;

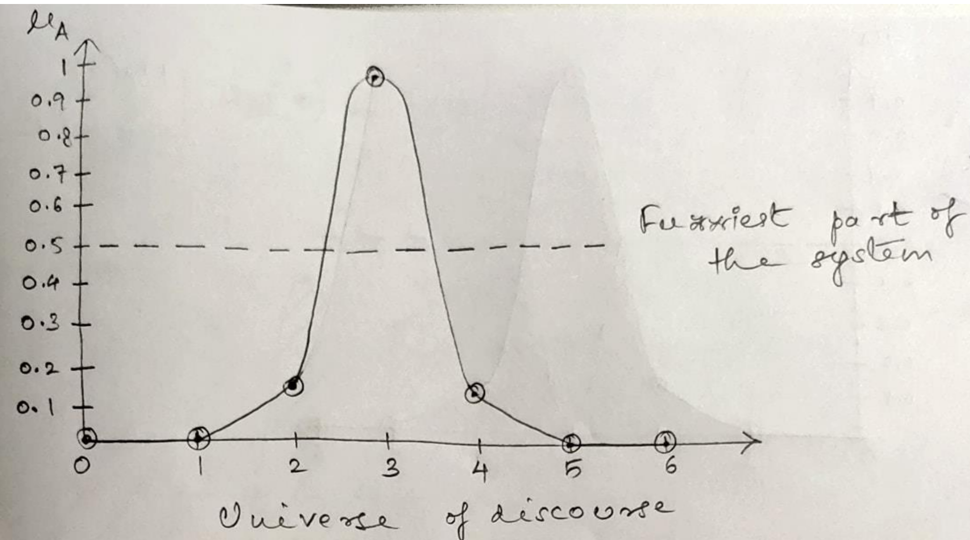
$$\mu_A(4) = e^{-2(4-3)^2} = e^{-2} = 0.135$$

When $x = 5$;

$$\mu_A(5) = e^{-2(5-3)^2} = e^{-2(4)} = 0.000335$$
$$\simeq 0$$

When $x = 6$;

$$\mu_A(6) = e^{-2(6-3)^2} = e^{-18} = 1.52 \times 10^{-8} \simeq 0.$$



Representing fuzzy measures M_1, M_2, M_3 on the graph:

i) $M_1 = \int_S f(x)\, dx$

where, $f(x) = \mu_A(x)$ for $\mu_A \leq 0.5$

$= 1 - \mu_A(x)$ for $\mu_A > 0.5$

When $x = 0$:

$\mu_A(0) = 0$ $\qquad$ $f(0) = 0$

When $x = 1$:

$\mu_A(1) = 0.00033$ $\qquad$ $f(1) = 0$

$\simeq 0$

When $x = 2$:

$\mu_A(2) = 0.135$ $\qquad$ $f(2) = 0.135$

When $x = 3$:

$\mu_A(3) = 1$ $\qquad$ $f(3) = 1 - \mu_A(3) = 1 - 1 = 0.$

When $x = 4$:

$\mu_A(4) = 0.135$ $\qquad$ $f(4) = 0.135$

When $x = 5$:

