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**Speakur: a case study of Web Component
encapsulation and synchronization**

APPROVED BY

SUPERVISING COMMITTEE:

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**Speakur: a case study of Web Component
encapsulation and synchronization**

by

Preston Brent Landers, B.A.

REPORT

Presented to the Faculty of the Graduate School of
The University of Texas at Austin
in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science in Engineering

THE UNIVERSITY OF TEXAS AT AUSTIN

May 2015

Dedicated to my wife Andrea and to my parents.

Acknowledgments

I wish to thank the multitudes of people who helped me. Time would fail me to tell of the multitudes of individuals ...

Speakur: a case study of Web Component encapsulation and synchronization

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The University of Texas at Austin, 2015

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This paper presents Speakur, a real-time commenting system for the mobile web, as a case study of W3C Web Component encapsulation and synchronization. Speakur illustrates how new HTML5 standards such as Web Components can help apply widely used software engineering principles such as encapsulation and composition to the realm of web authoring. Any web author can easily insert Speakur as a component on page (a custom HTML tag) to provide a realtime comments forum for that page/article without needing to run their own server component. It uses W3C Web Components (WC) standard to achieve encapsulation and separation from the containing page. Web Components are an upcoming HTML5 standard for writing custom HTML tags using techniques like templates and Shadow DOM.

The report will discuss this new technology of HTML5 Web Components, provide a case study of how to structure a real-world application using it along with embedding it in a larger system (possibly including native mobile apps), and

analyze and explain how to do efficient real-time component synchronization between web clients. The major research question is whether W3C Web Components provide a viable path towards the principles of encapsulation and easy code reuse which have been largely absent from web authoring so far. In other words, is this the future of the web? A subsidiary question is the most efficient way to achieve state synchronization between a large number of clients in the context of Web Components.

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Chapter 1

Introduction

This report describes Speakur, a real time discussion system for the mobile web, and presents a case study of applying W3C Web Components to achieve encapsulation and separation of concerns within the context of collaborative web authoring.

The potential componentization of web is one of the most exciting innovations in web development in years and mirrors the overall growth in the Service Oriented Architecture as a organizational and deployment concept. The conversion of dynamic web logic, not just inert snippets of HTML, into bundles of reusable, extendable, composable components enables web developers to move to a higher level of abstraction than was previously possible.

The componentization of the web will enable all sorts of interesting new composite services, mashups, and help broaden the potential pool of web developers. It will do this by allowing authors to publish easily reusable ‘widgets’ that can be easily juggled around and combined in novel ways that previously required a highly integrated (and hugely expensive) development model or lots of tedious glue code.

1.1 Structure of This Paper

The goal of this paper is to demonstrate the application of software engineering design patterns embodied in the W3C proposed Web Components standard such as encapsulation, composition, and automatic synchronization of application state. This paper attempts to explain many of the goals and principles of the Web Components initiative and show how a number of different technologies taken together help raise the overall level of abstraction for web authors and developers.

The Background section of this report provides an introduction some of the architectural problems inherent in modern web authoring and how Web Components (WC) attempts to address them. It also provides some background on software engineering design patterns that are embodied in Web Components such as encapsulation, composition, and inheritance, as well as technologies such as WebSockets and NoSQL databases. It describes some of the motivations behind Speakur and some of the specific software engineering problems that it is attempting to solve, such as the ability to provide a hassle-free way to host an embedded discussion forum inside an arbitrary web resource in a way that is fully encapsulated.

The Approach section details the specific structures and techniques used when constructing a Web Component, and describes the technology and software architecture choices that went into Speakur. It describes how Speakur uses Web Components to implement encapsulated functionality that is protected

The Implementation section describes the application of Web Component principles to the specific task of providing a flexible and suitably generic discussion forum / commenting system. It describes the overall architecture, code flow, and synchronization process. An important topic in this section is security: how can we implement a largely client-based system while maintaining some kind of data integrity?

This is followed by an Analysis section which discusses some of the outcomes as compared to the original goals and also looks at the impact of the selection of Web Components, Polymer, Firebase and some of the other architectural choices.

Finally, the Conclusion section

1.2 Source Code and Demonstration Resources

The source code for Speakur consists of HTML and Javascript code located in two git version control repositories. The first repository is the actual <speakur-discussion> element which is available for use by web authors, and the second contains additional demonstrations, a standalone application and a management console.

