THE KNUCKLE BALL

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THE PROBLEM

A soccer player is taking a penalty kick. He is aiming for a spot on the goal (11i, 7j, 2k) meters and needs to figure out how hard he must kick the ball to get it there.

He kicks the ball and notices that the ball doesn't go exactly where he aimed. Calculate the air resistance and its effect on the ball.

THE PROBLEM EXPLANATION

Soccer Player Jonah is aiming for a spot in the goal and is trying to figure out how hard he will have to kick it. He will be kicking the ball at 15 degrees using a technique called a knuckleball, which results in the ball having no spin as it travels through the air. He will be hitting the ball from rest, for the penalty kick.

First, we will calculate the velocity and time required without factoring for air resistance, then calculate for air resistance to figure out where it would exactly land.

ASSUMPTIONS/RELATED CONCEPTS

Assumptions

No wind for part 1

No spin on ball

Altitude 1000 ft above sea level

Goal is 2.438m by 7.31m

Ball is treated as particle in part 1

Related concepts

We will be using our knowledge of velocity, acceleration, displacement, and cartesian vectors

We will also use our newly acquired knowledge of air resistance and how it will affect the soccer ball

KNOWNS AND UNKNOWNS

Known:

The Dimensions of the Goal (11i, 7j, 2k)

Mass of Soccer Ball: 450g = 0.45Kg

Diameter of Soccer ball: 22cm = 0.22m

Air pressure: 101.325kPa (270M altitude, 70F)

Ball is kicked at 15 degrees

Unknown:

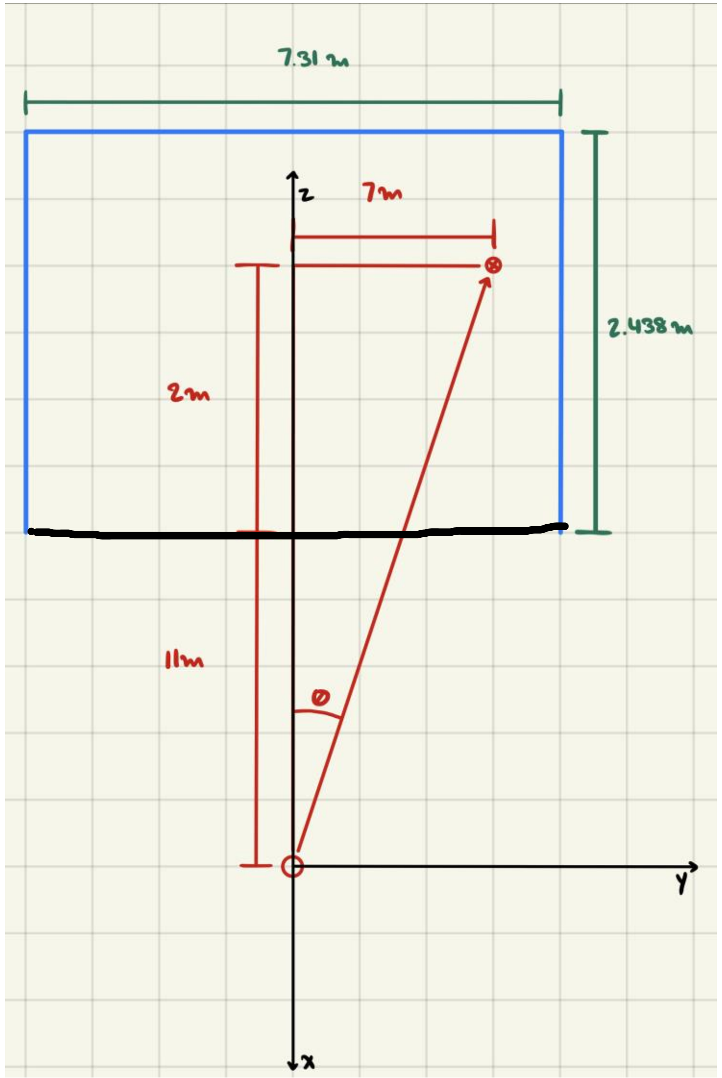
Air Resistance of Ball

Initial Velocity

Time to Reach Goal

SCHEMATICS

The recommended size of a Professional Soccer Goal is 8ft by 24 ft, which is 2.438m by 7.31m. Our goal is to figure out the initial velocity required of the ball to get it to the spot (11i, 7j, 2k)m and how long it would take.

