



# Golf Ball Simulator

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# Golf Club Information

| Club                       | Distance (Yards) | Wedge Angle (Degrees) | Wedge Mass (KG) | Shaft Length (M) |
|----------------------------|------------------|-----------------------|-----------------|------------------|
| Driver<br>(Long Range)     | 183              | 13                    | 0.18            | 1.2192           |
| 7-Iron<br>(Medium Range)   | 140              | 39                    | 0.258           | 1.2192           |
| Lob Wedge<br>(Short Range) | 65               | 61                    | 0.34            | 1.2192           |

Coefficient of Restitution between Golf Ball and Club: 0.83 (Max Limit for Golf)



Driver



7-Iron



Lob Wedge



# Information of Golf Ball

## Air Resistance Equations

- Most golf balls are of mass 0.04593kg
- Kinds of resistance to think about:
  - Rolling Friction and Air Resistance
- Dimples increase spin with air resistance, allowing ball to travel further
- Account for resistance in X and Y direction
- Surface area of Golf Ball with 150 dimples:  $0.7166\text{m}^2$
- ***We will be ignoring air resistance and spin***

$$R_x = \frac{1}{2}C_d\rho AV_x^2$$
$$R_y = \frac{1}{2}C_d\rho AV_y^2$$

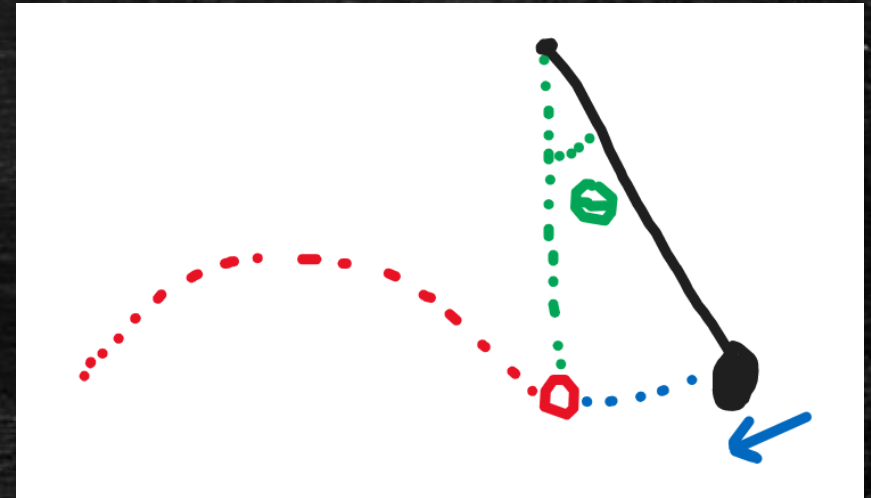
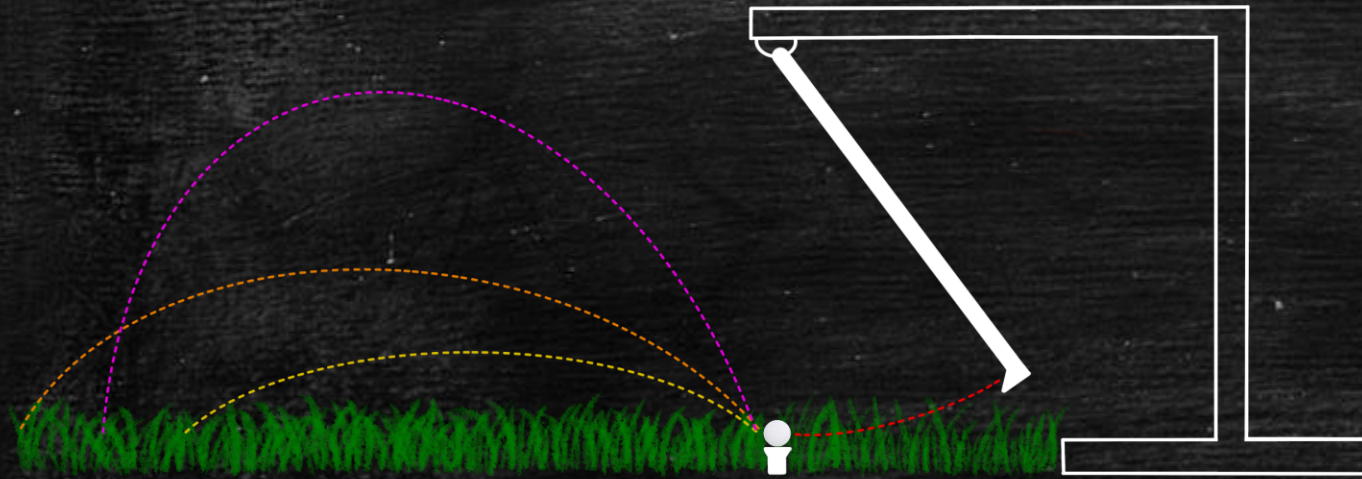
$C_D$  = Drag Coefficient  
 $\rho$  = Air density  
 $A$  = Surface Area  
 $V$  = Velocity



# The Machine

(What it looks like)

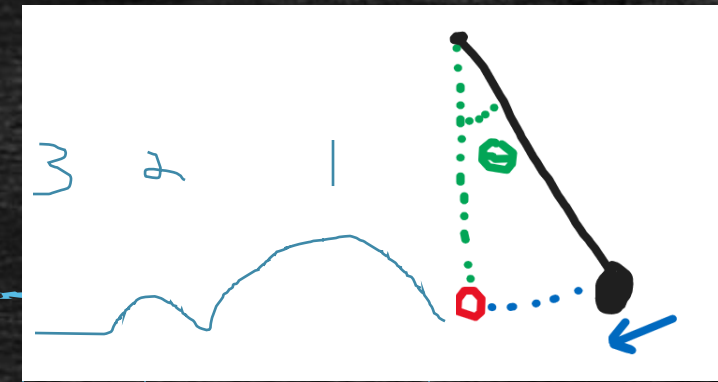
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# Golf Ball Movement Analysis (Projectile Motion)

- Ball weighs  $45.93\text{g} = 0.04593\text{kg}$
- The projectile motion in the x-direction will be the force from the golf club (cosine) and air resistance, and the y-direction will be the sine of the force from the golf club, and gravity,
- There will be an initial velocity from the club (stage 1), a midway velocity when the ball bounces once (stage 2), and a final velocity when the ball lands after bouncing in which it will come to a stop due to rolling friction (stage 3). (Look at drawing for image reference)
- Air resistance will always be moving against the golf ball, slowing it down
- The golf ball will be hit with the precise force needed to make the golf ball travel to the desired displacement



