Homework 10

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Work in groups of two. Include both of your names as a comment in the first line of each file. Then submit those files on canvas (once per group!).

1. Implement the function `function value = compSimpson(func,a,b,N)` that computes an approximation to $\int_a^b f(x)\ \mathrm{d}x$ using the composite Simpson rule with N intervals. You can look up the definition of the Simpson rule in the lecture notes.

   Submit `compSimpson.m`.

2. (a) Implement the Gauss quadrature $Q_G^2(f,a,b)$ for a function f on the interval $[a,b]$ as `function value = GaussQuadrature(f,a,b)`. Hint: you can test it by checking the value of $\int_1^2 e^x dx$ (it should give about 4.6697, see the lecture notes).

   (b) Implement composite Gauss quadrature using your function in a). The function should have the format `function value = compGauss(f,a,b,n)`, where n is the number of intervals.

   Hint: look at `compSimpson.m` and call the function from a) for each interval. You can check your code by testing that the value for the problem in a) is close to 4.6707742704 for large n.

   (c) Using $f(x) = 5 + \cos(10x)$ on $[1,5]$, verify that the composite Gauss quadrature rule converges with $O(h^4)$.

   Hint: compute a table for $n = 2^1, 2^2, 2^3, \ldots, 2^9$ with the columns $h = (b-a)/n$, the absolute error against the exact solution, and the rate the error reduces (`errrate = [0;errs(1:end-1)]./errs;` if `errs` is a column vector with the errors). You need to compute the exact value of $\int f(x)$ on $[1,5]$.

   (d) Determine the convergence orders of the composite Simpson and composite midpoint rule for the problem in c) in the same way.

   Submit: `GaussQuadrature.m`, `compGauss.m`, and `hw10q2.m` that produces the data tables for c) and d).

3. Prove (on paper) that the trapezoidal rule has is exact for polynomials up to degree $m = 1$ (show that the rule is not exact for quadratic functions).