Randomized Complete Block Design (RCBD)

$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$, Where $i = \#$ of treatments, $j = \#$ of blocks. (μ Grand Mean), (τ Treatment effect), (β Block effect).
Constraint: $\sum \tau_i = 0$, $\sum \beta_j = 0$.
Estimates: Grand Mean: $\hat{\mu} = \bar{y}_{..}$, Treatment Effect: $\hat{\tau}_i = \bar{y}_{i.} - \bar{y}_{..}$, Block Effect: $\hat{\beta}_i = \bar{y}_{.j} - \bar{y}_{..}$
Grand Smaple Average: $\bar{y}_{..} = \frac{1}{ab} \sum_{i=1}^{a} \sum_{j=1}^{b} y_{ij}$
Sample Mean for i^{th} Treatment (averaged over all blocks): $\bar{y}_{i.} = \frac{1}{b} \sum_{j=1}^{b} y_{ij}$
Sample Mean for j^{th} Block (averaged over all treatments): $\bar{y}_{.j} = \frac{1}{a} \sum_{i=1}^{a} y_{ij}$
Fitted y_{ij} Values: $\hat{y}_{ij} = \bar{y}_{i.} + \bar{y}_{.j} - \bar{y}_{..}$

RCBD - ANOVA

Pooled sample variance(est. σ^2): $s^2 = MSE$.
Test of Homogeneity of Treatment Effects:
$H_0: \tau_1 = \tau_2 = \cdots = \tau_a = 0$.
$H_a:$ not all $\tau_i = 0$.
Reject H_0 if $F > F_{\alpha, a-1, (a-1)(b-1)}$, or p-value $< \alpha$.

Source	df	SS	MS	F
Treatments	$a-1$	$SSTr$	$MSTr = \frac{SSTr}{a-1}$	$F = \frac{MSTr}{MSE}$
Blocks	$b-1$	SSB	$MSB = \frac{SSB}{b-1}$	$F = \frac{MSB}{MSE}$
Error	$(a-1)(b-1)$	SSE	$MSE = \frac{SSE}{(a-1)(b-1)}$	
Total	$ab-1$	SST		

C.I for μ_i: $\bar{y}_{i.} \pm t_{\alpha/2, df_{error}} \frac{s}{\sqrt{b}}$

Fisher's LSD Multiple Comparison Test: Treatment i and j are significantly different if$|\bar{y}_{i.} - \bar{y}_{j.}| > t_{\alpha/2, df_{error}} s\sqrt{\frac{2}{b}}$.
(Reject $H_0 : \tau_i = \tau_j$). **Fishers C.I.:** For all possible treatment differences, $\bar{y}_{i.} - \bar{y}_{k.} \pm t_{\alpha/2, df_{error}} s\sqrt{\frac{2}{b}}$.
Tukey's Multiple Comparison Test: Treatment i and j are significantly different if $|\bar{y}_{i.} - \bar{y}_{j.}| > q_{\alpha, a, df_{error}} \frac{s}{\sqrt{b}}$.
Tukey's C.I.: For all possible treatment differences, $\bar{y}_{i.} - \bar{y}_{k.} \pm q_{\alpha, a, df_{error}} \frac{s}{\sqrt{b}}$.

Two Factor Design

$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk}$. Constraint: $\sum \tau_i = 0$, $\sum \beta_j = 0$, and $\sum (\tau\beta)_{ij} = 0$ (for each fixed i,j).
Interaction: if $(\tau\beta)_{ij} = 0$ for every i,j then there is no interaction between factors A and B.
Estimates: $\bar{y}_{...} = \frac{1}{abn} \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} y_{ijk}$. $\bar{y}_{i..} = \frac{1}{bn} \sum_{j=1}^{b} \sum_{k=1}^{n} y_{ijk}$. $\bar{y}_{.j.} = \frac{1}{an} \sum_{i=1}^{a} \sum_{k=1}^{n} y_{ijk}$.
Fitted Values: $\hat{y}_{ijk} = \hat{\mu} + \hat{\tau}_i + \hat{\beta}_j + (\hat{\tau\beta})_{ij} = \bar{y}_{ij}$.

2 Factor - ANOVA

Note: $N = abn$
If $n = 1$ assume no interaction.
Test of significance of
AB Interaction should be done first.
If there is interaction it is not usesful
to test for significance of A and B.