The public documentation for web authors to use Speakur in their own sites can be found here:

Demonstrations of several web pages which show off embedded Speakur discussions are available at the following location:

Chapter 2

Background

When the Web was first created by Tim Berners-Lee in 1989, web pages were largely envisioned as single author documents, or at least written by a single cohesive team. The idea of gluing together a complex web application out of simpler components like snapping together Legos seemed like a far-off dream at best. The concept of creating your own HTML elements with customized behaviors seemed beyond the capabilities of the web browsers of the day like Mosaic and Netscape Navigator.

The rest of this text is just filler for now.

We are not going to look at the complete set of instructions contained in *Instructions for Preparation of Doctoral Dissertations and Dissertation Abstracts* or *Format For The Master's Thesis and Report* which can be obtained from the Office of Graduate Studies (OGS) or on their web page, <http://www.utexas.edu/ogs>. The doctoral Instructions I am using are dated March, 2001. The master's Format I am using is dated May, 2001.

Here we will look at a few instructions related to the arrangement of the dissertation, thesis, or report and a few other “technical” details, providing some examples of common L^AT_EX usage and some examples of not-so-common L^AT_EX

usage.

The following are just a couple of tests for the “quote” and “quotation” environments. The following paragraph is a quote.

This template package is provided and licensed “as is” without warranty of any kind, either expressed or implied, including, but not limited to, the implied warranties of merchantability and fitness for a particular purpose.

The following paragraph is a quotation.

This template package is provided and licensed “as is” without warranty of any kind, either expressed or implied, including, but not limited to, the implied warranties of merchantability and fitness for a particular purpose.

The OGS Instructions say prose quotations over four lines should be indented on the left. The Doctoral Degree Evaluator says the quote environment is the correct one to use.

2.1 Arrangement of Dissertation

Always remember that this “fake” dissertation is only intended to be a template for writing your own. Since the ultimate responsibility of making sure your dissertation meets the Graduate School’s requirements, however, lies only with you, you ***must*** get the current *Instructions for Preparation of Doctoral Dissertations*

and Dissertation Abstracts from the Office of Graduate Studies or their web page and check everything yourself. If you don't, you may have a very rude awakening from the Lynn Renegar, Doctoral Degree Evaluator (aka, "The Ruler Lady") at a most inopportune time.

Arrange your dissertation as follows (all sections are required unless said otherwise.

1. Fly Page (blank protective page). This page is **not** counted in the numbering. **Note:** This template does not insert a Fly Page; if you are printing an official copy, you must manually insert a blank piece of paper on your own. Electronic documents do not need a fly page.
2. Copyright Legend (optional) - See OGS Instructions Sample Form A. Begin counting **pretext** pages here, but **do not place a number on this page**.
3. Committee Certification of Approved Version. See OGS Instructions Sample Form B. This page is included in the pretext count, but there should be no page number on the page.
4. Title Page - See OGS Instructions Sample Form C. This page is counted, but there should not be a page number on this page.
5. Dedication and/or Epigraph (optional). Included in count, but not numbered.
6. Acknowledgments or Preface (optional) - Begin showing **pretext** page numbers with **lower case Roman numerals** at bottom of page.

7. Abstract (optional) - See OGS Instructions Sample Form D.
8. Table of contents - List ALL sections which follow it. There are too many different ways a table of contents may be done for the OGS to give examples in their Instructions booklet, but do be sure there is agreement between the major headings in your text and their designations in the Table of Contents (fortunately L^AT_EX does this for you automatically). Please ask the Doctoral Degree Evaluator for assistance if necessary.
9. List of Tables, List of Figures, List of Illustrations, Nomenclature, List of Supplemental Files (such as multimedia files) (optional).
10. Text. The text should be divided into chapters, books or sections. The first page is Arabic numeral “1”. All sections, **from the first page of text through Vita**, should be numbered consecutively.
11. If you group all Tables, Figures, or Illustrations in one place in your dissertation, the section should be placed here, immediately after the text and before any appendices (optional).
12. Appendix or Appendices (optional).
13. Glossary (optional) - this section may be placed either here or after the Table of Contents, in the area with List of Tables, List of Figures...
14. Bibliography - consult your supervisor about which recognized style to use.
15. Index (optional).