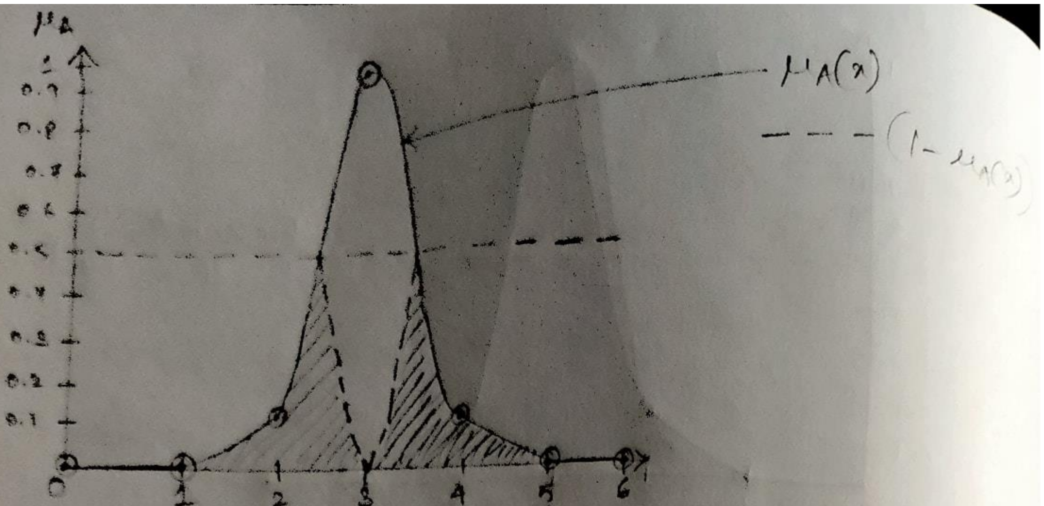
$\mu_A(5) = 0.00033$ $\qquad$ $f(5) = 0$

$\simeq 0$

When $x = 6$:

$\mu_A(6) = 0$ $\qquad$ $f(6) = 0.$

Graph for M_1:



