


A composite image featuring Elon Musk sitting on the nose of a SpaceX Falcon Heavy rocket. He is wearing a grey hoodie and jeans, holding a cowboy hat in his left hand. The rocket is white with the 'SPACEX' logo in blue. The background shows the Earth's horizon and a starry space sky.

Rockets and Fuel Exhaustion

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A photograph of a Space Shuttle launching from the launch pad. The shuttle is ascending vertically, leaving a large, bright orange and white plume of fire and smoke. To the left of the shuttle is a tall, dark service structure. To the right is a large, white building with a blue 'X' logo. The sky is clear and blue.

The Problem Statement



The Problem Explanation

Assumptions/Related Concepts

Assumptions

- Wind/Air resistance is negligible
- Earth is not spinning for the duration of the trip
- The rocket is treated as a particle
- Rocket thrust is constant
- Gravity is a part of our system

Related Concepts

- Mass/Flow Rate
- Projectile motion
- Linear Momentum
- Gravity
- Newton's second law



Knowns/Unknowns

Known

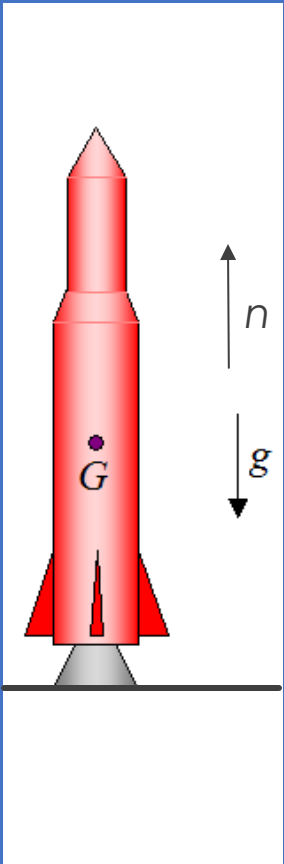
- Height (Rocket: 70m)
- Diameter (Rocket: 3.7m)
- Mass (Rocket: 549,054kg)
- Fuel ejected per second (1,451.5kg/s)
- Thrust time of stage 1 (162s)
- Thrust of Stage 1 (7686kN)
- Radius of earth (6731.009km)

Unknown

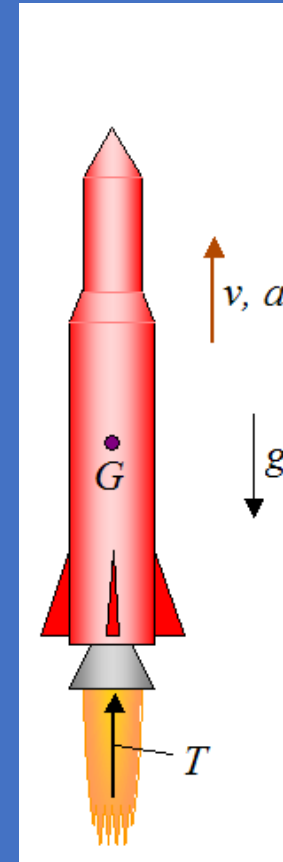
- Amount of fuel in rocket
- Change in gravitational acceleration
- Velocity at end of stage 1
- Acceleration at end of stage 1
- Distance traveled at the end of stage 1

Free Body Diagrams

Rocket Initial



Rocket Final



Definition of Variables

m

m_{dot}

v

a

v_{exh}

T

Re

$d = h$

Equations Used

$$v_f = v_i + v_e \ln \frac{M_i}{M_f}$$

Velocity equation ($v_i = 0$)

$$g_{(\text{altitude})} = g \left(\frac{r_e}{r_e + h} \right)^2$$

Acceleration of gravity
at given altitude above
sea level

$$\text{Thrust} = v_{\text{exh}} [dM / dt]$$

Thrust equation of
the rocket

$$\sum F_y: T - mg = ma$$

Sum of forces in
y-direction

$$F = ma$$

Acceleration equation

$$\dot{m} = \frac{dm}{dt}$$

Mass flow rate

$$d = v \cdot t$$

Distance equation

Calculations

- M = mass of rocket = 549,054kg
- M_e = mass of rocket exhaust that has already exited ->
 $1,451.5\text{kg/s} * 162\text{s} = 235143\text{kg}$ @ 162s (0 at 0s) = dme
 (Mass-Flow Rate)
- Thrust is given to us.

