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% Pset 2, question 3
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% Set seed
rng(1234);
N = 500;

% Generate X and e
X = randn(N,1);
e = randn(N,1);
Y = (X + 0.1*X.^2 + e) > 0;

% Part B: estimate misspecified Probit by ML

    beta_con = fminunc(@(b)Q(Y,X,b),[0 0]);

    % calculate standard errors

    % score outer product (meat)
    Xmatrix = [ones(N,1),X];
    LHat = Xmatrix * beta_con';
    scoreproduct = zeros(2,2);
    for i = 1:N
        score_i = (Y(i) * normpdf(LHat(i)) / normcdf(LHat(i))...
            - (1 - Y(i)) * normpdf(LHat(i)) / (1 - normcdf(LHat(i))));
        scoreproduct = scoreproduct + score_i * score_i * Xmatrix(i,:)' * Xmatrix(i,:);
    end
    scoreproduct = scoreproduct / N;

    % average hessian (bread)
    hessian = zeros(2,2);
    for i = 1:N
        y = Y(i);
        phi = normpdf(LHat(i));
        Phi = normcdf(LHat(i));

        firstpart = y * phi * (LHat(i) * Phi + phi) / Phi^2;
        secondpart = (1 - y) * phi * (phi - LHat(i) * (1 - Phi)) / ((1 - Phi)^2);
        hessian_i = (firstpart + secondpart) * Xmatrix(i,:)' * Xmatrix(i,:);
        hessian = hessian + hessian_i;
    end
    hessian = hessian / N;

    % report standard errors
    Omega = (1/sqrt(N)) * inv(hessian) * scoreproduct * inv(hessian);
    disp(['Coefficients a, b: '])
    disp(beta_con(1))
    disp(beta_con(2))

    disp('Standard errors a, b: ')
    disp(sqrt(Omega(1,1)))
    disp(sqrt(Omega(2,2)))

% c: Score test on hypothesis that b_2 = 0
% The following follows Wooldridge, pg. 570

    ghat = normpdf(LHat);
    Ghat = normcdf(LHat);
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    % LHS: u_i / sqrt(G * (1 - G))
    aux_lhs = (Y - Ghat) ./ sqrt(Ghat .* (1 - Ghat));

    % RHS: g_i / sqrt(G * (1 - G)) times x and z
    aux_RHS = bsxfun(@times,ghat ./ sqrt(Ghat .* (1 - Ghat)),[Xmatrix X.^2]);

    % Regress, obtain explained sum of squares
    aux_lhs_hat = aux_RHS * inv(aux_RHS' * aux_RHS) * aux_RHS' * aux_lhs;
    ESS = (aux_lhs_hat - mean(aux_lhs))' * (aux_lhs_hat - mean(aux_lhs));

    % The LM statistic has an asymptotic distribution of Chi squared 1 under the null
    disp(['LM Statistic: ' num2str(ESS)])
    disp(['P-value: ' num2str(1-chi2cdf(ESS,1))])

% d: Unconstrained models

    % Calculate unconstrained coefficients
    beta_unc = fminunc(@(b)Q(Y,X,b),[0 0 0])
    Xmatrix = [ones(N,1),X,X.^2];
    LHat = Xmatrix * beta_unc';

    % average (negative) hessian
    hessian = zeros(3,3);
    Phi = normcdf(LHat);
    phi = normpdf(LHat);

    for i = 1:N
        y = Y(i);
        phi = normpdf(LHat(i));
        Phi = normcdf(LHat(i));

        firstpart = y * phi * (LHat(i) * Phi + phi) / Phi^2;
        secondpart = (1 - y) * phi * (phi - LHat(i) * (1 - Phi)) / ((1 - Phi)^2);
        hessian_i = (firstpart + secondpart) * Xmatrix(i,:)' * Xmatrix(i,:);
        hessian = hessian + hessian_i;
    end
    hessian = hessian / N;

% Wald test
    g = beta_unc(1,3);
    gprime = [0 0 1];
    wald = N * g * inv((gprime * (1/sqrt(N)) * inv(hessian) * gprime')) * g;
    pvalue_d = 1 - chi2cdf(wald,1);
    disp(['P-value for Wald test on b2 = 0: ', num2str(pvalue_d)])
```