$$M_2 = \int_S \left| \mu_A(x) - \mu_{A1/2}(x) \right| dx$$

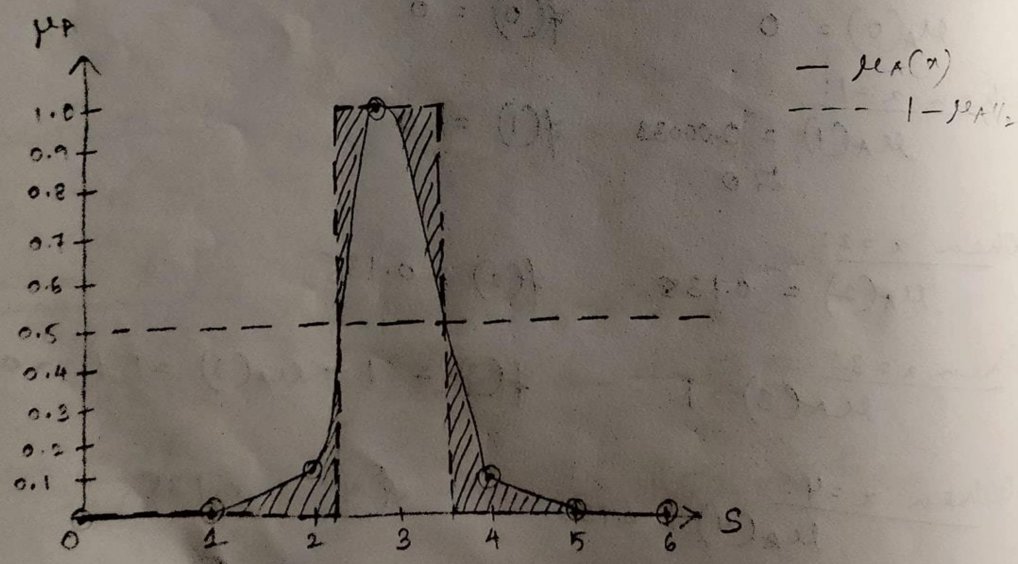
$\mu_{A1/2}$ = 1/2 cut of A

So,

$$\mu_{1/2} = 1 \quad \text{when } \mu_A(x) \geq 0.5$$

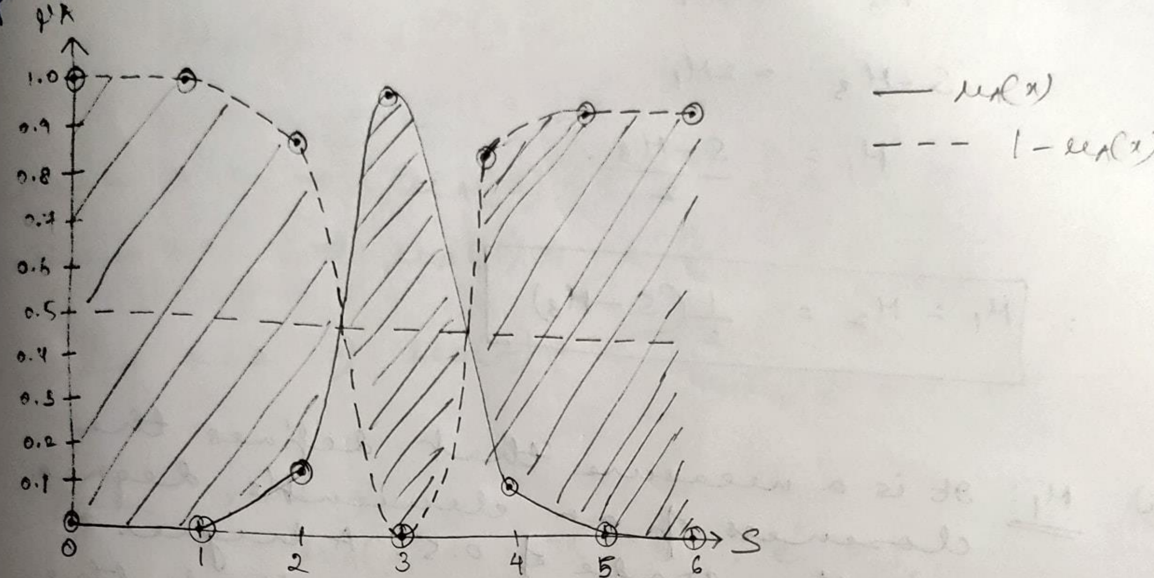
$$= 0 \quad \text{when } \mu_A(x) < 0.5$$

Graph for M_2:



$$M_3 = \int_S |\mu_A(x) - \mu_{\bar{A}}(x)|\, dx$$

graph of M_3:



(i) Relationship between M_1 and M_2:-

$$M_1 = \int_S f(x)\, dx \quad \text{where } f(x) = \begin{cases} \mu_A(x), & x \geq 0.5 \\ 1-\mu_A(x), & x < 0.5 \end{cases}$$

$$M_2 = \int_S |\mu_A(x) - \mu_{A1/2}(x)|\, dx$$

Since, $\mu_{A1/2} = 1$ when $\mu_A(x) \geq 0.5$

$$M_2 = \int_S |\mu_A(x) - 1|\, dx$$

$$M_1 = \int (1 - \mu_A(x))\, dx$$

And: $\mu_{A1/2} = 0$ when $\mu_A(x) < 0.5$

$$M_2 = \int_S \mu_A(x) \cdot dx$$

$$M_1 = \int_S \mu_A(x)\, dx$$

$$\therefore \boxed{M_1 = M_2}$$

Comparing graphs of M_3 and M_1 :-

$$\overline{M_3} = 2M_1$$

$$S - M_3 = 2M_1$$

$$M_1 = \frac{S - M_3}{2}$$

$$\therefore \boxed{M_1 = M_2 = \frac{1}{2}(S - M_3)}$$

(ii) **M_1**: It is a measure that defines the closeness of an element's degree to the grade of 0.5. A higher value of M1 indicates that the element's categorization into the set is less certain. Lower value indicates that it is more certain.

M2: This measure quantifies the width or spread of the fuzzy set around 0.5 (the ½-cut). Spread of membership reflects the level of fuzziness/uncertainty in the system.

A larger value of M2 indicates wider spread around ½ cut and higher ambiguity. Smaller value indicates less fuzziness.

M3: This measure indicates proximity of an element to the boundary between fuzzy set and its complement. It shows how close an element is to the edge of the set.

A larger value indicates that the element's membership degree is closer to the 'boundary and is more fuzzy. Smaller value shows that the degree of membership is more certain.

