Selmy: an adaptive API for cloud database services

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Selmy: an adaptive API for cloud database services

by

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REPORT

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Dedicated to my wife Andrea.

Acknowledgments

I wish to thank the multitudes of people who helped me. Time would fail me to tell of ...

Selmy: an adaptive API for cloud database services

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This document has the form of a "fake" doctoral dissertation in order to provide an example of such, but it is actually a copy of Miguel Lerma's documentation for the Mathematics Department Computer Seminar of 25 March 1998 updated in July 2001 and following by Craig McCluskey to meet the March 2001 requirements of the Graduate School.

This document and its source file show to write a Doctoral Dissertation using LaTeX and the utdiss2 package.

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Chapter 1

Introduction

This document deals with how to write a doctoral dissertation using LaTeX, and how to use the `utdiss2` package.

The latest version of this document/package can be obtained from `http://www.ph.utexas.edu/~laser/craigs_stuff/LaTeX/`.[1] If your installation of LaTeX is missing any style files used in this document (most likely with a `\usepackage{package-name.sty}` command at the beginning of disstemplate.tex), take a look at the link on this page to "Frequently Requested Style Files" or on the Comprehensive TeX Archive Network, `http://www.ctan.org`.

In case of any conflct between the requirements of the Office of Graduate Studies and what this document says to do, the requirements of the Office of Graduate Studies prevail.

---

[1]I will be transferring this page to the Office of Graduate Studies when I graduate. The new URL isn't defined yet, but I will place a "redirect" at this URL to send your browser to the correct location when the transition occurs.

1.1 History of This Package

In 1991 the `utdiss` package was written by Young U. Ryu in order to be used in the preamble of LaTeX doctoral dissertation files at the University of Texas at Austin. Since then some changes have occured, the most important one being the introduction of a new version of LaTeX called LaTeX 2_ε.

In order to partially adapt the utdiss package to this new version of LaTeX, Miguel Lerma introduced a few modifications in it, and his document, *How to Write a Doctoral Dissertation with LaTeX,* served as a test for it. His new package was called `utdiss1`.

With the significant changes in style introduced by the Graduate School in the Spring of 2001, as well as my need to write a dissertation myself, I extended Miguel Lerma's package to meet these new requirements. As in Miguel Lerma's case, this document serves as a test for it, but it is, in addition, intended as a template for others to use in writing their own dissertations. The new package is called `utdiss2`.

1.2 Revised Philosophy for This Package

Since the source file of this document is intended to be used by students writing their own dissertations, this document does not display all of the comments regarding usage of previous versions. It has, instead, transferred these comments to their respective places in the source file so someone editing their own copy of the source file to produce their own dissertation will see the comments

where they are needed. It may be helpful to print out a copy of the source file along with the PostScript version of the document so the two can be studied side-by-side.

Note: In spite of the effort to accommodate the package to the requirements of the University, it is not possible to guarantee that it will always work, and the author of the dissertation remains responsible for checking that such requirements are actually fulfilled by his/her final work.

The standard caveat applies:

This template package is provided and licensed "as is" without warranty of any kind, either expressed or implied, including, but not limited to, the implied warranties of merchantability and fitness for a particular purpose. Yadda, yadda, yadda, ...

In case of any problem with the use of `utdiss2`, send me email at `mccluskey@mail.utexas.edu`.

Chapter 2

Instructions for Preparing Dissertations, Theses, and Reports

We are not going to look at the complete set of instructions contained in *Instructions for Preparation of Doctoral Dissertations and Dissertation Abstracts* or *Format For The Master's Thesis and Report* which can be obtained from the Office of Graduate Studies (OGS) or on their web page, `http://www.utexas.edu/ogs`. The doctoral Instructions I am using are dated March, 2001. The master's Format I am using is dated May, 2001.

Here we will look at a few instructions related to the arrangement of the dissertation, thesis, or report and a few other "technical" details, providing some examples of common LaTeX usage and some examples of not-so-common LaTeX usage.

The following are just a couple of tests for the "quote" and "quotation" environments. The following paragraph is a quote.

