Lecture 1 | Criticality Safety

In this lecture, we discuss the topic of *criticality safety*, one of the most important physics-oriented aspects of nuclear engineering not specifically dealing with core physics, beginning with a brief overview. Thereafter, the chief physical concerns related to criticality safety are described and illustrated with a simple example, and a few notable accidents over the past several decades are described. Finally, computational aspects of criticality safety are discussed, and the topic of burnup credit is used as a case study to illustrate recent efforts in criticality safety analysis.

Overview

Nuclear criticality safety, as defined by the American National Standard ANSI/ANS 8.1 [**?**], is the "protection against the consequences of a criticality safety accident, preferably by prevention of the accident," and it defines a *criticality accident* as "the release of energy as a result of accdental production of a self-sustaining or divergent neutron chain reaction."

ANSI/ANS 8.1 provides general guidance for nuclear criticality safety applied to fissionable materials *outside* of reactors. In addition to ANSI/ANS 8.1, a number of more specialized standards exist, a few of which are summarized in Table 1.1. The interested student can request these standards from the library, but they tend to be expensive (and, admittedly, rather dry reading). ANSI/ANS 8.1 has been put on the 22.106 Stellar site.

The Nuclear Regulatory Commision (NRC) also offers guidance with respect to criticality safety. NRC documents are most often found in the NUREG series, published by the NRC itself, by contractors, or through international agreements.

Additionally, the NRC provides *regulation* through relevant portions of the Code of Federal Regulations (CFR). Both DOE and NRC share regulations under title 10, *i.e.* those regulations beginning with 10CFR.* The parts of 10CFR most relevant to

*That DOE and NRC share the same title is most likely due their common origin: the Atomic Energy Commision.

Table 1.1: Several ANSI/ANS Standards Applicable to Criticality Safety.

Number-Revised	Title
8.1-1998	Nuclear Criticality Safety in Operations with Fissionable Materials Outside Reactors
8.3-1997	Criticality Accident Alarm System
8.5-1996	Guide for Nuclear Criticality Safety in the Storage of Fissile Materials
8.17-2004	Criticality Safety Criteria for the Handling, Storage, and Transportation of LWR Fuel Outside Reactors
8.24-2007	Validation of Neutron Transport Methods for Nuclear Criticality Safety Calculations
8.27-2008	Burnup Credit for LWR Fuel

criticality safety are 10CFR-0, 1, 2, 20, and several between 50 and 75. As an exercise, the student is encouraged to find one or more of these regulations (or NUREG documents) related to criticality safety and explain its relevance.

As a historical note, nuclear criticality safety as a discipline is about as old as other nuclear engineering areas; it began with the large scale chemical processing at the K-25 gasseous diffusion plant in Oak Ridge, TN and the Hanford, WA site, both major components of the Manhatten Project. Of course, the initial work at Los Alamos generated much knowledge before the larger scale projects were begun, and in fact, it was a young Richard Feynman who carried much of that experience from Los Alamos to Oak Ridge in 194X at the bequest of Oppenheimer [**?**]. In Feynman's own words, he was told by Oppenheimer to tell the Oak Ridge folks (stubborn military types), "Los Alamos cannot accept the responsibility for the safety of the Oak Ridge plant unless they are full informed as to how it works." He delivered the message, and when they agreed to listen, he "told them all about neutrons, how they worked, da da, ta ta ta, there are too many neutrons together, you've got to keep the material apart, cadmium absorbs ..." and so forth. The Oak Ridge folks went back to the design board, and a "practice" was born.

Physical Concerns

When we analyze a system for criticality safety, what are the important characteristics the system? An analyst must understand *what* to analyze before computational techniques become useful.

Several features can be intuited by any student of reactor physics: the more fissile material one has, the easier it is to achieve criticality. That means increased *mass*, *concentration*, *enrichment*, or *volume* of a fissile material should bring a system closer to criticality (or past it, unfortunately).

But there are other factors: what neutrons do we like in our typical light water reactors? Thermal ones, of course, and to get those, we need *moderation*. Moreover, those same reactors feature a layer of water around the periphery, which *reflect* neutrons back into the core. To reduce power in a reactor (or to shut it down), we insert some level of *absorption* via control rods, which limits the *interaction* of fissile assemblies with one another.

