Math 343 - Lab 6

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Question 1

a)

Main and interaction effects

Effect	Coefficient($\hat{\tau}_1$)	Main Effect($-2\hat{\tau}_1$)
A	0.17	-0.34
B	5.67	-11.34
C	3.42	-6.84
AB	-0.83	1.66
AC	-4.42	8.84
BC	-1.42	2.84
ABC	-1.08	2.16

b)

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	7	1612.67	230.381	7.64	0.000
Linear	3	1051.50	350.500	11.62	0.000
A	1	0.67	0.667	0.02	0.884
B	1	770.67	770.667	25.55	0.000
C	1	280.17	280.167	9.29	0.008
2-Way Interactions	3	533.00	177.667	5.89	0.007
A*B	1	16.67	16.667	0.55	0.468
A*C	1	468.17	468.167	15.52	0.001
B*C	1	48.17	48.167	1.60	0.224
3-Way Interactions	1	28.17	28.167	0.93	0.348
A*B*C	1	28.17	28.167	0.93	0.348
Error	16	482.67	30.167		
Total	23	2095.33			

Figure 1: Anova table from Minitab.

The following effects are significant (p-value $< \alpha = 0.05$), B, C, AC, and BC.

c)

95% C.I. For the true main effect of B.

$$\begin{aligned}\hat{B} &\pm t_{\alpha/2, df_{error}} \cdot \frac{\sqrt{MSE}}{\sqrt{n \cdot 2^{k-2}}} \\ &\pm t_{0.025,16} \cdot \frac{\sqrt{30.167}}{\sqrt{3 \cdot 2^{3-2}}} \\ &\pm 2.120 \cdot \frac{\sqrt{30.167}}{\sqrt{3 \cdot 2^{3-2}}} \\ &\pm 4.753640093\end{aligned}$$

Thus, the confidence interval is $(0.916, 10.423)$ We are 95% confident that the true main effect of tool geometry (B), is between 0.916 and 10.423.

d)

e)

f)

Question 2

First we note that the contrast vectors will have $2^5 = 32$ values. The last 16 elements of the contrast vectors for A,C,E are as follows

$$\vec{A} = \begin{bmatrix} \vdots \\ - \\ + \\ - \\ + \\ - \\ + \\ - \\ + \\ \vdots \end{bmatrix}, \vec{C} = \begin{bmatrix} \vdots \\ - \\ - \\ - \\ - \\ + \\ + \\ + \\ + \\ \vdots \end{bmatrix}, \vec{E} = \begin{bmatrix} \vdots \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ \vdots \end{bmatrix}$$

Note that $\vec{A}$ alternates between $-$ and $+$, every other. $\vec{C}$ alternates between $-$ and $+$, every 4. $\vec{E}$ alternates between $-$ and $+$, every 16.

Thus, $\vec{AC} = \vec{A} \times \vec{C}$, is

$$\vec{AC} = \begin{bmatrix} \vdots \\ + \\ - \\ + \\ - \\ - \\ + \\ - \\ + \\ \vdots \end{bmatrix}$$

Finally, the last 16 elements of $A\vec{C}E$ is

$$A\vec{C}E = \vec{AC} \times \vec{E} = \begin{bmatrix} \vdots \\ + \\ - \\ + \\ - \\ - \\ + \\ - \\ + \\ + \\ - \\ + \\ - \\ - \\ + \\ - \\ + \end{bmatrix} \times \begin{bmatrix} \vdots \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \end{bmatrix} = \begin{bmatrix} \vdots \\ + \\ - \\ + \\ - \\ - \\ + \\ - \\ + \\ + \\ - \\ + \\ - \\ - \\ + \\ - \\ + \end{bmatrix}$$