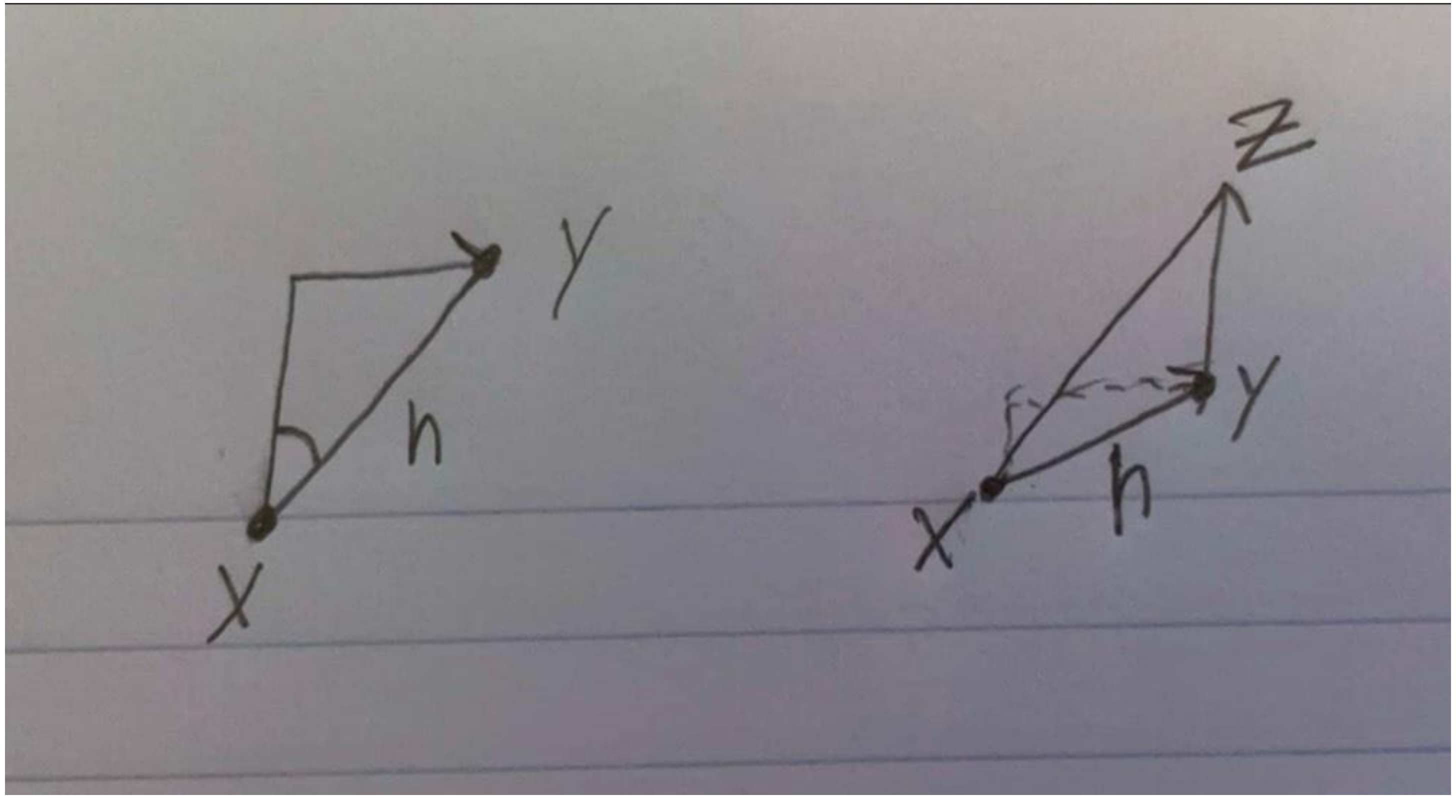


SCHEMATICS

Kinematics, no need for FBD

In order to make this 3D problem projectile motion, we need to create a new axis and make it easier to understand.