Since it's going in an arc:

$$v_x = v \cos(\theta)$$

$$v_y = v \sin(\theta)$$

To find theta:  $\arctan\left(\frac{v_y}{v_x}\right)$

And velocity is:  $\sqrt{v_x^2 + v_y^2}$

To find height:  $v_{yf}^2 = v_{yo}^2 + 2a_y d_y$

$$0 = (v \sin(\theta))^2 + 2gH$$

$$\frac{(v \sin(\theta))^2}{2g} = H$$

$$H = \frac{v^2 \sin^2(\theta)}{2g}$$

References: 7, 12, and B

To find range:  $R = v_x t$

$$R = v \cos(\theta) t \quad (\text{Note: Total time: } \frac{2v \sin(\theta)}{g})$$

$$R = v \cos(\theta) \frac{2v \sin(\theta)}{g}$$

$$R = v^2 \frac{(2 \sin(\theta) \cos(\theta))}{g} \quad (\text{Thanks to double angle formula... } \sin(2\theta) = 2 \sin(\theta) \cos(\theta))$$

$$R = v^2 \frac{\sin(2\theta)}{g}$$

Diagrams + Equations

| Horizontal            | Vertical                                  |
|-----------------------|---|
| $\Delta x = v_{ox} t$ | $\Delta y = v_{oy} t - \frac{1}{2} g t^2$ |
| $v_x = v_{ox}$        | $v_y = v_{oy} - g t$                      |

# Ball Projectile Motion Setup

$x = V_x * t$  <---- Sum of the forces in the x direction

$V_y$  = Velocity in the y direction

$V_x$  = Velocity in the x direction

$y = V_y * t + (\frac{1}{2})(g)(t^2)$  <---- Sum of the forces in the y direction

$V_y = V * \sin(\theta)$  <---- Velocity in y direction equation

$V_x = V * \cos(\theta)$  <---- Velocity in x direction equation

$V_f$  = velocity of ball after collision

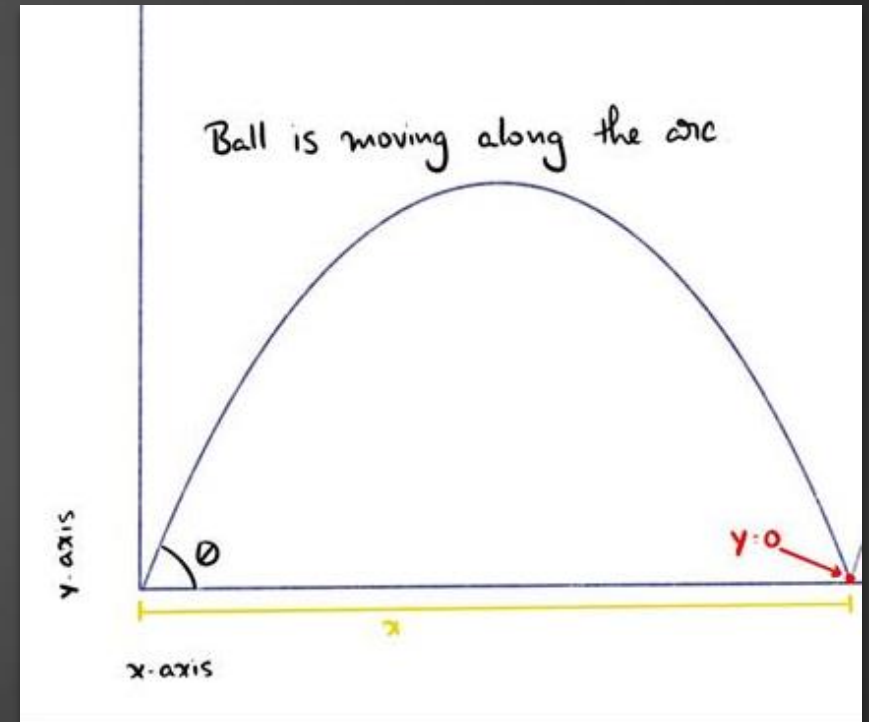
$\theta$  = Angle of the ball

$x$  = Distance in the x direction

$y$  = Distance in the y direction

$g$  = Gravity

$t$  = Time





# Ball Projectile Motion Equations/Information

(Stage 1: Trajectory of Golf Ball without Air Resistance)

Equations:

$$V_x = V \cdot \cos(\theta) \quad \leftarrow \text{Velocity in the x direction}$$

$$V_y = V \cdot \sin(\theta) \quad \leftarrow \text{Velocity in the y direction}$$

$$X = V_x \cdot t \quad \leftarrow \text{Displacement Equation in x direction}$$

$$Y = V_y \cdot t + .5 \cdot (g) \cdot t^2 \quad \leftarrow \text{Displacement Equation in y direction}$$

Knowns:

$$X, \theta : 183\text{m}, 13^\circ$$

Driver

$$X, \theta : 140\text{m}, 39^\circ$$

7-Iron

$$X, \theta : 65\text{m}, 61^\circ$$

Lob Wedge

$$g = -9.81 \text{ m/s}^2$$

Unknowns

$$V_x = ?$$

$$V_y = ?$$

$$V = ?$$

$$T = ?$$

# Ball Projectile Motion Calculations

(Stage 1: Trajectory of Golf Ball without Air Resistance)

Equations Window

$$V_x = V \cdot \cos(\theta)$$
$$V_y = V \cdot \sin(\theta)$$
$$x = V_x \cdot t$$
$$y = V_y \cdot t + .5 \cdot (g) \cdot t^2$$
$$x = 183[\text{m}]$$
$$\theta = 13[\text{deg}]$$
$$g = -9.81[\text{m/s}^2]$$
$$y = 0[\text{m}]$$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$g = -9.81 \text{ [m/s}^2]$     $t = 2.935 \text{ [s]}$     $\theta = 13 \text{ [deg]}$     $V = 63.99 \text{ [m/s]}$

$V_x = 62.35 \text{ [m/s]}$     $V_y = 14.4 \text{ [m/s]}$     $x = 183 \text{ [m]}$     $y = 0 \text{ [m]}$

No unit problems were detected.

Compilation time = 63 ms   Calculation time = 15 ms

Driver

Equations Window

$$V_x = V \cdot \cos(\theta)$$
$$V_y = V \cdot \sin(\theta)$$
$$x = V_x \cdot t$$
$$y = V_y \cdot t + .5 \cdot (g) \cdot t^2$$
$$x = 140[\text{m}]$$
$$\theta = 39[\text{deg}]$$
$$g = -9.81[\text{m/s}^2]$$
$$y = 0[\text{m}]$$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$g = -9.81 \text{ [m/s}^2]$     $t = 4.808 \text{ [s]}$     $\theta = 39 \text{ [deg]}$     $V = 37.47 \text{ [m/s]}$

$V_x = 29.12 \text{ [m/s]}$     $V_y = 23.58 \text{ [m/s]}$     $x = 140 \text{ [m]}$     $y = 0 \text{ [m]}$

No unit problems were detected.

Compilation time = 62 ms   Calculation time = 16 ms

7-Iron

Equations Window

$$V_x = V \cdot \cos(\theta)$$
$$V_y = V \cdot \sin(\theta)$$
$$x = V_x \cdot t$$
$$y = V_y \cdot t + .5 \cdot (g) \cdot t^2$$
$$x = 65[\text{m}]$$
$$\theta = 61[\text{deg}]$$
$$g = -9.81[\text{m/s}^2]$$
$$y = 0[\text{m}]$$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$g = -9.81 \text{ [m/s}^2]$     $t = 4.889 \text{ [s]}$     $\theta = 61 \text{ [deg]}$     $V = 27.42 \text{ [m/s]}$

$V_x = 13.29 \text{ [m/s]}$     $V_y = 23.98 \text{ [m/s]}$     $x = 65 \text{ [m]}$     $y = 0 \text{ [m]}$

No unit problems were detected.