> This template package is provided and licensed "as is" without warranty of any kind, either expressed or implied, including, but not limited to, the implied warranties of merchantability and fitness for a particular purpose.

The following paragraph is a quotation.

> This template package is provided and licensed "as is" without warranty of any kind, either expressed or implied, including, but not limited to, the implied warranties of merchantability and fitness for a particular purpose.

The OGS Instructions say prose quotations over four lines should be indented on the left. The Doctoral Degree Evaluator says the quote environment is the correct one to use.

2.1 Arrangement of Dissertation

Always remember that this "fake" dissertation is only intended to be a template for writing your own. Since the ultimate responsibility of making sure your dissertation meets the Graduate School's requirements, however, lies only with you, you ***must*** get the current *Instructions for Preparation of Doctoral Dissertations and Dissertation Abstracts* from the Office of Graduate Studies or their web page and check everything yourself. If you don't, you may have a very rude awakening from the Lynn Renegar, Doctoral Degree Evaluator (aka, "The Ruler Lady") at a most inopportune time.

Arrange your dissertation as follows (all sections are required unless said otherwise.

1. Fly Page (blank protective page). This page is **not** counted in the numbering. **Note:** This template does not insert a Fly Page; if you are printing an

official copy, you must manually insert a blank piece of paper on your own. Electronic documents do not need a fly page.

2. Copyright Legend (optional) - See OGS Instructions Sample Form A. Begin counting **pretext** pages here, but **do not place a number on this page.**

3. Committee Certification of Approved Version. See OGS Instructions Sample Form B. This page is included in the pretext count, but there should be no page number on the page.

4. Title Page - See OGS Instructions Sample Form C. This page is counted, but there should not be a page number on this page.

5. Dedication and/or Epigraph (optional). Included in count, but not numbered.

6. Acknowledgments or Preface (optional) - Begin showing **pretext** page numbers with **lower case Roman numerals** at bottom of page.

7. Abstract (optional) - See OGS Instructions Sample Form D.

8. Table of contents - List ALL sections which follow it. There are too may different ways a table of contents may be done for the OGS to give examples in their Instructions booklet, but do be sure there is agreement between the major headings in your text and their designations in the Table of Contents (fortunately LaTeX does this for you automatically). Please ask the Doctoral Degree Evaluator for assistance if necessary.

9. List of Tables, List of Figures, List of Illustrations, Nomenclature, List of Supplemental Files (such as multimedia files) (optional).

10. Text. The text should be divided into chapters, books or sections. The first page is Arabic numeral "**1**". All sections, **from the first page of text through Vita,** should be numbered consecutively.

11. If you group all Tables, Figures, or Illustrations in one place in your dissertation, the section should be placed here, immediately after the text and before any appendices (optional).

12. Appendix or Appendices (optional).

13. Glossary (optional) - this section may be placed either here or after the Table of Contents, in the area with List of Tables, List of Figures...

14. Bibliography - consult your supervisor about which recognized style to use.

15. Index (optional).

16. Vita - This should be a brief biographical sketch of the author. List in the Table of Contents. See OGS Instructions Sample Form E.

2.2 Other Requirements

2.2.1 Margins

The dissertation, after printing, should have left and top margins of 1 1/2 inches, and the right and bottom margins should be 1 1/4 inches. These margins

should be consistent throughout the dissertation - including all pages in the appendix. **All page numbers must be *at least* one inch from the edges of the page**. Headers are rarely used in dissertations; if you are considering using them, check with the Doctoral Degree Evaluator first to be sure they will be accepted.

2.2.2 Spacing and Page Arrangement

The document should be double-spaced or space-and-a-half. Exceptions to double-spacing are: the Table of Contents, Lists of Tables, Tables, Figures, Graphs, Captions, Footnotes, Endnotes, Appendices, Glossary, Bibliography and Index; these may be single-spaced. Paragraph indentations are usually five to ten spaces. Prose quotations over four lines should be in block quote (double or single spaced, indented on the left). Do not use quotation marks if the quotation is indented except for quotations within the block quote. Please refer to a style manual for more detailed instructions.

Be sure that each new chapter or major section (i.e., Appendix, Bibliography, Vita) begins on a new page.