- Compared to M1, M2 focuses more on the distribution of membership ~~functions~~ values and the width of it around the 1/2-cut. M1 focuses on proximity to the midpoint and element's categorisation within the set.

iii) Given: $\lambda = 1$, $a = 3$, $n = 2$

$$S = [0, 6]$$

$$\mu_A(x) = e^{-\lambda (x-a)^n}$$

$$= e^{-1(x-3)^2}$$

when $x = 0 \rightarrow \mu_A(0) = e^{-1(9)} = 0.00012 \simeq 0$

when $x = 1 \Rightarrow \mu_A(1) = e^{-1(1-3)^2} = e^{-4} = 0.018$

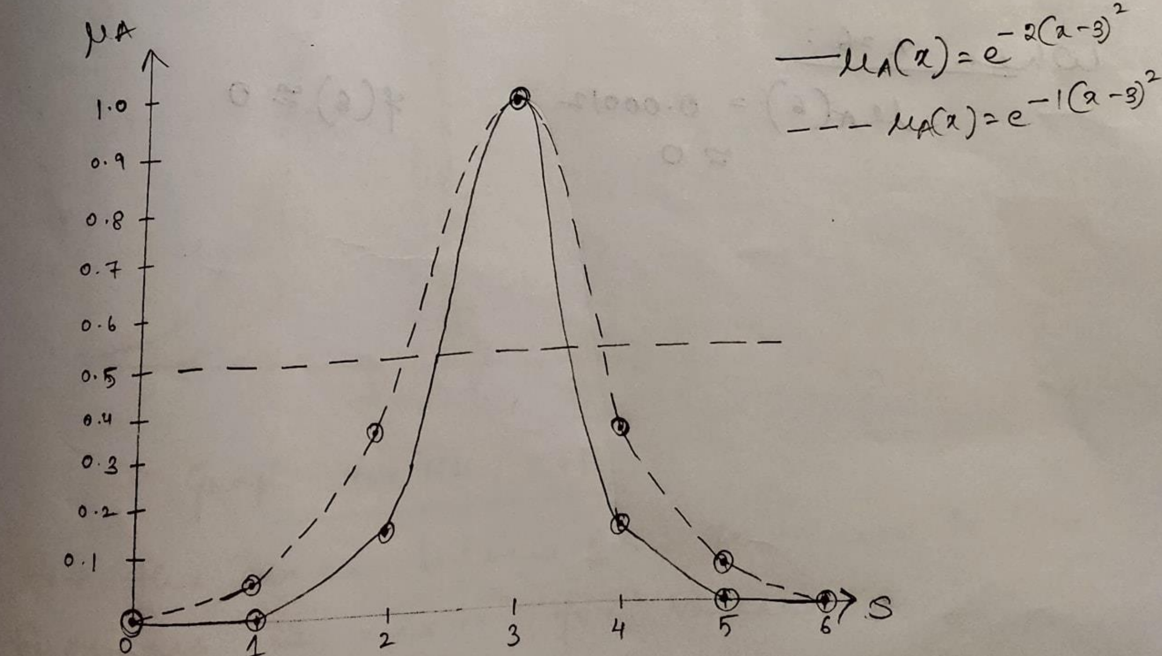
when $x = 2 \Rightarrow \mu_A(2) = e^{-1(2-3)^2} = e^{-1(+1)} = e^{-1} = 0.367$

when $x = 3 \Rightarrow$
$\mu_A(3) = e^{-1(3-3)^2} = e^{0} = 1$

when $x = 4 \Rightarrow \mu_A(4) = e^{-1(4-3)^2} = e^{-1} = 0.367$

when $x = 5 \Rightarrow \mu_A(5) = e^{-1(5-3)^2} = e^{-4} = 0.018$

when $x = 6 \Rightarrow \mu_A(6) = e^{-1(6-3)^2} = e^{-9} = 0.00012 \simeq 0.$



$$M_1 = \int_S f(x)\, dx$$

where,

$$f(x) = \begin{cases} \mu_A(x) & , \mu_A(x) \leq 0.5 \\ 1 - \mu_A(x) & , \mu_A(x) > 0.5 \end{cases}$$

