mui_240a2_pset2_ipynb

November 8, 2015

1 Problem Set #2, Preston Mui

2 Calorie Demand

In [19]:
```
# Preparation to use the dataset
using DataFrames, Gadfly
data = readtable("RPS_calorie_data.out");
```

2 Using the power series basis, construct a new basis that is orthogonal to the design points ($K = 12$)

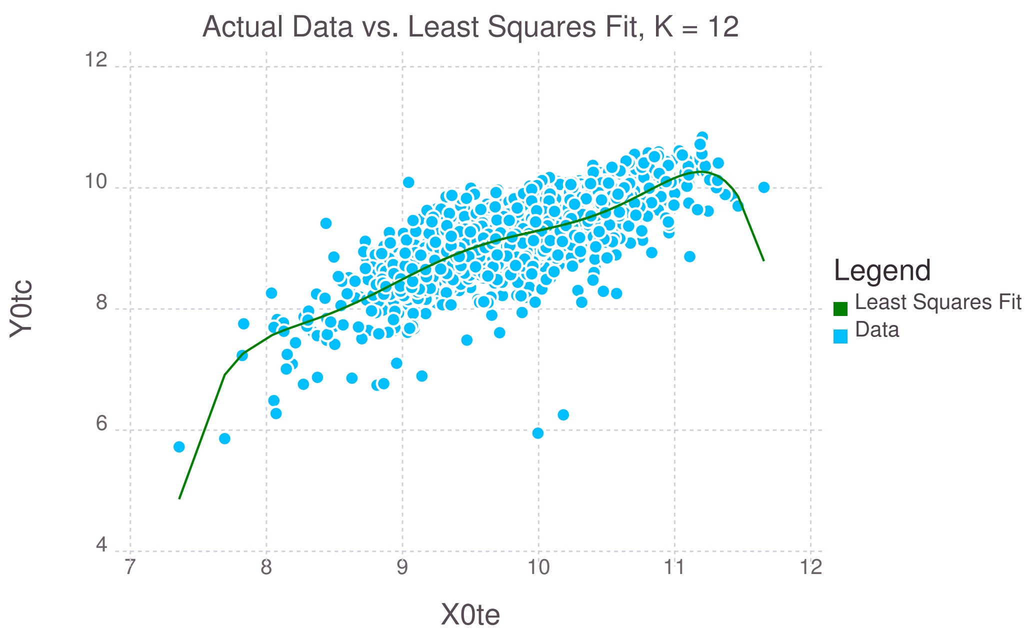
In [20]:
```
# Extract the matrices from the data
K = 12
Y = convert(Array,data[:Y0tc])
X = zeros(size(data)[1],K)

# Construct the basis vectors
for k = 1:K
    X[:,k] = data[:X0te].^(k-1)
end

# Function for Gram-Schmidt
function GSO(X::Array{Float64})
    # Orthogonalization
    K = size(X)[2]
    W = copy(X)
    for k = 1:K
        for i = 1:k-1
            W[:,k] -= (dot(X[:,k],W[:,i]) / dot(W[:,i],W[:,i])) * W[:,i]
        end
    end
    # Normalization
    for k = 1:K
        W[:,k] = W[:,k] / sqrt((dot(W[:,k],W[:,k])/length(W)[1]))
    end
    return W
end;
```

Let W_i denote the $K \times 1$ vector of orthonormal basis functions for household i. Compute the Least Squares Fit:

In [21]:
```
# MLE Estimate of basis function coefficients
W = GSO(X)
```

```julia
theta = W'*Y / length(W)[1]
m = W * theta
plot(layer(x=X[:,2], y=m, Geom.line,Theme(default_color=colorant"green")),
    layer(x=X[:,2], y=Y, Geom.point),
    Guide.XLabel("X0te"),
    Guide.YLabel("Y0tc"),
    Guide.Title("Actual Data vs. Least Squares Fit, K = $K"),
    Guide.manual_color_key("Legend",["Least Squares Fit","Data"],["green","deepskyblue"]))
```

Out[21]:

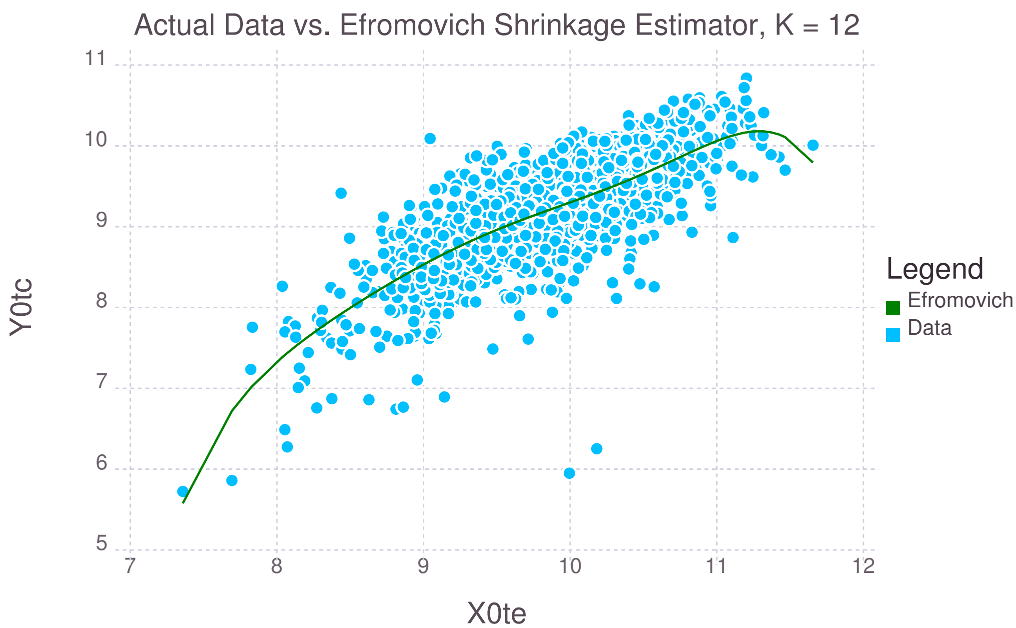


3 Using the shrinkage estimator, estimate $m(X_i)$. Plot this on the unsmoothed data.

In [22]:
```julia
# MLE of Variance
sigma2 = norm(Y-m) / (length(Y)- K)

# Shrinkage Coefficients
function efromovich(Z::Array{Float64,1},var::Float64,N::Int64)
    shrinkage = similar(Z)
    for k = 1:length(shrinkage)
        shrinkage[k] = max(1 - (var/N)/Z[k]^2,0)
    end
    return shrinkage
end

# Plot the Efromovich
muse = W*(efromovich(theta,sigma2,length(Y)).*theta)
```

```
plot(layer(x=X[:,2], y=muse, Geom.line,Theme(default_color=colorant"green")),
    layer(x=X[:,2], y=Y, Geom.point),
    Guide.XLabel("X0te"),
    Guide.YLabel("Y0tc"),
    Guide.Title("Actual Data vs. Efromovich Shrinkage Estimator, K = $K"),
    Guide.manual_color_key("Legend",["Efromovich","Data"],["green","deepskyblue"]))
```

Out[22]:



Comments: The Efromovich Shrinkage Estimator is very close to the Maximum Likelihood Estimator. The shrinkage coefficients are very close to 1.

4 Compute the Soft Threshold Estimate Consider the SURE Risk of the soft threshold estimator $\hat{\theta}_k$. Say, without loss of generality, that the Z_k are ordered such that $|Z_1| \leq |Z_2| \leq \cdots \leq |Z_K|$. Then, say that $\lambda \in [Z_m, Z_{m+1})$ for some $m \in 1, \cdots, K$. Then,