2.3 Master's Theses and Reports

Always remember that this "fake" thesis or report — assuming you have followed the instructions in the next chapter about how to format it as such — is only intended to be a template for writing your own. Since the ultimate responsibility of making sure your thesis or report meets the Graduate School's requirements, however, lies only with you, you ***must*** get the current *Format For The Master's*

Thesis and Report from the Office of Graduate Studies or their web page and check everything yourself. If you don't, you may have a very rude awakening from the Mike Feissli, Master's Degree Evaluator at a most inopportune time.

That said, the formatting requirements for Dissertations and Reports and Theses are very similar. They are, however, ***not*** identical. The primary differences are in the ordering of the title and signature pages and where the optional index is inserted. For Master's Theses and Reports, the Title Page must be in front of the Signature Page. For Master's Theses and Reports, ***nothing*** is permitted to come between the bibliography and the vita; the index, if used, must be before the bibliography. If you want to use an index, talk with Mike Feissli before your deadline to verify that its inclusion is acceptable. The index can be removed by commenting out one line with a percent sign, if necessary, for producing the "official" copy of your thesis or report, and then inserted for copies for your advisor and you by removing the percent sign.

Chapter 3

How to Use the utdiss2 Package

3.1   Preamble

The preamble of the document starts like this:

```
\documentclass[12pt]{report}
\usepackage{utdiss2}
```

The first line declares "`report`" as the document class, with an option of 12pt for the character size, which is slightly greater that usual (the default is 10pt), but is what the Office of the Graduate School (OGS) recommends. You may include other options, as in any other LaTeX document.

The second line loads the `utdiss2` package, which contains a set of commands intended to produce a document fulfilling the official requirements for a doctoral dissertation or master's thesis or report. Besides that, you may include other packages. For instance:

```
\usepackage{amsmath,amsthm,amsfonts,amscd}
```

for mathematical symbols, or,

```
\usepackage{draftcopy}
```

to have a large "watermark" across each page of your document that says, "DRAFT."

The next few commands in the preamble are required.

`\author{Craig William McCluskey}` Replace my name in the command by your full, official University name. Make it combination of lower and upper-cases.

`\address{9905 Chukar Circle\\ Austin, Texas 78758}` Replace my address with your *permanent* (not local) address. Use \\ to separate address lines.

`\title{Writing a Doctoral Dissertation with \LaTeX{} at the University of Texas at Austin}` Replace the name of this document in this command by your dissertation title. If the title consists of more than one line, it should be in inverted pyramid form. You may have to specify the line breakings by \\ commands.

`\supervisor[Isaac Newton]{Johannes Kepler}` This document has two supervisors listed. See the source file (disstemplate.tex) for information on how to have only one supervisor. This command can be broken across lines as it is in the source file and as the `\committeemembers` command is shown below.

```
\committeemembers
        [Erwin Schr\"odinger]
        [Albert Einstein]
        [Charles Townes]
        {Arthur Schawlow}
```

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This document shows four non-supervisor committee members. See the source file (disstemplate.tex) for information on how to have a different number.

`\previousdegrees{B.S.}` Replace B.S. with your previous degree.

The next few commands in the preamble are optional.

```
%\graduationmonth{...}
%\graduationyear{...}
%\typist{...}
```

Their use is documented in the source file.

At this point, if you are writing a master thesis or report you must use the optional `\degree` and `\degreeabbr` commands and specify

```
%\degree{MASTER OF ARTS}
%\degreeabbr{M.A.}
%\masterreport
%\masterthesis
```

as documented in the source file. By default the document is formated as a *dissertation*[1]

The default spacing for both text and quoted text is doublespaced. That can be changed with the following self-explanatory commands:

---

[1]The command `\dissertation` is also provided for symmetry.

```
\oneandonehalfspacing

\singlespacing

\oneandonehalfspacequote

\singlespacequote
```

Some versions of LaTeX in combination with some types of printers produce printed output that has incorrect vertical margins. The command `\topmargin 0.125` is provided to allow easy adjustment if it's needed.

If there are 10 or more sections, 10 or more subsections for a section, etc., you need to make an adjustment to the Table of Contents with the command `\longtocentry`. This command allocates the proper horizontal space for double-digit numbers.