So, by combining the X-Y axis, it can now be considered a 3d problem (H-Axis)



INITIAL VELOCITY EQUATION

T = ? seconds

Vi = ? m/s

Z = 2m, X = 11m, Y = 7m

Theta = 15 degrees

$h = sqrt(X^2 + Y^2)$

$h = V_i*cos(Theta)*t + (½)*(a)*t$^2 $a = 0$

$Z = V_i*sin(Theta)*t + (½)*(g)*t$^2 $g = -9.81$

CALCULATIONS (NO AIR RESISTANCE)

Time

h = sqrt($11^2 + 7^2$) = 13.038m = sqrt(170)m

Sqrt(170) = V_i*cos(15)*(t) + (½)*(0)*$(t)^2$

2 = V_i*sin(15)*(t) + (½)*(-9.81)*$(t)^2$

Vi = sqrt(170)/(cos(15)*(t))

2 = (sqrt(170)/(cos(15)*(t)))*sin(15)*(T) + (½)*(-9.81)*$(T)^2$

T = 0.55 s

Velocity

T = 0.55 s

Sqrt(170) = V_i*cos(15)*(t) + (½)*(0)*(t)^2

Sqrt(170) = V^i*cos(15)*(.55) + (½)*(0)*(.55)

Vi = 24.54 m/s

REAL WORLD COMPARISON

In a game of professional soccer, the average penalty kick goes flying towards the goal at a speed of 70 miles per hour (31.29 m/s). Given where the shot must be made from, that means that the ball can reach the net in less than .400 seconds.

Considering our player is not a professional athlete, a kick with speeds 24.54 m/s and a time of .55 seconds is reasonable.

THE ACTUAL SITUATION

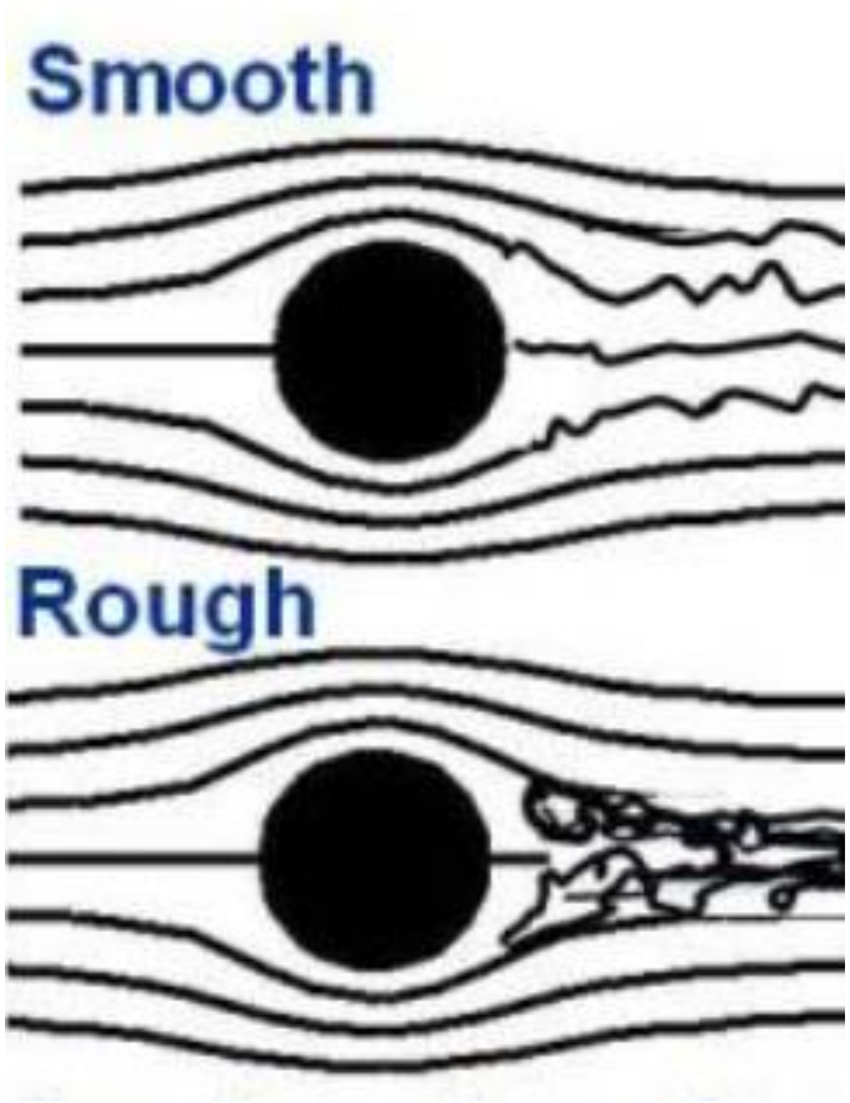
In theory, as long as we kick the football with the speed of Vi at 15°, the football must be able to sit on the coordinates of (11i, 7j, 2k). However, since we cannot ignore the influence of air resistance on the ball, the ball cannot accurately land on the established coordinates. So where will the ball land?

