MULTIVARIATE COINTEGRATION TESTS AND THE LAW OF ONE PRICE IN INTERNATIONAL WHEAT MARKETS

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The multivariate cointegration testing procedures recently developed by Johansen are used to evaluate the law of one price (LOP) for prices in five international wheat markets. Efficient arbitrage and trade activities should ensure that individual wheat prices in spatially separated markets are linked through a common long-run equilibrium. The results indicate that the LOP fails as a long-run equilibrium relationship when transportation costs are ignored. However, if wheat prices are adjusted for freight rates, the LOP is fully supported.

The law of one price (LOP) is an important component of most, if not all, models of international trade (Officer 1986). The LOP asserts that efficient trade and arbitrage activities will ensure that prices in spatially separated markets, once adjusted for exchange rates and transportation costs, will be equalized. Assuming absolute adherence to the LOP allows the definition of a single "representative price" common to all trade regions.

Recent advances in time-series modeling techniques have brought about a number of investigations of cointegration relationships among economic variables (see, for example, Ardeni; Campbell and Shiller; and Goodwin and Schroeder). Most analyses of cointegration have utilized the bivariate tests of Engle and Granger. However, Engle and Granger's bivariate tests have recently been recognized as being subject to a number of serious limitations (which are discussed below). In contrast, this investigation describes and applies the multivariate techniques recently developed by Johansen, and Johansen and Juselius for evaluating cointegration relationships among a group of variables.

The LOP has been studied extensively in international commodity markets. The empirical evidence has overwhelmingly concluded that adherence to the LOP is limited. Officer (1989) notes that "of 40 tests, 28 reject the LOP, eight have mixed findings, and only four unambiguously cannot reject the LOP" (p. 24). Crouhy-Veyrac, Crouhy, and Melitz argue that the negative conclusions of many LOP tests can be attributed to a neglect of transportation costs. Goodwin, Grennes, and Wohlgenant (1990a, 1990b) also point to transportation costs as reasons for rejecting the LOP in conventional tests. In addition, they argue that delivery lags and price expectations may play an important role in tests of the LOP. In a bivariate cointegration testing framework, Ardeni concludes that the LOP fails as a long-run equilibrium relationship in international commodity markets. In international wheat markets, Ardeni's results indicate that the LOP is supported in four of 10 bivariate wheat price comparisons.[1]

Ardeni has argued that conventional regression tests of the LOP may have misrepresented or ignored the time-series properties of individual price data series. Such properties may have important implications for statistical tests of the LOP. In particular, ignoring serial correlation in an empirical test of the LOP may yield inferential biases and inconsistencies. Furthermore, empirical tests that use price differentials may suffer because such differencing transformations and filters are ad hoc in nature and may be inappropriate for a given price series. As an alternative, Ardeni utilizes the bivariate two-step cointegration testing techniques of Engle and Granger. However, the cointegration tests of Engle and Granger are

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[1]Ardeni evaluated the LOP using five bivariate comparisons between the U.S. Gulf export price, two Australian export prices, and the Canadian export price. Each test was repeated with the right and left-hand side variable designations reversed. This yielded a total of 10 tests of cointegration, of which four supported the LOP.

limited: (1) by the fact that cointegration considerations are confined to pair-wise comparisons; and (2) because such tests require one of the two prices to be designated as exogenous. In addition, concerns have been raised about the potential for small-sample biases in parameter estimates obtained from such two-step procedures (Banerjee *et al.*). Finally, Engle and Granger's testing procedures do not have well-defined limiting distributions,[2] and as a result, do not offer straightforward testing procedures (Hall).

Multivariate cointegration tests have recently been introduced by Johansen, and Johansen and Juselius. The multivariate approach suggests a maximum likelihood estimation procedure that provides estimates of all the cointegrating vectors existing among a group of variables. Johansen's maximum likelihood approach utilizes test statistics that have an exact limiting distribution that is a function of a single parameter. In this light, Johansen's multivariate cointegration testing procedures may offer significant advantages over the bivariate two-step approach of Engle and Granger.