3.2   Document

Next comes the actual text. It could be a sequence of chapters divided into sections, subsections, etc., all in the main file:

```
\chapter{...}    % The first chapter. The

                 % \chapter command is of the form

                 % \chapter[..]{..} or \chapter{..} where

   ... text ...  % [...] is the entry in table of contents

                 % and {...} is the chapter heading printed

                 % in the body of the document.
\section{...}    %

                 % IMPORTANT: If your chapter heading consists
```

```
                      % of more than one line, it will be auto-

     ... text ...     % matically broken into separate lines.

                      % If you don't like the way LaTeX breaks the

                      % chapter heading into lines, however, use

\section{...}         % `\newheadline' command to break lines.

                      % NEVER USE \\ IN SECTIONAL (E.G., CHAPTER,

     ... text ...     % SECTION, SUBSECTION, SUBSUBSECTION) HEADINGS!

                      %

\chapter{...}         % This is Chapter 2.

     ... text ...

\section{...}

     ... text...

\subsection{...}

     ... more text ...

\subsubsection{...}

     ... more text ...

\appendix             % The appendix begins here.

% \appendices         % If more than one appendix chapters,

                      % use \appendices instead of \appendix

\chapter{...}         % First appendix chapter, i.e., Appendix A.

\section{...}         % This is appendix section A.1.

     .................
```

Or, the chapters can be written in different files like this document and be loaded by \include commands:

```
\include{chapter-introduction}

\include{chapter-instructions}

\include{chapter-howtouse}

\include{chapter-makingbib}

\include{chapter-tables+figs}

\include{chapter-math}

\appendices

\index{Appendices@\emph{Appendices}}%

\include{chapter-appendix1}

\include{chapter-appendix2}

\include{chapter-appendix3}
```

Having the chapters in separate files makes the main .tex file simpler and allows chapters to be easily re-ordered (just swap the order of the include commands) or left (commented) out for draft copies.

Note: If you have only one appendix, in addition to using \appendix instead of \appendices, you must leave out the \chapter definition at the start of the appendix's text. Putting it in will cause the insertion of an extra page with only the word Appendix on it and will cause the appendix to be labeled Appendix 1, both of which are poor form if there is only one appendix.

If you are writing a short dissertation that does not require chapters, you need to use the command \nochapters just before the first section:

```
\nochapters


\section{...}      % First section.

    ... text ...
\section{...}      % Second section.

    ... text ...

      (...)
```

Next comes the bibliography. It can be made by hand like this:

```
\begin{thebibliography}{foo}

\bibitem ...

\end{thebibliography}
```

Or it can also be generated with BiBTₑX, as explained in chapter 4.

Finally the vita is produced like this:

```
\begin{vita}

    % Insert your brief biographical sketch here.

    % Your permanent address and the name of the

    % typist(s) are generated automatically.
\end{vita}
```

Chapter 4

Making the Bibliography with BiBT_EX

BiBT_EX allows one to generate automatically the bibliography from a database of bibliographic items. You need to do the following:

1. Create the bibliographic database, which is a file whose name ends in `.bib`. Let us call it `diss.bib`. Entries in this file are like this:

   ```
   @BOOK{knuth:tb,
     author = "Donald K. Knuth",
     title = "The \TeXbook",
     publisher = "Addison-Wesley",
     year = "1984",
   }
   @TECHREPORT{poorten:sp,
     author = "Alf~J.~van der Poorten",
     title = "Some problems of recurrent interest",
     institution = "School of Mathematics and Physics,
                    Macquarie University",
     address = "North Ryde, Australia 2113",
   ```

```
    number = "81-0037",

    month = "August",

    year = "1981",

  }

  @ARTICLE{erdos:oap,

   author = "Paul Erd{\"o}s and Paul Turan",

   title = "On a problem in the theory of uniform

           distribution, {I}",

   journal = "Indag. Math.",

   volume = "10",

   year = "1948",

   pages = "370--378",

  }
```

2. Include a \bibliographystyle command in your LaTeX file, say

   \bibliographystyle{plain} and a \bibliography command to load the
   bibliography, in this case \bibliography{diss}, at the point of your docu-
   ment where the bibliography should be inserted.