EQUATIONS (AIR RESISTANCE)

R(Newtons) = 0.5 * D * A * rho * V^2

D = drag coefficient, A = cross sectional area m^2, rho = air density, v = velocity

We know that whether the wind is steady or not will have a great impact on the air resistance, so we only consider the steady wind speed state here



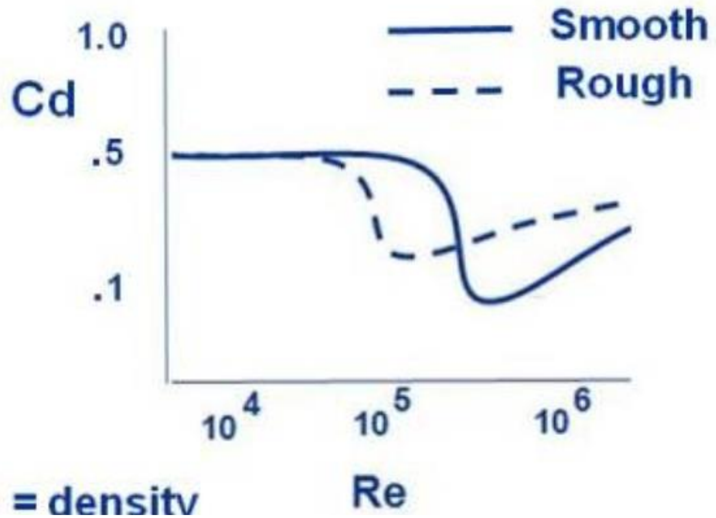
THE SURVEY DATA

$$\text{Reynolds Number} = Re = \frac{\rho V d}{\mu} \sim 3.0 \times 10^5$$

$$\text{Drag Coefficient} = Cd = \frac{2D}{\rho V^2 A} = .25$$

We compared the wind resistance coefficients of different soccer balls at different speeds at standard atmospheric pressure during the World Cup.

By comparison, we have just obtained Vi, which is about 88km/h, so we adopt the wind resistance coefficient of D=0.25



