Randomized Complete Block Design (RCBD)

$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$, Where $i =$# of treatments, $j =$# of blocks. (μ Grand Mean), (τ Treatment effect), (β Block effect).
Constraint: $\sum \tau_i = 0$, $\sum \beta_j = 0$.
Estimates: Grand Mean: $\hat{\mu} = \bar{y}_{..}$, Treatment Effect: $\hat{\tau}_i = \bar{y}_{i.} - \bar{y}_{..}$, Block Effect: $\hat{\beta}_i = \bar{y}_{.j} - \bar{y}_{..}$
Grand Smaple Average: $\bar{y}_{..} = \frac{1}{ab}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}$
Sample Mean for i^{th} Treatment (averaged over all blocks): $\bar{y}_{i.} = \frac{1}{b}\sum_{j=1}^{b} y_{ij}$
Sample Mean for j^{th} Block (averaged over all treatments): $\bar{y}_{.j} = \frac{1}{a}\sum_{i=1}^{a} y_{ij}$
Fitted y_{ij} Values: $\hat{y}_{ij} = \bar{y}_{i.} + \bar{y}_{.j} - \bar{y}_{..}$

RCBD - ANOVA

Source	df	SS	MS	F
Treatments	$a-1$	$SSTr$	$MSTr = \frac{SSTr}{a-1}$	$F = \frac{MSTr}{MSE}$
Blocks	$b-1$	SSB	$MSB = \frac{SSB}{b-1}$	$F = \frac{MSB}{MSE}$
Error	$(a-1)(b-1)$	SSE	$MSE = \frac{SSE}{(a-1)(b-1)}$	
Total	$ab-1$	SST		

Pooled sample variance(est. σ^2): $s^2 = MSE$.
Test of Homogeneity of Treatment Effects:
H_0: $\tau_1 = \tau_2 = \cdots = \tau_a = 0$.
H_a: not all $\tau_i = 0$.
Reject H_0 if $F > F_{\alpha,a-1,(a-1)(b-1)}$, or p-value $< \alpha$.

C.I for μ_i: $\bar{y}_{i.} \pm t_{\alpha/2,df_{error}}\frac{s}{\sqrt{b}}$

Fisher's LSD Multiple Comparison Test: Treatment i and j are significantly different if$|\bar{y}_{i.} - \bar{y}_{j.}| > t_{\alpha/2,df_{error}} s\sqrt{\frac{2}{b}}$. (Reject $H_0 : \tau_i = \tau_j$). **Fishers C.I.:**For all possible treatment differences, $\bar{y}_{i.} - \bar{y}_{k.} \pm t_{\alpha/2,df_{error}} s\sqrt{\frac{2}{b}}$.
Tukey's Multiple Comparison Test: Treatment i and j are significantly different if $|\bar{y}_{i.} - \bar{y}_{j.}| > q_{\alpha,a,df_{error}}\frac{s}{\sqrt{b}}$.
Tukey's C.I.: For all possible treatment differences, $\bar{y}_{i.} - \bar{y}_{k.} \pm q_{\alpha,a,df_{error}}\frac{s}{\sqrt{b}}$.

Two Factor Design

$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk}$. Constraint: $\sum \tau_i = 0$, $\sum \beta_j = 0$, and $\sum(\tau\beta)_{ij} = 0$ (for each fixed i,j).
Interaction: if $(\tau\beta)_{ij} = 0$ for every i,j then there is no interaction between factors A and B.
Estimates: $\bar{y}_{...} = \frac{1}{abn}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}$. $\bar{y}_{i..} = \frac{1}{bn}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}$. $\bar{y}_{.j.} = \frac{1}{an}\sum_{i=1}^{a}\sum_{k=1}^{n} y_{ijk}$.
Fitted Values: $\hat{y}_{ijk} = \hat{\mu} + \hat{\tau}_i + \hat{\beta}_j + \hat{(\tau\beta)}_{ij} = \bar{y}_{ij}$.

2 Factor - ANOVA

Source	df	SS	MS	F
Factor A	$a-1$	SSA	$MSA = \frac{SSA}{a-1}$	$F = \frac{MSA}{MSE}$
Factor B	$b-1$	SSB	$MSB = \frac{SSB}{b-1}$	$F = \frac{MSB}{MSE}$
AB Interaction	$(a-1)(b-1)$	$SSAB$	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$F = \frac{MSAB}{MSE}$
Error	$N-ab$	SSE	$MSE = \frac{SSE}{N-ab}$	
Total	$N-1$	SST		

Note: $N = abn$
If $n = 1$ assume no interaction.
Test of significance of AB Interaction should be done first.
If there is interaction it is not usesful to test for significance of A and B.

Test of Significance of AB Interaction: H_0: $(\tau\beta)_{11} = (\tau\beta)_{12} = \cdots = (\tau\beta)_{ab} = 0$. H_a: not all $(\tau\beta)_{ij}$ are the same.
Reject H_0 if $F > F_{\alpha,(a-1)(b-1),N-ab}$, or p-value $< \alpha$.
If H_0 is rejected, conclude that there exists interaction between Factors A and B at significance level α.

Case 1: Interaction between A and B

It is meaningful to test for the significance of the main effect of A and the main effect of B.
May combine the interaction sum of squares with the error sum of squares.
Anova: df: $df_{error} = abn - a - b + 1$, **SSE:** $SSE` = SSAB + SSE$, **MSE:** $MSE` = \frac{SSE`}{abn-a-b+1}$.
Note that F values must be updated with the new $MSE`$ before testing.

Case 2: No interaction between A and B