Compilation time = 62 ms   Calculation time = 16 ms

Lob Wedge

| Club      | Ball Velocity | Angle      | Time in Flight | Distance |
|-----------|---------------|------------|----------------|----------|
| Driver    | 64 m/s        | 13 degrees | 2.935 seconds  | 183 m    |
| 7-Iron    | 37.47 m/s     | 39 degrees | 4.81 seconds   | 140 m    |
| Lob Wedge | 27.42 m/s     | 61 degrees | 4.89 seconds   | 65 m     |



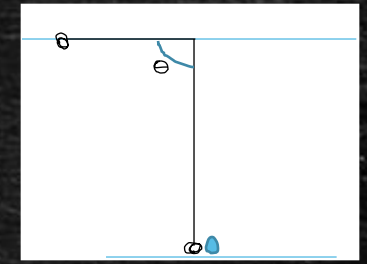
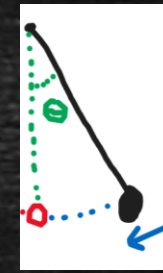
# Stage 1: Numbers Explained

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- With the known values of distance, the angle of the ball's trajectory, gravity, and the final y position of the ball, we were able to calculate the balls initial/final velocity, the velocity in the x and y component, and the total amount of time it takes the ball to travel from the initial position to the final position (given the ball travels in a straight line)
- With the initial/final velocity of the ball, we can now work backwards to find the needed velocity of the golf club before making contact with the ball, and then after making contact with the ball.



# Stage 0 Golf Club Movement Analysis (Pendulum motion)



- Our simulator will follow a pendulum path, being pushed down and making contact with the golf ball when  $\theta = \theta$ .
- We will ignore air resistance, as well as any pendulum movement right after impact with the ball as it will not matter in this simulator.
- We only need the mass of the wedge, the length of the stick, the angle away from the ball, and gravity for the pendulum.
  - The shape of the head of the club will not impact the calculations

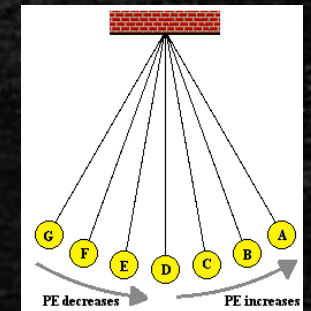
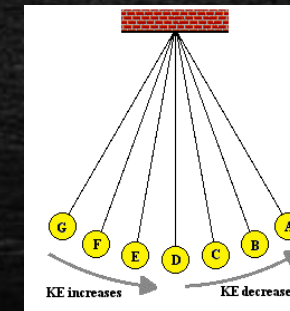
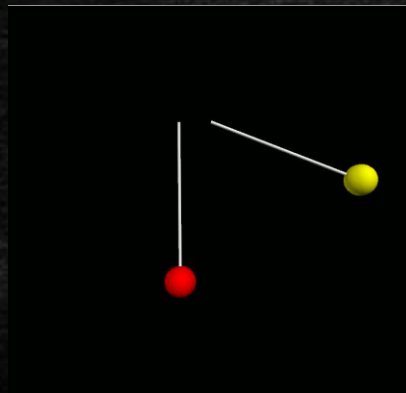
Conservation of energy also applies to pendulum motion given that there is no air resistance

$$KE_i + PE_i = KE_f + PE_f$$

$$KE = \frac{1}{2} * m * (v^2)$$

$$PE = m * g * h$$

$$h = L - L * \cos(\theta)$$





# Pendulum Gravity Equations/Information

Stage 0: Pendulum Swing

- Assuming swing speed is constant and pendulum motion only adds to overall velocity
- We will be adding the gravitational velocity of the swing to the final velocity

## Equations:

$KE = PE$  <---- Conservation of Energy

$PE = m * g * h$  <---- Potential Energy

$KE = .5 * m * v^2$  <---- Kinetic Energy

## Knowns:

$m = 18 \text{ kg}$  Driver

$m = .258 \text{ kg}$  7-Iron

$m = .34 \text{ kg}$  Lob Wedge

$g = -9.81 \text{ m/s}^2$

$h = 1.219 \text{ m}$

$\theta = 90$

## Unknowns:

$KE = ?$

$PE = ?$

$v = ?$

# Pendulum Gravity Calculation

## Stage 0: Pendulum Swing

Equations Window

KE = PE  
PE = m\*g\*h  
KE = .5\*m\*v^2  
  
m = .18[kg]  
g = 9.81[m/s^2]  
h = 1.219[m]  
theta = 90[deg]

Solution

Main  
Unit Settings: SI C kPa kJ mass deg  
g = 9.81 [m/s^2]    h = 1.219 [m]    KE = 2.153 [J]    m = 0.18 [kg]  
PE = 2.153 [J]    θ = 90 [deg]    v = 4.89 [m/s]  
Driver

Equations Window

KE = PE  
PE = m\*g\*h  
KE = .5\*m\*v^2  
  
m = .258[kg]  
g = 9.81[m/s^2]  
h = 1.219[m]  
theta = 90[deg]

Solution

Main  
Unit Settings: SI C kPa kJ mass deg  
g = 9.81 [m/s^2]    h = 1.219 [m]    KE = 3.085 [J]    m = 0.258 [kg]  
PE = 3.085 [J]    θ = 90 [deg]    v = 4.89 [m/s]  
7-Iron

Equations Window

KE = PE  
PE = m\*g\*h  
KE = .5\*m\*v^2  
  
m = .34[kg]  
g = 9.81[m/s^2]  
h = 1.219[m]  
theta = 90[deg]

Solution

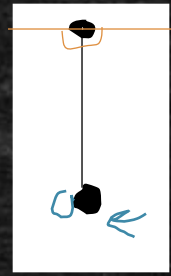
Main  
Unit Settings: SI C kPa kJ mass deg  
g = 9.81 [m/s^2]    h = 1.219 [m]    KE = 4.066 [J]    m = 0.34 [kg]  
PE = 4.066 [J]    θ = 90 [deg]    v = 4.89 [m/s]  
Lob Wedge

| Club      | Angle of pendulum (degrees) | Mass (Kg) | Velocity of Ball (m/s) | Club Length (m) |
|-----------|-----------------------------|-----------|------------------------|-----------------|
| Driver    | 90                          | 0.18      | 4.89                   | 1.2192          |
| 7-Iron    | 90                          | 0.258     | 4.89                   | 1.2192          |
| Lob Wedge | 90                          | 0.34      | 4.89                   | 1.2192          |



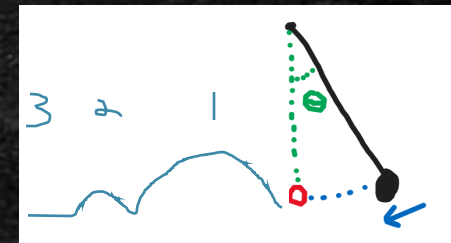
# Momentum transfer from wedge to golf ball