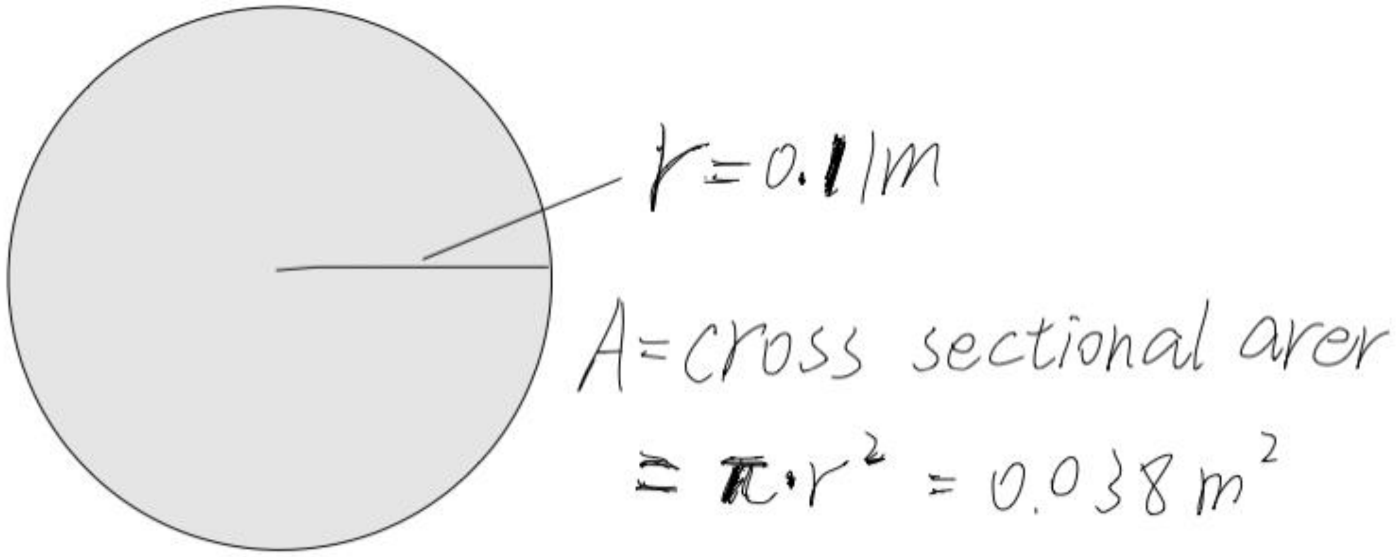
THE SURVEY DATA

The cross-sectional area of the football

$A=0.038m^2$

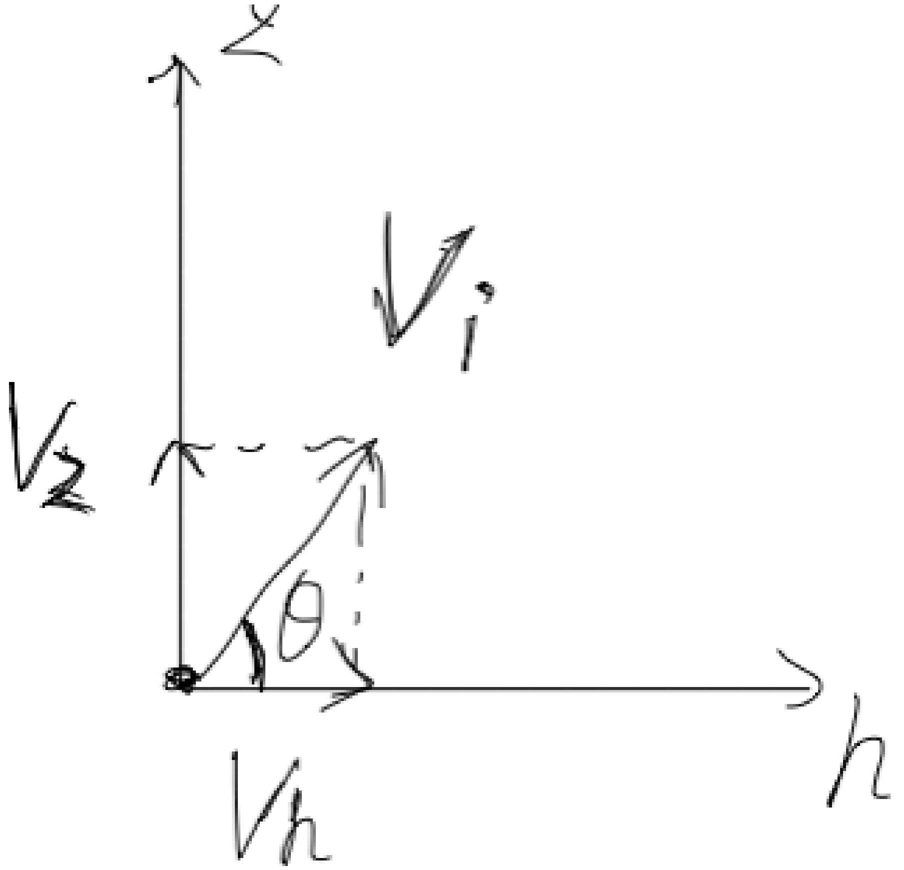
By looking up the table, the air density we selected was 1.293 kg/m^3

Rho=1.293 kg/m^3



R(Newtons) = 0.5 * D * A * rho * V^2

$R=0.5*0.15*0.038*1.293V^2=0.0037V^2$



Wind resistance is in the opposite direction of speed.We decompose the velocities into the H axis and the Z axis.

We study the motion in two directions separately, and then there is wind resistance in both directions separately

The magnitude and direction of the wind resistance are changing all the time. According to our existing knowledge, we cannot establish an accurate mathematical model, so we stipulate that the magnitude and direction of the wind resistance are always equal to the initial moment.