16. Vita - This should be a brief biographical sketch of the author. List in the Table of Contents. See OGS Instructions Sample Form E.

2.2 Other Requirements

2.2.1 Margins

The dissertation, after printing, should have left and top margins of 1 1/2 inches, and the right and bottom margins should be 1 1/4 inches. These margins should be consistent throughout the dissertation - including all pages in the appendix. **All page numbers must be *at least one inch from the edges of the page*.** Headers are rarely used in dissertations; if you are considering using them, check with the Doctoral Degree Evaluator first to be sure they will be accepted.

2.2.2 Spacing and Page Arrangement

The document should be double-spaced or space-and-a-half. Exceptions to double-spacing are: the Table of Contents, Lists of Tables, Tables, Figures, Graphs, Captions, Footnotes, Endnotes, Appendices, Glossary, Bibliography and Index; these may be single-spaced. Paragraph indentations are usually five to ten spaces. Prose quotations over four lines should be in block quote (double or single spaced, indented on the left). Do not use quotation marks if the quotation is indented except for quotations within the block quote. Please refer to a style manual for more detailed instructions.

Be sure that each new chapter or major section (i.e., Appendix, Bibliography, Vita) begins on a new page.

2.3 Master's Theses and Reports

Always remember that this “fake” thesis or report — assuming you have followed the instructions in the next chapter about how to format it as such — is only intended to be a template for writing your own. Since the ultimate responsibility of making sure your thesis or report meets the Graduate School's requirements, however, lies only with you, you *must* get the current *Format For The Master's Thesis and Report* from the Office of Graduate Studies or their web page and check everything yourself. If you don't, you may have a very rude awakening from the Mike Feissli, Master's Degree Evaluator at a most inopportune time.

That said, the formatting requirements for Dissertations and Reports and Theses are very similar. They are, however, *not* identical. The primary differences are in the ordering of the title and signature pages and where the optional index is inserted. For Master's Theses and Reports, the Title Page must be in front of the Signature Page. For Master's Theses and Reports, *nothing* is permitted to come between the bibliography and the vita; the index, if used, must be before the bibliography. If you want to use an index, talk with Mike Feissli before your deadline to verify that its inclusion is acceptable. The index can be removed by commenting out one line with a percent sign, if necessary, for producing the “official” copy of your thesis or report, and then inserted for copies for your advisor and you by removing the percent sign.

Chapter 3

How to Use the utdiss2 Package

3.1 Preamble

The preamble of the document starts like this:

```
\documentclass[12pt]{report}  
\usepackage{utdiss2}
```

The first line declares “report” as the document class, with an option of 12pt for the character size, which is slightly greater than usual (the default is 10pt), but is what the Office of the Graduate School (OGS) recommends. You may include other options, as in any other L^AT_EX document.

The second line loads the utdiss2 package, which contains a set of commands intended to produce a document fulfilling the official requirements for a doctoral dissertation or master’s thesis or report. Besides that, you may include other packages. For instance:

```
\usepackage{amsmath,amsthm,amsfonts,amscd}
```

for mathematical symbols, or,

```
\usepackage{draftcopy}
```

to have a large “watermark” across each page of your document that says, “DRAFT.”

The next few commands in the preamble are required.

`\author{Craig William McCluskey}` Replace my name in the command by your full, official University name. Make it combination of lower and upper-cases.

`\address{9905 Chukar Circle\\ Austin, Texas 78758}` Replace my address with your **permanent** (not local) address. Use `\\` to separate address lines.

`\title{Writing a Doctoral Dissertation with \LaTeX{} at the University of Texas at Austin}` Replace the name of this document in this command by your dissertation title. If the title consists of more than one line, it should be in inverted pyramid form. You may have to specify the line breakings by `\\` commands.

`\supervisor[Isaac Newton]{Johannes Kepler}` This document has two supervisors listed. See the source file (`disstemplate.tex`) for information on how to have only one supervisor. This command can be broken across lines as it is in the source file and as the `\committeemembers` command is shown below.

`\committeemembers`

`[Erwin Schr\"odinger]`

`[Albert Einstein]`

`[Charles Townes]`

`{Arthur Schawlow}`

This document shows four non-supervisor committee members. See the source file (disstemplate.tex) for information on how to have a different number.

`\previousdegrees{B.S.}` Replace B.S. with your previous degree.

The next few commands in the preamble are optional.