The objective of this article is to evaluate the LOP as a long-run equilibrium condition in international wheat markets. Specifically, the LOP is tested in a multivariate framework for wheat export prices at the U.S. Gulf, Canadian Pacific ports, and Australia and wheat import prices in Japan and Rotterdam. Efficient international trade and arbitrage should ensure that a single cointegration relationship exists among these prices.

Multivariate Cointegration Tests and the LOP

Conventional tests of the LOP typically posit an equilibrium relationship between two markets in which price changes in one market are reflected by equilibrating changes in the other market. Such an equilibrium relationship is typically written as:

$$p_t^1 = \alpha + \beta p_t^2 + e_t, \qquad (1)$$

where p_t^1 and p_t^2 are commodity prices, stated in levels or in logarithmic terms, in the two alternative markets. Adherence to the LOP is typically assumed to be satisfied if β is not significantly different from one. Such conventional regression tests suffer from several weaknesses. By definition, prices are jointly determined in integrated markets. This raises the issue of simultaneity biases in such bivariate comparisons. Another important point is that prices may vary in a nonsynchronous manner within the band created by transportation costs (Goodwin, Grennes, and Wohlgenant 1990b). In this case, any value of β could be consistent with adherence to the LOP. Finally, as Stock notes, if p_t^1 and p_t^2 are nonstationary but trend together, conventional statistical procedures cannot be used to provide reliable hypothesis tests regarding the values of α and β. In particular, while the parameter estimates are consistent, the estimated standard errors are not. In this light, it may be impossible to conduct explicit hypothesis tests for the estimated parameters of conventional regression models.

Ardeni suggests the use of bivariate cointegration tests to overcome some of the weaknesses inherent in conventional tests of the LOP. If p_t^1 and p_t^2 are each nonstationary and require *d* differencing transformations to produce a stationary series, they are said to be integrated of order *(d)*. If the two price series can be linearly combined to produce a single series that is integrated of order *(d-b)*, they are said to be cointegrated of order *(d,b)*. Thus, if p_t^1 and p_t^2 are each integrated of order *(1)*, but their combination in a regression of equation (1) produces a residual series e_t that is stationary, p_t^1 and p_t^2 are cointegrated of order *(1,1)*. Cointegration of the two price series implies a long-run parity relationship whereby deviations from the LOP have a stable mean of zero.

[2]Engle and Granger provide critical values for their seven bivariate cointegration test statistics for samples of n=100. Engle and Yoo further investigate properties of three of the bivariate cointegration test statistics and provide critical values for samples of n=200.

In spite of the improvements over conventional LOP testing approaches offered by the cointegration testing framework of Engle and Granger, such tests are constrained by the limitations mentioned above and by the fact that the tests are limited to pair-wise price comparisons. In contrast, price relationships in international commodity markets involve the simultaneous determination of several individual market prices; and thus, it may be more natural to consider joint cointegration tests. This raises the issue of whether individual market prices are determined by some common driving fundamentals or whether each market's price reacts to its own particular set of fundamentals or forcing variables (Baillie and Bollerslev). The LOP suggests that a set of p international prices for a common good should possess a single common cointegrating vector or, equivalently, should possess p-1 stochastic trends (unit-roots). The multivariate cointegration tests developed by Johansen and the analogous common stochastic trend tests of Stock and Watson are ideally suited to address international price linkages in a multivariate framework.

Consider a vector of p time-ordered variables X_t, where X_t is defined by a general polynomial distributed lag model:

$$X_t = \pi_1 X_{t-1} + \pi_2 X_{t-2} + \cdots + \pi_k X_{t-k} + \epsilon_t, \quad t = 1,\ldots,T, \tag{2}$$

where ϵ_t is a p dimensional vector of i.i.d. Gaussian random errors with zero mean and a variance matrix Ω. The number of unique cointegrating vectors, r, is given by the rank of the impact matrix:

$$\Pi = I - \pi_1 - \pi_2 - \ldots - \pi_k, \tag{3}$$

where I is a $p \times p$ identity matrix. If the impact matrix is of rank r where $r < p$, there are r stationary linear (cointegration) combinations of the variables in X_t and p - r common stochastic trends. If $r = 0$, there are no cointegrating vectors and there are p stochastic trends. It is useful to define two $p \times r$ matrices, α and β, such that:

$$\Pi = \alpha\beta' \tag{4}$$

Although X_t is nonstationary, the linear combinations $\beta' X_t$ are stationary. Thus, the columns of β form the r unique cointegrating vectors and the space spanned by β is the cointegration space.