These key chacteristics, easily remembered via the acronym MAGICMERV [**?**], are equally applicable to nuclear criticality safety.

To illustrate several of the characteristics listed, consider Figure 1.1.

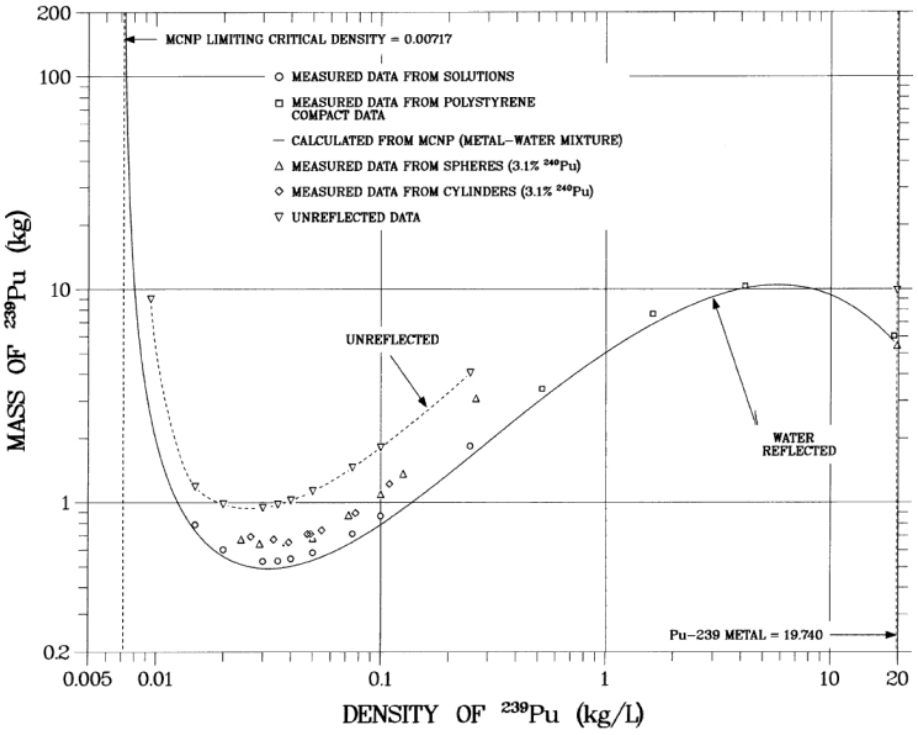

Figure 1.1: Pu-239 solution critical mass. (Borrowed from Martin, J. Physics for Radiation Protection: A Handbook, Wiley (2016).

Accidents

What exactly *is* a criticality accident? In the simplest terms, it is an excursion, an uncontrolled chain reaction. To help understand accidents both qualitatively and quantitatively, let's review a few ideas from reactor kinetics. Recall that *reactivity* measures a departure from criticality, and is most often expressed via

$$\rho = \frac{k-1}{k}\,. \tag{1.1}$$

Postive ρ denotes a supercritical state, negative ρ a subcritical state, and $\rho = 0$ is equivalent to $k = 1$ or a critical state. We can further differentiate between a *delayed critical* state and a *prompt critical* state. The latter occurs when $\rho > \beta$, where β is the delayed neutron fraction. This situation is to be avoided in all but a few specific experimental situations, for in the prompt critical regime, the growth of the neutron population occurs on timescales close to the prompt neutron lifetime (say 10^{-8} to 10^{-4} seconds in many systems of interest).

To get an idea of the orders of magnitude we deal with in an accident situation, consider a critical system with a constant neutron population of just one neutron. Suddenly, a perturbation brings the system into a prompt critical state for a short time Δt. During this time, the population grows as $n \propto n_o e^{(\rho-\beta)t/\Lambda}$. Assuming $\Lambda = 10^{-5}$ seconds, $\rho - \beta = 0.002$, and the perturbation lasts two tenths of second[†], we estimate that the number of neutrons produced is roughly

$$\begin{aligned} N &= \int_0^{0.5} dt e^{(\rho-\beta)t/\Lambda} \\ &= \frac{10^{-5}}{0.002}(e^{40}-1) \\ &\approx 1 \cdot 10^{15}\,. \end{aligned} \tag{1.2}$$

