Homework #1

Professor: Pat Kline

Students: Christina Brown, Sam Leone, Peter McCrory, Preston Mui

Identification I (OLS)

Identification II (A Structural Labor Supply Model)

Identification III (Mixture of Normals)

Quantile Treatment Effects

Iterated Projections

Prove the law of iterated projections

$$E^*[Y_i|X_i] = E^*[E^*[Y_i|X_i, Z_i]|X_i]$$

Proof.
Define $W_i' = \begin{bmatrix} X_i' & Z_i' \end{bmatrix}$. Recall that the linear projection of W_i onto Y_i requires that, for $\beta = E[W_i W_i']^{-1} E[W_i Y_i]$, the following must hold:

$$E[W_i(Y_i - W_i'\beta)] = 0.$$

This further implies that

$$E[X_i \underbrace{(Y_i - W_i'\beta)}_{\equiv u_i}] = 0$$

Thus,

$$\begin{aligned}
E^*[Y_i|X_i] &= X_i' E[X_i X_i'] E[X_i Y_i] \\
&= X_i' E[X_i X_i']^{-1} E[X_i(W_i'\beta + u_i)] \\
&= X_i' E[X_i X_i']^{-1} E[X_i W_i'\beta] + X_i' E[X_i X_i']^{-1} E[X_i u_i] \\
&= X_i' E[X_i X_i']^{-1} E[X_i E^*[Y_i|X_i, Z_i]] \\
&= E^*[E^*[Y_i|X_i, Z_i]|X_i]
\end{aligned}$$

The third equality follows from the linearity of the projection function. The second term in the third equality is equal to zero by the observation made above. The fourth follows from the definition $E^*[Y_i|X_i, Z_i] = W_i\beta$. □

FWL Theorem

Weighted Average Derivative Properties