## Analysis (Pendulum motion/Linear Momentum)

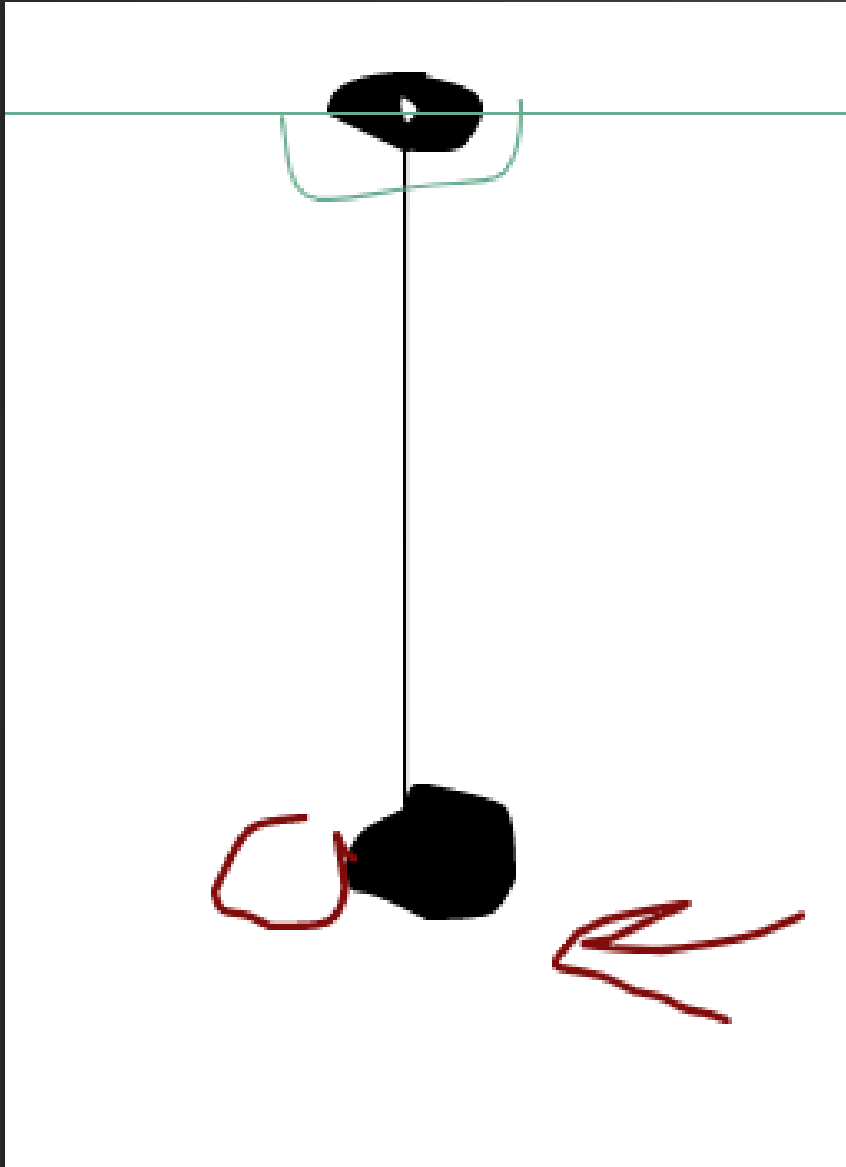


- The energy gets transferred from the wedge to the ball on the moment of contact
  - Causing the ball to launch into the far distance with velocity inflicted from the wedge
  - Concept of Linear momentum and work-energy make this easy
  - Finding the velocity of the pendulum (club) at 90 degrees will give us the velocity the ball is hit at
- Pendulum length is 1.2192 meters for each swing, and mass changes based on the club used
  - weight change with the club, but theta final is 90.
  - Club must have another force, gravity isn't enough (the swing assists the initial velocity)
  - From initial hit to first bounce, ball will travel total displacement wanted

| Club      | Angle of pendulum (degrees) | Mass (Kg) | Displacement wanted (m) | Club Length (m) |
|-----------|-----------------------------|-----------|-------------------------|-----------------|
| Driver    | 90                          | 0.18      | 183                     | 1.2192          |
| 7-Iron    | 90                          | 0.258     | 140                     | 1.2192          |
| Lob Wedge | 90                          | 0.34      | 65                      | 1.2192          |



References: 12, B and C



# Momentum Transfer Setup

$m_c$  = Mass of Club

$m_b$  = Mass of ball

$v_{ci}$  = Velocity of Club initial

$v_{bi}$  = Velocity of ball initial

$v_{cf}$  = Velocity of club final

$v_{bf}$  = Velocity of ball final

$COR = (v_{bf} - v_{cf}) / (v_{ci} - v_{bi})$  <---- Coefficient of Restitution Equation

$m_c * v_{ci} + m_b * v_{bi} = m_c * v_{cf} + m_b * v_{bf}$  <---- Conservation of Momentum



# Momentum Transfer of Golf Club Swing to Golf Ball

## Equations/Information

(Stage 0: Ball and Club Collision, before Ball Projectile motion)

### Equations:

$M_c v_{ci} + m_b v_{bi} = m_c v_{cf} + m_b v_{bf}$  <---- Conservation of momentum

$COR = (v_{bf} - v_{cf}) / (v_{ci} - v_{bi})$  <---- Coefficient of Restitution

### Knowns:

$M_c, V_{bf}$ : 0.18kg, 64m/s Driver

$M_c, V_{bf}$ : 0.258kg, 37.47m/s 7-Iron

$M_c, V_{bf}$ : 0.34 kg, 27.42m/s Lob Wedge

$m_b = 0.04593$

$v_{bi} = 0 \text{ m/s}$

$COR = .83$

### Unknowns:

$v_{cf} = ?$

$v_{ci} = ?$

# Velocity of Golf Club Swing Calculations

(Stage 0: Ball and Club Collision, before Ball Projectile motion)

Equations Window

$$m_c \cdot v_{ci} + m_b \cdot v_{bi} = m_c \cdot v_{cf} + m_b \cdot v_{bf}$$
$$COR = (v_{bf} - v_{cf}) / (v_{ci} - v_{bi})$$
  
$$m_c = .18[\text{kg}]$$
$$m_b = .04593[\text{kg}]$$
  
$$v_{bf} = 64[\text{m/s}]$$
$$v_{bi} = 0[\text{m/s}]$$
  
$$COR = .83$$

"Mass of Club"

"Velocity of Ball"

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$COR = 0.83$  $m_b = 0.04593 [\text{kg}]$  $m_c = 0.18 [\text{kg}]$  $v_{bf} = 64 [\text{m/s}]$  $v_{bi} = 0 [\text{m/s}]$  $v_{cf} = 27.57 [\text{m/s}]$  $v_{ci} = 43.9 [\text{m/s}]$

No unit problems were detected.

