Homework 05

MATH 5600 Timo Heister, heister@math.utah.edu

Work in groups of two. Include both of your names as a comment in the first line of each file. Then submit those files on canvas (once per group!).

1. Write a function `a = fit_line(x,y)` that computes the line of best fit to a given set of points (x_i, y_i) given as a vector of x coordinates and a vector of y coordinates and returns a vector $a = [a_0, a_1]$ of coefficients that describe the line of best fit $L(x) = a_0 + a_1 \cdot x$.

2. Using the function `fit_line` compute the line, exponential, and power fit to the data points given in the file `hw05q2_base.m` (on canvas). Create a plot that shows the points and the three curves. Include a legend. Also print out the equations with coefficients for the three curves.

   Bonus: compute the misfit of the three fits to see confirm which one is "best".

   Submit: `fit_line.m` and `hw05q2.m` that produces the plot.

3. Find Taylor series approximations using n=1, 2, and 4 to the function $f(x) = 1 + e^x$ at x=0.7, using the expansion point $x_0 = 1$. For each n, compute the approximation, find an upper bound on the error using Taylor's theorem, and compare it to the actual error.

4. Submit `hw05q4.m` for a) and do b) on paper.

   (a) Determine the error and convergence rate for the forward difference method for the derivative of the function $f(x) = \sin(x)$ at $x_0 = 0$ and at $x_0 = 1$ by creating a table in MATLAB (see example from class).

   (b) The order you will see is different for one of the expansion points than what the theory gives us. Create the Taylor expansion for $\sin(x)$ up to $n = 6$. Use this to explain what is happening in a).