Practice Exam 2

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1. You are given the following nonlinear system:

$$\begin{aligned} 1 &= xy \\ 2 &= x^2 \end{aligned}$$

   Which method would use use to solve the system numerically? State the equation to compute the approximation x^{n+1} and execute one step with $x^0 = (1, 0)$.

2. You are given the fixed point iteration $g(x) = x^2$. Find an interval I in which g has a unique fixed point and where the fixed point iteration converges for any starting value. Is the convergence linear or quadratic?

3. State the pros/cons of Bisection, Newton, and Secant method.

4. a) Compute the eigenvalues and one eigenvector for each eigenvalue for the matrix

$$A = \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix}.$$

   b) State a numerical algorithm to compute eigenvalue(s) for a matrix. Apply two iterations of the algorithm to the matrix A above. Use the infinity norm and start with $x_0 = (1, 1)$.

   c) What value will the method in b) return for the eigenvalue of the matrix A in a)?

5. You are given the quadrature rule

$$\int_a^b f(x)dx \approx Q(f) = \frac{b-a}{3}\left(f(a) + f\left(\frac{a+b}{2}\right) + f(b)\right).$$

   (a) Determine the order m of this quadrature rule. For this, prove that $Q(f)$ integrates polynomials up to degree m exactly and show that it is not exact for all polynomials of degree $m+1$ (find a counterexample).

   (b) Would you use this quadrature over the Simpson rule? Why/why not?

6. a) State the formula for a composite quadrature rule $Q^c(f, a, b)$ to approximate the integral $\int_a^b f(x)\,\mathrm{d}x$ given a quadrature $Q(f, a, b)$ that approximates f on $[a, b]$. Draw a picture. Make sure you define x_i and specify the correct loop bounds.

   b) Derive a bound for the error in a)

   c) What is the reason for using composite quadrature rules?

7. a) Given a set of points (x_i, y_i), what is the definition of an interpolating function?

   b) You are given the following data points: $(1, -1), (2, 3), (3, 0), (4, 1)$. Construct (don't solve) a linear system that determines the coefficients of the interpolating polynomial of degree three.

   c) You are given a large number of points (say $n > 10$) that contain noisy data. What kind of interpolating function from lecture would you use/not use and why?