
# Rockets and Fuel Exhaustion

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# The Problem Statement

# The Problem Explanation

# Assumptions/Related Concepts

## Assumptions

- Wind/Air resistance is negligible
- Earth is not spinning for the duration of the trip
- The rocket is treated as a particle
- Rocket thrust is constant
- Gravity is a part of our system

## Related Concepts

- Mass/Flow Rate
- Projectile motion
- Linear Momentum
- Gravity
- Newton's second law

# Knowns/Unknowns

## Known

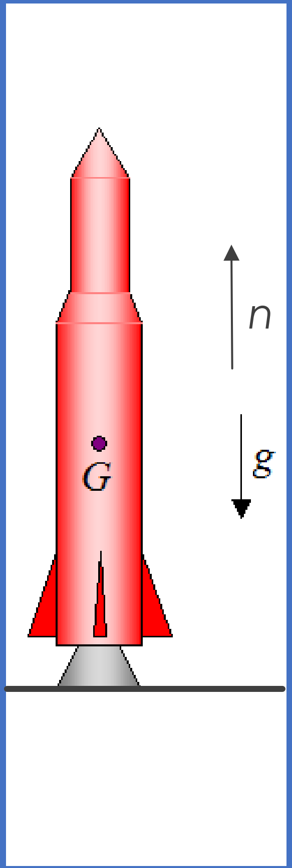
- Height (Rocket: 70m)
- Diameter (Rocket: 3.7m)
- Mass (Rocket: 549,054kg)
- Fuel ejected per second (1,451.5kg/s)
- Thrust time of stage 1 (162s)
- Thrust of Stage 1 (7686kN)
- Radius of earth (6731.009km)

## Unknown

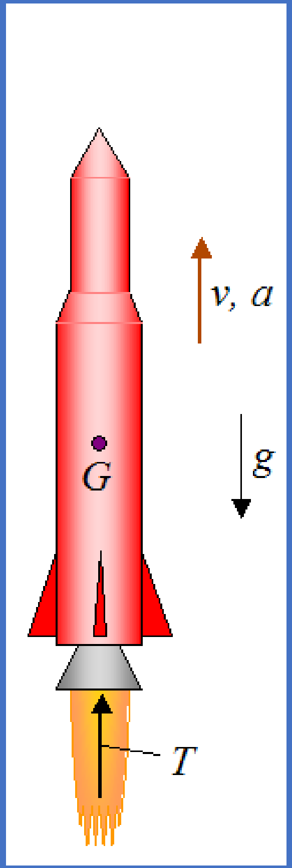
- Amount of fuel in rocket
- Change in gravitational acceleration
- Velocity at end of stage 1
- Acceleration at end of stage 1
- Distance traveled at the end of stage 1

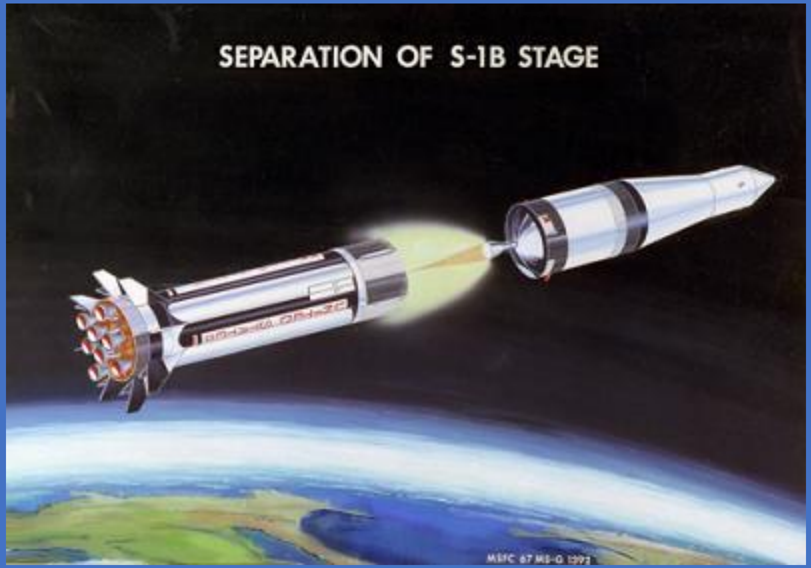
# Free Body Diagrams

## Rocket Initial




## Rocket Final



# Definition of Variables

| | | | |
|---|---|---|---|
| $m$ | $m_{dot}$ | $V$ | $a$ |
| $V_{exh}$ | $T$ | $Re$ | $d = h$ |

