Math 343 - Lab 6

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Question 1

a)

Main and interaction effects

Effect	Coefficient($\hat{\tau}_1$)	Main Effect($-2\hat{\tau}_1$)
A	0.17	-0.34
B	5.67	-11.34
C	3.42	-6.84
AB	-0.83	1.66
AC	-4.42	8.84
BC	-1.42	2.84
ABC	-1.08	2.16

b)

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	7	1612.67	230.381	7.64	0.000
Linear	3	1051.50	350.500	11.62	0.000
A	1	0.67	0.667	0.02	0.884
B	1	770.67	770.667	25.55	0.000
C	1	280.17	280.167	9.29	0.008
2-Way Interactions	3	533.00	177.667	5.89	0.007
A*B	1	16.67	16.667	0.55	0.468
A*C	1	468.17	468.167	15.52	0.001
B*C	1	48.17	48.167	1.60	0.224
3-Way Interactions	1	28.17	28.167	0.93	0.348
A*B*C	1	28.17	28.167	0.93	0.348
Error	16	482.67	30.167		
Total	23	2095.33			

Figure 1: Anova table from Minitab.

The following effects are significant (p-value $< \alpha = 0.05$), B, C, AC, and BC.

c)

95% C.I. For the true main effect of B.

$$\begin{aligned}\hat{B} &\pm t_{\alpha/2,df_{error}} \cdot \frac{\sqrt{MSE}}{\sqrt{n \cdot 2^{k-2}}} \\ &\pm t_{0.025,16} \cdot \frac{\sqrt{30.167}}{\sqrt{3 \cdot 2^{3-2}}} \\ &\pm 2.120 \cdot \frac{\sqrt{30.167}}{\sqrt{3 \cdot 2^{3-2}}} \\ &\pm 4.753640093\end{aligned}$$

Thus, the confidence interval is $(0.916, 10.423)$ We are 95% confident that the true main effect of tool geometry (B), is between 0.916 and 10.423.

d)

From the half normal plot of the standardized effects we can se that B, C, and AC are signiificant.

Regression Equation

Life Hours = 40.83 + 5.67 B + 3.42 C - 4.42 AC

Figure 2: Regression Equation from Minitab.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	1519.00	506.33	17.57	0.000
B	1	770.67	770.67	26.74	0.000
C	1	280.17	280.17	9.72	0.005
AC	1	468.17	468.17	16.25	0.001
Error	20	576.33	28.82		
Lack-of-Fit	4	93.67	23.42	0.78	0.557
Pure Error	16	482.67	30.17		
Total	23	2095.33			

Figure 3: Anova table from Minitab.

e)

The values (df, SS, MS) for B, C and AC are the same in both ANOVA tables. The other effects have been added to the error values (df, SS, MS). More precisely, MSE for the ANOVA in d is:

$$\text{MSE (part d)} = \text{MSE (part b)} + \text{MSA} + \text{MSAB} + \text{MSBC} + \text{MSABC}$$

The degrees of freedom follow the same pattern:

$$df_{error}(\text{part d}) = df_{error}(\text{part b}) + df_A + df_{AB} + df_{BC} + df_{ABC}$$

f)

