Math 343 - Homework 4

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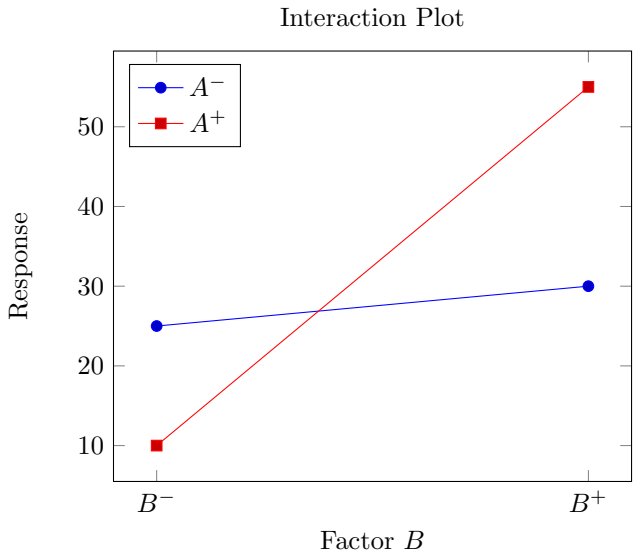
May 13, 2023

Question 1

a)



Since the Lines in the interaction plot are not parallel, this indicates interaction.

b)

The Main Effect of A is:

$$\begin{aligned} A &= \bar{y}_{A^+} - \bar{y}_{A^-} \\ &= \frac{\mu_{22} + \mu_{21}}{2} - \frac{\mu_{12} + \mu_{11}}{2} \\ &= \frac{55 + 10}{2} - \frac{30 + 25}{2} \\ &= 5 \end{aligned}$$

The Main Effect of B is:

$$\begin{aligned} B &= \bar{y}_{B^+} - \bar{y}_{B^-} \\ &= \frac{\mu_{12} + \mu_{22}}{2} - \frac{\mu_{11} + \mu_{21}}{2} \\ &= \frac{30 + 55}{2} - \frac{25 + 10}{2} \\ &= 25 \end{aligned}$$

The Interaction Effect of A and B is:

$$
\begin{aligned}
AB &= \frac{\mu_{22} + \mu_{11}}{2} - \frac{\mu_{21} + \mu_{12}}{2} \\
&= \frac{55 + 25}{2} - \frac{10 + 30}{2} \\
&= 20
\end{aligned}
$$

Question 2

a)

Two-way ANOVA: y versus, A, B
Note that calculated values are bold.

Source	DF	SS	MS	F	P
A	1	0.322	-	-	-
B	-	80.554	40.2771	4.59	-
Interaction	-	-	-	-	-
Error	12	105.327	8.7773	-	-
Total	17	231.551			

b)

c)

d)

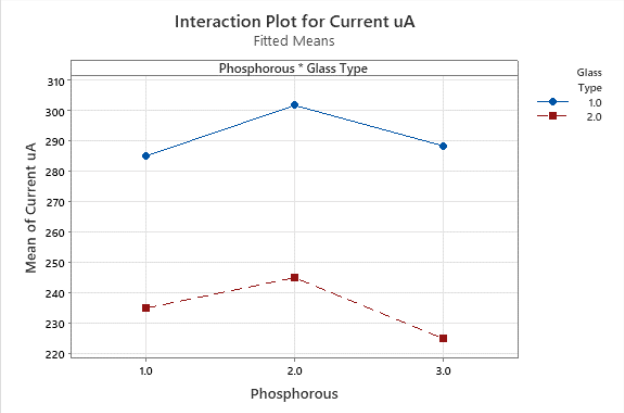
Question 3

b)

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Phosphorous Type	2	933.3	466.7	8.84	0.004
Glass Type	1	14450.0	14450.0	273.79	0.000
Phosphorous Type*Glass Type	2	133.3	66.7	1.26	0.318
Error	12	633.3	52.8		
Total	17	16150.0			

(a) Anova table from Minitab.

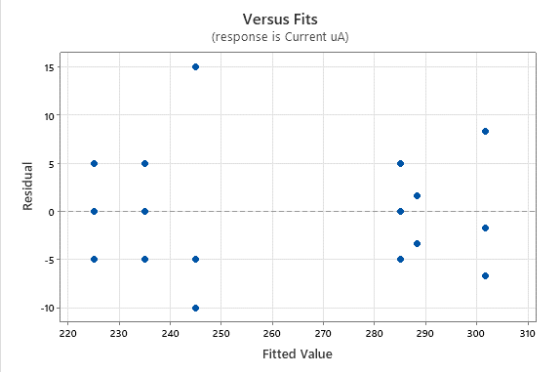


(b) Interaction plot from Minitab.

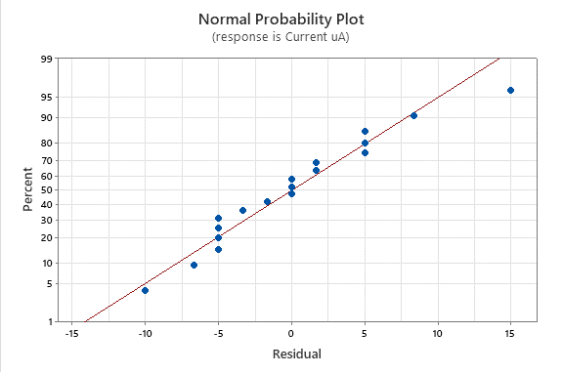
H_0: $(\tau\beta)_{11} = (\tau\beta)_{12} = (\tau\beta)_{22} = (\tau\beta)_{21} = 0$
H_a: At least one $(\tau\beta)_{ij}$ is different.

Since the p-value for the interaction of the two factors is $0.318 > \alpha = 0.05$, we can conclude the following. There is not enough evidence to support the hypotheis that at least one $(\tau\beta)_{ij}$ is different, ie, there does not exist interaction between the two factors.

c)



(a) Residuals versus fits plot from Minitab.



(b) Normal probability plot from Minitab.

The residual plot does not seem to indicate heteroskedasticity. The normal probability plot appears to follow a stright line, indicating normality. These two plots indicate that the model assumptions are satified.

d)

H_0: $\tau_1 = \tau_2 = \tau_3 = 0$
H_a: At least one τ_i is different.

Since the p-value is $0.004 < \alpha = 0.05$, we can conclude the following.
There is enough statistic evidence to support at least one τ_i being different.
a Tukey's test for pairwise comparison

Phosphorous Type	N	Mean
1	6	$\frac{285+235}{2} = 260$
2	6	$\frac{301.67+245}{2} = 273.335$
3	6	$\frac{288.33+225}{2} = 256.665$

The difference in means are:

Difference of Types	Difference
$2-1$	$273.335 - 260 = 13.335$
$3-1$	$256.665 - 260 = -3.335$
$3-2$	$256.665 - 273.335 = -16.67$

e)

Question 4

Question 5

Question 6

Question 7