# Equations Used

$$v_f = v_i + v_e \ln \frac{M_i}{M_f}$$

Velocity equation (Vi = 0)

$$g_{(\text{altitude})} = g\left(\frac{r_e}{r_e + \text{h}}\right)^2$$

Acceleration of gravity at given altitude above sea level

$$\text{Thrust} = v_{exh} \, [\, dM / dt \,]$$

Thrust equation of the rocket

$$\sum F_y : \; T - mg = ma$$

Sum of forces in y-direction

$$F = ma$$

Acceleration equation

$$\dot{m} = \frac{dm}{dt}$$

Mass flow rate

$$d = v \cdot t$$

Distance equation

# Calculations

- M = mass of rocket = 549,054kg

- Me = mass of rocket exhaust that has already exited ->

  1,451.5kg/s * 162s = 235143kg @ 162s (0 at 0s) = dme (Mass-Flow Rate)

- Thrust is given to us. $\text{Thrust} = v_{exh}\,[\, dM / dt \,]$ -> T = 7686kN

$$P_{final} = P_{init}$$

$$m\,v + m\,dv + |dm|\,v - |dm|\,v_e = m\,v + |dm|\,v$$

$$m\,v + m\,dv + |dm|\,v - |dm|\,v_e = m\,v + |dm|\,v$$

$$m\,dv - |dm|\,v_e = 0$$

$$m\,dv + dm\,v_e = 0$$

$$m\,dv = -\,v_e\,dm$$

$$dv = -\,v_e\,[\,dm/m\,]$$

$$dv = -\,v_e\,\frac{dm}{m}$$

$$\int dv = -\,v_e \int \frac{dm}{m}$$

$$\int_{v_i}^{v_f} dv = -\,v_e \int_{M_i}^{M_f} \frac{dm}{m}$$

$$v\Big]_{v_i}^{v_f} = -\,v_e\,\ln m\Big]_{M_i}^{M_f}$$

$$v_f - v_i = -\,v_e\,[\,\ln M_f - \ln M_i\,]$$

$$v_f - v_i = v_e\,(\ln M_i - \ln M_f)$$

$$v_f = v_i + v_e\,\ln \frac{M_i}{M_f}$$

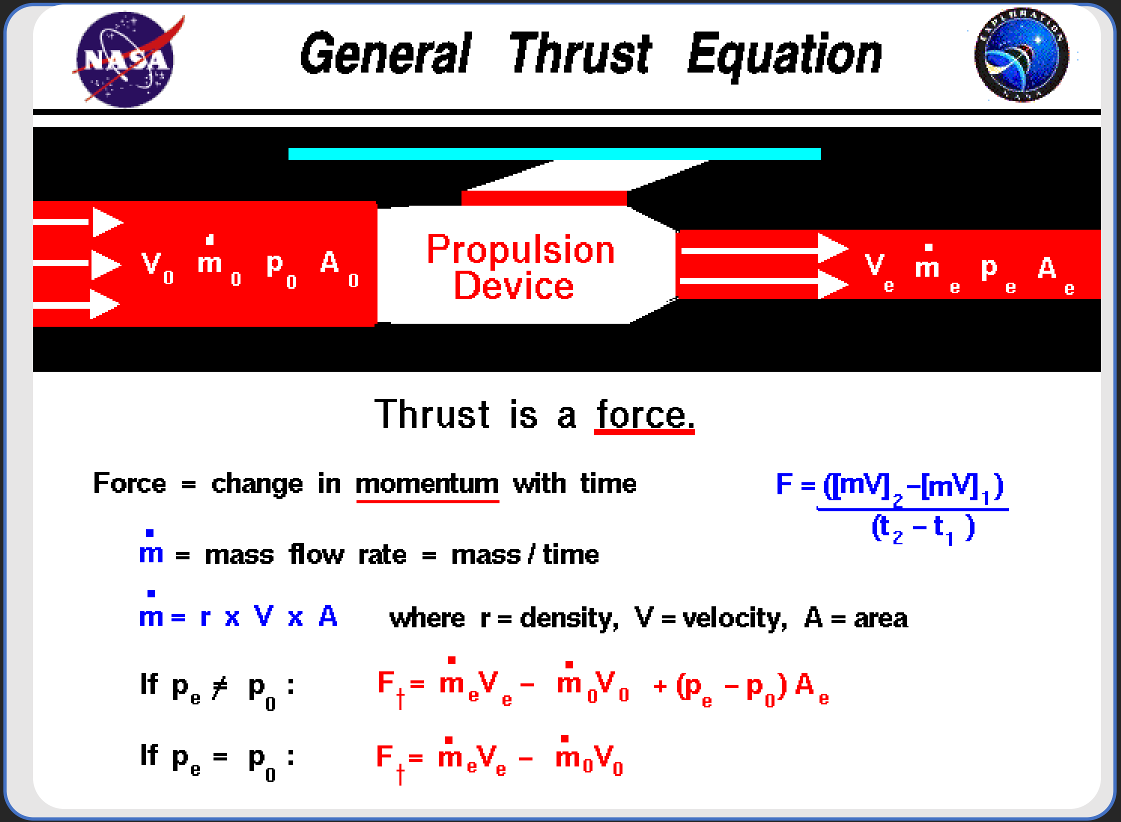
This is the concept of conservation of linear momentum to solve for the velocity