   The document at this point will look like this:

   ```
   \bibliographystyle{plain}
   \bibliography{diss}
   ```

3. Run LaTeX on your main file, say foo.tex: latex foo. This generates an
   auxiliary file foo.aux with a list of \cite references.

4. Run BiBTEX on your file: `bibtex foo`. BiBTEX reads the auxiliary file, looks up the bibliographic database (`diss.bib`), and writes a `.bbl` file with the bibliographic information formated according to the bibliographic style file (`.bst`, say `plain.bst`) specified. Messages about resources used and error messages are written to a `.blg` file (in the case of this template, disstemplate.blg).

5. Run LATEX again: `latex foo`, which now reads the `.bbl` reference file.

6. Run LATEX for a third time: `latex foo`, resolving all references.

This includes all bibliographic items that have been cited in the document with a `\cite` command. In order to include non cited items in the bibliography, use the command `\nocite`. For example, `\nocite{knuth:tb}` anywhere in the document (after `\begin{document}`) includes in the bibliography the item with label `knuth:tb`. In order to include *all* items of the bibliographic database, use the command `\nocite{*}`.

Chapter 5

Making Tables and Including Figures

The *tabular* environment allows us to create complex tables and figures, and draw boundaries around and within it. The following example illustrates this:

Table 5.1: An example of a table.

Gegenwart			*Imperfekt*			*Perfekt*		
ich	bin		ich	war		ich	bin	gewesen
du	bist		du	warst		du	bist	gewesen
er			er			er		
sie	ist		sie	wart		sie	ist	gewesen
es			es			es		
wir	sind		wir	waren		wir	sind	gewesen
ihr	seid		ihr	wart		ihr	seid	gewesen
sie	sind		sie	waren		sie	sind	gewesen
Sie	sind		Sie	waren		Sie	sind	gewesen

Note: The assistance of Herr Professor Lothar Frommhold
in generating this table of German declensions
is gratefully acknowledged.

This table was created with the following sequence of commands:

```
\begin{table}[h]

\begin{center}

\caption{An example of a table.}
```

```latex
\vskip 10pt

\begin{tabular}{|ll|l|ll|l|lll|}

\cline{1-2} \cline{4-5} \cline{7-9}

\multicolumn{2}{|c|} {\textsl{Gegenwart}} & &

\multicolumn{2}{|c|} {\textsl{Imperfekt}} & &

\multicolumn{3}{|c|} {\textsl{Perfekt}} \\

\cline{1-2} \cline{4-5} \cline{7-9}

ich & bin  & & ich & war   & & ich & bin  & gewesen \\

du  & bist & & du  & warst & & du  & bist & gewesen \\

er  &      & & er  &       & & er  &      &         \\

sie & ist  & & sie & wart  & & sie & ist  & gewesen \\

es  &      & & es  &       & & es  &      &         \\

\cline{1-2} \cline{4-5} \cline{7-9}

wir & sind & & wir & waren & & wir & sind & gewesen \\

ihr & seid & & ihr & wart  & & ihr & seid & gewesen \\

sie & sind & & sie & waren & & sie & sind & gewesen \\

\cline{1-2} \cline{4-5} \cline{7-9}

Sie & sind & & Sie & waren & & Sie & sind & gewesen \\

\cline{1-2} \cline{4-5} \cline{7-9}

\end{tabular} \\[10pt]

Note: The assistance of Herr Professor Lothar Frommhold \\

in generating this table of German declensions \\

is gratefully acknowledged.
```

```
\vskip -20pt
\end{center}
\end{table}
\index{commands!environments!table}%
```

The argument h indicates the position for the table, in this case "here if possible". Other values of this argument are: t (top of the page), b (bottom of the page), p (on the page of floats) and H (HERE! - requires using the package float.sty. Note: When this option is used, LaTeX ignores all of its formatting rules and does what you say, putting the entire float exactly where it is defined. Check your output to make sure it is what you want! If you are having trouble with LaTeX wanting to put a figure that's larger than roughly half-a-page, as well as all of the figures following it, at the end of a chapter, try using the command \clearpage immediately following the large figure — and maybe a \newpage later.) It is possible to combine several arguments, such as ht ("here if possible, otherwise on top of the page"). The default is tbp.

