Homework 06

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Work in groups of two. Include both of your names as a comment in the first line of each file. Then submit those files on canvas (once per group!).

1. Create a single script `hw05q1.m` that generates the output (tables and figure).

   (a) Use Forward Difference, Backward Difference, and Centered Difference to approximate the function $f(x) = e^{(x^2)}$ at the point $x_0 = 1$ with values $h = 0.5, 0.25, \dots, 2^{-10}$.

   (b) Then compute the absolute error (what is the exact value?) for each of the three methods and show it as a table with columns h, err_{FD}, err_{BD}, err_{CD}.

   (c) Now create a log-log plot for each of those columns (x axis is h, y axis is the error). Is the result what you expect?

   (d) Compute the error rate for each of the columns (ratio between two consecutive entries) and show it as a table with columns h, $rate_{FD}$, $rate_{BD}$, $rate_{CD}$.

   If `err` is a row vector containing your errors, you can use the expression
   `rate=[0, err(1:end-1)./err(2:end)];`

2. Create a single script `hw06q2.m`.

   (a) Use the centered 3 point difference approximation to approximate $f''(x)$ of $f(x) = \exp(\sin(x))$ at $x = 1$ with values $h = 0.5, 0.25, \dots, 2^{-10}$.

   (b) Compute the absolute error and show a table with columns: h, error, and ratio between errors.

   (c) Conclude (using a printf statement), what power of h the method converges with ($O(h)$, $O(h^2)$, ...).

3. Submit `finite_difference_laplace.m` and `hw06q3.m`:

   (a) Create a function
   `function [x,u]=finite_difference_laplace(func,a,b,N,u_a,u_b)`
   that solves the boundary value problem

   $$\begin{aligned} u''(x) &= f(x), \quad a < x < b \\ u(a) &= u_a \\ u(b) &= u_b \end{aligned}$$

   on the interval $[a, b]$ cut into $N - 1$ equal pieces and returns the points $a = x_1 < x_2 < \dots < x_N = b$ in the vector `x` and the function values of the solution u at these points in the vector `u`. The first parameter `func` is a function handle representing $f(x)$.

   (b) Use the code in a) to solve $u''(x) = -x \cdot \cos(x)$ with $u(1) = -1$ and $u(9) = 5$ for different values of N: $N = 6$, $N = 12$, and $N = 24$. Plot the three solutions together with the exact solution in a single plot. Include a legend. Hint: make sure that your boundary values are correct and that your solution for $N = 24$ is very close to the exact solution.

(c) Determine the convergence rate of the method (in terms of h) by generating a table with values of h and the error (use the maximum of the absolute value of the difference between the exact and the solution computed in b) in every point x_i. Conclude (using a printf statement), what power of h the method converges with ($O(h)$, $O(h^2)$, $\ldots$).