# Calculations

$$\text{Thrust} = v_{exh} \, [\, dM / dt \,]$$

$$v_f = v_i + v_e \ln \frac{M_i}{M_f}$$

- V = u*ln(549054/(549054-1451.5*T))+0
- U = ve (velocity of the exhaust gases relative to the rocket, which is constant)
- Mdot = 1451.5kg/s (Mass flow rate)
- $F_t$ = thrust = 7686kN
- Ve = 7686/1451.5 = 5.295
- V = 5.295 *ln(549054/(549054-1451.5*T))



# Calculations

Units

| | |
|---|---|
| g[km/s] | gravity |
| h[km] | height |
| F[kN] | Sum Force y |
| a[km/s^2] | acceleration |
| v[km/s] | velocity |
| t[s] | time |

## Initial

$$g = 0.00980665\left(\frac{6371.009}{6371.009+h}\right)^2$$

$g = 0.00980665$

$$h = v \cdot t$$

$h = 0$

$$F = 7686 \cdot t - ((549054 - 1451.5 \cdot t) \cdot g)$$

$F = -5384.3804091$

$$a = \frac{F}{(549054 - 1451.5 \cdot t)}$$

$a = -0.00980665$

$$v = 5.295 \cdot \ln\left(\frac{549054}{549054 - 1451.5 \cdot t}\right)$$

$v = 0$

$t = 0$

0 162

## Final

$$g = 0.00980665\left(\frac{6371.009}{6371.009+h}\right)^2$$

$g = 0.0084816696256$

$$h = v \cdot t$$

$h = 479.579487421$

$$F = 7686 \cdot t - ((549054 - 1451.5 \cdot t) \cdot g)$$

$F = 1242469.51061$

$$a = \frac{F}{(549054 - 1451.5 \cdot t)}$$

$a = 3.95803113177$

$$v = 5.295 \cdot \ln\left(\frac{549054}{549054 - 1451.5 \cdot t}\right)$$

$v = 2.96036720631$

$t = 162$

0 162

# Simulation

$$g = 0.00980665\left(\frac{6371.009}{6371.009 + h}\right)^2$$

$g = 0.00980665$

$$h = v \cdot t$$

$h = 0$

$$F = 7686 \cdot t - \left(\left(549054 - 1451.5 \cdot t\right) \cdot g\right)$$

$F = -5384.3804091$

$$a = \frac{F}{\left(549054 - 1451.5 \cdot t\right)}$$

$a = -0.00980665$

$$v = 5.295 \cdot \ln\left(\frac{549054}{549054 - 1451.5 \cdot t}\right)$$

$v = 0$

$t = 0$

0 162

# Conclusion

Distance from Surface at 162s: 479.580km

Acceleration at 162s: 3.958km/s^2

Velocity at 162s: 2.960km/s

# Realistic scenario

- Do not include
  - Drag/air resistance
  - the rotation of the earth
  - change in thrust with changing pressure and altitude

However, our scenario emulates the goal of the stage one rocket:

The Earth's atmosphere is about 300 miles (480 km) thick, but most of the atmosphere (about 80%) is within 10 miles (16 km) of the surface of the Earth. There is no exact place where the atmosphere ends; it just gets thinner and thinner, until it merges with outer space.

With the predicted height of 479.58 km, Rocket -> Space

# Sources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