`%\graduationmonth{...}`

`%\graduationyear{...}`

`%\typist{...}`

Their use is documented in the source file.

At this point, if you are writing a master thesis or report you must use the optional `\degree` and `\degreeabbr` commands and specify

`%\degree{MASTER OF ARTS}`

`%\degreeabbr{M.A.}`

`%\masterreport`

`%\masterthesis`

as documented in the source file. By default the document is formatted as a *dissertation*¹

The default spacing for both text and quoted text is doublespaced. That can be changed with the following self-explanatory commands:

¹The command `\dissertation` is also provided for symmetry.

```

\oneandonehalfspacing
\singlespacing
\oneandonehalfspacequote
\singlespacequote

```

Some versions of LaTeX in combination with some types of printers produce printed output that has incorrect vertical margins. The command `\topmargin 0.125` is provided to allow easy adjustment if it's needed.

If there are 10 or more sections, 10 or more subsections for a section, etc., you need to make an adjustment to the Table of Contents with the command `\longtoctentry`. This command allocates the proper horizontal space for double-digit numbers.

3.2 Document

Next comes the actual text. It could be a sequence of chapters divided into sections, subsections, etc., all in the main file:

```

\chapter{...}    % The first chapter. The
                  % \chapter command is of the form
                  % \chapter[...]{...} or \chapter{...} where
... text ...    % [...] is the entry in table of contents
                  % and {...} is the chapter heading printed
                  % in the body of the document.

\section{...}    %
                  % IMPORTANT: If your chapter heading consists

```



```

% of more than one line, it will be auto-
... text ... % matically broken into separate lines.
% If you don't like the way LaTeX breaks the
% chapter heading into lines, however, use
\section{...} % '\newheadline' command to break lines.
% NEVER USE \\ IN SECTIONAL (E.G., CHAPTER,
... text ... % SECTION, SUBSECTION, SUBSUBSECTION) HEADINGS!
%
\chapter{...} % This is Chapter 2.
... text ...
\section{...}
... text...
\subsection{...}
... more text ...
\subsubsection{...}
... more text ...
\appendix % The appendix begins here.
% \appendices % If more than one appendix chapters,
% use \appendices instead of \appendix
\chapter{...} % First appendix chapter, i.e., Appendix A.
\section{...} % This is appendix section A.1.
.....

```

Or, the chapters can be written in different files like this document and be loaded by `\include` commands:

```
\include{chapter-introduction}  
\include{chapter-instructions}  
\include{chapter-howtouse}  
\include{chapter-makingbib}  
\include{chapter-tables+figs}  
\include{chapter-math}  
\appendices  
\index{Appendices@\emph{Appendices}}%  
\include{chapter-appendix1}  
\include{chapter-appendix2}  
\include{chapter-appendix3}
```

Having the chapters in separate files makes the main `.tex` file simpler and allows chapters to be easily re-ordered (just swap the order of the include commands) or left (commented) out for draft copies.

Note: If you have only one appendix, in addition to using `\appendix` instead of `\appendices`, you must leave out the `\chapter` definition at the start of the appendix's text. Putting it in will cause the insertion of an extra page with only the word `Appendix` on it and will cause the appendix to be labeled `Appendix 1`, both of which are poor form if there is only one appendix.

If you are writing a short dissertation that does not require chapters, you need to use the command `\nochapters` just before the first section:

```
\nochapters

\section{...}      % First section.
    ... text ...

\section{...}      % Second section.
    ... text ...

    (...)
```

Next comes the bibliography. It can be made by hand like this:

```
\begin{thebibliography}{foo}
\bibitem ...
\end{thebibliography}
```

Or it can also be generated with BiB_TE_X, as explained in chapter 4.