Figure 5.1 is a typical example of inclusion of a figure contained in an encapsulated PostScript file. In order to use it, it is necessary to include the command \usepackage{psfig} at the beginning of the document.

You can see the commands that generated this figure in the source file. Look for the line \begin{figure}[htb] % Imported eps example.

The command that imports the file is \psfig, and it also controls its size (height and width), and can rotate the figure (angle).

Figure 5.1: An example of an imported jpg file.

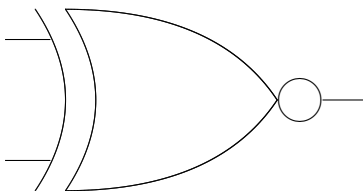Figures can also be drawn by using LaTeX commands. Figure 5.2 is an example (taken from [?]).

Figure 5.2: An example of a picture

The commands that generated this picture are in the source file following the line \begin{figure}[htb] % Picture example.

The commands used have rather obvious meanings. In particular, the command \qbezier draws a quadratic Bezier curve, defined by its two ending points, and a third point (whose coordinates are in the middle) that is used as control

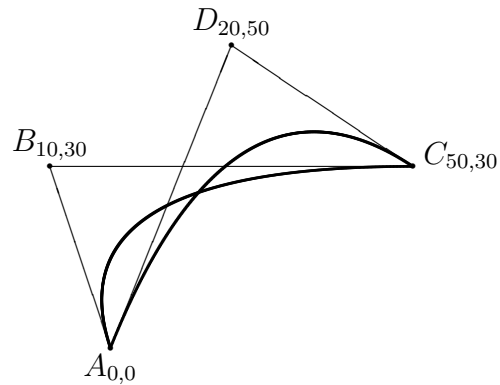point. Figure 5.3 illustrates the effect of the control point:

Figure 5.3: Bezier curves

This figure has been generated with the following commands:

```
\begin{figure}[htb] % Bezier curves example.
\begin{center}
   \setlength{\unitlength}{.8mm}
   \begin{picture}(55,55)(-15,0)
      \linethickness{1pt}
      \qbezier(0,0)(-10,30)(50,30)
      \qbezier(0,0)(20,50)(50,30)
      \thinlines
      \put(0,0){\line(-1,3){10}}
      \put(50,30){\line(-1,0){60}}
      \put(0,0){\line(2,5){20}}
      \put(50,30){\line(-3,2){30}}
      \put(0,0){\circle*{1}}
```

```
        \put(0,-1){\makebox(0,0)[t]{$A_{0,0}$}}

        \put(-10,30){\circle*{1}}

        \put(-10,31){\makebox(0,0)[b]{$B_{10,30}$}}

        \put(50,30){\circle*{1}}

        \put(58,29){\makebox(0,0)[b]{$C_{50,30}$}}

        \put(20,50){\circle*{1}}

        \put(20,51){\makebox(0,0)[b]{$D_{20,50}$}}

    \end{picture}

\caption{Bezier curves}

\label{f:qb}

\end{center}

\end{figure}
```

Chapter 6

An Example of Mathematical Writing

6.1 Generalized Fatou's Lemma

Here we show an application of the following lemma:

Lemma 6.1.1 (Generalized Fatou's Lemma). *Let A be a Dedekind ring and F a rational series in $A[[X]]$, i.e., $F = p/q$ for some $p, q \in A[X]$. Then there exist two polynomials $P, Q \in A[X]$ such that $F = P/Q$, where P and Q are relatively prime and $Q(0) = 1$.*

Proof. See [**?**], p. 15, theorem 1.3. □

Theorem 6.1.2. *Let $\{c_n\}_{n=-\infty}^{\infty}$ a set of elements from K such that $c_n \in k'$ for every $n \geq n_0$, and verifying the following recurrence relation of order M:*

$$c_n = r_1 \, c_{n-1} + r_2 \, c_{n-2} + \cdots + r_M \, c_{n-M} \tag{6.1}$$

for every $n \in \mathbb{Z}$, where $r_1, r_2, \ldots, r_M$ are in K, $r_M \neq 0$. Then:

(i) The coefficients $r_1, r_2, \ldots, r_M$ are in k', and for every $n \in \mathbb{Z}$, $c_n \in k'$.