Compilation time = 78 ms   Calculation time = 15 ms

Driver

Equations Window

$$m_c \cdot v_{ci} + m_b \cdot v_{bi} = m_c \cdot v_{cf} + m_b \cdot v_{bf}$$
$$COR = (v_{bf} - v_{cf}) / (v_{ci} - v_{bi})$$
  
$$m_c = .258[\text{kg}]$$
$$m_b = .04593[\text{kg}]$$
  
$$v_{bf} = 37.47[\text{m/s}]$$
$$v_{bi} = 0[\text{m/s}]$$
  
$$COR = .83$$

"Mass of Club"

"Velocity of Ball"

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$COR = 0.83$  $m_b = 0.04593 [\text{kg}]$  $m_c = 0.258 [\text{kg}]$  $v_{bf} = 37.47 [\text{m/s}]$  $v_{bi} = 0 [\text{m/s}]$  $v_{cf} = 17.45 [\text{m/s}]$  $v_{ci} = 24.12 [\text{m/s}]$

No unit problems were detected.

Compilation time = 62 ms   Calculation time = 16 ms

7-Iron

Equations Window

$$m_c \cdot v_{ci} + m_b \cdot v_{bi} = m_c \cdot v_{cf} + m_b \cdot v_{bf}$$
$$COR = (v_{bf} - v_{cf}) / (v_{ci} - v_{bi})$$
  
$$m_c = .34[\text{kg}]$$
$$m_b = .04593[\text{kg}]$$
  
$$v_{bf} = 27.42[\text{m/s}]$$
$$v_{bi} = 0[\text{m/s}]$$
  
$$COR = .83$$

"Mass of Club"

"Velocity of Ball"

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$COR = 0.83$  $m_b = 0.04593 [\text{kg}]$  $m_c = 0.34 [\text{kg}]$  $v_{bf} = 27.42 [\text{m/s}]$  $v_{bi} = 0 [\text{m/s}]$  $v_{cf} = 13.3 [\text{m/s}]$  $v_{ci} = 17.01 [\text{m/s}]$

No unit problems were detected.

Compilation time = 78 ms   Calculation time = 15 ms

Lob Wedge

| Club      | Mass of Club | Club Velocity | Mass of Ball | Ball Velocity Final |
|-----------|--------------|---------------|--------------|---------------------|
| Driver    | 0.18 kg      | 43.9 m/s      | .04593 kg    | 64 m/s              |
| 7-Iron    | 0.258 kg     | 24.12 m/s     | .04593 kg    | 37.47 m/s           |
| Lob Wedge | 0.34 kg      | 17.01 m/s     | .04593 kg    | 27.42 m/s           |



# Stage 2: Knowns and Unknowns for Bounce One and Resulting Projectile Motion



## Equations:

$COR = (v_{bf} - v_{cf}) / (v_{ci} - v_{bi})$  <----- Coefficient of Restitution

$V_x = V * \cos(\theta)$  <--- Velocity in the x direction

$V_y = V * \sin(\theta)$  <--- Velocity in the y direction

$X = V_x * t$  <--- Displacement on the x axis

$Y = V_y * t + .5 * (g) * t^2$  <--- Displacement on the y axis

(V<sub>gf</sub> and V<sub>gi</sub> are 0 because ground does not move)

$$e = 0.120 \quad v_{1z'} > 20 \text{ m/s}$$

$$e = 0.510 - 0.0375v_{1z'} + 0.000903v_{1z'}^2 \quad v_{1z'} \leq 20 \text{ m/s}$$

Angle of impact stays the same from initial hit to bounce hit

Using the logic of bounces and retained energy, the ball will retain the percentage of the COR

Example: With COR of 0.12, ball retains 12% of velocity

## Knowns:

$$COR_{\text{bounce 1}} = 0.12$$

$$v_{bi}, \theta : -14.4 \text{ m/s}, 13^\circ$$

$$v_{bi}, \theta : -23.98 \text{ m/s}, 39^\circ$$

$$v_{bi}, \theta : -23.58 \text{ m/s}, 61^\circ$$

$$v_{gi} = 0 \text{ m/s}$$

$$v_{gf} = 0 \text{ m/s}$$

$$g = -9.81 \text{ m/s}^2$$

Driver

7-Iron

Lob Wedge

## Unknowns:

$$V_{bf} = ? \quad (V_{bf} = V_y)$$

$$V = ?$$

# Stage 2: Right After Impact Calculations



## Collision with ground (bounce 1)

## Projectile motion after bounce ( $v_{bf} = V_y$ )

Equations Window

$COR = (v_{bf} - v_{gf}) / (v_{gi} - v_{bi})$

$v_{bi} = -14.4[m/s]$   
 $v_{gi} = 0[m/s]$   
 $v_{gf} = 0[m/s]$   
 $COR = .12$

"Velocity in y"

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$COR = 0.12$     $v_{bf} = 1.728 [m/s]$     $v_{bi} = -14.4 [m/s]$     $v_{gf} = 0 [m/s]$     $v_{gi} = 0 [m/s]$

No unit problems were detected.

Compilation time = 78 ms   Calculation time = 15 ms

Driver

Equations Window

$V_x = V \cdot \cos(\theta)$   
 $V_y = V \cdot \sin(\theta)$

$x = V_x \cdot t$   
 $y = V_y \cdot t + .5 \cdot (g) \cdot t^2$

$V_y = 1.728[m/s]$   
 $\theta = 13[deg]$   
 $g = -9.81[m/s^2]$   
 $y = 0[m]$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$g = -9.81 [m/s^2]$     $t = 0.3523 [s]$     $\theta = 13 [deg]$     $V = 7.682 [m/s]$   
 $V_x = 7.485 [m/s]$     $V_y = 1.728 [m/s]$     $x = 2.637 [m]$     $y = 0 [m]$

No unit problems were detected.

Compilation time = 79 ms   Calculation time = 15 ms

Driver

Equations Window

$COR = (v_{bf} - v_{gf}) / (v_{gi} - v_{bi})$

$v_{bi} = -23.58[m/s]$   
 $v_{gi} = 0[m/s]$   
 $v_{gf} = 0[m/s]$   
 $COR = .12$

"Velocity in y"

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$COR = 0.12$     $v_{bf} = 2.83 [m/s]$     $v_{bi} = -23.58 [m/s]$     $v_{gf} = 0 [m/s]$     $v_{gi} = 0 [m/s]$

No unit problems were detected.

Compilation time = 63 ms   Calculation time = 31 ms

7-Iron

Equations Window

$V_x = V \cdot \cos(\theta)$   
 $V_y = V \cdot \sin(\theta)$

$x = V_x \cdot t$   
 $y = V_y \cdot t + .5 \cdot (g) \cdot t^2$

$V_y = 2.83[m/s]$   
 $\theta = 39[deg]$   
 $g = -9.81[m/s^2]$   
 $y = 0[m]$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$g = -9.81 [m/s^2]$     $t = 0.577 [s]$     $\theta = 39 [deg]$     $V = 4.497 [m/s]$   
 $V_x = 3.495 [m/s]$     $V_y = 2.83 [m/s]$     $x = 2.016 [m]$     $y = 0 [m]$

No unit problems were detected.