Finally the vita is produced like this:

```
\begin{vita}
    % Insert your brief biographical sketch here.
    % Your permanent address and the name of the
    % typist(s) are generated automatically.
\end{vita}
```

Chapter 4

Making the Bibliography with BiB_TE_X

BiB_TE_X allows one to generate automatically the bibliography from a database of bibliographic items. You need to do the following:

1. Create the bibliographic database, which is a file whose name ends in .bib.

Let us call it `diss.bib`. Entries in this file are like this:

```
@BOOK{knuth:tb,  
  author = "Donald K. Knuth",  
  title = "The \TeXbook",  
  publisher = "Addison-Wesley",  
  year = "1984",  
}  
  
@TECHREPORT{poorten:sp,  
  author = "Alf~J.~van der Poorten",  
  title = "Some problems of recurrent interest",  
  institution = "School of Mathematics and Physics,  
                Macquarie University",  
  address = "North Ryde, Australia 2113",
```

```

    number = "81-0037",
    month = "August",
    year = "1981",
}
@ARTICLE{erdos:oap,
  author = "Paul Erd{\o}s and Paul Turan",
  title = "On a problem in the theory of uniform
          distribution, {I}",
  journal = "Indag. Math.",
  volume = "10",
  year = "1948",
  pages = "370--378",
}

```

2. Include a `\bibliographystyle` command in your \LaTeX file, say

`\bibliographystyle{plain}` and a `\bibliography` command to load the bibliography, in this case `\bibliography{diss}`, at the point of your document where the bibliography should be inserted.

The document at this point will look like this:

```

\bibliographystyle{plain}
\bibliography{diss}

```

3. Run \LaTeX on your main file, say `foo.tex`: `latex foo`. This generates an auxiliary file `foo.aux` with a list of `\cite` references.

4. Run BiB_TE_X on your file: `bibtex foo`. BiB_TE_X reads the auxiliary file, looks up the bibliographic database (`diss.bib`), and writes a `.bbl` file with the bibliographic information formatted according to the bibliographic style file (`.bst`, say `plain.bst`) specified. Messages about resources used and error messages are written to a `.blg` file (in the case of this template, `disstemplate.blg`).
5. Run L^AT_EX again: `latex foo`, which now reads the `.bbl` reference file.
6. Run L^AT_EX for a third time: `latex foo`, resolving all references.

This includes all bibliographic items that have been cited in the document with a `\cite` command. In order to include non cited items in the bibliography, use the command `\nocite`. For example, `\nocite{knuth:tb}` anywhere in the document (after `\begin{document}`) includes in the bibliography the item with label `knuth:tb`. In order to include *all* items of the bibliographic database, use the command `\nocite{*}`.

Chapter 5

Making Tables and Including Figures

The *tabular* environment allows us to create complex tables and figures, and draw boundaries around and within it. The following example illustrates this:

Table 5.1: An example of a table.

<i>Gegenwart</i>	<i>Imperfekt</i>	<i>Perfekt</i>
ich bin	ich war	ich bin gewesen
du bist	du warst	du bist gewesen
er	er	er
sie ist	sie wart	sie ist gewesen
es	es	es
wir sind	wir waren	wir sind gewesen
ihr seid	ihr wart	ihr seid gewesen
sie sind	sie waren	sie sind gewesen
Sie sind	Sie waren	Sie sind gewesen

Note: The assistance of Herr Professor Lothar Frommhold in generating this table of German declensions is gratefully acknowledged.

This table was created with the following sequence of commands:

```
\begin{table}[h]
\begin{center}
\caption{An example of a table.}
```

```

\vskip 10pt
\begin{tabular}{|l|l|l|l|l|}
\cline{1-2} \cline{4-5} \cline{7-9}
\multicolumn{2}{|c|} {\textsl{Gegenwart}} & & & \\
\multicolumn{2}{|c|} {\textsl{Imperfekt}} & & & \\
\multicolumn{3}{|c|} {\textsl{Perfekt}} & & \\
\cline{1-2} \cline{4-5} \cline{7-9}
ich & bin & & ich & war & & ich & bin & gewesen \\
du & bist & & du & warst & & du & bist & gewesen \\
er & & & er & & & er & & \\
sie & ist & & sie & wart & & sie & ist & gewesen \\
es & & & es & & & es & & \\
\cline{1-2} \cline{4-5} \cline{7-9}
wir & sind & & wir & waren & & wir & sind & gewesen \\
ihr & seid & & ihr & wart & & ihr & seid & gewesen \\
sie & sind & & sie & waren & & sie & sind & gewesen \\
\cline{1-2} \cline{4-5} \cline{7-9}
Sie & sind & & Sie & waren & & Sie & sind & gewesen \\
\cline{1-2} \cline{4-5} \cline{7-9}
\end{tabular} \vskip 10pt

Note: The assistance of Herr Professor Lothar Frommhold \\
in generating this table of German declensions \\
is gratefully acknowledged.