Suppose a worker is about a half a meter from the neutron source, so that the fluence is $\Phi \approx 10^{15}/(4 \cdot \pi \cdot 50^2)$ n/cm^2. Looking up neutron fluence-to-dose equivalent factors (see 10CFR-20 [‡]) suggests that a high energy (1 MeV) fission fluence of $27 \cdot 10^6$ n/cm^2 corresponds to a dose of 1 rem. Hence, our excursion yields a dose of around 1200 rem (12 Sv). For perspective, a dose of 4 Sv is lethal roughly half the time. Of course, this is a crude example, but it gives a very clear picture of the issues at hand.

[†] the average human reaction time is about 200 milliseconds. See `http://www.humanbenchmark.com/tests/reactiontime/stats.php`

[‡] `http://www.nrc.gov/reading-rm/doc-collections/cfr/part020/part020-1004.html`

In fact, the example above is not too far different than one of the first documented criticality accidents from August of 1945. Figure 1.2 shows a 6.2 kg plutonium sphere coated with nickel and surrounded by blocks of tungsten carbide for reflection. In the accident, an experimenter was placing blocks to achieve criticality. While placing the last block, the detector reading suggested that the block would actual produce a supercritical state. Unfortunately, the experimenter dropped the brick onto the assembly, yielding a prompt supercritical excursion of 10^{16} fissions before he was able to remove the brick. A later study estimated the resulting dose to be 510 rem, which proved to be fatal some 28 days later. The same assembly would be involved in a second fatal accident just months later, where the experimenter handled one of two beryllium hemisphere reflectors with his thumb (in a hole in the hemisphere) and a screwdriver, a procedure Feynman dubbed "tickling the dragon's tail." The screwdriver, holding the reflectors apart, slipped and caused an excursion that led to the experimenter's death 9 days later and significant doses to observers; see Figure 1.3 for a recreation of the experiment.

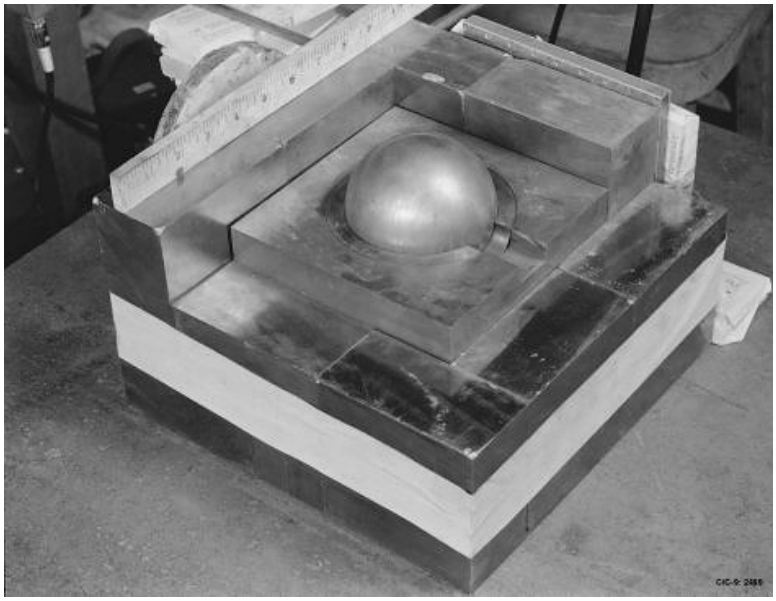

Figure 1.2: Plutonium sphere reflected by tungsten-carbide bricks.

A second type of accident involves processing operations in which fissile materials are in solution. The first domestic processing accident occured at the Y-12 plant in Oak Ridge, TN. Y-12 produces parts from HEU for use in nuclear weapons.

The accident occured in a portion of the plant used to recover HEU from waste

Figure 1.3: Recreation of the "tickling the dragon's tail" experiment.

material and place it in tanks of favorable geometry. These tanks were to be emptied, cleaned, and leak-tested with water. Before the leak-testing began, however, uranium solution had leaked from the process stream into the piping below the storage tanks (and actually into one of the tanks, as its release valve had been let open). When the other tanks, full of water, were emptied, the uranium solution and water accumulated in a 55 gallon drum. A nearby worker first noticed dark yellow fumes followed by a blue flash. Eight workers received significant doses, though none died as a result of acute effects.

