Math 343 - Homework 4

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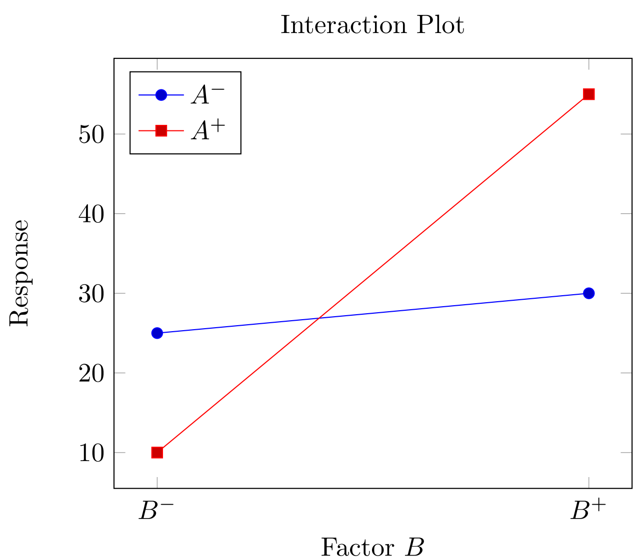
Question 1

a)



Since the Lines in the interaction plot are not parallel, this indicates interaction.

b)

The Main Effect of A is:

$$\begin{aligned} A &= \bar{y}_{A^+} - \bar{y}_{A^-} \\ &= \frac{\mu_{22} + \mu_{21}}{2} - \frac{\mu_{12} + \mu_{11}}{2} \\ &= \frac{55 + 10}{2} - \frac{30 + 25}{2} \\ &= 5 \end{aligned}$$

The Main Effect of B is:

$$\begin{aligned} B &= \bar{y}_{B^+} - \bar{y}_{B^-} \\ &= \frac{\mu_{12} + \mu_{22}}{2} - \frac{\mu_{11} + \mu_{21}}{2} \\ &= \frac{30 + 55}{2} - \frac{25 + 10}{2} \\ &= 25 \end{aligned}$$

The Interaction Effect of A and B is:

$$\begin{aligned} AB &= \frac{\mu_{22}+\mu_{11}}{2} - \frac{\mu_{21}+\mu_{12}}{2} \\ &= \frac{55+25}{2} - \frac{10+30}{2} \\ &= 20 \end{aligned}$$

Question 2

a)

Two-way ANOVA: y versus, A, B
Note that calculated values are bold.

Source	DF	SS	MS	F	P
A	1	0.322	**0.322**	**0.0366**	**0.098**
B	**2**	80.554	40.2771	4.59	**0.171**
Interaction	**2**	**45.348**	**22.674**	**2.5832**	**0.031**
Error	12	105.327	8.7773		
Total	17	231.551			

b)

3 levels where used for Factor B.

c)

18 total observations of the experiment were performed.

d)

Interaction:
H_0: $(\tau\beta)_{11} = (\tau\beta)_{12} = (\tau\beta)_{22} = (\tau\beta)_{21} = (\tau\beta)_{13} = (\tau\beta)_{23} = 0$
H_a: At least one $(\tau\beta)_{ij}$ is different.

Since the p-value for the interaction of the two factors is $0.031 < \alpha = 0.05$, we can conclude the following. There is enough evidence to support the hypotheis that at least one $(\tau\beta)_{ij}$ is different, ie, there exists interaction between the two factors.

Factor A:
H_0: $\tau_1 = \tau_2 = 0$
H_a: At least one τ_i is different.

Since the p-value is $0.098 > \alpha = 0.05$, we can conclude the following.
There is not enough statistic evidence to support at least one τ_i being different.

Factor B:
H_0: $\beta_1 = \beta_2 = \beta_3 = 0$
H_a: At least one β_i is different.

Since the p-value is $0.171 > \alpha = 0.05$, we can conclude the following.
There is not enough statistic evidence to support at least one β_i being different.

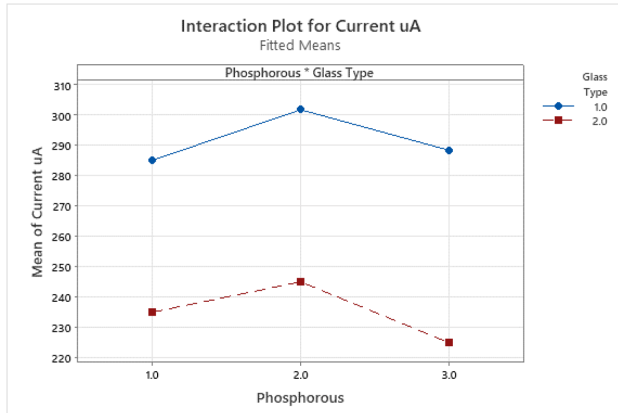
Question 3

b)

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Phosphorous Type	2	933.3	466.7	8.84	0.004
Glass Type	1	14450.0	14450.0	273.79	0.000
Phosphorous Type*Glass Type	2	133.3	66.7	1.26	0.318
Error	12	633.3	52.8		
Total	17	16150.0			

(a) Anova table from Minitab.