```



```

\vskip -20pt
\end{center}
\end{table}
\index{commands!environments!table}%

```

The argument `h` indicates the position for the table, in this case “here if possible”. Other values of this argument are: `t` (top of the page), `b` (bottom of the page), `p` (on the page of floats) and `H` (HERE! - requires using the package `float.sty`. Note: When this option is used, LaTeX ignores all of its formatting rules and does what you say, putting the entire float exactly where it is defined. Check your output to make sure it is what you want! If you are having trouble with LaTeX wanting to put a figure that’s larger than roughly half-a-page, as well as all of the figures following it, at the end of a chapter, try using the command `\clearpage` immediately following the large figure — and maybe a `\newpage` later.) It is possible to combine several arguments, such as `ht` (“here if possible, otherwise on top of the page”). The default is `tbp`.

Figure 5.1 is a typical example of inclusion of a figure contained in an encapsulated PostScript file. In order to use it, it is necessary to include the command `\usepackage{psfig}` at the beginning of the document.

You can see the commands that generated this figure in the source file. Look for the line `\begin{figure}[htb] % Imported eps example`.

The command that imports the file is `\psfig`, and it also controls its size (height and width), and can rotate the figure (angle).



Figure 5.1: An example of an imported jpg file.

Figures can also be drawn by using \LaTeX commands. Figure 5.2 is an example (taken from [?]).

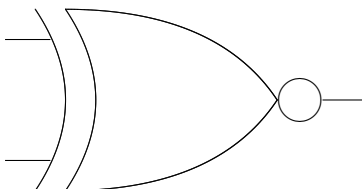


Figure 5.2: An example of a picture

The commands that generated this picture are in the source file following the line `\begin{figure}[htb] % Picture example.`

The commands used have rather obvious meanings. In particular, the command `\qbezier` draws a quadratic Bezier curve, defined by its two ending points, and a third point (whose coordinates are in the middle) that is used as control

point. Figure 5.3 illustrates the effect of the control point:

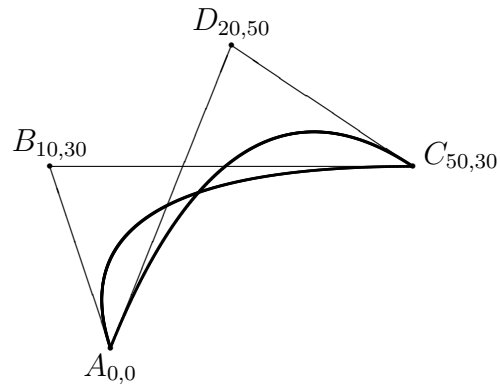


Figure 5.3: Bezier curves

This figure has been generated with the following commands:

```
\begin{figure}[htb] % Bezier curves example.
\begin{center}
\setlength{\unitlength}{.8mm}
\begin{picture}(55,55)(-15,0)
\linethickness{1pt}
\q bezier(0,0)(-10,30)(50,30)
\q bezier(0,0)(20,50)(50,30)
\thinlines
\put(0,0){\line(-1,3){10}}
\put(50,30){\line(-1,0){60}}
\put(0,0){\line(2,5){20}}
\put(50,30){\line(-3,2){30}}
\put(0,0){\circle*{1}}
\end{picture}
\end{center}
```

```

\put(0,-1){\makebox(0,0)[t]{$A_{0,0}$}}
\put(-10,30){\circle*{1}}
\put(-10,31){\makebox(0,0)[b]{$B_{10,30}$}}
\put(50,30){\circle*{1}}
\put(58,29){\makebox(0,0)[b]{$C_{50,30}$}}
\put(20,50){\circle*{1}}
\put(20,51){\makebox(0,0)[b]{$D_{20,50}$}}
\end{picture}
\caption{Bezier curves}
\label{f:qb}
\end{center}
\end{figure}

```

Chapter 6

An Example of Mathematical Writing

6.1 Generalized Fatou's Lemma

Here we show an application of the following lemma:

Lemma 6.1.1 (Generalized Fatou's Lemma). *Let A be a Dedekind ring and F a rational series in $A[[X]]$, i.e., $F = p/q$ for some $p, q \in A[X]$. Then there exist two polynomials $P, Q \in A[X]$ such that $F = P/Q$, where P and Q are relatively prime and $Q(0) = 1$.*

