Test_case move_end_effector()

input:

joint j1 (45, 1, (0,0,0), (0,1,0), True)

joint j2 (45, 2, (0,0,0), crossProd((cos45, 0, sin45), (0,1,0)), True)

joint j3 (-30, 3, (0.5, 0.7071, 0.5), crossProd((cos45, 0, sin45), (0,1,0)), False)

joint j4 (+90, 4, (0.8415, 0.8365, 0.8415), crossProd((cos45, 0, sin45), (0,1,0)), False)

joint j5 (0, 4, (0.795, 1.078, 0.795), crossProd((0, 1, 0), (cos45, 0, sin45)), False)

j_list = { j1, j2, j3, j4, j5 }

link l1 (1, 1, j3, j2)

link l2 (0.5, 2, j4, j3)

link l3 (0.25, 3, j5, j4)

l_list = { l1, l2, l3 }

Pos target = (0.5, 0.8, 0.5)

double thresh = 0.10

Computation:

i == 0

if (j1.stat == True) True then:

j1.pos3 = (0,0,0)

i == 1

if (j2.stat == True) True then:
    J2.jPos = (0,0,0)

i == 2

if (j3.stat == True) False then: X

else: then:
    break from loop

---

j == 2 , K == 0

y = J2.jPos.y + l1.length
  = 0 + 1 = 1

J3.jPos = (0, y, 0) = (0, 1, 0)

j == 3 , K == 1

y = J3.jPos.y + l2.length
  = 1 + 0.5 = 1.5

J3.jPos = (0, y, 0) = (0, 1.5, 0)

j == 4 , K == 2

y = J4.jPos.y + l3.length
  = 1.5 + 0.25 = 1.75

J5.jPos = (0, y, 0) = (0, 1.75, 0)

Vect<Pos> pl = target = {(0.5, 0.8, 0.5)}

angle = 0°,

r = target.z / target.x = 0.5 / 0.5 = 1.0

angle:

$angle = \tan^{-1}(r) = \tan^{-1}(1.0) = 0.785398$

---

if (target.x > 0) True then:

if (angle > 0) True then:

$angle = angle * \frac{180}{\pi} = 0.785398 * 180/\pi = 45°$

$angle = 45°$

$axis = (0, 1, 0)$

$$rot\_matrix = \begin{bmatrix} \cos45 + 0.0*(1-\cos45) & 0.0*(1-\cos45) - 0*\sin45 & 0.0*(1-\cos45) + 1.0*\sin45 \\ 1.0*(1-\cos45) + 0*\sin45 & \cos45 + 1.0*(1-\cos45) & 1.0*(1-\cos45) - 0*\sin45 \\ 0.0*(1-\cos45) - 1*\sin45 & 0.0*(1-\cos45) + 0*\sin45 & \cos45 + 0.0*(1-\cos45) \end{bmatrix}$$

$$= \begin{bmatrix} \cos45 & 0 & \sin45 \\ 0 & 1 & 0 \\ -\sin45 & 0 & \cos45 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}/2 & 0 & +\sqrt{2}/2 \\ 0 & 1 & \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix}$$

Pos target_2d = rot_matrix × target

$$= \begin{pmatrix} 0.5 \\ 0.8 \\ 0.5 \end{pmatrix} \times \begin{bmatrix} \sqrt{2}/2 & 0 & +\sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix}$$

$$= 0.5 \cdot \begin{pmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix} + 0.8 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 0.5 \begin{pmatrix} -\sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix}$$

$$= \begin{pmatrix} 0.5\sqrt{2}/2 + 0.0.8 + 0.5 \cdot \sqrt{2}/2 \\ 0.5 \cdot 0 + 0.8 + 0 \\ 0.5\sqrt{2}/2 \quad 0.0.8 + 0.5\sqrt{2}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{2}/2 \\ 0.8 \\ 0 \end{pmatrix}$$