Johansen shows that the maximum likelihood estimate of the space spanned by β corresponds to the r largest squared canonical correlations between the residuals of X_{t-k} and ΔX_t corrected for the effects of ΔX_{t-1} through ΔX_{t-k+1}. Johansen suggests a straightforward approach for hypothesis testing regarding the number of cointegrating vectors and for obtaining maximum likelihood estimates of the cointegrating vector. Under Johansen's approach, the following vector autoregressive models are estimated:

$$\begin{aligned} \Delta X_t &= \Sigma_{i=1}^{k-1} \Gamma_{0i}\, \Delta X_{t-i} + \upsilon_{0t}, \\ X_{t-k} &= \Sigma_{i=1}^{k-1} \Gamma_{1i}\, \Delta X_{t-i} + \upsilon_{1t}. \end{aligned} \tag{5}$$

The residual vectors υ_{0t} and υ_{1t} are then used to construct two likelihood ratio test statistics that can be used to determine the number of unique cointegrating vectors in X_t. The first test statistic, known as the trace test, evaluates the null hypothesis that there are at most r cointegrating vectors and is given by:

$$\tau_{trace} = -T\, \Sigma_{i=r+1}^{p} \ln(1-\lambda_i), \tag{6}$$

where $\lambda_{r+1}, \ldots \lambda_p$ denotes the p-r smallest canonical correlations of υ_{0t} with respect to υ_{1t}.[3] The second likelihood ratio test, known as the maximal eigenvalue test, evaluates the null hypothesis that there are exactly r cointegrating vectors in X_t and is given by:

$$\tau_{max} = -T\, \ln\left(1-\lambda_{r+1}\right). \tag{7}$$

[3]Equivalently, $\lambda_{r+1}, \ldots, \lambda_p$ are the p-r smallest eigenvalues of $S_{10} S_{00}^{-1} S_{01}$ with respect to S_{11}, where S_{ij} is the second moment matrix defined by $S_{ij} = T^{-1}(\upsilon_{it}\upsilon_{jt}')$.

Each test has as its alternative, the case of g > r cointegrating vectors. Johansen and Juselius suggest that the trace test may lack power relative to the maximal eigenvalue test.

In short, a straightforward approach to testing the LOP in a multivariate context is presented by Johansen and Juselius's cointegration tests. Adherence to the LOP implies a single cointegrating vector among the group of prices being evaluated.

Empirical Application and Results

The LOP was evaluated in a multivariate context for monthly wheat price data covering the period from January 1978 through December 1989 for five important international wheat markets.[4] The data were chosen to include major importing and exporting markets. Export (fob) prices included the U.S. Gulf market for No. 2 Dark Northern Spring Wheat (14 percent protein), the Canadian Pacific market for No. 1 Western Red Spring Wheat (13.5 percent protein), and the Australian export market for Australian Standard White (ASW) Wheat. Import (cif) prices included the Rotterdam and Japanese import markets for U.S. origin No. 2 Dark Northern Spring Wheat.[5] The prices were collected from selected issues of the International Wheat Council's (IWC) *World Wheat Statistics* and from unpublished data obtained from the IWC. All prices were quoted in U.S. dollar terms. The Japanese and Australian prices had a small number of missing observations that were replaced with predicted values from a regression of the individual prices on prices in closely related markets.[6] Hathaway notes that most agricultural trade is priced in U.S. dollars because of the dominant position of U.S. agricultural exports and the dollar. It is important to note that quality differences that may influence price linkages may exist in these individual wheats across markets.