Proof. See [?], p. 15, theorem 1.3. □

Theorem 6.1.2. *Let $\{c_n\}_{n=-\infty}^{\infty}$ a set of elements from K such that $c_n \in k'$ for every $n \geq n_0$, and verifying the following recurrence relation of order M :*

$$c_n = r_1 c_{n-1} + r_2 c_{n-2} + \cdots + r_M c_{n-M} \quad (6.1)$$

for every $n \in \mathbb{Z}$, where r_1, r_2, \dots, r_M are in K , $r_M \neq 0$. Then:

- (i) The coefficients r_1, r_2, \dots, r_M are in k' , and for every $n \in \mathbb{Z}$, $c_n \in k'$.*
- (ii) If $c_n \in \mathcal{O}_{k',v}$ for every $n \geq n_0$, then the coefficients r_1, r_2, \dots, r_M are all in $\mathcal{O}_{k',v}$.*

Proof.

(i) Let C_n and R be the matrices:

$$C_n = \begin{pmatrix} c_n & c_{n+1} & \cdots & c_{n+M-1} \\ c_{n+1} & c_{n+2} & \cdots & c_{n+M} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n+M-1} & c_{n+M} & \cdots & c_{n+2M-2} \end{pmatrix} \quad (6.2)$$

and

$$R = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ r_M & r_{M-1} & r_{M-2} & \cdots & r_1 \end{pmatrix} \quad (6.3)$$

We have that $C_{n+1} = R C_n$. Since the recurrence relation is of order M , C_n is non singular. On the other hand, $R = C_{n+1} C_n^{-1}$. Since the elements of C_n are in k' for $n \geq n_0$, the entries of R , and those of R^{-1} , will be in k' . Since $C_{n-1} = R^{-1} C_n$, we get that the entries of C_n will be in k' also for $n < n_0$.

(ii) For each $t \geq n_0$ define the formal power series

$$F_t(X) = \sum_{n=0}^{\infty} c_{t+n} X^n \quad (6.4)$$

which is in $\mathcal{O}_{k',v}[[X]]$. We have $F_t(X) = p_t(X)/q(X)$, where $p_t(X), q(X) \in k'[X]$ are the following:

$$p_t(X) = \sum_{j=0}^{M-1} \left(c_{t+j} - \sum_{i=1}^j r_i c_{t+j-i} \right) X^j \quad (6.5)$$

$$q(X) = 1 - r_1 X - r_2 X^2 - \cdots - r_M X^M \quad (6.6)$$

This can be checked by multiplying $F_t(X)$ by $q(X)$ and using the recurrence relation, which gives $F_t(X) q(X) = p_t(X)$ (see [?]).

Now we will prove that $p_t(X)$ and $q(X)$ are relatively prime. To do so, we will see that they cannot have any common root (in $\overline{k'}$). In fact, assume that α is a common root of $p_{t_0}(X)$ and $q(X)$ for some $t_0 \geq n_0$, i.e.: $p_{t_0}(\alpha) = q(\alpha) = 0$. Since $q(0) = 1$, then $\alpha \neq 0$. Now we have:

$$X F_{t_0+1}(X) = F_{t_0}(X) - c_{t_0} \quad (6.7)$$

so:

$$\begin{aligned} X p_{t_0+1}(X) &= X q(X) F_{t_0+1}(X) \\ &= q(X) (F_{t_0}(X) - c_{t_0}) = p_{t_0}(X) - c_{t_0} q(X) \end{aligned} \quad (6.8)$$

Hence $p_{t_0+1}(\alpha) = 0$, which means that α is also a root of $p_{t_0+1}(X)$. By induction we get that $p_t(\alpha) = 0$ for every $t \geq t_0$. Grouping the terms of $p_t(X)$ with respect to $c_t, c_{t+1}, \dots, c_{t+M-1}$, we get:

$$p_t(X) = \sum_{j=0}^{M-1} a_j(X) c_{t+j} \quad (6.9)$$

where

$$a_j(X) = X^j \left(1 - \sum_{i=1}^{M-j-1} r_i X^i \right) \quad (6.10)$$

Note that $a_0(X), a_1(X), \dots, a_{M-1}(X)$ do not depend on t . On the other hand $p_t(\alpha) = 0$ implies

$$\sum_{j=0}^{M-1} a_j(\alpha) c_{t+j} = 0 \quad (6.11)$$

for every $t \geq t_0$. Note that $a_{M-1}(\alpha) = \alpha^{M-1} \neq 0$, so $a_0(\alpha), a_1(\alpha), \dots, a_{M-1}(\alpha)$ are not all zero, and (6.11) means that the columns of the matrix C_{t_0} are linearly

dependent, so $\det C_{t_0} = 0$, which contradicts the fact that C_{t_0} is non singular. Hence, the hypothesis that $p_t(X)$ and $q(X)$ have a common root has to be false. This proves that $p_t(X)$ and $q(X)$ are relatively prime.