Pos target_2d = $\begin{pmatrix} \sqrt{2}/2 & 0.8 & 0 \end{pmatrix}$

---

fwd_backward_pass

J5. J Pos = target_2d

Vector<Pos> pl

i == 5 , j == 3

if (fwd J5.stat == True) False then: X

$$uv = (J4.J Pos, - J5.J Pos) / \| J4.J Pos - J5.J Pos \|$$

$$= \frac{((0, 1.5, 0) - (0.\sqrt{2}/2, 0.8, 0))}{\| ((0.5, 1.5, 0) - (\sqrt{2}/2, 0.8, 0)) \|}$$

$$UV = \frac{(-\sqrt{2}/2,\ 0.7,\ 0)}{\|(-\sqrt{2}/2,\ 0.7,\ 0)\|}$$

$$= \frac{(-\sqrt{2}/2,\ 0.7,\ 0)}{\sqrt{(\sqrt{2}/2)^2 + (0.7)^2 + (0)^2}} = \frac{(0-\sqrt{2}/2,\ 0.7,\ 0)}{\sqrt{0.99}}$$

$$= (-0.71067,\ 0.70352,\ 0) = UV$$

$$Pos\,jP = UV \cdot l3.length + J5.jPos$$

$$= (-0.71067,\ 0.70352,\ 0) \cdot 0.25 + (0.\sqrt{2}/2,\ 0.8,\ 0)$$

$$= (0.52944,\ 0.97588,\ 0) = J41.JPos$$

$i == 41$, $j == 2$

if (J4.stat == True) False then X

$$UV = \frac{(J3.JPos - J41.JPos)}{\|J3.JPos - J41.JPos\|}$$

$$= \frac{(0, 1, 0) - (0.52944, 0.97588, 0)}{\|(0, 1, 0) - (0.52944, 0.97588, 0)\|}$$

$$= \frac{(-0.52944,\ 0.02412,\ 0)}{\|(-0.52944,\ 0.02412,\ 0)\|}$$

$$= \frac{(-0.52944,\ 0.02412,\ 0)}{\sqrt{-0.52944^2 + 0.02412^2 + 0^2}}$$

$= (-0.99896, 0.04551036, 0) = uv$

Pos jp = uv · l2.length + J4.JPos

$= (-0.99896, 0.04551036, 0) \cdot 0.5 + (0.52444, 0.47588, 0)$

$= (0.02996, 0.998635, 0) = J8.JPos$

i == 3 , j == 1

if (J3.stat == True) False : then X

$$uv = \frac{(J2.JPos - J3.JPos)}{\| J2.JPos - J3.JPos \|}$$

$$= \frac{((0,0,0) - (0.02996, 0.998635, 0))}{\| (0,0,0) - (0.02996, 0.998635, 0) \|}$$

$$= \frac{(-0.02996, -0.998635, 0)}{\| (-0.02996, -0.998635, 0) \|}$$

$$= \frac{(-0.02996, -0.998635, 0)}{\sqrt{(0.02996^2 + 0.998635^2 + 0^2)}}$$

$= (-0.029987, -0.99955, 0)$

$\equiv uv$

Pos jP = uV · l1.length + J3.jPos

= (-0.029987, -0.99955, 0) · 1 + (0.02996, 0.998635, 0)

= (0.000027746, -0.000915, 0)

= J2.jPos

---

i == 2 , j == 0

if (J2.stat == True) True : then:

break out of loop

---

i == 0 , count == -1

if (J1.stat == True) True then:

J1.jPos = Pos (0,0,0)

count ++

i == 1 , count == 0

if (J2.stat == True) True then:

J2.jPos = Pos (0,0,0)

count ++

i == 2 , count == 1

True False

if (J3.stat == False) then: X

else:

break out of loop

$i = count = 1$, $j = = 0$

$$uv = \frac{(J3.JPos - J2.JPos)}{\| (J3.JPos - J2.JPos) \|}$$

$$= \frac{(0.02996, +0.998635, 0) - (0.00002746, 0.00915, 0)}{\| (0.02996, +0.998635, 0) - (0.00002746, -0.00915, 0) \|}$$

$$= \frac{(0.02993254, 0.99772, 0)}{\| (0.02993254, 0.99772, 0) \|}$$

$$= \frac{(0.02993254, 0.99772, 0)}{\sqrt{0.02993254^2 + 0.99772^2 + 0^2}}$$

$$= (0.029987, 0.9995486, 0) = uv$$

$$Pos\ jP = uv \cdot l1.length + J2.JPos$$

$$= (0.029987, 0.995486, 0) \cdot 1 + (0.00002746, -0.00915, 0)$$

$$= (0.03001446, 0.996401, 0) = J2.jPos$$

$i = 2$, $j = = 1$

$$uv = \frac{(J4.JPos - J3.JPos)}{\| (J4.JPos - J3.JPos) \|}$$

$$= \frac{(0.52944, 0.97588, 0) - (0.02996, 0.998635, 0)}{\| (0.52944, 0.97588, 0) - (0.02996, 0.998635, 0) \|}$$