Compilation time = 79 ms   Calculation time = 31 ms

7-Iron

Equations Window

$COR = (v_{bf} - v_{gf}) / (v_{gi} - v_{bi})$

$v_{bi} = -23.98[m/s]$   
 $v_{gi} = 0[m/s]$   
 $v_{gf} = 0[m/s]$   
 $COR = .12$

"Velocity in y"

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$COR = 0.12$     $v_{bf} = 2.878 [m/s]$     $v_{bi} = -23.98 [m/s]$     $v_{gf} = 0 [m/s]$     $v_{gi} = 0 [m/s]$

No unit problems were detected.

Compilation time = 62 ms   Calculation time = 16 ms

Lob Wedge

Equations Window

$V_x = V \cdot \cos(\theta)$   
 $V_y = V \cdot \sin(\theta)$

$x = V_x \cdot t$   
 $y = V_y \cdot t + .5 \cdot (g) \cdot t^2$

$V_y = 2.878[m/s]$   
 $\theta = 61[deg]$   
 $g = -9.81[m/s^2]$   
 $y = 0[m]$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$g = -9.81 [m/s^2]$     $t = 0.5867 [s]$     $\theta = 61 [deg]$     $V = 3.291 [m/s]$   
 $V_x = 1.595 [m/s]$     $V_y = 2.878 [m/s]$     $x = 0.936 [m]$     $y = 0 [m]$

No unit problems were detected.

Compilation time = 78 ms   Calculation time = 15 ms

Lob Wedge



# Stage 2: Final Collision With Ground Before Rolling

$$e = 0.510 - 0.0375v_{1z'} + 0.000903v_{1z'}^2, \quad v_{1z'} \leq 20 \text{ m/s}$$

## Equations:

$$\text{COR} = .51 - 0.0375*V + .000903*(V^2) \quad \text{---- Coefficient of Restitution}$$

$$\text{COR} = (v_{bf} - v_{gf}) / (v_{gi} - v_{bi}) \quad \text{---- Coefficient of restitution}$$

$$V_x = V * \cos(\theta) \quad \text{---- Velocity in the x direction}$$

$$V_y = V * \sin(\theta) \quad \text{---- Velocity in the y direction}$$

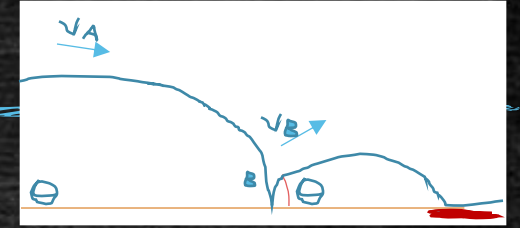
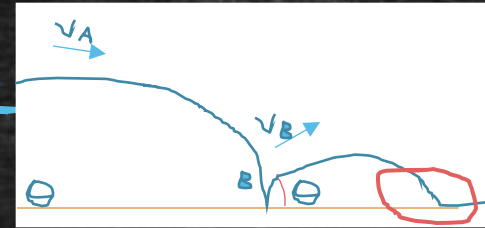
$$X = V_x * t \quad \text{---- displacement in the x direction}$$

$$Y = V_y * t + .5 * (g) * t^2 \quad \text{---- displacement in the y direction}$$

## Unknowns:

$$\text{COR} = ? \quad X = ?$$

$$V_{bf,x,y} = ? \quad T = ?$$



## Knowns:

$$V_{\text{total}}, V_{bi,y}, \theta : \text{Driver}$$

$$7.682 \text{ m/s}, -1.728 \text{ m/s}, 13^\circ$$

$$V_{\text{total}}, V_{bi,y}, \theta : \text{7-Iron}$$

$$4.497 \text{ m/s}, -2.83 \text{ m/s}, 39^\circ$$

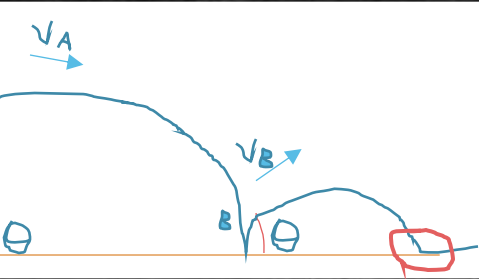
$$V_{\text{total}}, V_{bi,y}, \theta : \text{Lob Wedge}$$

$$3.291 \text{ m/s}, -2.878 \text{ m/s}, 61^\circ$$

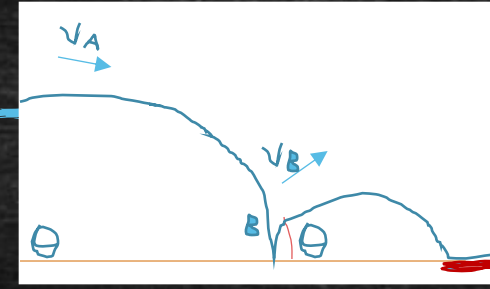
$$V_{gi} = 0 \text{ m/s}$$

$$V_{gf} = 0 \text{ m/s}$$

# Stage 2: Final Collision with Ground Before Rolling Calculations



## Collision with ground (2)



## Roll after Collision (2)

$V_y$  neglected, thus the other variables that would have been the resulting projectile motion are also neglected

Equations Window

"Velocity in y"

$$COR = (v_{bf} - v_{gf}) / (v_{gi} - v_{bi})$$

$$COR = .51 - .0375 \cdot V + .000903 \cdot (V^2)$$

$V = 7.682 \text{ [m/s]}$   
 $v_{bi} = -1.728 \text{ [m/s]}$   
 $v_{gi} = 0 \text{ [m/s]}$   
 $v_{gf} = 0 \text{ [m/s]}$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$COR = 0.2752$        $V = 7.682 \text{ [m/s]}$        $v_{bf} = 0.4756 \text{ [m/s]}$   
 $v_{bi} = -1.728 \text{ [m/s]}$        $v_{gf} = 0 \text{ [m/s]}$        $v_{gi} = 0 \text{ [m/s]}$

Driver

Equations Window

$V_x = V \cdot \cos(\theta)$   
 $V_y = V \cdot \sin(\theta)$   
 $x = V_x \cdot t$   
 $y = V_y \cdot t + .5 \cdot (g) \cdot t^2$   
 $V_y = .4756 \text{ [m/s]}$   
 $\theta = 13 \text{ [deg]}$   
 $g = -9.81 \text{ [m/s}^2]$   
 $y = 0 \text{ [m]}$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$g = -9.81 \text{ [m/s}^2]$        $t = 0.69696 \text{ [s]}$        $\theta = 13 \text{ [deg]}$        $V = 2.144 \text{ [m/s]}$   
 $V_x = 2.06 \text{ [m/s]}$        $V_y = 0.4756 \text{ [m/s]}$        $x = 0.1597 \text{ [m]}$        $y = 0 \text{ [m]}$

No unit problems were detected.  
 Compilation time = 78 ms      Calculation time = 16 ms

Driver

Equations Window

"Velocity in y"

$$COR = (v_{bf} - v_{gf}) / (v_{gi} - v_{bi})$$

$$COR = .51 - .0375 \cdot V + .000903 \cdot (V^2)$$

$V = 4.497 \text{ [m/s]}$   
 $v_{bi} = -2.83 \text{ [m/s]}$   
 $v_{gi} = 0 \text{ [m/s]}$   
 $v_{gf} = 0 \text{ [m/s]}$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$COR = 0.3596$        $V = 4.497 \text{ [m/s]}$        $v_{bf} = 1.018 \text{ [m/s]}$   
 $v_{bi} = -2.83 \text{ [m/s]}$        $v_{gf} = 0 \text{ [m/s]}$        $v_{gi} = 0 \text{ [m/s]}$