The empirical tests of cointegration must be preceded by a test of nonstationarity for the individual variables under consideration. Specifically, unit root tests must be used to determine the order of integration of each variable. The Dickey-Fuller test is generated from the following regression:

$$\Delta X_t = \alpha + \phi X_{t-1} + \Sigma_{i=1}^{p} \theta_i \Delta X_{t-i} + \epsilon_t, \quad (8)$$

where p is large enough to ensure that the residual ϵ_t is white noise. The Dickey-Fuller test statistic is the ratio of ϕ to its standard error. The null hypothesis of a unit root is rejected for values of ϕ that are negative and significantly different from zero using the critical values calculated by Dickey and Fuller. The Dickey-Fuller tests for each of the wheat price series are presented in Table 1. The test is conducted for the levels and first-differences of each series. In every case, a single unit root is indicated in that the null hypothesis of a unit root is not rejected for the level series, but is rejected for the first-differenced series. Confirmation that each series is integrated of order (1) allows the researcher to proceed to the multivariate cointegration tests.

Johansen and Juselius's multivariate trace and maximal eigenvalue cointegration tests were applied to the five wheat prices. Likelihood ratio tests indicated a lag order of two for the first VAR system implied by (5).[7] The importance of including an intercept term in the VAR equations has been addressed by Dickey and Rossana. Following their suggested approach, intercepts were included in the first system of equations and the second set of equations was

[4]Because Johansen and Juselius only provide critical values for tests of four or less cointegrating vectors, the application of their tests is constrained to considerations of cointegration for five or fewer variables.

[5]The U.S., Canadian, and Australian prices utilized in this analysis are very similar to those used by Ardeni. An exception is that the U.S. export price is for Dark Northern Spring Wheat; whereas, Ardeni used the U.S. price for Hard Red Winter Wheat. Because of quality differences, the Dark Northern Spring Wheat price may be more comparable to the Canadian Red Spring Wheat price; and thus, may be preferable. Another distinction is that the prices used in this study are monthly market averages; whereas, Ardeni used unit values for several markets.

[6]In particular, 11 missing Japanese prices were replaced with predicted values from a regression of Japanese prices on Rotterdam prices. A single observation of the Australian price was replaced by the predicted value from a regression of the Australian price on the U.S. price.

[7]A consideration of alternative lag lengths did not alter the results of the tests discussed below.

Table 1. Dickey-Fuller Unit-Root Testing Results

Price	Dickey-Fuller Test Statistic[a]	Augmented Dickey-Fuller Test Statistic[a]	Lag Order (p)[b]
	Levels		
Canada	-2.654	-2.625	2
U.S.	-2.643	-2.517	2
Australia	-1.908	-2.044	2
Rotterdam	-2.669	-2.391	3
Japan	-2.433	-2.412	2
t-Adjusted Rotterdam	-2.837	-2.567	2
t-Adjusted Japan	-2.544	-2.458	2
	First-Differenced		
Canada	-6.728*	-6.058*	2
U.S.	-7.903*	-7.459*	2
Australia	-6.938*	-5.720*	2
Rotterdam	-8.311*	-7.133*	2
Japan	-8.314*	-7.511*	2
t-Adjusted Rotterdam	-8.587*	-7.415*	2
t-Adjusted Japan	-8.970*	-8.173*	2

[a]Critical values for n=100 are -2.89 for α=.05 and -3.50 for α=.01 (Fuller). An asterisk indicates rejection of the null hypothesis of a unit root at the .01 level.

[b]Lag orders for augmented tests chosen using the minimum value of Akaike's Final Prediction Error (FPE).

augmented by including a sixth equation that regressed a column vector of ones on the lagged differentials. The two sets of residuals υ_{0t} and υ_{1t} were then used to compute the canonical correlations for use in the two multivariate cointegration tests.

Table 2 contains the results of the multivariate cointegration tests. Given the suspected sensitivity of LOP tests to transportation costs, the tests were conducted on two alternative models. The first, designated Model I, used the fob export and cif import prices directly. The second, designated Model II, corrected the Rotterdam and Japan cif import prices for transportation costs by subtracting the per-unit monthly average freight rates for wheat for trade from the U.S. Gulf to Rotterdam and Japan. The freight rates were collected from the IWC's *World Wheat Statistics* and from the IWC's *Market Report* series. The cointegration tests are evaluated at the .05 level of significance.