$$= \frac{(0.499418, -0.022755, 0)}{\|(0.499418, -0.022755, 0)\|}$$

$$= \frac{(0.499418, -0.022755, 0)}{\sqrt{0.499418^2 + 0.022755^2 + 0^2}}$$

$$= (0.9989638, -0.045510, 0) = uv$$

$$pos\,ij = uv.l2.length + j3.jPos$$

$$= (0.9989638, -0.045510, 0)\,0.5 + (0.02996, 0.998635, 0)$$

$$= (0.5294419, 0.97588, 0)$$

$$= j4.jPos$$

$$i == 3, \quad j == 2$$

$$uv = \frac{(j5.jPos - j4.jPos)}{\|(j5.jPos - j4.jPos)\|}$$

$$= \frac{(\sqrt{2}/2, 0.8, 0) - (0.5294419, 0.97588, 0)}{\|(\sqrt{2}/2, 0.8, 0) - (0.5294419, 0.97588, 0)\|}$$

$$= \frac{(0.177665, -0.17588, 0)}{\|(0.177665, -0.17588, 0)\|}$$

$$= \frac{(0.177665, -0.17588, 0)}{\sqrt{0.177665^2 + 0.17588^2 + 0^2}}$$

$$= (0.710668, -0.7035278, 0) = UV$$

Pos j8 = UV • l3.length + J4.JPos

$$= (0.710668, -0.7035278, 0) \cdot 0.25 + (0.5294419, 0.97588, 0)$$

$$= (0.707109, 0.799998, 0) = J5.JPos$$

end fwd backward pass

double mag = 0

Vector diff_V = target_2d − J5.JPos

$$= (\sqrt{2}/2, 0.8, 0,) - (0.707109, 0.799998, 0)$$

$$= (-0.0000002188, 0.000002, 0)$$

mag = || (0.0000002188, 0.000002, 0) ||

$$= \sqrt{(0.0000002188)^2 + (0.000002)^2 + 0^2}$$

$$= 0.0000029643$$

if (mag < thresh)

= if (0.0000029643 < 0.1) True then:

end of while loop

Jlist = { J1.JPos, J2.JPos, J3.JPos, J4.JPos, J5.JPos }

target = (0.5, 0.8, 0.5)

angle = 0

r = (target.z / target.x) = 0.5/0.5 = 1.0

angle = $\tan^{-1}(r) = \tan^{-1}(1.0) = 0.785$

if (target.x > 0) True then:

angle if(angle > 0) True then:

angle = angle x $180°/\pi = 0.785 \cdot 180/\pi = 45°$

---

angle = 45°

target = (0.5, 0.8, 0.5)

axis = (0, 1, 0)

---

rot_matrix =

cos45 + 0.0·(1-cos45),	0.1·(1-cos-45) - 0·sin-45,	0·0·(1-cos-45) + 1·sin-45
1.0·(1-cos45) + 0·sin-45,	cos45 + 1.1·(1-cos-45),	1.0·(1-cos-45) - 0·sin(45)
0.1·(1-cos-45) - 1·sin-45,	0.1·(1-cos-45) + 0·sin-45,	cos-45 + 0.0·(1-cos-45)

=

cos-45	0	sin-45
0	1	0
-sin-45	0	cos-45

$$= \begin{bmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix}$$

$i = 1$

$J1.JPos = rot\_matrix \times J1.JPos$

$$= \begin{bmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} * \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0 \cdot \begin{pmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix} + 0 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} -\sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad J1.JPos = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$i == 2$

$J2.JPos = J2.JPos \times rot\_matrix$

$$= \begin{bmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0 \cdot \begin{pmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix} + 0 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} -\sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad J2.JPos = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$i == 3$

$J3.JPos = J3.JPos \times rot\_matrix$

$$= \begin{bmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} \cdot \begin{pmatrix} 0.02496 \\ 0.998635 \\ 0 \end{pmatrix} =$$

$$0.02996 \cdot \begin{pmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix} + 0.998635 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} -\sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix}$$

$$= \begin{pmatrix} 0.02996 \cdot \sqrt{2}/2 + 0.998635 \cdot 0 + 0 \cdot -\sqrt{2}/2 \\ 0 \cdot 0.02996 \cdot , + 0.998635 \cdot 1 + 0 \cdot 0 \\ 0.02996 \cdot \sqrt{2}/2 + 0.998635 \cdot 0 + 0 \cdot \sqrt{2}/2 \end{pmatrix} = \begin{pmatrix} 0.02118 \\ 0.998635 \\ 0.02996\sqrt{2} \end{pmatrix}$$

