Homework 04

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Work in groups of two. Include both of your names as a comment in the first line of each file. Then submit those files on canvas (once per group!).

0. (do not submit!) For the matrix `A=delsq(numgrid('L',k*k));` for $k = 5, 6, 7, 8$ compute cond(A), and cond($P^{-1}A$) where $P = LL'$ is the sparse Cholesky factorization computed by `L=ichol(A, struct('type','ict','droptol',1e-3));` Does this explain the iteration numbers we saw for CG with or without preconditioner for this example? (see `inclass_pcgdemo.m`).

1. Prove that for a matrix norm $\|A\|_M$ induced by a given vector norm $\|x\|_m$ the following estimates hold for any matrices A, B, and vectors x:

$$\|Ax\|_m \leq \|A\|_M \|x\|_m$$
$$\|AB\|_M \leq \|A\|_M \|B\|_M$$

   Hint: Use the definition for the induced matrix norm.

2. Let $Ax = b$ and $A\hat{x} = \hat{b}$. Show that

$$\frac{\|x - \hat{x}\|}{\|x\|} \leq \text{cond}(A) \frac{\|b - \hat{b}\|}{\|b\|}.$$

   This is Theorem 14 in the lecture notes.

3. Let $A = \begin{pmatrix} 2 & 4 \\ -3 & -6.001 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

   (a) Solve $Ax = b$ using backslash.

   (b) Change the second entry of b to 3.01 and solve for the new solution.

   (c) Compute the relative difference between the solutions.

   (d) Does this agree with the Theorem above? Explain why/why not.

4. We are now looking at the system $Mx = b$ with

$$M = \begin{pmatrix} 10^{-14} & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \qquad b = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

   a) Compute the solution x to this system with MATLAB's "backslash", `MakeLU`, and `MakePLU`.

   b) Compute the error of each of the three methods using `norm(x-xref)` where `xref` is the analytical solution (what is the exact solution to this system? Compute it by hand!).

   c) Explain the difference in the errors!

   Note: Please submit `hw04q4.m` for a) and b) and write c) on paper.