(ii) If $c_n \in \mathcal{O}_{k',v}$ for every $n \geq n_0$, then the coefficients $r_1, r_2, \ldots, r_M$ are all in $\mathcal{O}_{k',v}$.

Proof.

(i) Let C_n and R be the matrices:

$$C_n = \begin{pmatrix} c_n & c_{n+1} & \cdots & c_{n+M-1} \\ c_{n+1} & c_{n+2} & \cdots & c_{n+M} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n+M-1} & c_{n+M} & \cdots & c_{n+2M-2} \end{pmatrix} \tag{6.2}$$

and

$$R = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ r_M & r_{M-1} & r_{M-2} & \cdots & r_1 \end{pmatrix} \tag{6.3}$$

We have that $C_{n+1} = R\, C_n$. Since the recurrence relation is of order M, C_n is non singular. On the other hand, $R = C_{n+1}\, C_n^{-1}$. Since the elements of C_n are in k' for $n \geq n_0$, the entries of R, and those of R^{-1}, will be in k'. Since $C_{n-1} = R^{-1}\, C_n$, we get that the entries of C_n will be in k' also for $n < n_0$.

(ii) For each $t \geq n_0$ define the formal power series

$$F_t(X) = \sum_{n=0}^{\infty} c_{t+n}\, X^n \tag{6.4}$$

which is in $\mathcal{O}_{k',v}[[X]]$. We have $F_t(X) = p_t(X)/q(X)$, where $p_t(X), q(X) \in k'[X]$ are the following:

$$p_t(X) = \sum_{j=0}^{M-1} \left(c_{t+j} - \sum_{i=1}^{j} r_i\, c_{t+j-i} \right) X^j \tag{6.5}$$

$$q(X) = 1 - r_1\, X - r_2\, X^2 - \cdots - r_M\, X^M \tag{6.6}$$

This can be checked by multiplying $F_t(X)$ by $q_t(X)$ and using the recurrence relation, which gives $F_t(X)\, q(X) = p_t(X)$ (see [?]).

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Now we will prove that $p_t(X)$ and $q(X)$ are relatively prime. To do so, we will see that they cannot have any common root (in $\overline{k'}$). In fact, assume that α is a common root of $p_{t_0}(X)$ and $q(X)$ for some $t_0 \geq n_0$, i.e.: $p_{t_0}(\alpha) = q(\alpha) = 0$. Since $q(0) = 1$, then $\alpha \neq 0$. Now we have:

$$X\, F_{t_0+1}(X) = F_{t_0}(X) - c_{t_0} \tag{6.7}$$

so:

$$X\, p_{t_0+1}(X) = X\, q(X)\, F_{t_0+1}(X)$$
$$= q(X)\,(F_{t_0}(X) - c_{t_0}) = p_{t_0}(X) - c_{t_0}\, q(X) \tag{6.8}$$

Hence $p_{t_0+1}(\alpha) = 0$, which means that α is also a root of $p_{t_0+1}(X)$. By induction we get that $p_t(\alpha) = 0$ for every $t \geq t_0$. Grouping the terms of $p_t(X)$ with respect to $c_t, c_{t+1}, \ldots, c_{t+M-1}$, we get:

$$p_t(X) = \sum_{j=0}^{M-1} a_j(X)\, c_{t+j} \tag{6.9}$$

where

$$a_j(X) = X^j \left(1 - \sum_{i=1}^{M-j-1} r_i\, X^i\right) \tag{6.10}$$

Note that $a_0(X), a_1(X), \ldots, a_{M-1}(X)$ do not depend on t. On the other hand $p_t(\alpha) = 0$ implies

$$\sum_{j=0}^{M-1} a_j(\alpha)\, c_{t+j} = 0 \tag{6.11}$$

for every $t \geq t_0$. Note that $a_{M-1}(\alpha) = \alpha^{M-1} \neq 0$, so $a_0(\alpha), a_1(\alpha), \ldots, a_{M-1}(\alpha)$ are not all zero, and (6.11) means that the columns of the matrix C_{t_0} are linearly

dependent, so $\det C_{t_0} = 0$, which contradicts the fact that C_{t_0} is non singular. Hence, the hypothesis that $p_t(X)$ and $q(X)$ have a common root has to be false. This proves that $p_t(X)$ and $q(X)$ are relatively prime.