7-Iron

Equations Window

$V_x = V \cdot \cos(\theta)$   
 $V_y = V \cdot \sin(\theta)$   
 $x = V_x \cdot t$   
 $y = V_y \cdot t + .5 \cdot (g) \cdot t^2$   
 $V_y = 1.018 \text{ [m/s]}$   
 $\theta = 39 \text{ [deg]}$   
 $g = -9.81 \text{ [m/s}^2]$   
 $y = 0 \text{ [m]}$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$g = -9.81 \text{ [m/s}^2]$        $t = 0.2976 \text{ [s]}$        $\theta = 39 \text{ [deg]}$        $V = 1.618 \text{ [m/s]}$   
 $V_x = 1.257 \text{ [m/s]}$        $V_y = 1.018 \text{ [m/s]}$        $x = 0.2699 \text{ [m]}$        $y = 0 \text{ [m]}$

No unit problems were detected.  
 Compilation time = 78 ms      Calculation time = 31 ms

7-Iron

Equations Window

"Velocity in y"

$$COR = (v_{bf} - v_{gf}) / (v_{gi} - v_{bi})$$

$$COR = .51 - .0375 \cdot V + .000903 \cdot (V^2)$$

$V = 3.291 \text{ [m/s]}$   
 $v_{bi} = -2.878 \text{ [m/s]}$   
 $v_{gi} = 0 \text{ [m/s]}$   
 $v_{gf} = 0 \text{ [m/s]}$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$COR = 0.3964$        $V = 3.291 \text{ [m/s]}$        $v_{bf} = 1.141 \text{ [m/s]}$   
 $v_{bi} = -2.878 \text{ [m/s]}$        $v_{gf} = 0 \text{ [m/s]}$        $v_{gi} = 0 \text{ [m/s]}$

Lob Wedge

Equations Window

$V_x = V \cdot \cos(\theta)$   
 $V_y = V \cdot \sin(\theta)$   
 $x = V_x \cdot t$   
 $y = V_y \cdot t + .5 \cdot (g) \cdot t^2$   
 $V_y = 1.141 \text{ [m/s]}$   
 $\theta = 61 \text{ [deg]}$   
 $g = -9.81 \text{ [m/s}^2]$   
 $y = 0 \text{ [m]}$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$g = -9.81 \text{ [m/s}^2]$        $t = 0.2526 \text{ [s]}$        $\theta = 61 \text{ [deg]}$        $V = 1.305 \text{ [m/s]}$   
 $V_x = 0.6325 \text{ [m/s]}$        $V_y = 1.141 \text{ [m/s]}$        $x = 0.1471 \text{ [m]}$        $y = 0 \text{ [m]}$

No unit problems were detected.  
 Compilation time = 63 ms      Calculation time = 15 ms

Lob Wedge



## Stage 2: Numbers Explained

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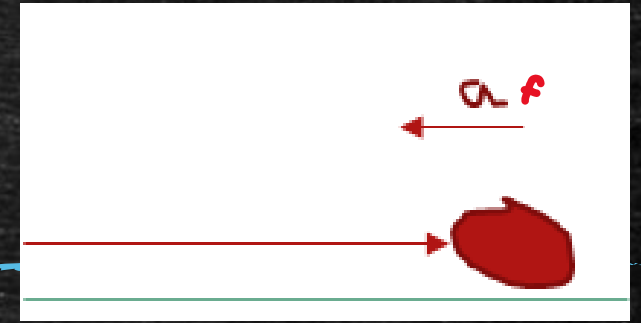
- We were able to calculate:
  - The collision of the ball hitting the ground
  - The projectile motion of the ball resulting from the bounce off the ground  
(Velocity, displacement, time)
  - The collision of the ball hitting the ground the second time, which will be used when calculating how far the ball rolls after this collision, given that the velocity in the y direction is neglected after collision.



# Golf Ball Coming to a Stop

## Equations/Information

Stage 3: Rolling to a Stop



- Coefficient of rolling friction for grass: 0.20 *(Frictional Characteristics of Roadside Grass Types)*
  - We are assuming that the coefficient of friction is constantly moving against the ball
  - It will move in a straight line, decreasing in speed until friction finally stops it
  - We will be taking the velocity from the first bounce when it finishes, and friction will cause a constant deceleration
  - Ball does not slip, and grass is treated as flat ground

### Equations:

$V_{fin}^2 - V_{ini}^2 = 2 * a * x$  <--- Equation to solve for distance

$x = V_{ini} * t + 0.5 * a * t^2$  <--- Equation to solve for time

$a = F/m = -F_f/m = (0.2 * m * g)/m = 0.2 * g$  <--- Acceleration due to friction

References: 6, E, and F

### Knowns:

$V_i = 2.06 \text{ m/s}$

Driver

$V_i = 1.257 \text{ m/s}$

7-Iron

$V_i = 0.6325 \text{ m/s}$

Lob Wedge

$V_f = 0 \text{ m/s}$

COF = 0.2

$g = -9.81 \text{ m/s}^2$

### Unknowns:

$A = ?$

$X = ?$

$T = ?$



# Golf Ball Coming to a Stop Calculations

## Stage 3: Rolling to a Stop

Equations Window

COF = .2  
 $v_i = 2.06[\text{m/s}]$   
 $v_f = 0[\text{m/s}]$   
 $g = -9.81[\text{m/s}^2]$   
 $a = \text{COF} \cdot g$   
 $v_f^2 = v_i^2 + 2 \cdot a \cdot x$   
 $x = v_i \cdot t + .5 \cdot a \cdot t^2$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$a = -1.962 [\text{m/s}^2]$   
 $t = 1.049 [\text{s}]$   
 $x = 1.081 [\text{m}]$

COF = 0.2  
 $v_f = 0 [\text{m/s}]$

$g = -9.81 [\text{m/s}^2]$   
 $v_i = 2.06 [\text{m/s}]$

Driver

Equations Window

COF = .2  
 $v_i = 1.257[\text{m/s}]$   
 $v_f = 0[\text{m/s}]$   
 $g = -9.81[\text{m/s}^2]$   
 $a = \text{COF} \cdot g$   
 $v_f^2 = v_i^2 + 2 \cdot a \cdot x$   
 $x = v_i \cdot t + .5 \cdot a \cdot t^2$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$a = -1.962 [\text{m/s}^2]$   
 $t = 0.6413 [\text{s}]$   
 $x = 0.4027 [\text{m}]$

COF = 0.2  
 $v_f = 0 [\text{m/s}]$

$g = -9.81 [\text{m/s}^2]$   
 $v_i = 1.257 [\text{m/s}]$

7-Iron

Equations Window

COF = .2  
 $v_i = .6325[\text{m/s}]$   
 $v_f = 0[\text{m/s}]$   
 $g = -9.81[\text{m/s}^2]$   
 $a = \text{COF} \cdot g$   
 $v_f^2 = v_i^2 + 2 \cdot a \cdot x$   
 $x = v_i \cdot t + .5 \cdot a \cdot t^2$