Source	df	SS	MS	F
Factor A	$a-1$	SSA	$MSA = \frac{SSA}{a-1}$	$F = \frac{MSA}{MSE}$
Factor B	$b-1$	SSB	$MSB = \frac{SSB}{b-1}$	$F = \frac{MSB}{MSE}$
AB Interaction	$(a-1)(b-1)$	$SSAB$	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$F = \frac{MSAB}{MSE}$
Error	$N-ab$	SSE	$MSE = \frac{SSE}{N-ab}$	
Total	$N-1$	SST		

Test of Significance of AB Interaction: $H_0: (\tau\beta)_{11} = (\tau\beta)_{12} = \cdots = (\tau\beta)_{ab} = 0$. $H_a:$ not all $(\tau\beta)_{ij}$ are the same.
Reject H_0 if $F > F_{\alpha, (a-1)(b-1), N-ab}$, or p-value $< \alpha$.
If H_0 is rejected, conclude that there exists interaction between Factors A and B at significance level α.

Case 1: No Interaction between A and B

It is meaningful to test for the significance of the main effect of A and the main effect of B.
May combine the interaction sum of squares with the error sum of squares.
Anova: **df:** $df_{error} = abn - a - b + 1$, **SSE:** $SSE' = SSAB + SSE$, **MSE:** $MSE' = \frac{SSE'}{abn-a-b+1}$.
Note that F values must be updated with the new MSE' before testing.
Tukeys C.I.: For $\tau_i - \tau_{i'}$ (or β). Equivalent to FWE $= \alpha$ for hypothesis test.
Factor A: $\bar{y}_{i..} - \bar{y}_{i'..} \pm q_{\alpha, a, df_{error}} \sqrt{\frac{MSE}{bn}}$. **Factor B:** $\bar{y}_{.j.} - \bar{y}_{.j'.} \pm q_{\alpha, b, df_{error}} \sqrt{\frac{MSE}{an}}$
Fisher's C.I.: For $\tau_i - \tau_{i'}$ (or β). Equivalent to α for hypothesis test.
Factor A: $\bar{y}_{i..} - \bar{y}_{i'..} \pm t_{\alpha/2, df_{error}} \sqrt{\frac{2MSE}{bn}}$. **Factor B:** $\bar{y}_{.j.} - \bar{y}_{.j'.} \pm t_{\alpha/2, df_{error}} \sqrt{\frac{2MSE}{an}}$

Case 2: Interaction between A and B

Not meaningful to test significance of main effect of A and B. Treat each individual factor-level combonation of ith level of A, and j^{th} level of B, as a treatment. So there are ab treatments. Test every pair of treatments using Tukey's or Fisher's.

Test: H_0: $\mu_{11} = \mu_{12} = \cdots = \mu_{ab}$ using the F test for CRD.

Tukey's C.I.: $\bar{y}_{ij} - \bar{y}_{i'j'} \pm q_{\alpha, ab, abn-ab} \sqrt{\frac{MSE}{n}}$

Fisher's C.I.: $\bar{y}_{ij} - \bar{y}_{i'j'} \pm t_{\alpha/2, abn-ab} \sqrt{\frac{2MSE}{n}}$

2^k Factorial Design Using the $[-1, +1]$ Notation

2 Factor - ANOVA

Std Order	A	B	AB	C	AC	BC	ABC
(1)	−	−	+	−	+	+	−
a	+	−	−	−	−	+	+
b	−	+	−	−	+	−	+
ab	+	+	+	−	−	−	−
c	−	−	+	+	−	−	+
ac	+	−	−	+	+	−	−
bc	−	+	−	+	−	+	−
abc	+	+	+	+	+	+	+

Note: $A \times B = AB$ and so on.
Factor interactions: There will be k main effects and $\binom{k}{2}$ two-factor interactions, $\binom{k}{3}$ three-factor interactions.

Contrasts:
Estimated main effect: $\frac{1}{2^{k-1}} \cdot \sum_{i=1}^{2^k} c_i \bar{y}_i$

Model: Standard error: $se(\hat{\beta}) = \sqrt{\frac{\sigma^2}{n2^k}} = \sqrt{\frac{MSE}{n2^k}}$. Confidence interval: $\hat{\beta} \pm t_{\alpha/2, df_{error}} \cdot se(\hat{\beta})$