Randomized Complete Block Design (RCBD)

$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$, Where $i =$# of treatments, $j =$# of blocks. (μ Grand Mean), (τ Treatment effect), (β Block effect).
Constraint: $\sum \tau_i = 0$, $\sum \beta_j = 0$.
Estimates: Grand Mean: $\hat{\mu} = \bar{y}_{\cdot\cdot}$, Treatment Effect: $\hat{\tau}_i = \bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot}$, Block Effect: $\hat{\beta}_i = \bar{y}_{\cdot j} - \bar{y}_{\cdot\cdot}$
Grand Smaple Average: $\bar{y}_{\cdot\cdot} = \frac{1}{ab}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}$
Sample Mean for i^{th} Treatment (averaged over all blocks): $\bar{y}_{i\cdot} = \frac{1}{b}\sum_{j=1}^{b} y_{ij}$
Sample Mean for j^{th} Block (averaged over all treatments): $\bar{y}_{\cdot j} = \frac{1}{a}\sum_{i=1}^{a} y_{ij}$
Fitted y_{ij} Values: $\hat{y}_{ij} = \bar{y}_{i\cdot} + \bar{y}_{\cdot j} - \bar{y}_{\cdot\cdot}$

RCBD - ANOVA

Source	df	SS	MS	F
Treatments	$a-1$	$SSTr$	$MSTr = \frac{SSTr}{a-1}$	$F = \frac{MSTr}{MSE}$
Blocks	$b-1$	SSB	$MSB = \frac{SSB}{b-1}$	$F = \frac{MSB}{MSE}$
Error	$(a-1)(b-1)$	SSE	$MSE = \frac{SSE}{(a-1)(b-1)}$	
Total	$ab-1$	SST		

Pooled sample variance(est. σ^2): $s^2 = MSE$.
Test of Homogeneity of Treatment Effects:
H_0: $\tau_1 = \tau_2 = \cdots = \tau_a = 0$.
H_a: not all $\tau_i = 0$.
Reject H_0 if $F > F_{\alpha,a-1,(a-1)(b-1)}$, or p-value $< \alpha$.

C.I for μ_i: $\bar{y}_{i\cdot} \pm t_{\alpha/2,df_{error}} \frac{s}{\sqrt{b}}$

Fisher's LSD Multiple Comparison Test: Treatment i and j are significantly different if$|\bar{y}_{i\cdot} - \bar{y}_{j\cdot}| > t_{\alpha/2,df_{error}} s\sqrt{\frac{2}{b}}$.
(Reject $H_0 : \tau_i = \tau_j$). **Fishers C.I.:**For all possible treatment differences, $\bar{y}_{i\cdot} - \bar{y}_{k\cdot} \pm t_{\alpha/2,df_{error}} s\sqrt{\frac{2}{b}}$.
Tukey's Multiple Comparison Test: Treatment i and j are significantly different if $|\bar{y}_{i\cdot} - \bar{y}_{j\cdot}| > q_{\alpha,a,df_{error}} \frac{s}{\sqrt{b}}$.
Tukey's C.I.: For all possible treatment differences, $\bar{y}_{i\cdot} - \bar{y}_{k\cdot} \pm q_{\alpha,a,df_{error}} \frac{s}{\sqrt{b}}$.

Two Factor Design

$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk}$. Constraint: $\sum \tau_i = 0$, $\sum \beta_j = 0$, and $\sum(\tau\beta)_{ij} = 0$ (for each fixed i,j).
Interaction: if $(\tau\beta)_{ij} = 0$ for every i,j then there is no interaction between factors A and B.
Estimates: $\bar{y}_{\cdots} = \frac{1}{abn}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}$. $\bar{y}_{i\cdot\cdot} = \frac{1}{bn}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}$. $\bar{y}_{\cdot j\cdot} = \frac{1}{an}\sum_{i=1}^{a}\sum_{k=1}^{n} y_{ijk}$.
Fitted Values: $\hat{y}_{ijk} = \hat{\mu} + \hat{\tau}_i + \hat{\beta}_j + (\hat{\tau\beta})_{ij} = \bar{y}_{ij\cdot}$

2 Factor - ANOVA

Source	df	SS	MS	F
Factor A	$a-1$	SSA	$MSA = \frac{SSA}{a-1}$	$F = \frac{MSA}{MSE}$
Factor B	$b-1$	SSB	$MSB = \frac{SSB}{b-1}$	$F = \frac{MSB}{MSE}$
AB Interaction	$(a-1)(b-1)$	$SSAB$	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$F = \frac{MSAB}{MSE}$
Error	$N-ab$	SSE	$MSE = \frac{SSE}{N-ab}$	
Total	$N-1$	SST		

Note: $N = abn$
If $n = 1$ assume no interaction.
Test of significance of AB Interaction should be done first. If there is interaction it is not usesful to test for significance of A and B.

Test of Significance of AB Interaction: H_0: $(\tau\beta)_{11} = (\tau\beta)_{12} = \cdots = (\tau\beta)_{ab} = 0$. H_a: not all $(\tau\beta)_{ij}$ are the same.
Reject H_0 if $F > F_{\alpha,(a-1)(b-1),N-ab}$, or p-value $< \alpha$.
If H_0 is rejected, conclude that there exists interaction between Factors A and B at significance level α.

