Summer 2022: Data Analysis
Homework II
Due: TBA
Submit through Canvas

Instructions: Provided solutions to these questions using this template. Include graphics with your solutions. Put all code an appendix to this homework. Use the *verbatim* command to leave code unchanged.

1. Suppose that a shipping company models the weight of packages with the PDF

$$f(x) = \frac{70}{69X^2} \qquad\qquad 1 < x < 70.$$

a) Verify $f(x)$ is a proper density.
Solution:
By inspection, we see $f(x) \geq 0$ for all $x \in (0, 1)$. Now we must show the density integrates to unity. Note that

$$\begin{aligned} \int_{-\infty}^{\infty} f(x)dx &= \int_{0}^{\infty} \frac{70}{69x^2} dx \\ &= 1 \end{aligned}$$

b) Get the CDF of X.
Solution:

$$\begin{aligned} P(X \geq x) = F(x) &= \int_{0}^{x} f(t)dx \\ F(x) &= \int_{1}^{x} \frac{70}{69t^2} \\ &= \frac{70}{69}\left[1 - \frac{1}{x}\right] \end{aligned}$$

c) Find the chance a randomly selected package weights at least 20 pounds.
Solution:

$$P\left(X \geq 20\right)$$

$$
\begin{aligned}
P\left(X \geq 20\right) &= 1 - P\left(X \leq 20\right) \\
&= 1 - F(20) \\
&= 1 - \frac{70}{69}\left[1 - \frac{1}{x}\right] \\
&= 1 - \frac{70}{69}\left[1 - \frac{1}{20}\right]
\end{aligned}
$$

d) Get μ
Solution:

$$
\begin{aligned}
&= \int_0^\infty xf(x)dx \\
&= \int_0^\infty x\frac{70}{69x^2}dx \\
&= \frac{70}{69}\int_0^{70} \frac{1}{x}dx \\
&= \frac{70}{69}\left[ln(x)\right|_{x=1}^{70} \\
&= \frac{70}{69}\left[ln(70) - ln(1)\right] \\
&= 4.31
\end{aligned}
$$

e) If shipping cost is \$5 per pound, get E[shipping cost per package].
Solution:
To calculate the average price of a package we multiply \$5 by 4.31.

$$\$5 \times 4.31 = \$21.55$$

2. Make plots of the normal PDF when

a) $\mu = 5$ and $\sigma^2 = 1$.
Solution:

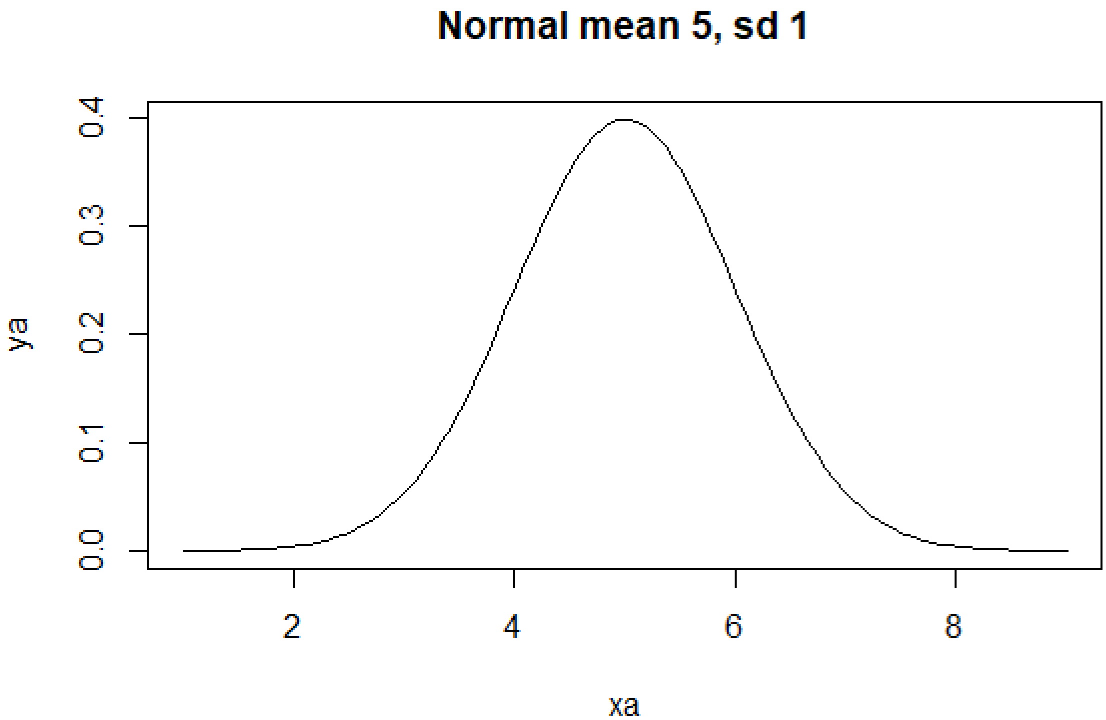

Figure 1: Mean = 5, Variance = 1

b) $\mu = 1$ and $\sigma^2 = 5$.
Solution:

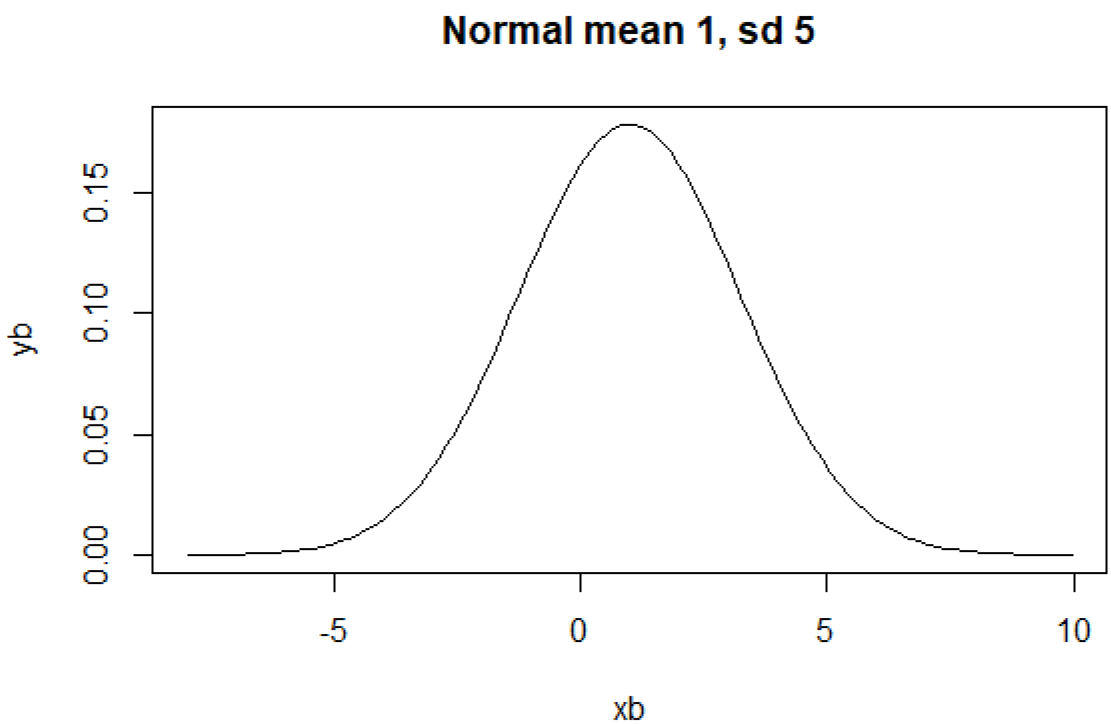

Figure 2: Mean = 1, Variance = 5

c) $\mu = 0$ and $\sigma^2 = 1$.
Solution:

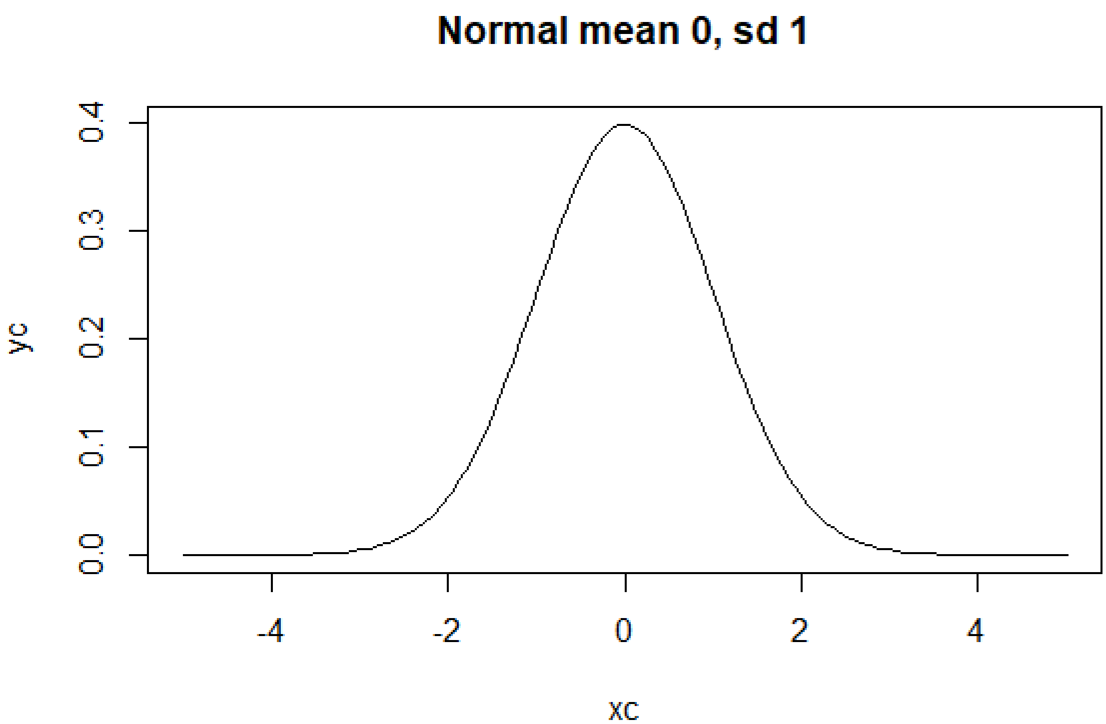

Figure 3: Mean = 0, Variance = 1

d) $\mu = 0$ and $\sigma^2 = 0.1$.
Solution:

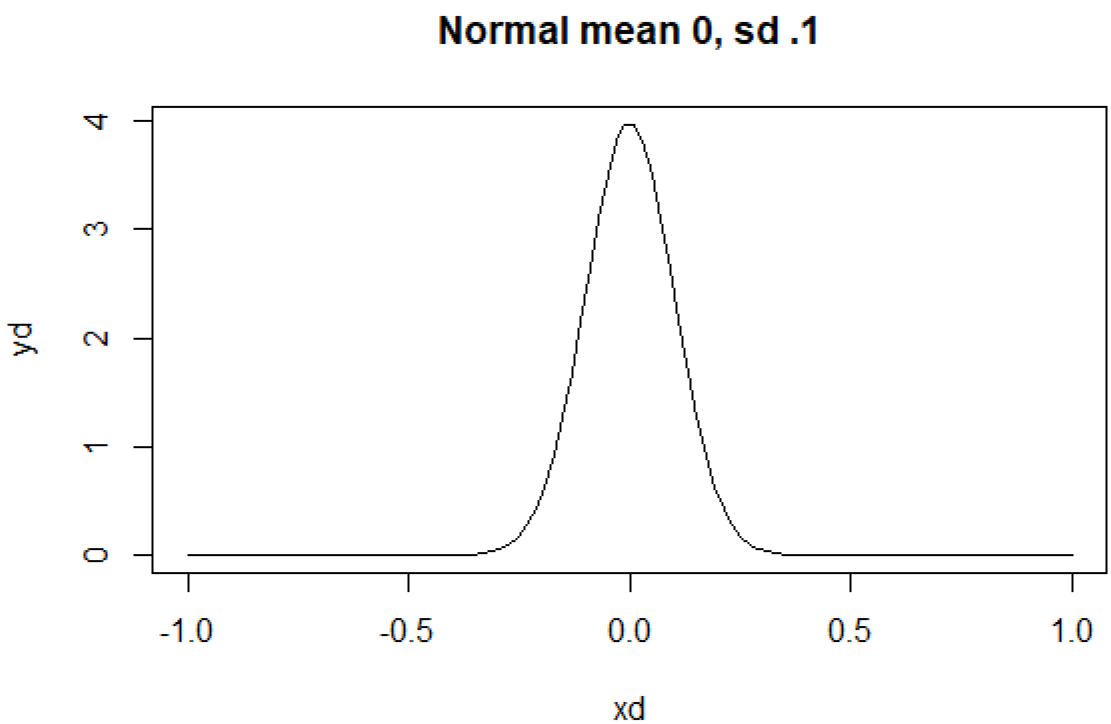

Figure 4: Mean = 0, Variance = .1

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```{r}

xb <- seq(1-sqrt(5)*4, 1+sqrt(5)*4, length = 100)

yb <- dnorm(xb, mean = 1, sd = sqrt(5))

plot(xb,yb, type = 'l', main = 'Normal mean 1, sd 5')
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Extra Credit

Let $\Gamma(\alpha) := \int_0^\infty x^{\alpha-1}e^{-x}dx$ for $\alpha > 0$. Show that $\Gamma(1/2) = \sqrt{\pi}$.