By (generalized Fatou's) lemma 6.1.1, and taking into account that $\mathcal{O}_{k',v}$ is a Dedekind ring, we get that there exist two relatively prime polynomials $P_t(X)$ and $Q_t(X)$ in $\mathcal{O}_{k',v}[X]$ such that $F_t(X) = P_t(X)/Q_t(X)$ and $Q_t(0) = 1$. Hence: $p_t(X) Q_t(X) = q(X) P_t(X)$. By unique factorization of polynomials in $k'[X]$, there is a $u \in k'$ such that $P_t(X) = u \, p_t(X)$ and $Q_t(X) = u \, q_t(X)$. Since $Q_t(0) = q(0) = 1$, we get that $u = 1$, so $P_t(X) = p_t(X)$ and $Q_t(X) = q(X)$. Hence, the coefficients of $q(X)$ are in $\mathcal{O}_{k',v}$.

$\square$

6.2 Other Examples of Mathematical Writing

6.2.1 An Example of a Commutative Diagram

The following is an example of a commutative diagram. It requires the `amscd` package.

$$
\begin{CD}
S^{W_\Lambda} \otimes T @>j>> T \\
@VVV @VV{\operatorname{End} P}V \\
(S \otimes T)/I @= (Z \otimes T)/J
\end{CD}
$$

That diagram has been made with the following commands:

```
\newcommand{\End}{\operatorname{End}}
```

```
\begin{CD}
S^{{\mathcal{W}}_\Lambda}\otimes T   @>j>>   T\\
@VVV                                @VV{\End P}V\\
(S\otimes T)/I              @=    (Z\otimes T)/J
\end{CD}
```

6.2.2   Using AMS Fonts

To use AMS fonts it is necessary to choose from an assortment of LaTeX packages. For instance the command \usepackage{amsfonts} calls in the *amsfonts* package, which provides blackboard bold letters (e.g. $\mathbb{R}$) and some math symbols. A superset of that package is *amssymb*. Other packages are *eufrak* for Frankfurt letters (e.g. $\mathfrak{R}$) and *eucal* for Euler script (e.g. $\mathcal{R}$). Consult the LaTeX documentation about this subject for additional information.

Appendices

Appendix A

Lerma's Appendix

The source LaTeX file for this document is no longer quoted in its entirety in the output document. A LaTeX file can include its own source by using the command `\verbatiminput{\jobname}`.

Appendix B

My Appendix #2

B.1 The First Section

This is the first section. This is the second appendix.

B.2 The Second Section

This is the second section of the second appendix.

B.2.1 The First Subsection of the Second Section

This is the first subsection of the second section of the second appendix.

B.2.2 The Second Subsection of the Second Section

This is the second subsection of the second section of the second appendix.

B.2.2.1 The First Subsubsection of the Second Subsection of the Second Section

This is the first subsubsection of the second subsection of the second section of the second appendix.

B.2.2.2 The Second Subsubsection of the Second Subsection of the Second Section

This is the second subsubsection of the second subsection of the second section of the second appendix.

Appendix C

My Appendix #3

C.1 The First Section

This is the first section. This is the third appendix.

C.2 The Second Section

This is the second section of the third appendix.

Vita

Preston Brent Landers was born in Arlington, Texas and attended high school in Incline Village, Nevada. He received his Bachelor of Arts in English from the University of Texas at Austin. He works as a software engineer for Journyx, Inc.[*] in Austin, Texas and began graduate studies at the University of Texas at Austin in August 2012.

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This report was typeset with LaTeX[†] by the author.

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[*] `http://www.journyx.com`

[†] LaTeX is a document preparation system developed by Leslie Lamport as a special version of Donald Knuth's TeX Program.