EQUATIONS AND CALCULATION

Simplified after wind resistance: $R=2.23N$

H axis: $h = Vi*\cos(Theta)*t + (½)*(a1)*t^2$

$a1 = R*\cos(theta)/m=4.79\ m/s^2$

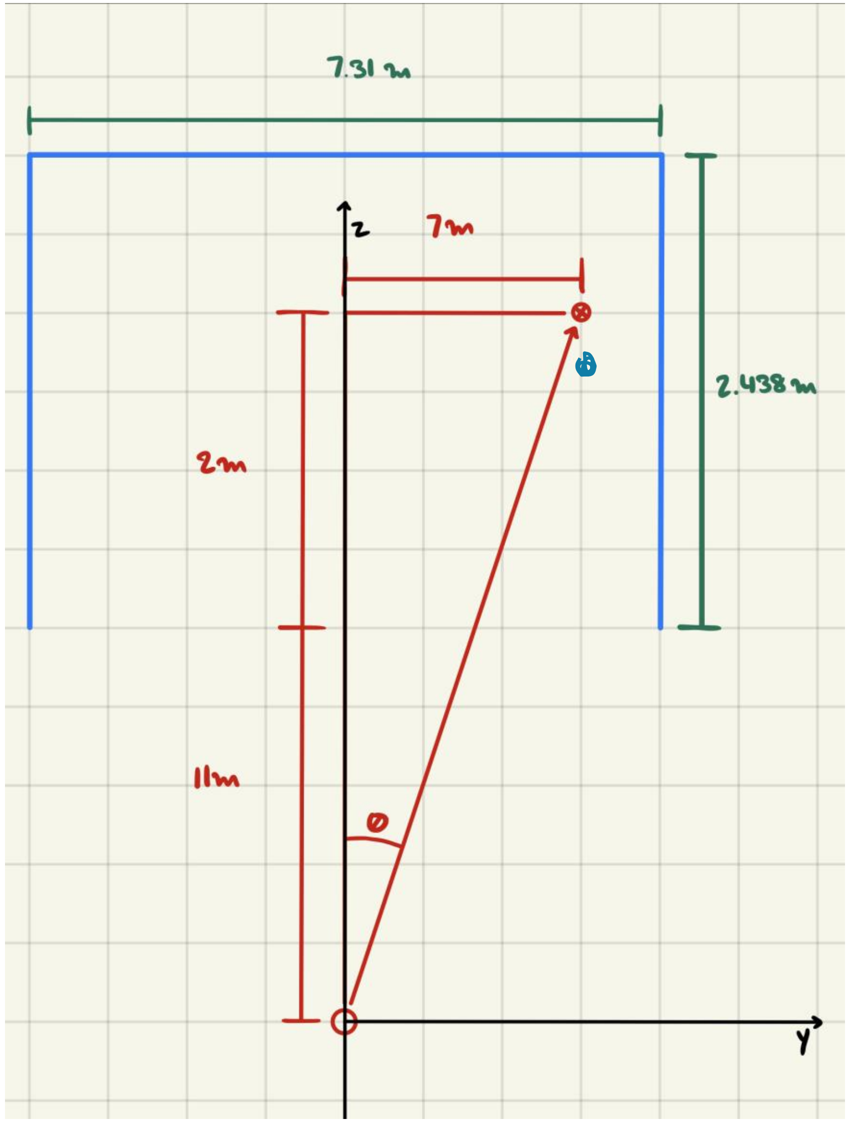
Z axis: $Z = Vi*\sin(Theta)*t + (½)*(a2)*t^2$

$a2= -(R*\sin(theta)/m+g)=11.09m/s^2$

Solution: $t=0.585s$ $Z=1.82m$

CONCLUSION

Given simplified wind resistance, the ball will land at the coordinate point(11i, 7j, 1.82k).



SOURCES

https://www.grc.nasa.gov/WWW/K-12/airplane/socdrag.html (Air resistance)

https://www.insidescience.org/news/aerodynamics-soccer-ball (Soccer ball motion)

https://www.youtube.com/watch?v=lHJ42Yv7p8w (Knuckle Ball Tutorial)

https://www.neurotrackerx.com/post/science-penalty-kicks (Data on penalty kicks)

https://en.wikipedia.org/wiki/Dynamics_%28mechanics%29 (The Class)