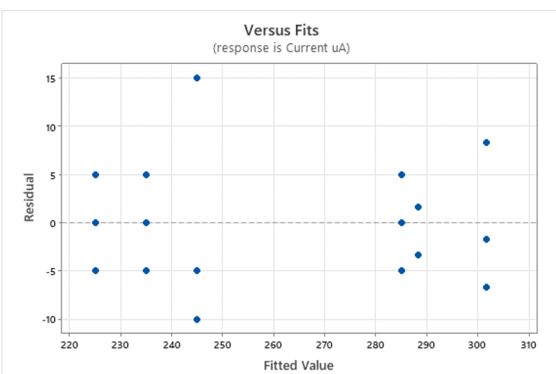


(b) Interaction plot from Minitab.

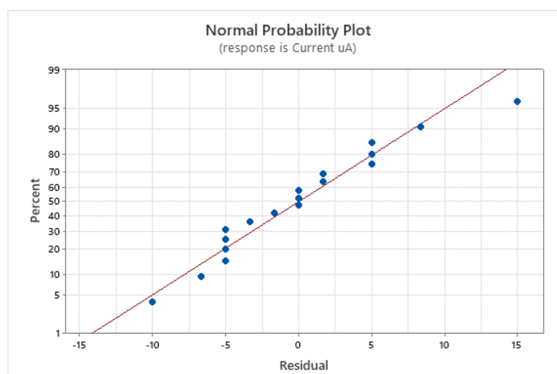
H_0: $(\tau\beta)_{11} = (\tau\beta)_{12} = (\tau\beta)_{22} = (\tau\beta)_{21} = 0$
H_a: At least one $(\tau\beta)_{ij}$ is different.

Since the p-value for the interaction of the two factors is $0.318 > \alpha = 0.05$, we can conclude the following. There is not enough evidence to support the hypotheis that at least one $(\tau\beta)_{ij}$ is different, ie, there does not exist interaction between the two factors.

c)



(a) Residuals versus fits plot from Minitab.



(b) Normal probability plot from Minitab.

The residual plot does not seem to indicate heteroskedasticity. The normal probability plot appears to follow a stright line, indicating normality. These two plots indicate that the model assumptions are satified.

d)

H_0: $\tau_1 = \tau_2 = \tau_3 = 0$
H_a: At least one τ_i is different.

Since the p-value is $0.004 < \alpha = 0.05$, we can conclude the following.
There is enough statistic evidence to support at least one τ_i being different.

Tukey's test for pairwise comparison
First we calulate the means of the Phosphorous Types:

Phosphorous Type	Mean
1	$\frac{285+235}{2} = 260$
2	$\frac{301.67+245}{2} = 273.335$
3	$\frac{288.33+225}{2} = 256.665$

Then we calculate α from the $FWE = 0.05$:

$\alpha = 1 - (1 - 0.05)^{\frac{1}{6}} = 0.0085$

Then note that:

$$\begin{aligned} q_{\alpha,a,df_{error}}\sqrt{\frac{MSE}{b \cdot n}} &= q_{0.0085,3,12}\sqrt{\frac{52.8}{2 \cdot 6}} \\ &= 5.17 \cdot 2.0976 \\ &= 10.85 \end{aligned}$$

The difference in means are:

Difference of Types	Difference	95%Simultaneous C.I.
$2 - 1$	$273.335 - 260 = 13.335$	$(2.48, 24.16)$
$3 - 1$	$256.665 - 260 = -3.335$	$(-14.18, 7.51)$
$3 - 2$	$256.665 - 273.335 = -16.67$	$(-33.34, -5.81)$

Since the C.I. for the difference in means 3 and 1 contains 0, we can say that the means are not significantly different. The line graph representing the data is as follows:

Phosphorous Type **(3)** **(1)** **(2)**

e)

For a Fisher's LSD 95% confidence interval for difference in mean between Phosphor Type 1 and Phosphorous Type 2. We can say the 2 treatment means are significantly different if:

$$\begin{aligned} |\bar{y}_i - \bar{y}_j| &> t_{\alpha/2,N-a}\sqrt{MSE \cdot \left(\frac{1}{n_i} + \frac{1}{n_j}\right)} \\ |260 - 273.33| &> t_{0.025,18-3}\sqrt{5.28 \cdot \left(\frac{2}{6}\right)} \\ 13.33 &> 2.131 \cdot 1.326 \\ 13.33 &> 2.825 \end{aligned}$$

Therefore, via Fisher's LSD method, The means for Phosphorous Types 1 and 2 are significantly different.

