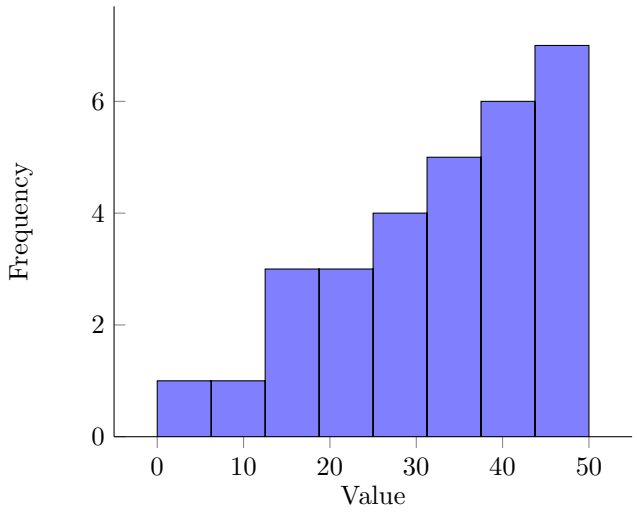
Math 343 - Homework 2

Preston Duffield
Western Washington University
April 14, 2023

Question 1

Data with a rightward skew would produce a normal probability plot with a positive curvature. Below is an example of a histogram that would produce a positively curved normal probability plot.



Question 2

H_0: $\mu_1 - \mu_2 = 10$
H_a: $\mu_1 - \mu_2 > 10$

$$\begin{aligned} Z &= \frac{\bar{y}_1 - \bar{y}_2 - 10}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \\ &= \frac{162.5 - 155 - 10}{\sqrt{\frac{1^2}{10} + \frac{1^2}{12}}} \\ &= -5.838 \end{aligned}$$

By observing that $Z_\alpha = 3.09$ we can conclude the following. There is not enough statistical evidence to support the hypothesis that $\mu_1 - \mu_2 = 10$, ie, the breaking strength of plastic 1 exceeds that of plastic 2 by at least 10 psi. Therefore, based on the sample information, they should not use plastic 1.

$$\begin{aligned} 100(1-\alpha)\text{ C.I} = \bar{y}_1 - \bar{y}_2 \pm Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \\ 7.5 \pm 3.29\sqrt{\frac{1^2}{10} + \frac{1^2}{12}} \\ 7.5 \pm 1.40 \end{aligned}$$

We are 99% confident that the true value of $\mu_1 - \mu_2$ is between 6.1 and 8.9. This is consistent with our above test which conlucded that the difference was not greater than 10.

Question 3

a)

Since the P-value $> \alpha$ we can conclude the following. There is enough statistical evidence to support the hypothesis that both of the variances are equal.

b)

Since the P-value $= 0.962 > \alpha$ we can conclude the following. There is enough statistical evidence to support the hypothesis that the two means are equal.

c)

Note that for both Type 1 and Type 2, the hypothesis we will test is as follows,
H_0: The data are drawn from a normal distribution.
H_a: The data are not drawn from a normal distribution.

Test

Null hypothesis	$H_0: \sigma_1 / \sigma_2 = 1$
Alternative hypothesis	$H_1: \sigma_1 / \sigma_2 \neq 1$
Significance level	$\alpha = 0.05$

	Test			
Method	Statistic	DF1	DF2	P-Value
Bonett	0.00	1		0.963
Levene	0.00	1	18	1.000

Figure 1: The output of the test for two variances from Minitab.

Test

Null hypothesis	$H_0: \mu_1 - \mu_2 = 0$
Alternative hypothesis	$H_1: \mu_1 - \mu_2 \neq 0$

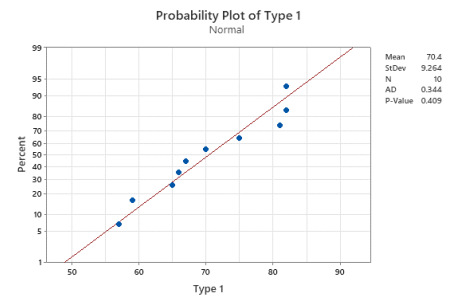
T-Value	DF	P-Value
0.05	18	0.962

Figure 2: The output of the two sample t test from Minitab. Assuming equal variances.

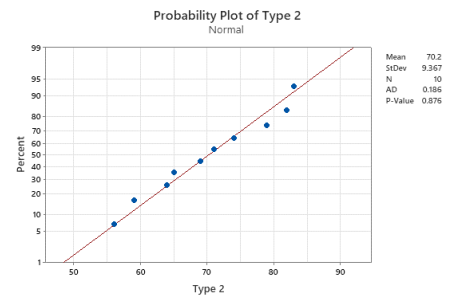
Type 1 Since the P-value $= 0.409 > \alpha$ we can conclude the following. The evidence of the data is consistent with the hypothesis that the data are drawn from a normal distribution.

Type 2 Similarly, since the P-value $= 0.876 > \alpha$ we can conclude the following. The evidence of the data is consistent with the hypothesis that the data are drawn from a normal distribution.

Question 4



(a) Minitab output showing the probability plot of type 1.



(b) Minitab output showing the probability plot of type 2.

Question 5

a/b)

Test

Null hypothesis	H_0: μ_difference = 0
Alternative hypothesis	H_1: μ_difference ≠ 0

T-Value	P-Value
0.43	0.674

Figure 4: The output of the paired t test from Minitab.

Since the P-value = 0.674 > α we can conclude the following. There is enough statistical evidence to support the hypothesis that the two means are equal.

c)

Estimation for Paired Difference

Mean	StDev	SE Mean	95% CI for μ_difference
0.000250	0.002006	0.000579	(-0.001024, 0.001524)

μ_difference: population mean of (Caliper 1 - Caliper 2)

Figure 5: The output of the paired t test containing the confidence interval from Minitab.

From the above confidence interval we can conclude the following. We are 95% confident that the true difference between the population means is between -0.001 and 0.001.

We can also note that the confidence interval contains 0, which is consistent with our hypothesis test.

Question 6

a)

Listing 1: R output of Shapiro-Wilk test on Birth Order: 1

```
Shapiro-Wilk normality test

data:  b1
W = 0.84597, p-value = 0.05201
```

Since the P-value = 0.05201 $> \alpha$ we can conclude the following. The evidence of the data is consistent with the hypothesis that the data are drawn from a normal distribution.

Listing 2: R output of Shapiro-Wilk test on Birth Order: 1

```
Shapiro-Wilk normality test

data:  b2
W = 0.92972, p-value = 0.4452
```

Since the P-value = 0.4452 $> \alpha$ we can conclude the following. The evidence of the data is consistent with the hypothesis that the data are drawn from a normal distribution.

b)

Listing 3: R output of a paired t test

```
Paired t-test

data:  b1 and b2
t = -0.36577, df = 9, p-value = 0.723
alternative hypothesis:
    true mean difference is not equal to 0
95 percent confidence interval:
    -0.3664148  0.2644148
sample estimates:
mean difference
    -0.051
```

The confidence intercal on the difference in mean score leads us to the following conclusion. We are 95% confident that the true difference in the population means is between -0.36 and 0.26. Since the confidence interval contains 0, we can also conclude that the two sample means may be equal.

c)

H_0: $\mu_1 = \mu_2$
H_a: $\mu_1 \neq \mu_2$

Since the P-value = $0.723 > \alpha$ we can conclude the following. There is enough statistical evidence to support the hypothesis that the sample means are equal, ie, $\mu_1 = \mu_2$.

Question 7

Question 8