<u>When $x = 0$:</u>

$\mu_A(0) = 0$ $\quad$ $f(0) = 0$

<u>When $x = 1$:</u>

$\mu_A(1) = 0.018$ $\quad$ $f(1) = 0.018$

<u>When $x = 2$:</u>

$\mu_A(2) = 0.367$ $\quad$ $f(2) = 0.367$

<u>When $x = 3$:</u>

$\mu_A(3) = 1$ $\quad$ $f(3) = 1 - 1 = 0$

<u>When $x = 4$:</u>

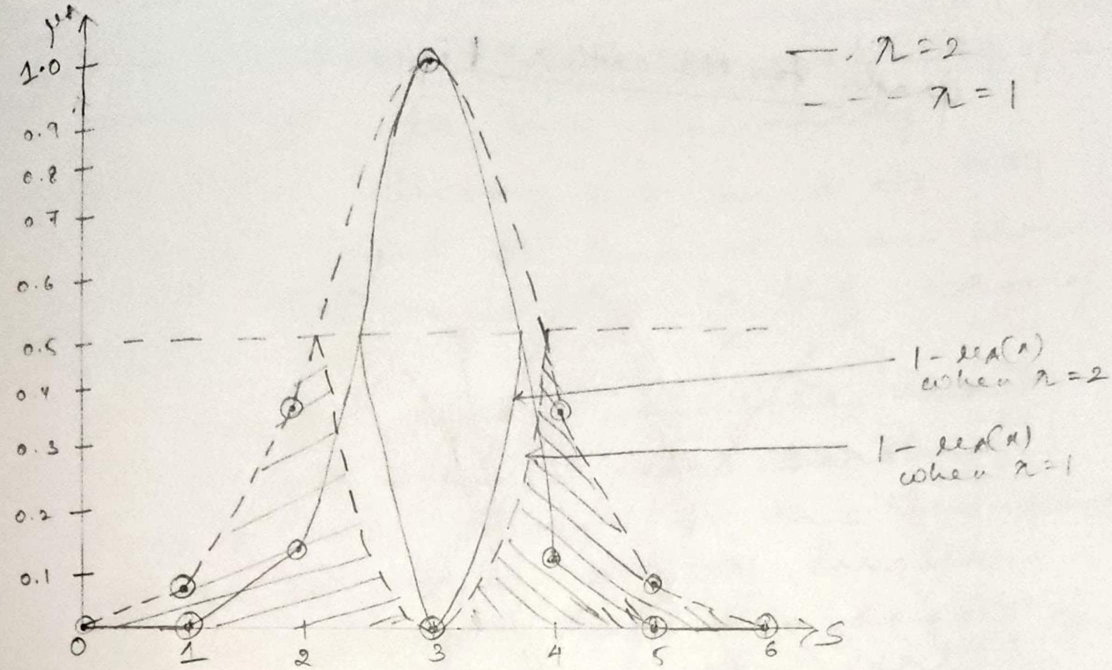
$\mu_A(4) = 0.367$ $\quad$ $f(4) = 0.367$