$$= \underline{J3.JPos}$$

$i == 3$

$J4.JPos = J4.JPos \times$ rot_matrix

$$= \begin{pmatrix} 0.52944119 \\ 0.97588 \\ 0 \end{pmatrix} \times \begin{bmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix}$$

$$= 0.52944119 \cdot \begin{pmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix} + 0.97588 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} -\sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{2}/2 \cdot 0.52944119 + 0.97588 \cdot 0 + 0 \cdot -\sqrt{2}/2 \\ 0 \cdot 0.52944119 \cdot 0 + 0.97588 \cdot 1 + 0 \cdot 0 \\ 0.52944119 \cdot \sqrt{2}/2 + 0 \cdot 0.97588 + 0 \cdot \sqrt{2}/2 \end{pmatrix}$$

$$= \begin{pmatrix} 0.37437 \\ 0.97588 \\ 0.37437 \end{pmatrix}, \quad J4.JPos = \begin{pmatrix} 0.37437 \\ 0.97588 \\ 0.374137 \end{pmatrix}$$

i == 4 ,

J5.JPos × Rot_matrix

$$= \begin{pmatrix} 0.707109 \\ 0.799998 \\ 0 \end{pmatrix} \times \begin{bmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix}$$

$$= 0.707109 \cdot \begin{pmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix} + 0.799998 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} -\sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix}$$

$$= \begin{pmatrix} 0.707109 \cdot \sqrt{2}/2 + 0 \cdot 0.799998 + 0 \cdot -\sqrt{2}/2 \\ 0 \cdot 0.707109 + 0.799998 \cdot 1 + 0 \cdot 0 \\ 0.707109 \cdot \sqrt{2}/2 + 0.799998 \cdot 0 + 0 \cdot \sqrt{2}/2 \end{pmatrix}$$

$$= \begin{pmatrix} 0.5000015 \\ 0.799998 \\ 0.5000015 \end{pmatrix} = J5.JPos$$

J5.JPos = (0.5000015, 0.799998, 0.5000015)

end of computation

output:

J1.JPos = (0.37437, 0.97588, 0.37437)

J2.JPos = (0, 0, 0)

J3.JPos = (0.02118, 0.999635, 0.02118)

J4.JPos = (0.37437, 0.97588, 0.37437)

Result

Success

Test_case root_arm()

input:

joint j1(45°, 1, (0,0,0), (0,1,0), False True)
joint j2(45°, 2, (0,0.7071,0), crossprod((0,1,0), (cos45, 0, sin45)), True)
joint j3(-30°, 3, (0.5, 0.7071, 0.5), crossprod((0,1,0), (cos45, 0, sin45)), False)
joint j4(+90°, 4, (0.8415, 0.8365, 0.8415), crossprod((0,1,0), (cos45, 0, sin45)), False)
joint j5(0°, 5, (0.745, 1.078, 0.745), crossprod((0,1,0), (cos45, 0, sin45)), False)

jlist = {j1, j2, j3, j4, j5}

link l1 (1, 1, j3, j2)
link l2 (0.5, 2, j4, j3)
link l3 (0.25, 3, j5, j4)
llist = {l1, l2, l3}

computation:

i == 0

if (j1.stat = True) True then:

j1.jpos = (0,0,0)

i == 1

if (j2.stat == True), True then:

j2.jpos = (0,0,0)

i == 2

if (j3.stat == True), False then: X

else: then:

break from loop

i == i = 2, K = 0

Then

$y = j2.jPos.y + l1.length$

$= 0 + 1m = 1$

$j3.jPos = (0, Y, 0) = (0, 1, 0)$

$i == 3, \quad K == 1$

$y = j3.jPos.y + l2.length =$

$1 + 0.5 = 1.5$

$j4.jPos = (0, Y, 0) = (0, 1.5, 0)$

$i == 4, \quad K == 2$

$y = j4.jPos.y + l3.length =$

$1.5 + 0.25 = 1.75$

$j5.jPos = (0, Y, 0,) = (0, 1.75, 0)$

end of computation

output:

$j1.jPos = (0,0,0)$

$j2.jPos = (0,0,0)$

$j3.jPos = (0,1.0,0)$

$j4.jPos = (0,1.5,0)$

$j5.jPos = (0,1.75,0)$

Result

success