Homework 03

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Work in groups of two. Include both of your names as a comment in the first line of each file. Then submit those files on canvas (once per group!).

0. (do not submit!)

   Compute the LU decomposition with pivoting using `[L,U,P] = lu(A);` and then use the result to solve $Ax = b$ (for now use backslash instead of forward/back substitution) for

$$\begin{pmatrix} 0 & 1 & 4 \\ 5 & 6 & 0 \\ 2 & 4 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}.$$

1. Test back substitution (see canvas) by solving the linear system $Ax = b$ with

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 & 9 \\ 0 & 0 & 10 & 11 & 12 \\ 0 & 0 & 0 & 13 & 14 \\ 0 & 0 & 0 & 0 & 15 \end{pmatrix}, b = \begin{pmatrix} 2.4 \\ 12 \\ -12 \\ -14 \\ -15 \end{pmatrix}$$

   Write a script to print the solution x and check the computation by also printing the residual vector $r = Ax - b$ and its norm $\|r\|_2$ using `norm(r)`.

2. Implement forward substitution (`function x=ForwardSubstitution(A,b)`) that solves the linear system $Ax = b$ if A is a lower triangular matrix.

   Test the algorithm with the following linear system:

$$\begin{pmatrix} 2 & 0 & 0 \\ -1 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 17 \\ 0 \end{pmatrix}$$

3. Write a script that solves the linear system $Ax = b$ using `MakeLU`, `ForwardSubstitution`, and `BackSubstition` only. Print the residual norm (see question 1) and timing for the decomposition and the solves for different problem sizes.

   Use `A=full(delsq(numgrid('B',k)));` with $k = 5^2, 6^2, 7^2$ and a vector b containing all ones.

   Bonus: how does backslash perform if you solve directly and remove `full` from the definition of A?

4. Implement `function [L,U,P]=MakePLU(A)` that implements LU decomposition with partial pivoting to compute $LU = PA$. The algorithm should swap the row with the largest absolute value in the current column to the diagonal (you can only look at rows below the current one!). Test it with the matrices

$$A = \begin{pmatrix} 0 & 1 & 4 \\ 2 & 4 & 6 \\ 5 & 6 & 0 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 0 & 1 & 2 & 3 \\ -3 & 0 & 0 & 1 \\ 0 & 1 & 1/5 & 3 \\ 2 & 5 & 1 & 7 \end{pmatrix}.$$

Note: of course you may not use the existing `lu(A)` function.