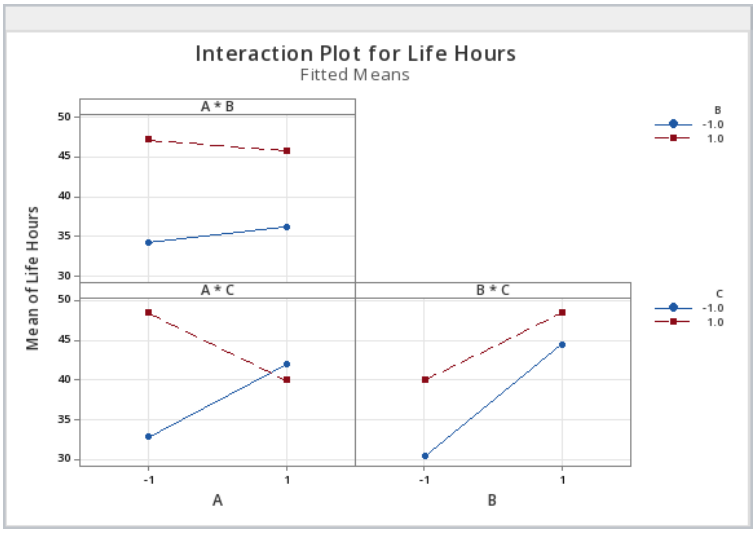

Figure 4: Interaction Plot from Minitab.

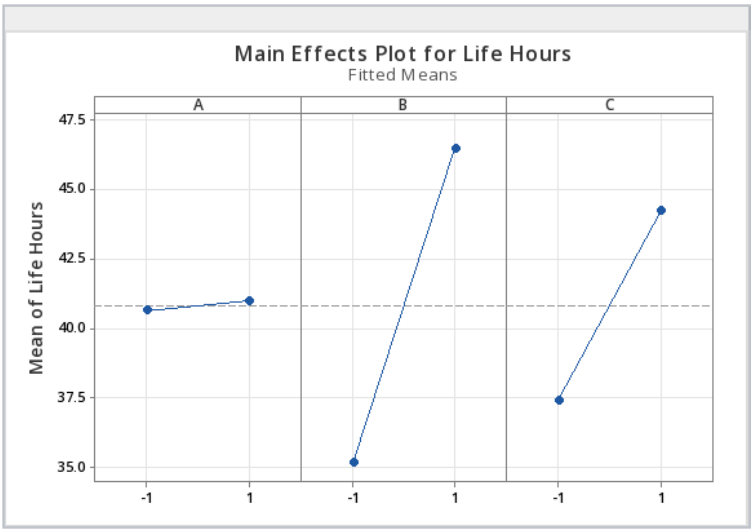

Figure 5: Main Effects Plot from Minitab.

To maximize life hours I would select, A low, B high, C high. This is because B, and C high both have a significant positive effect on life hours so should be high. Therefore, A must be low due to the interaction of AC being significant.

Question 2

First we note that the contrast vectors will have $2^5 = 32$ values. The last 16 elements of the contrast vectors for A,C,E are as follows

$$\vec{A} = \begin{bmatrix} \vdots \\ - \\ + \\ - \\ + \\ - \\ + \\ - \\ + \\ \vdots \end{bmatrix}, \vec{C} = \begin{bmatrix} \vdots \\ - \\ - \\ - \\ - \\ + \\ + \\ + \\ + \\ \vdots \end{bmatrix}, \vec{E} = \begin{bmatrix} \vdots \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ \vdots \end{bmatrix}$$

Note that $\vec{A}$ alternates between $-$ and $+$, every other. $\vec{C}$ alternates between $-$ and $+$, every 4. $\vec{E}$ alternates between $-$ and $+$, every 16.

Thus, $\vec{AC} = \vec{A} \times \vec{C}$, is

$$\vec{AC} = \begin{bmatrix} \vdots \\ + \\ - \\ + \\ - \\ - \\ + \\ - \\ + \\ \vdots \end{bmatrix}$$

Finally, the last 16 elements of $\vec{ACE}$ is

$$\vec{ACE} = \vec{AC} \times \vec{E} = \begin{bmatrix} \vdots \\ + \\ - \\ + \\ - \\ - \\ + \\ - \\ + \\ + \\ - \\ + \\ - \\ - \\ + \\ - \\ + \end{bmatrix} \times \begin{bmatrix} \vdots \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \end{bmatrix} = \begin{bmatrix} \vdots \\ + \\ - \\ + \\ - \\ - \\ + \\ - \\ + \\ + \\ - \\ + \\ - \\ - \\ + \\ - \\ + \end{bmatrix}$$