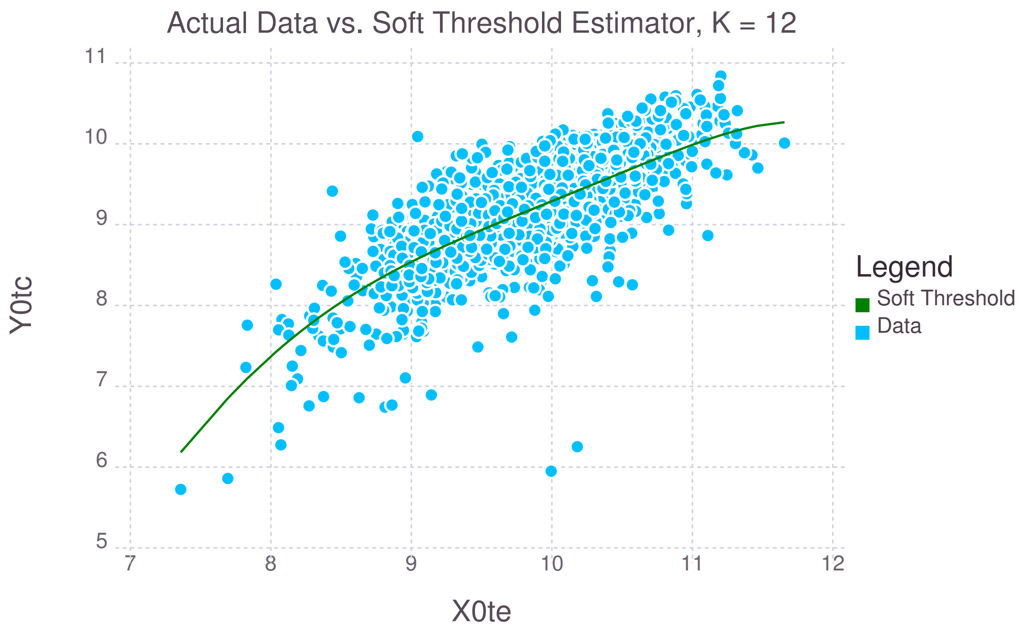
$$\hat{R}_{SURE} = K\frac{\sigma^2}{N} - 2m\frac{\sigma^2}{N} + \sum_{k=1}^{m} Z_k^2 + (K-m)\lambda^2$$

This is increasing in λ on this interval, so the risk-minimizing choice of λ on this interval is $\lambda = |Z_m|$. So, to minimize risk, I will iterate through $m = 1, \cdots, K$ and compare estimates of risk for $\lambda = Z_m$ for each m.

In [23]:
```
# Function to calculate RSURE of Wasserman Soft Threshold using the MLE's of Z and Sigma
function risk_wasserman(Z::Array{Float64,1}, lambda::Float64, var::Float64, N::Int64)
    K = length(Z)
    risk = K * var / N
    for k = 1:length(Z)
```

```
            if abs(Z[k]) <= lambda
                risk += Z[k]^2 - 2 * var / N
            else
                risk += lambda^2
            end
        end
        return risk
    end

    # Create Wasserman Soft Threshold
    function wasserman(Z::Array{Float64,1}, var::Float64, N::Int64)

        # Find lambda to minimize risk
        lambda_choices = sort(abs(Z))
        risks = similar(Z)
        for m = 1:K
            risks[m] = risk_wasserman(Z,lambda_choices[m],var,N)
        end
        lambda = lambda_choices[findmin(risks)[2]]

        # Create Wasserman estimator
        theta_w = similar(Z)
        for k = 1:length(Z)
            theta_w[k] = sign(Z[k]) * (abs(Z[k]) - lambda) * (abs(Z[k]) > lambda)
        end
        return theta_w
    end

    # Plot the Soft Threshold Estimator on the Data
    msoft = W*wasserman(theta,sigma2,length(Y))
    plot(layer(x=X[:,2], y=msoft, Geom.line,Theme(default_color=colorant"green")),
        layer(x=X[:,2], y=Y, Geom.point),
        Guide.XLabel("X0te"),
        Guide.YLabel("Y0tc"),
        Guide.Title("Actual Data vs. Soft Threshold Estimator, K = $K"),
        Guide.manual_color_key("Legend",["Soft Threshold","Data"],["green","deepskyblue"]))
```

Out[23]:



2.1 Income and Geography

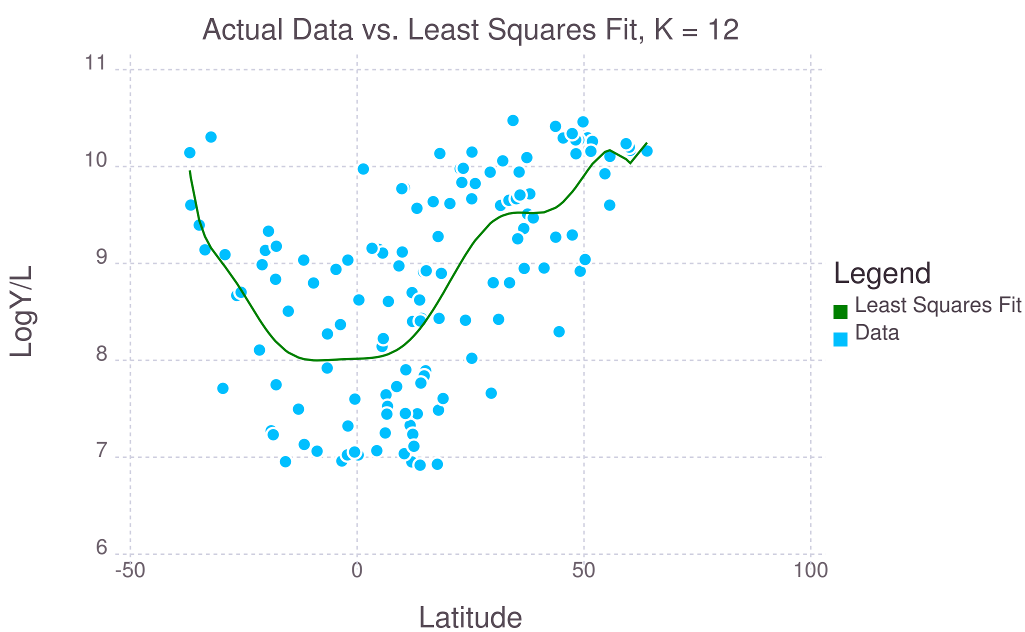
```
In [24]: # Code to read in the data
         hjfile = open("HallJones400.asc")
         hjdata = readlines(hjfile)

         # Read in logYL and Latitude, skipping any observation that is
         logYL = Array(Float64,0)
         latitude = Array(Float64,0)
         for line = 210:361
             if isnan(parse(Float64,hjdata[line][46:54]))
                 continue
             end
             push!(logYL,parse(Float64,hjdata[line][46:54]))
             push!(latitude,parse(Float64,hjdata[line][196:203]))
         end
```

2 Using the power series basis, construct an Orthogonal Basis

```
In [25]: # Construct the basis vectors
         K = 12
         X = zeros(length(latitude),K)
         for k = 1:K
             X[:,k] = latitude.^(k-1)
         end

         W = GSO(X)
```

```
theta = inv(W'*W) * W' * logYL
m = W * theta

plot(layer(x=X[:,2], y=m, Geom.line,Theme(default_color=colorant"green")),
    layer(x=X[:,2], y=logYL, Geom.point),
    Guide.XLabel("Latitude"),
    Guide.YLabel("LogY/L"),
    Guide.Title("Actual Data vs. Least Squares Fit, K = $K"),
    Guide.manual_color_key("Legend",["Least Squares Fit","Data"],["green","deepskyblue"]))
```

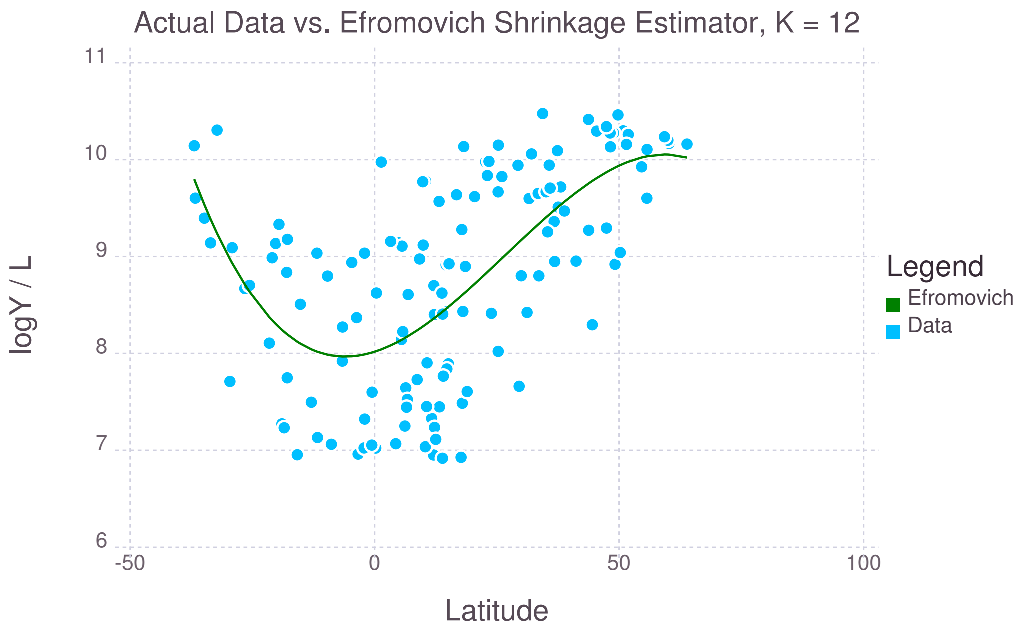
Out[25]:



3 Use the Efromovich Shrinkage Estimator

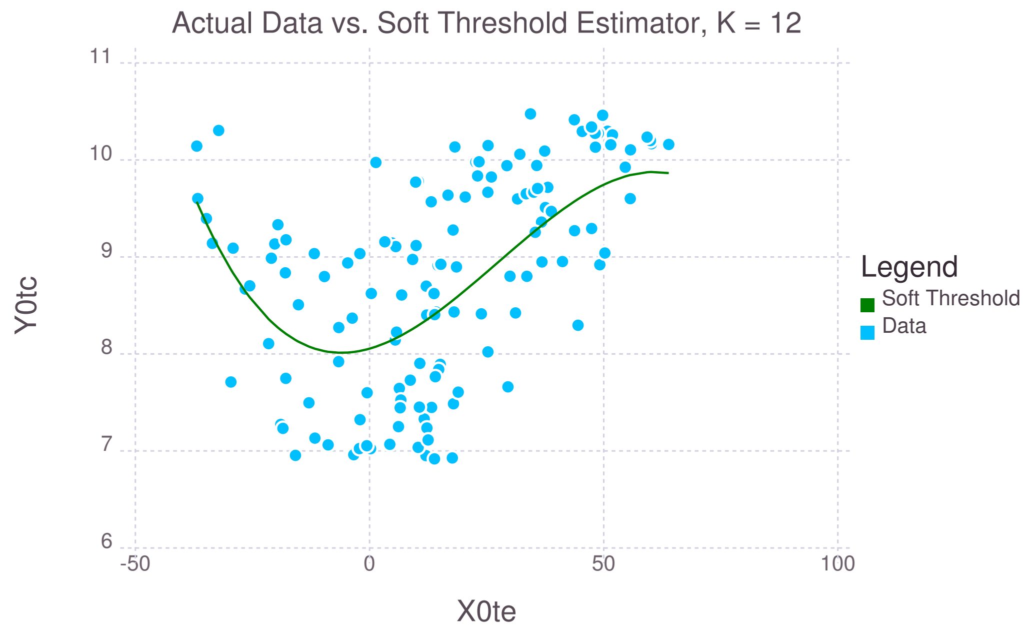
In [26]:
```
# MLE of Variance
sigma2 = norm(logYL - m) / (length(logYL)- K)

# Plot data
muse = W*(efromovich(theta,sigma2,length(logYL)).*theta)
plot(layer(x=X[:,2], y=muse, Geom.line,Theme(default_color=colorant"green")),
    layer(x=X[:,2], y=logYL, Geom.point),
    Guide.XLabel("Latitude"),
    Guide.YLabel("logY / L"),
    Guide.Title("Actual Data vs. Efromovich Shrinkage Estimator, K = $K"),
    Guide.manual_color_key("Legend",["Efromovich","Data"],["green","deepskyblue"]))
```

Out[26]:



4 Soft Threshold Estimate

```
In [27]: # Plot the Soft Threshold Estimator on the Data
         msoft = W*wasserman(theta,sigma2,length(logYL))
         plot(layer(x=X[:,2], y=msoft, Geom.line,Theme(default_color=colorant"green")),
             layer(x=X[:,2], y=logYL, Geom.point),
             Guide.XLabel("X0te"),
             Guide.YLabel("Y0tc"),
             Guide.Title("Actual Data vs. Soft Threshold Estimator, K = $K"),
             Guide.manual_color_key("Legend",["Soft Threshold","Data"],["green","deepskyblue"]))

Out[27]:
```


Actual Data vs. Soft Threshold Estimator, K = 12
Y0tc
X0te
Legend
Soft Threshold
Data
11
10
9
8
7
6
-50
0
50
100