Question 4

a)

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Drill Speed	1	0.14822	0.148225	57.01	0.000
Feed Rate	3	0.09250	0.030833	11.86	0.003
Drill Speed*Feed Rate	3	0.04187	0.013958	5.37	0.026
Error	8	0.02080	0.002600		
Total	15	0.30340			

Figure 3: Anova table from Minitab.

b)

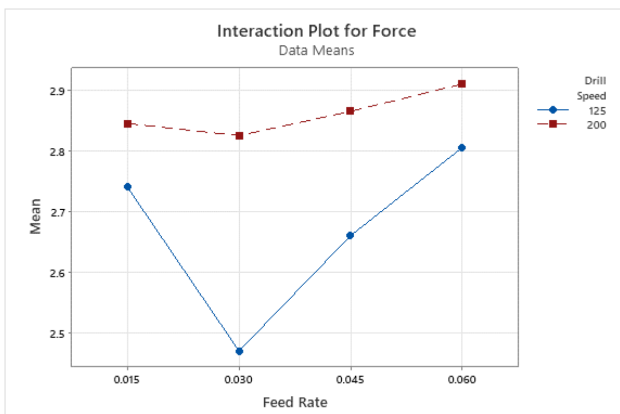


Figure 4: Interaction plot from Minitab.

Based on the interaction plot there does appear to be interaction between the 2 factors.

c)

H_0: There does not exist interaction between the two factors.
H_a: There does exist interaction between the two factors.

Since the p-value for the interaction of the two factors is $0.026 < \alpha = 0.05$, we can conclude the following. There is enough evidence to support the hypotheis that there does exist interaction between the two factors.

d)

Tukeys test for pairwise comparison at FWE = 0.05

(125 .030) (125 .045) (125 .015) (125 .060) (200 .030) (200 .015) (200 .045) (200 .060)

e)

Fisher's LSD 95% C.I for the difference in treatment means
The mean for Drill Speed = 125, Feed Rate = 0.015, $\bar{y}_{125,0.015} = 2.740$.
The mean for Drill Speed = 200, Feed Rate = 0.045, $\bar{y}_{200,0.045} = 2.865$.
The absolute value of their difference is $|2.740 - 2.865| = 0.125$.

The two treatment means are significantly different if:

$$\begin{aligned} 0.125 &> t_{\alpha/2,df_{error}}\sqrt{\frac{2MSE}{n}} \\ &> t_{0.025,8}\sqrt{\frac{2\cdot 0.0026}{2}} \\ &> 2.306\cdot 0.0509 \\ &> 0.1173 \end{aligned}$$

Therfore, the two treatment means $\bar{y}_{125,0.015}$ and $\bar{y}_{200,0.045}$ are significantly different at $\alpha = 0.05$

Question 5

d)

For the model: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2 + \epsilon$
The coefficients are:

Coefficient	Value	P-value	Signifcant at $\alpha = 0.05$?
β_0	-977.5	0.000	Yes
β_1	-10.64	0.013	Yes
β_2	2.028	0.000	Yes
$\beta_2 2$	-0.001040	0.000	Yes
$\beta_1 2$	0.01060	0.016	Yes

Since each coefficient is significant at $\alpha = 0.05$. The model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2 + \epsilon$ is supported by the experiment.

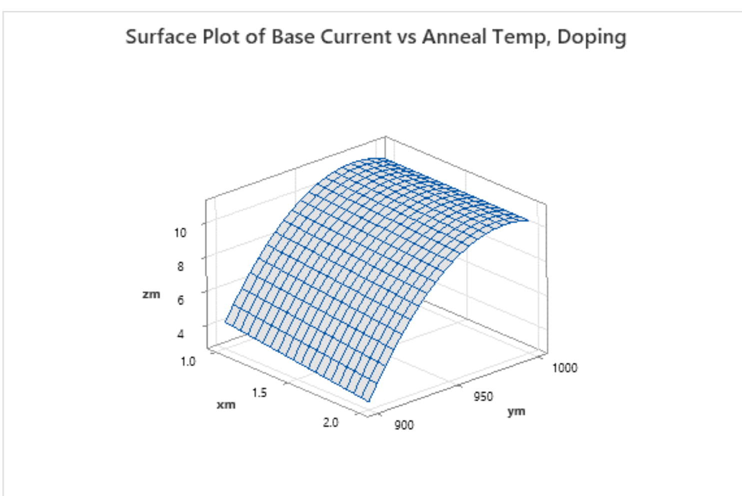


Figure 5: Surface plot from Minitab.