By (generalized Fatou's) lemma 6.1.1, and taking into account that $\mathcal{O}_{k',v}$ is a Dedekind ring, we get that there exist two relatively prime polynomials $P_t(X)$ and $Q_t(X)$ in $\mathcal{O}_{k',v}[X]$ such that $F_t(X) = P_t(X)/Q_t(X)$ and $Q_t(0) = 1$. Hence: $p_t(X) Q_t(X) = q(X) P_t(X)$. By unique factorization of polynomials in $k'[X]$, there is a $u \in k'$ such that $P_t(X) = u p_t(X)$ and $Q_t(X) = u q_t(X)$. Since $Q_t(0) = q(0) = 1$, we get that $u = 1$, so $P_t(X) = p_t(X)$ and $Q_t(X) = q(X)$. Hence, the coefficients of $q(X)$ are in $\mathcal{O}_{k',v}$.

□

6.2 Other Examples of Mathematical Writing

6.2.1 An Example of a Commutative Diagram

The following is an example of a commutative diagram. It requires the `amscd` package.

$$\begin{array}{ccc} S^{\mathcal{W}_\Lambda} \otimes T & \xrightarrow{j} & T \\ \downarrow & & \downarrow \text{End } P \\ (S \otimes T)/I & \xlongequal{\quad} & (Z \otimes T)/J \end{array}$$

That diagram has been made with the following commands:

```
\newcommand{\End}{\operatorname{End}}
```



```

\begin{CD}
S^{\{\mathcal{W}\}_\Lambda}\otimes T @>j>> T\\
@VVV @VV{\text{End } P}V\\
(S\otimes T)/I @= (Z\otimes T)/J
\end{CD}

```

6.2.2 Using AMS Fonts

To use AMS fonts it is necessary to choose from an assortment of \LaTeX packages. For instance the command `\usepackage{amsfonts}` calls in the *amsfonts* package, which provides blackboard bold letters (e.g. \mathbb{R}) and some math symbols. A superset of that package is *amssymb*. Other packages are *eufrak* for Frankfurt letters (e.g. \mathfrak{R}) and *eucal* for Euler script (e.g. \mathcal{R}). Consult the \LaTeX documentation about this subject for additional information.

Appendices

Appendix A

Lerma's Appendix

The source \LaTeX file for this document is no longer quoted in its entirety in the output document. A \LaTeX file can include its own source by using the command `\verbatiminput{\jobname}`.

Appendix B

My Appendix #2

B.1 The First Section

This is the first section. This is the second appendix.

B.2 The Second Section

This is the second section of the second appendix.

B.2.1 The First Subsection of the Second Section

This is the first subsection of the second section of the second appendix.

B.2.2 The Second Subsection of the Second Section

This is the second subsection of the second section of the second appendix.

B.2.2.1 The First Subsubsection of the Second Subsection of the Second Section

This is the first subsubsection of the second subsection of the second section of the second appendix.

B.2.2.2 The Second Subsubsection of the Second Subsection of the Second Section

This is the second subsubsection of the second subsection of the second section of the second appendix.

Appendix C

My Appendix #3

C.1 The First Section

This is the first section. This is the third appendix.

C.2 The Second Section

This is the second section of the third appendix.

Vita

Preston Brent Landers was born in Texas and attended high school on the Nevada side of Lake Tahoe. He received his Bachelor of Arts in English from the University of Texas at Austin. He works as a software engineer for Journyx, Inc.* in Austin, Texas and began graduate studies in Software Engineering at the University of Texas at Austin in August 2012.

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This report was typeset with \LaTeX^\dagger by the author.

*<http://www.journyx.com>

† \LaTeX is a document preparation system developed by Leslie Lamport as a special version of Donald Knuth's \TeX Program.