<u>When $x = 5$:</u>

$\mu_A(5) = 0.018$ $\quad$ $f(5) = 0.018$

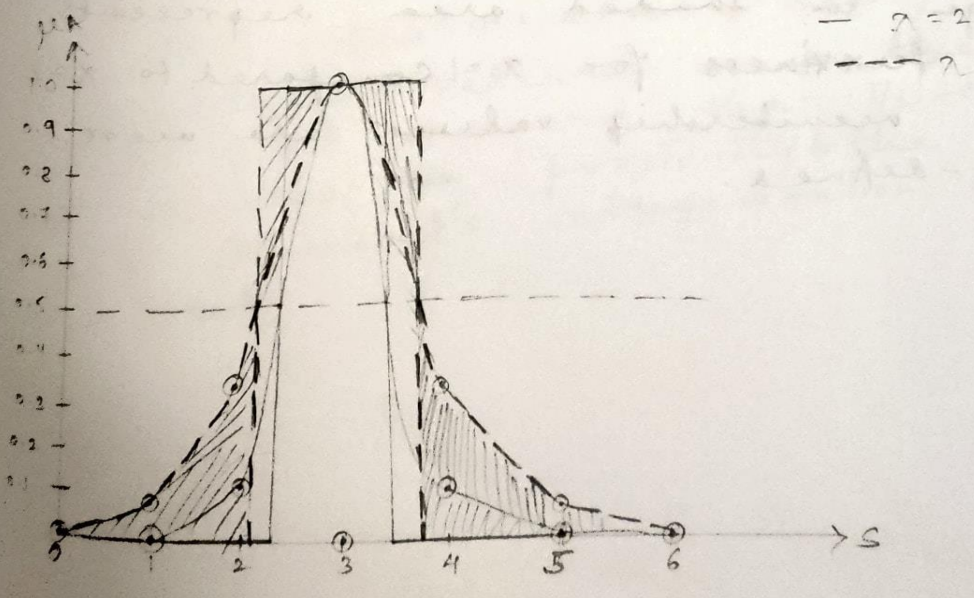
<u>When $x = 6$:</u>

$\mu_A(6) = 0.00012 \simeq 0$ $\quad$ $f(6) \simeq 0$



Graph for M1 when $x=1$.

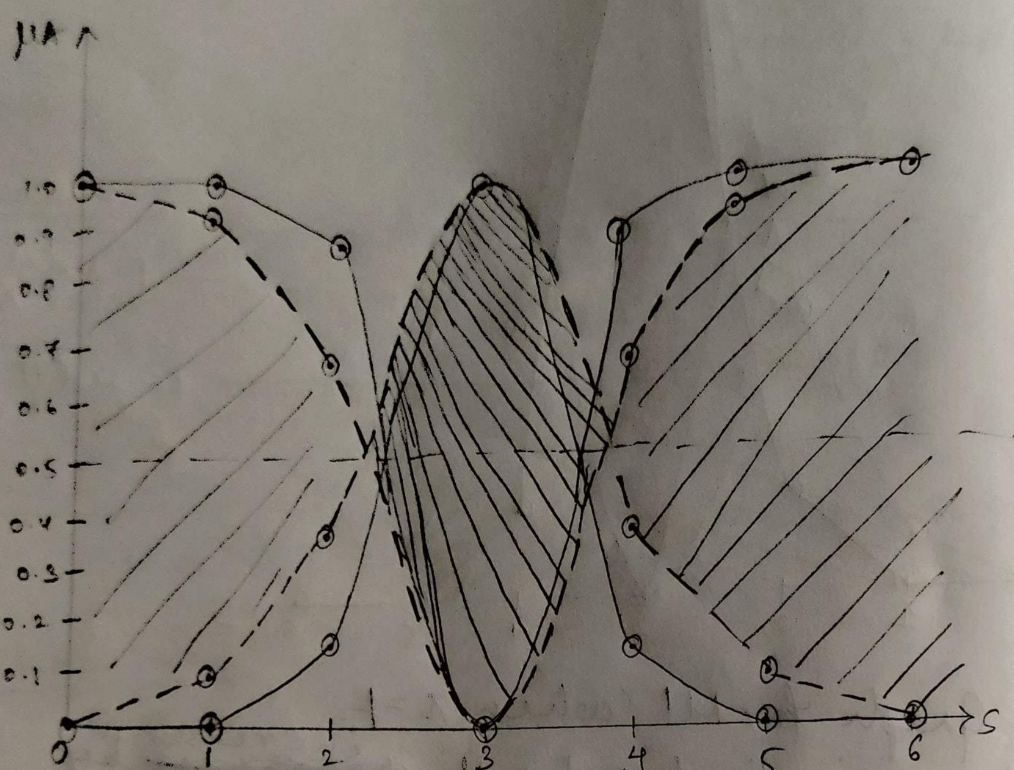
→ Fuzziness indicated by M_1 increases by reducing x.



Graph for M2, $x=1$.

For the area below 0.5, fuzziness is more. Fuzziness is less for values greater than 0.5.

Graph for MS with $x = 1$:



Change in shaded area represents less fuzziness for $x=1$ compared to $x=2$.