Math 343 - Homework 1

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Question 1

a)

b)

c)

d)

Question 2

First we note that,

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim N(0,1)$$

Then we can derive that,

$$\begin{aligned}
1 - \alpha &= P(-t_{\alpha,n-1} \leq \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \leq t_{\alpha,n-1}) \\
&= P(-t_{\alpha,n-1}\frac{S}{\sqrt{n}} \leq \bar{X} - \mu_0 \leq t_{\alpha,n-1}\frac{S}{\sqrt{n}}) \\
&= P(-\bar{X} - t_{\alpha,n-1}\frac{S}{\sqrt{n}} \leq -\mu_0 \leq -\bar{X} + t_{\alpha,n-1}\frac{S}{\sqrt{n}}) \\
&= P(\bar{X} + t_{\alpha,n-1}\frac{S}{\sqrt{n}} \geq \mu_0 \geq \bar{X} - t_{\alpha,n-1}\frac{S}{\sqrt{n}}) \\
&= P(\bar{X} - t_{\alpha,n-1}\frac{S}{\sqrt{n}} \leq \mu_0 \leq \bar{X} + t_{\alpha,n-1}\frac{S}{\sqrt{n}})
\end{aligned}$$

Therefore, the confidence interval for one population mean μ in the case where the population variance σ^2 is unknown can be described as

$$\bar{X} \pm t_{\alpha,n-1}\frac{S}{\sqrt{n}}$$

Question 3

The P-value can be obtained in R using the following code, where t_0 varies.

Listing 1: Calculating the P-value for a t_0 value

```
# Define the t_0 value, degrees of freedom, and tail of the distribution
t_0 <- 2.48
df <- 10
tail <- 2

# Calculate the P-value using the "pt" function
p_val <- pt(t_0, df, tail)

# Print the P-value
print(p_val)
```

a)

When $t_0 = 2.48$, the P-value is 0.637.

b)

When $t_0 = 3.55$, the P-value is 0.869.

c)

When $t_0 = 2.00$, the P-value is 0.478.

Question 4

a)

Test

Null hypothesis $H_0: \mu = 225$
Alternative hypothesis $H_1: \mu > 225$

T-Value	P-Value
0.67	0.257

Figure 1: The output of the 1-sample t test from Minitab.

b)

Since the p-value (0.257) is greater than the significance level (0.05), we fail to reject the null hypothesis. That is, there is not have enough statistical evidence to support that the mean repair time exceeds 225 hours.

c)

d)

Descriptive Statistics

N	Mean	StDev	SE Mean	95% Lower Bound for μ
16	241.5	98.7	24.7	198.2

μ: population mean of Hours

Figure 2: The Descriptive Statistics of the 1-sample t test from Minitab.

Question 5

a)

b)

c)

d)

e)

Question 6

Question 7