Problem 4

$$\max(0, x+y-1)$$

Checking the above on 4 T-norm operator properties

1) Boundary conditions -

i) Assume $x = x_1$ and $y = 0$
$\max(0, x_1 + 0 - 1) = 0$

ii) Assume $x = x_1$ and $y = 1$
$\max(0, x_1 + 1 - 1) = x$

Both boundary conditions satisfy

2) Commutativity -

$$\max(0, x+y-1) = \max(0, y+x-1)$$

3) Associativity -

LHS $= \max(0, x + \max(0, y+z-1) - 1)$

RHS $= \max(0, \max(0, x+y-1) + z - 1)$

Case 1 - $y+z-1 > 0$ & $x+y-1 > 0$

so,

LHS = $\max(0,\ x+y+z-2)$

RHS = $\max(0,\ x+y+z-2)$

LHS = RHS

<u>Case 2</u> – $x+y-1 \leq 0$ and $y+z-1 \leq 0$

LHS = $\max(0,\ x-1)$

RHS = $\max(0,\ z-1)$

since x and z range between $[0,1]$ so both LHS and RHS will always be zero.

<u>Case 3</u> – $x+y-1 > 0$ and $y+z-1 \leq 0$

RHS = $\max(0,\ x+y+z-2)$

LHS = $\max(0,\ x-1)$

$\rightarrow$ LHS will alway be zero since x is $[0,1]$

For RHS,

$\max(0,\ x+y+z-1-1)$

From our initial condition $y+z-1 \leq 0$

$$\max(0, \underbrace{x}_{[0,1]} + \underbrace{y+z-1}_{\leq 0} \underbrace{-1}_{[1]})$$

we see from above the range of values will be ~~from~~ ≤ 0.

hence $\max(0, \underbrace{x+y+z-1-1}_{\leq 0}) \equiv 0$

$\therefore$ LHS = RHS for all 3 cases,
So the operator follows associativity

4) Monotonicity -

assume $x \leq x'$ and $y \leq y'$

$\therefore \quad x + y \leq x' + y'$

adding -1 on both sides keeps the equality same

$$x+y-1 \leq x'+y'-1$$

$$\max(0, x+y-1) \leq \max(0, x'+y'-1)$$

Hence the above operator is T-norm since it satisfies all 4 properties.

To determine it's t-conorm (i.e. s-norm) we can use DeMorgan's law -

$$a S b = 1 - (1-a) T (1-b)$$

$\therefore$ the operator becomes

$$= 1 - \max(0, 1-x+1-y-1)$$

$$= 1 - \max(0, 1-(x+y))$$

So, $1 - \max(0, 1-(x+y))$ is the corresponding t-conorm or s-norm for the given operator.