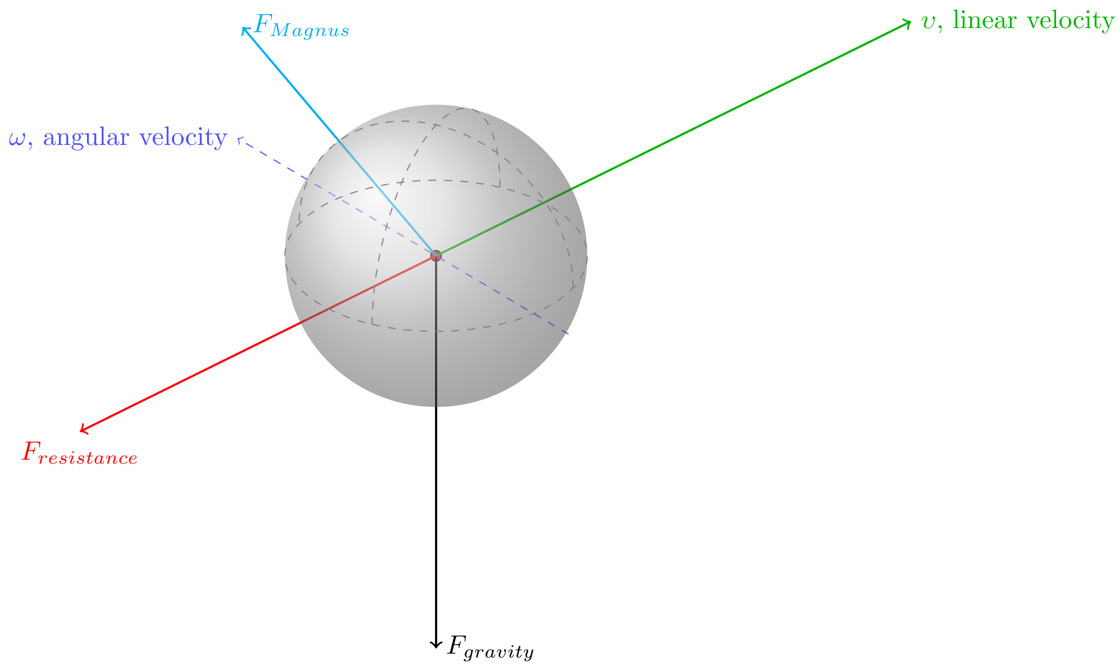
Ball's aerodynamics



Main forses

Gravity force

Gravity, also called gravitation, in mechanics, the universal force of attraction acting between all matter. On Earth all bodies have a weight, or downward force of gravity, proportional to their mass, which Earth's mass exerts on them. Gravity is measured by the acceleration that it gives to freely falling objects. At Earth's surface the acceleration of gravity is about 9.81 metres per second per second.

$$F_{gravity} = mg,\ g \approx 9.81\, m/s^2$$

Resistance force

Air friction, or resistance force, is an example of fluid friction. Unlike the standard model of surface friction, such friction forces are velocity dependent. For higher velocites the frictional drag is approximately proportional to the square of the velocity:

$$F_{resistance} = \alpha v^2,\ \alpha \approx 0.000151$$

Magnus force

The Magnus force occurs when the ball rotates in the surrounding gas. Rotation increases the speed of the gas on one side of the ball and decreases the speed of the gas on the other side. Based on the fact that the positional pressure at a high flow rate is small, and the low flow rate is large, the difference of the lateral pressures of the rotating ball is introduced, representing the Magnus force. The direction of the Magnus force is perpendicular to the axis of rotation and the direction of the ball's motion, so it basically changes the flight velocity direction.

$$F_{Magnus} = \beta \left[\vec{\omega} \times \vec{v}\right],\ \beta \approx 0.0000185$$

Dynamics of the system

Newton's laws of motion:

$$m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -\alpha \begin{bmatrix} \dot{x}^2 \\ \dot{y}^2 \\ \dot{z}^2 \end{bmatrix} - \beta \left[\vec{\omega} \times \vec{v} \right] - m \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

$$\left[\vec{\omega} \times \vec{v} \right] = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_x & \omega_y & \omega_z \\ \dot{x} & \dot{y} & \dot{z} \end{bmatrix} = \begin{bmatrix} 0 \cdot \dot{x} - \omega_z \dot{y} + \omega_y \dot{z} \\ \omega_z \dot{x} + 0 \cdot \dot{y} - \omega_x \dot{z} \\ -\omega_y \dot{x} + \omega_x \dot{y} + 0 \cdot \dot{z} \end{bmatrix}$$

$$m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = - \begin{bmatrix} \alpha \dot{x}^2 - \beta \omega_z \dot{y} + \beta \omega_y \dot{z} \\ \beta \omega_z \dot{x} + \alpha \dot{y}^2 - \beta \omega_x \dot{z} \\ -\beta \omega_y \dot{x} + \beta \omega_x \dot{y} + \alpha \dot{z}^2 \end{bmatrix} - m \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$