Problem ①

a) Claim: Let $p, q \geq 2$ be integer multiple of 3. Let c, d be non-neg integers such that $(c-d)$ is not a multiple of 3. There is no such integer that
$n \equiv c \bmod p$ $\quad n \equiv d \bmod q$

$p, q \geq 2$
$p = 3k$ $k \in \mathbb{Z}$
$q = 3L$ $L \in \mathbb{Z}$

a)

p	q	c	d	$3 \mid p$	$3 \mid q$	$c-d$	$3 \mid (c-d)$	n	$n \equiv c \bmod p$	$n \equiv d \bmod q$	Claim
6	9	4	2	T(2)	T(3)	2	F	F	F	F	T
12	15	8	3	T(4)	T(5)	5	F	F	F (8)	F (3)	T
3	6	5	3	T(1)	T(6)	2	F	F	F	F	T

b) There is such value of n such that $n \equiv c \bmod p$
$n \equiv d \bmod q$
Let $p, q \geq 2$ be integers of 3
Let c, d be non-negative ints such that $(c-d)$ is not a multiple of 3

Problem ①

ⓐ Claim: Let $p, q \geq 2$ be integers multiple of 3. Let c, d be non-negative integers such that $(c-d)$ is not a multiple of 3. There is no such integer that

$n \equiv c \bmod p$ $\quad n \equiv d \bmod q$

$p, q, \geq 2$
$p = 3K$ $K \in \mathbb{Z}$
$q = 3L$ $L \in \mathbb{Z}$

ⓐ

p	q	c	d	3|p	3|q	c-d	3|(c-d)	n	n ≡ c mod p	n ≡ d mod q	Claim
6	9	4	2	T(2)	T(3)	2	F	F	F	F	T
12	15	8	3	T(4)	T(5)	5	F	F	F (8)	F (3)	T
3	6	5	3	T(1)	T(6)	2	F	F	F	F	T

ⓑ There is such value of n such that $n \equiv c \bmod p$
$n \equiv d \bmod q$

Let $p, q \geq 2$ be integers of 3
Let c, d be non-negative ints such that $(c-d)$ is not a multiple of 3

Problem ① cont.

$n \equiv c \bmod p$ $\quad n \equiv d \bmod q$

$n = pK + c$ $\quad n = ql + d$

(c)

Mathematical statement	Reason this statement is True (from approved List)
BWOC Suppose there is an n such that $n \equiv c \bmod p$ & $n \equiv d \bmod q$	since this is the negatiation of the claim
$\Rightarrow$ $n = pK + c$, $K \in \mathbb{Z}$ and $n = ql + d$, $l \in \mathbb{Z}$	by def of mod applied to each claim
$\Rightarrow$ $pK + c = ql + d$, $K, l \in \mathbb{Z}$	by substitution
$\Rightarrow$ $ql - pK = c - d$, $K, l \in \mathbb{Z}$	by algebra
$p = 3(a)$, $a \in \mathbb{Z}$ $\Rightarrow$ $(3a)l - (3b)(K) = c - d$, $a, b, l, K \in \mathbb{Z}$	by def of multiple of 3
$q = 3(b)$, $b \in \mathbb{Z}$ $\Rightarrow$ $3(al - bK) = c - d$	by algebra
$\Rightarrow$ $3(m) = c - d$, $m \in \mathbb{Z}$	by product & diff of integers is an integer
$\Rightarrow$ $c - d$ is a multiple of 3	by def of multiple of 3
$\Rightarrow$ contradiction	because $c - d$ is not a multiple of 3

Problem 2 Continued

	mathematical statement	Reason this statement is true
c) BWOC 1 Suppose	and $(A \times B) \subseteq (B \times C)$ is $A - C \neq \emptyset$	since this is the negation of our claim
2 $\Rightarrow$	$B \neq \emptyset$	by def of "$\times$"
3 $\Rightarrow$	$a \in A$, $a \notin C$	by def of "$-$"
4 $\Rightarrow$	$\langle a, b \rangle \in A \times B$	by def of "$\times$"
5 $\Rightarrow$	$\langle a, b \rangle \in B \times C$	by def of "$\subseteq$"
6 $\Rightarrow$	$a \in B$, $b \in C$	by def of "$\times$"
7 $\Rightarrow$	$\langle a, a \rangle \in (A \times B)$	by def of "$\times$"
8 $\Rightarrow$	$\langle a, a \rangle \in (B \times C)$	by def of "$\subseteq$"
9 $\Rightarrow$	$a \in B$, $a \in C$	by def of "$\times$"
10 $=$	$a \in C$ contradicts	because in step 3 we said $a \notin C$

d)

A	B	C	$A \times B$	$B \times C$	$A - C$	$(A \times B) \subseteq (B \times C)$	$A - C = \emptyset$
a	$\emptyset$	ℓ	$\{a\}$	$\{\ell\}$	$\{a\}$	F	F

Problem 2

Claim Let B be a non empty set, if (

if $(A \times B) \subseteq (B \times C)$ then $A - C = \emptyset$

$= \underbrace{(A \times B) \subseteq (B \times C)}_{p} \Rightarrow \underbrace{(A - C) = \emptyset}_{q}$ $\quad p \Rightarrow q \xrightarrow{\text{negation}} p \wedge \neg q$

a) $(A \times B) \subseteq (B \times C) \wedge (A - C) \neq \emptyset$ ⇐ negation

b) The claim breaks at step 8 because the statement says x is not in C but (x, y) is in C which makes no sense

c)

Problem 3

a) The negation of this statement is that there is at least one irrational number and one rational number such that their sum is not irrational

b)

	mathematical statement	reason for this statement is true
BWOC Suppose	$X+Y=S,\ X,S \in \mathbb{Q},\ Y \in \mathbb{I}$	negation of claim
$\Rightarrow$	$\frac{n_x}{d_x}+Y=\frac{n_s}{d_s}$, $n_s\, n_x\, d_s\, d_x \in \mathbb{Z}$, $d_x, d_s \neq 0$	by def of rational
$\Rightarrow$	$Y=\frac{n_s}{d_s}-\frac{n_x}{d_x}$ $n_s\, n_x\, d_s\, d_x \in \mathbb{Z}$, $d_s, d_x \neq 0$	by simple algebra
$\Rightarrow$	$Y=\frac{n_s d_x - n_x d_s}{(d_s)(d_x)}$ $n_s\, n_x\, d_s\, d_x \in \mathbb{Z}$, $d_s, d_x \neq 0$	by subtraction of fractions
$\Rightarrow$	$Y=\frac{a-b}{d}$ $a, b, d \in \mathbb{Z}$, $d \neq 0$	by def of product of ints is an int
$\Rightarrow$	$Y=\frac{n}{d}$ $n, d \in \mathbb{Z}$, $d \neq 0$	by def of difference of ints is an int

$\Rightarrow$ contradict because we said Y is an irrational number