Case 1: No Interaction between A and B

It is meaningful to test for the significance of the main effect of A and the main effect of B.
May combine the interaction sum of squares with the error sum of squares.
Anova: df: $df_{error} = abn - a - b + 1$, **SSE:** $SSE' = SSAB + SSE$, **MSE:** $MSE' = \frac{SSE'}{abn-a-b+1}$.
Note that F values must be updated with the new MSE' before testing.
Tukeys C.I.: For $\tau_i - \tau_{i'}$ (or β). Equivalent to FWE $= \alpha$ for hypothesis test.
Factor A: $\bar{y}_{i\cdot\cdot} - \bar{y}_{i'\cdot\cdot} \pm q_{\alpha,a,df_{error}}\sqrt{\frac{MSE}{bn}}$. **Factor B:** $\bar{y}_{\cdot j\cdot} - \bar{y}_{\cdot j'\cdot} \pm q_{\alpha,b,df_{error}}\sqrt{\frac{MSE}{an}}$
Fisher's C.I.: For $\tau_i - \tau_{i'}$ (or β). Equivalent to α for hypothesis test.
Factor A: $\bar{y}_{i\cdot\cdot} - \bar{y}_{i'\cdot\cdot} \pm t_{\alpha/2,df_{error}}\sqrt{\frac{2MSE}{bn}}$. **Factor B:** $\bar{y}_{\cdot j\cdot} - \bar{y}_{\cdot j'\cdot} \pm t_{\alpha/2,df_{error}}\sqrt{\frac{2MSE}{an}}$

Case 2: Interaction between A and B

Not meaningful to test significance of main effect of A and B. Treat each individual factor-level combonation of i^{th} level of A, and j^{th} level of B, as a treatment. So there are ab treatments. Test every pair of treatments using Tukey's or Fisher's.
Test: H_0: $\mu_{11} = \mu_{12} = \cdots = \mu_{ab}$ using the F test for CRD.
Tukey's C.I.: $\bar{y}_{ij} - \bar{y}_{i^{\text{`}}j^{\text{`}}} \pm q_{\alpha,ab,abn-ab}\sqrt{\frac{MSE}{n}}$
Fisher's C.I.: $\bar{y}_{ij} - \bar{y}_{i^{\text{`}}j^{\text{`}}} \pm t_{\alpha/2,abn-ab}\sqrt{\frac{2MSE}{n}}$

2^k Factorial Design Using the $[-1,+1]$ Notation

2 Factor - ANOVA

Std Order	A	B	AB	C	AC	BC	ABC
(1)	−	−	+	−	+	+	−
a	+	−	−	−	−	+	+
b	−	+	−	−	+	−	+
ab	+	+	+	−	−	−	−
c	−	−	+	+	−	−	+
ac	+	−	−	+	+	−	−
bc	−	+	−	+	−	+	−
abc	+	+	+	+	+	+	+

Note: $A \times B = AB$ and so on.
Factor interactions: There will be k main effects and $\binom{k}{2}$ two-factor interactions, $\binom{k}{3}$ three-factor interactions.
Contrasts:
Estimated main effect: $\frac{1}{2^{k-1}} \cdot \sum_{i=1}^{2^k} c_i\bar{y}_i$
Standard Error: $se(\hat{\text{Effect}}) = \sqrt{\frac{MSE}{n2^{k-2}}} = \frac{s}{\sqrt{n2^{k-2}}}$
C.I.: $\hat{\text{Effect}} \pm t_{\alpha/2,df_{error}} \cdot se(\hat{\text{Effect}})$

Model: Standard error: $se(\hat{\beta}) = \sqrt{\frac{\sigma^2}{n2^k}} = \sqrt{\frac{MSE}{n2^k}}$. Confidence interval: $\hat{\beta} \pm t_{\alpha/2,df_{error}} \cdot se(\hat{\beta})$
Hypothesis Test: H_0: True effect = x (usually 0). H_a: True effect $\neq$ ($<$ *or* $>$) x.
t test: $t = \frac{\hat{\text{Effect}}-x}{se(\hat{\text{Effect}})} = \frac{\sqrt{n2^{k-2}}\cdot(\hat{\text{Effect}})}{\sqrt{MSE}}$. Reject H_0 if $|t| > t_{\alpha/2,df_{error}}$ or p-value $< \alpha$.
F test: $F = t^2$. Reject if $F > F_{\alpha,1,df_{error}}$ or p-value $< \alpha$.
Reject H_0, conclude that the main effect of Effect is significant.
Sum of Squares: $SS_{effect} = (n2^{k-2})(\hat{\text{Effect}})^2$.
Notes on Anova:
Main Effects: $\hat{A} = 2\hat{\tau}_2 = -2\hat{\tau}_1$.
From the Coefficients from Minitab: $\hat{\tau}_1$ is Coef, $se(\hat{\tau}_1)$ is SE Coef.
From the Coefficients from Minitab: $(\tau\hat{\beta}\gamma)_{111}$ is Coef, $se((\tau\hat{\beta}\gamma)_{111})$ is SE Coef.
From the Coded Coefficients from Minitab: $\hat{\tau}_2$ is Coef, $se(\hat{\tau}_2)$ is SE Coef.
From the Coded Coefficients from Minitab: $(\tau\hat{\beta}\gamma)_{222}$ is Coef, $se((\tau\hat{\beta}\gamma)_{222})$ is SE Coef.

For 2^2 Design: Sum of Squares of Effects: $SS_{contrast} = \frac{\left(\sum_{i=1}^4 c_i\bar{y}_i\right)^2}{\frac{1}{n}\sum_{i=1}^4 c_i^2}$. $SS_{effect} = n(\frac{1}{2}\sum_{i=1}^4 c_i\bar{y}_i)^2 = n(\text{Main Effect})^2$.