

## Homework #2

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### 3. Matlab Probit DGP

- a) (Matlab program attached)
- b) The ML point estimates of  $\hat{\beta}^{con} = (\hat{\alpha}, \hat{b})$  are  $(-0.0175, 0.9159)$ . The standard errors are  $(0.0918, 0.1301)$ .
- c) Score test: Regressing the restricted model's generalized residuals, given by

$$gres_i = \frac{Y_i \phi(X_i' \hat{\beta}^{con})}{\Phi(X_i' \hat{\beta}^{con})} - \frac{(1 - Y_i) \phi(X_i' \hat{\beta}^{con})}{1 - \Phi(X_i' \hat{\beta}^{con})}$$

on  $X_i^2$  yields a coefficient of 0.0039 with a p-value of effectively 0, rejecting the null of 0.

- d) The unrestricted model yields point estimates of  $\hat{\beta}^{unc} = (\hat{\alpha}, \hat{b}, \hat{b}_2) = (-0.0435, 0.9175, 0.0428)$ . The wald test, which tests  $g(\beta) = 0$ , where

$$g(\beta) \equiv (0, 0, 1) \cdot \beta = b_2$$
$$G(\beta) \equiv \frac{\partial g(\beta)}{\partial \beta} = (0, 0, 1)$$

and the Wald statistic is given by

$$N \cdot \hat{b}_2 \cdot \left( G \cdot H^{-1} \cdot G' \right)^{-1} \cdot \hat{b}_2$$

where  $H$  is the average Hessian at the ML estimate. Here, the Wald evaluates to 0.3336, which has a p-value of 0.5636 under the  $\chi^2$  distribution with d.f. 1, much higher than the 0 obtained in part b.