The results for Model I indicate that there are no unique cointegrating vectors; and thus, that there are five stochastic trends among the five uncorrected wheat prices. The trace and maximal eigenvalue tests produce identical conclusions. Such a finding is unfavorable to the LOP as it implies that no stationary long-run equilibrium relationship exists among the five wheat prices. In light of the importance of transportation costs to international price

Table 2. Multivariate Cointegration Testing Results for World Wheat Prices

Test	Null Hypothesis[a]	Cointegration Test Statistic	Critical Value[b]
Model I: Uncorrected Wheat Price			
Trace Test			
	H_0: r = 0	66.188	69.977
	H_0: r ≤ 1	39.298	48.419
	H_0: r ≤ 2	18.375	31.256
	H_0: r ≤ 3	4.385	17.844
	H_0: r ≤ 4	.229	8.083
Maximal Eigenvalue Test			
	H_0: r = 0	26.891	33.262
	H_0: r = 1	20.923	27.341
	H_0: r = 2	13.990	21.279
	H_0: r = 3	4.155	14.595
	H_0: r = 4	.229	8.083
Model II: Wheat Prices Corrected for Transport Costs			
Trace Test			
	H_0: r = 0	76.324*[c]	69.977
	H_0: r ≤ 1	42.792	48.419
	H_0: r ≤ 2	21.598	31.256
	H_0: r ≤ 3	11.844	17.844
	H_0: r ≤ 4	4.858	8.083
Maximal Eigenvalue Test			
	H_0: r = 0	33.532*	33.262
	H_0: r = 1	21.194	27.341
	H_0: r = 2	9.755	21.279
	H_0: r = 3	6.986	14.595
	H_0: r = 4	4.858	8.083

[a]Both tests have as their alternatives: H_a: r > n.

[b]Critical values are for the 95 percent quantile and are taken from Johansen and Juselius.

[c]An asterisk indicates rejection of the null hypothesis at the .05 level.

linkages and the volatility of freight rates over the period of study,[8] this result is not surprising. Goodwin, Grennes, and Wohlgenant (1990a) found that a failure to consider transportation costs in empirical tests produced results unfavorable to the LOP, but that when transportation costs were explicitly recognized, the LOP received strong support.

The results for Model II, for which the cif import prices were adjusted for transporta-

[8]Freight rates for wheat trade between the U.S. Gulf and Japan ranged from $13 per metric ton in January 1976 to $44 per metric ton in April 1980. Rates between the U.S. Gulf and Rotterdam ranged from $4 per metric ton in February 1976 to $23 per metric ton in September 1978.

tion costs, are much more encouraging for the LOP. At the .05 level, these results indicate that a single cointegration relationship exists among the corrected wheat prices. Equivalently, the results indicate that there are only four stochastic trends among the five wheat prices. Again, the trace and maximal eigenvalue tests yield identical results. The results are entirely consistent with efficiently linked prices in international wheat markets.[9]

In all, the results confirm the earlier findings of Goodwin, Grennes, and Wohlgenant (1990a) which suggested strong support for the LOP when transportation costs are explicitly recognized. The results verify efficiently linked prices in international wheat markets and thus provide strong support for the LOP in international wheat markets.

Concluding Remarks

This analysis evaluates adherence to the law of one price in international wheat markets. Efficient international commodity arbitrage and trade should ensure that a unique cointegration relationship exists among international prices for a common good. In this light, prices from five important international wheat markets are evaluated using the multivariate cointegration tests of Johansen and Juselius. The analysis is conducted in two segments. The first utilized fob export and cif import prices directly. The second segment uses international freight rates for wheat to correct the cif import prices for transportation costs. The results indicate a lack of an equilibrium cointegration relationship for the uncorrected prices, but a single cointegration relationship among the transportation-adjusted prices. This finding is consistent with the results of earlier analyses (Goodwin, Grennes, and Wohlgenant 1990a) which emphasizes the important role of transportation costs in empirical tests of the LOP. The results are consistent with efficiently linked prices in international wheat markets and offer support for the LOP in international wheat markets.

[9]Because the cointegration tests may be influenced by structural change in the price relationships, the cointegration tests were repeated for various sub-groups of the data. The results of these alternative tests were consistent with those presented in Table 2 and are available from the author on request.

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