Question 6

a)

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	82.037	0.815	100.71	0.000	
Recoded A					
-1	-7.037	0.815	-8.64	0.001	1.00
Recoded B					
-1	-12.862	0.815	-15.79	0.000	1.00
Recoded A*Recoded B					
-1 -1	2.013	0.815	2.47	0.069	1.00

Figure 6: Coefficients from Minitab.

Regression Equation

Extraction (%) = 82.037 - 7.037 Recoded A_-1 + 7.037 Recoded A_1 - 12.862 Recoded B_-1
+ 12.862 Recoded B_1 + 2.013 Recoded A*Recoded B_-1 -1
- 2.013 Recoded A*Recoded B_-1 1 - 2.013 Recoded A*Recoded B_1 -1
+ 2.013 Recoded A*Recoded B_1 1

Figure 7: Regression equation from Minitab.

b)

The estimated main effect of A is:

$$\begin{aligned} A &= 2\hat{\beta}_1 \\ &= 2(10.05) \\ &= 20.10 \end{aligned}$$

The estimated main effect of B is:

$$\begin{aligned} B &= 2\hat{\beta}_2 \\ &= 2(21.70) \\ &= 43.40 \end{aligned}$$

The estimated interaction effect of A and B is:

$$\begin{aligned} AB &= 2\hat{\beta}_3 \\ &= 2(8.05) \\ &= 16.10 \end{aligned}$$

c)

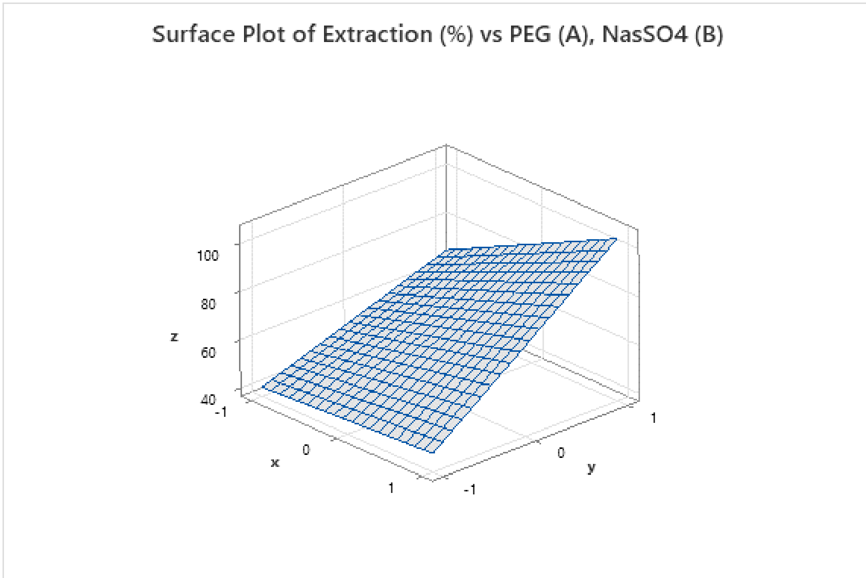


Figure 8: Regression equation from Minitab.

Since there is very little curvature, interaction is likely very small. However, there is curvature, so the interaction may be significant.

Question 7

In terms of these means, $\mu_{(1)}, \mu_a, \mu_b$, and μ_{ab} are the overall mean, the effect of factor A, the effect of factor B, and the interaction effect respectively:
$\mu_{(1)} = (\mu_{11} + \mu_{21} + \mu_{12} + \mu_{22})/4$,
$\mu_a = (\mu_{21} + \mu_{22})/2 - (\mu_{11} + \mu_{12})/2$,
$\mu_b = (\mu_{12} + \mu_{22})/2 - (\mu_{11} + \mu_{21})/2$,
$\mu_{ab} = (mu_{11} + mu_{22})/2 - (mu_{21} + mu_{12})/2$.

The intercept β_0 is the expected value of y when all factors are at their low level (i.e., $x_1 = x_2 = -1$), which corresponds to μ_{11}. However, we must also account for the changes in y due to the other terms in the regression model, which are non-zero when the factors are at their low level. In other words,
$\beta_0 = \mu_{11} + \beta_1 + \beta_2 + \beta_3$.

Expressing β_1, β_2, and β_3 in terms of the factor-level means, we can substitute these expressions into the equation for β_0:
$\beta_1 = \mu_a/2$,
$\beta_2 = \mu_b/2$,
$\beta_3 = \mu_{ab}/2$.

Therefore, $\beta_0 = \mu_{11} + \mu_a/2 + \mu_b/2 + \mu_{ab}/2$, which simplifies to:

$$\beta_0 = \mu_{(1)} - \mu_{ab}/2.$$