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$a = -1.962 [\text{m/s}^2]$   
 $t = 0.3226 [\text{s}]$   
 $x = 0.102 [\text{m}]$

COF = 0.2  
 $v_f = 0 [\text{m/s}]$

$g = -9.81 [\text{m/s}^2]$   
 $v_i = 0.6325 [\text{m/s}]$

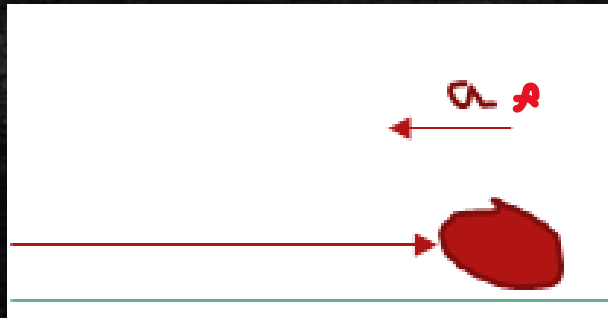
Lob Wedge

|                   | Driver   | 7-Iron   | Lob Wedge |
|-------------------|----------|----------|-----------|
| Distance Traveled | 1.081[m] | .4027[m] | .102[m]   |

## Stage 3: Numbers Explained

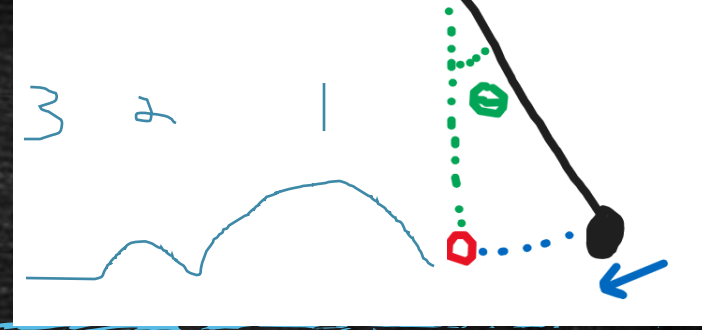
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- Given the ball is rolling on the ground, we were able to calculate how far and how long the ball rolled before stopping due to friction.
- We were able to calculate:
  - The distance the ball rolls before stopping
  - The time it takes for the ball to stop
  - The deceleration due to friction





# Table of Golf Shots



| Club                 | Stage 0<br>Before initial hit (Club)<br>(V) | Stage 1<br>After initial hit<br>(V, X, T) | Stage 2<br>After bounce<br>(V) | Stage 2<br>Right after<br>landing<br>(V, X, T) | Stage 3<br>Once stopped<br>(A, X, T) | Total<br>(X, T) |
|----------------------|---|---|--------------------------------|--|--------------------------------------|-----------------|
| Long<br>(Driver)     | 43.9[m/s]                                   | 64[m/s]                                   | 7.682[m/s]                     | 2.06[m/s]                                      | -1.962[m/s <sup>2</sup> ]            | 186.754[m]      |
|                      |   | 183[m]                                    |                                | 2.637[m]                                       | 1.081[m]                             |                 |
|                      |   | 2.935[s]                                  |                                | 0.3523[s]                                      | 1.049[s]                             | 4.34[s]         |
| Medium<br>(7-Iron)   | 24.12[m/s]                                  | 37.47[m/s]                                | 4.497[m/s]                     | 1.257[m/s]                                     | -1.962[m/s <sup>2</sup> ]            | 142.42[m]       |
|                      |   | 140[m]                                    |                                | 2.016[m]                                       | 0.4027[m]                            | 6.03[s]         |
|                      |   | 4.81[s]                                   |                                | 0.577[s]                                       | 0.6413[s]                            |                 |
| Short<br>(Lob-Wedge) | 17.01[m/s]                                  | 27.42[m/s]                                | 3.291[m/s]                     | 0.6325[m/s]                                    | -1.962[m/s <sup>2</sup> ]            | 66.038[m]       |
|                      |   | 65[m]                                     |                                | 0.936[m]                                       | 0.102[m]                             | 5.8[s]          |
|                      |   | 4.89[s]                                   |                                | 0.5867[s]                                      | 0.3226[s]                            |                 |

# Conclusion/Finale

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We have successfully created a semi-realistic simulation of the trajectory of a golf ball after being hit by a given golf club.

Our simulation can allow for any type of golf club given the average distance the golf ball travels and the angle of the face of the golf club.

Our simulation's answers, however, do not fully portray real life as air-resistance and spin are not accounted for.



# Realistic Scenario (With Air Resistance)

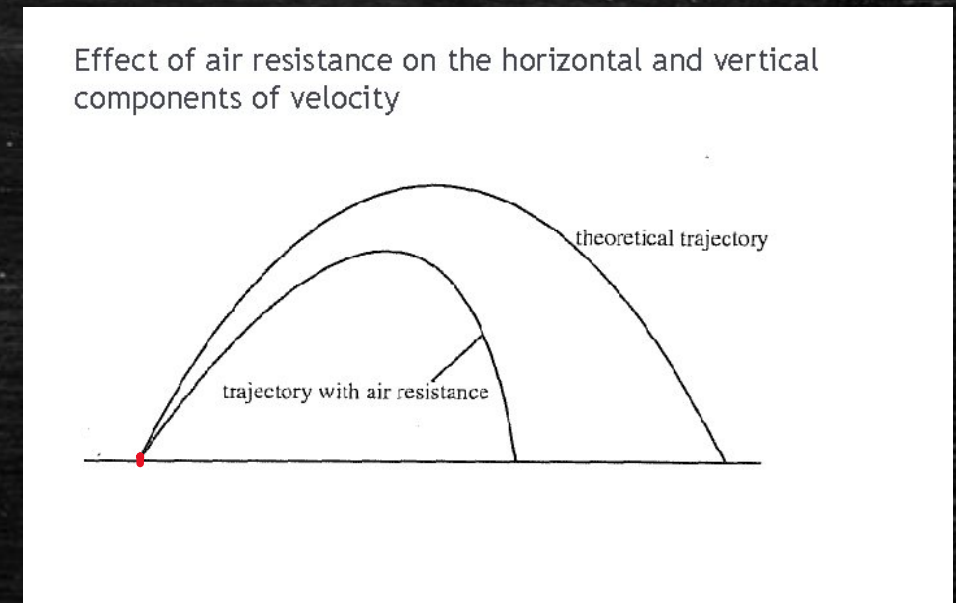
Overall, our calculations have replicated real life to the best of their ability with the initial parameters. Our conclusions do have a degree of error, which we address below; however, our simulation does emulate real-life golf swings to a certain degree.

What we didn't include:

- Air Resistance
- Rotation of the Ball
- The ball bounced more than once
- The ball travels in three dimensions, but we treated it as a two-dimensional trajectory

Impact of Air Resistance:

- The projectile motion of the ball would be different, which would also impact the collision with the ground:
  - The ball wouldn't have traveled as far
  - The angle of the ball as it hits the ground would have been steeper



Reference: J



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