$$\text{Thrust} = v_{\text{exh}} [dM / dt]$$

$$\rightarrow T = 7686\text{kN}$$

$$P_{\text{final}} = P_{\text{init}}$$

$$m v + m dv + |dm| v - |dm| v_e = m v + |dm| v$$

$$m v + m dv + |dm| v - |dm| v_e = m v + |dm| v$$

$$m dv - |dm| v_e = 0$$

$$m dv + dm v_e = 0$$

$$m dv = - v_e dm$$

$$dv = - v_e [dm/m]$$

$$dv = - v_e \frac{dm}{m}$$

$$\int dv = - v_e \int \frac{dm}{m}$$

$$\int_{v_i}^{v_f} dv = - v_e \int_{M_i}^{M_f} \frac{dm}{m}$$

$$\left[v \right]_{v_i}^{v_f} = - v_e \left[\ln m \right]_{M_i}^{M_f}$$

$$v_f - v_i = - v_e [\ln M_f - \ln M_i]$$

$$v_f - v_i = v_e (\ln M_i - \ln M_f)$$

$$v_f = v_i + v_e \ln \frac{M_i}{M_f}$$


This is the concept of conservation of linear momentum to solve for the velocity

Calculations


$$\text{Thrust} = v_{\text{exh}} \left[\frac{dM}{dt} \right]$$

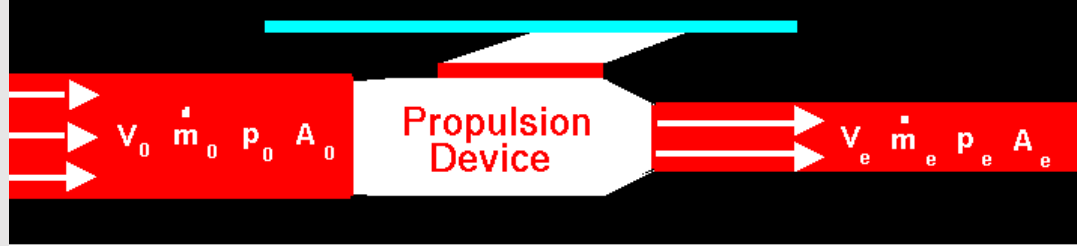
$$v_f = v_i + v_e \ln \frac{M_i}{M_f}$$

- $V = u \cdot \ln(549054 / (549054 - 1451.5 \cdot T)) + 0$
- $U = v_e$ (velocity of the exhaust gases relative to the rocket, which is constant)
- $\dot{M} = 1451.5 \text{ kg/s}$ (Mass flow rate)
- $F_t = \text{thrust} = 7686 \text{ kN}$
- $v_e = 7686 / 1451.5 = 5.295$
- $V = 5.295 \cdot \ln(549054 / (549054 - 1451.5 \cdot T))$



General Thrust Equation





Thrust is a force.

Force = change in momentum with time $F = \frac{([mV]_2 - [mV]_1)}{(t_2 - t_1)}$

\dot{m} = mass flow rate = mass / time

$\dot{m} = \rho \times V \times A$ where ρ = density, V = velocity, A = area

If $p_e \neq p_0$: $F_t = \dot{m}_e V_e - \dot{m}_0 V_0 + (p_e - p_0) A_e$

If $p_e = p_0$: $F_t = \dot{m}_e V_e - \dot{m}_0 V_0$

Calculations

Units

g[km/s]	gravity
h[km]	height
F[kN]	Sum Force y
a[km/s^2]	acceleration
v[km/s]	velocity
t[s]	time

Initial

$$g = 0.00980665 \left(\frac{6371.009}{6371.009 + h} \right)^2$$

$g = 0.00980665$

$$h = v \cdot t$$

$h = 0$

$$F = 7686 \cdot t - ((549054 - 1451.5 \cdot t) \cdot g)$$

$F = -5384.3804091$

$$a = \frac{F}{(549054 - 1451.5 \cdot t)}$$

$a = -0.00980665$

$$v = 5.295 \cdot \ln \left(\frac{549054}{549054 - 1451.5 \cdot t} \right)$$

$v = 0$

$$t = 0$$

0

162

Final

$$g = 0.00980665 \left(\frac{6371.009}{6371.009 + h} \right)^2$$

$g = 0.0084816696256$

$$h = v \cdot t$$

$h = 479.579487421$

$$F = 7686 \cdot t - ((549054 - 1451.5 \cdot t) \cdot g)$$

$F = 1242469.51061$

$$a = \frac{F}{(549054 - 1451.5 \cdot t)}$$

$a = 3.95803113177$

$$v = 5.295 \cdot \ln \left(\frac{549054}{549054 - 1451.5 \cdot t} \right)$$

$v = 2.96036720631$

$$t = 162$$

0

162

Simulation

The simulation interface consists of a vertical list of seven rows, each containing an equation and a corresponding value box. The rows are numbered 1 through 7 on the left. Each equation has a close button (X) in the top right corner. The value boxes are located at the bottom right of each equation's row.

- Row 1: Equation $g = 0.00980665 \left(\frac{6371.009}{6371.009 + h} \right)^2$. Value box: $g = 0.00980665$.
- Row 2: Equation $h = v \cdot t$. Value box: $h = 0$.
- Row 3: Equation $F = 7686 \cdot t - ((549054 - 1451.5 \cdot t) \cdot g)$. Value box: $F = -5384.3804091$.
- Row 4: Equation $a = \frac{F}{(549054 - 1451.5 \cdot t)}$. Value box: $a = -0.00980665$.
- Row 5: Equation $v = 5.295 \cdot \ln \left(\frac{549054}{549054 - 1451.5 \cdot t} \right)$. Value box: $v = 0$.
- Row 6: Control row with a play button icon, the text $t = 0$, a slider bar from 0 to 162 with a blue dot at 0, and a close button (X).
- Row 7: A hand cursor icon.

Conclusion



Distance from Surface at
162s: 479.580km



Acceleration at 162s:
 3.958km/s^2



Velocity at 162s: 2.960km/s

Realistic scenario

- Do not include
 - Drag/air resistance
 - the rotation of the earth
 - change in thrust with changing pressure and altitude

However, our scenario emulates the goal of the stage one rocket:

The Earth's atmosphere is about 300 miles (480 km) thick, but most of the atmosphere (about 80%) is within 10 miles (16 km) of the surface of the Earth. There is no exact place where the atmosphere ends; it just gets thinner and thinner, until it merges with outer space.

With the predicted height of 479.58 km, Rocket -> Space

Sources

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