Unfortunately, criticality accidents are not a thing of the past. A recent example happened in Tokai-mura, Japan at a uranium conversion plant. The accident occurred when workers placed 16.6 kg of 18.8% enriched uranium into a tank designed to contain no more than 2.4 kg of uranium at such high enrichments. The accident was the first of its kind in Japan, and the first resulting in casualties. The accident is somewhat unique in that criticality was maintained intermittently for roughly 20 hours before effective action was taken.

A rather complete history of criticality accidents in the U.S. and around the world is contained in the latest edition of *A Review of Criticality Accidents* from Los Alamos. This document is a really great piece of nuclear history, and students are highly encouraged to skim through the many accidents covered.

Computational Aspects

The intent of this section to provide the reader with an overview of criticality safety analysis validation. A brief review of regulations and guidance pertinent to criticality safety of fissile materials outside reactors is given, with a particular emphasis on requirements for ensuring subcritical conditions. The traditional approach to bias determination is discussed, and one specific method used in practice is described and demonstrated in an illustrative example. Subsequently, modern S/U-based validation techniques are discussed, and an illustrative example is provided and related to the traditional approach.

Subcritical Limits

As noted above, ANSI/ANS-8.1 provides guidance for avoiding criticality accidents during handling of fissionable material outside of reactors [2]. The standard provides basic safety principles in addition to several limits for simple systems of single isotopes. Specifically, the standard defines a *subcritical limit* to be "the limiting value assigned to a controlled parameter that results in a subcritical system under specified conditions. The parameter limit allows for uncertainties in the calculations and experiemental data used in the derivation but not for contingencies."

These limits are *absolute maxima*, and since they do not include contingencies, in practice, adminstrative margins are employed. A typical value, as we'll see below, tends to be 5% on k_{eff} .

Table 1.2 gives several examples of single control parameters used in criticality safety analysis. Note, while ANSI/ANS 8.1 specifies limits in terms of a single parameter, in some cases, multiple parameter limits are also employed. The standard suggests a few examples, and cites the technical literature for further guidance. However, while less conservative, multiple parameter limits require additional adminstrative margins and may be harder to use in validation.

Table 1.2: Example single parameters and subcritical limits.

parameter	example limit
fissile mass	0.78 kg ^{235}U in UO_2NO_3
dimension (width, volume, etc.)	6.2 L ^{235}U in UO_2NO_3
concentration	11.6 g/L ^{235}U in UO_2NO_3
enrichment	5% ^{235}U in UO_2
fissile mass	20.1 kg ^{235}U in UO_2 (w/ $\rho \leq 18.81$ g/cc)

Criticality Safety Analysis Validation

While the single parameter limits provide useful guidance, two questions naturally arise. First, how does one actually establish these limits? And second, how does one ensure subcriticality in systems that are much more complex than, for example, a sphere? The answer in both cases is by using computational methods validated against experiment.

In addition to the single parameter limits, ANSI/ANS-8.1 provides requirements for ensuring computational methods used in criticality safety analyses are both valid and within the "area of applicability" for specific applications. The standard defines the area of applicability as "limiting ranges of material compositions, geometric arrangements, neutron energy spectra, and other relevant parameters ... within which the bias of the calculational method is established" [2]. In other words, an *application* is the system of interest, such as a spent fuel canister for shipment and disposal. A computational method is *applicable* if it is verified against a set of *experiments* that are similar to the application with respect to composition, geometry, and so forth.

A *computational bias* is the systematic discrepancy between and experimental data and calculated results. Biases have associated uncertainties, which quantify the accuracy and precision of calculated values and the uncertainty in measured data.

When computational tools are used in criticality safety analyses, the standard requires that the bias be established. Qualitatively, the bias must be determined via correlating data from critical experiments to computational models of the same experiments using the tool to be validated. Typically, the measured and computed values of k_{eff} are related, but other physical quantities may also be used. The bias is used to normalize a particular code within its area of applicability so that its results, after applying the bias, may be used to predict criticality within the bias uncertainty.

Another American National Standard, ANSI/ANS-8-17, further explicates use of computational tools by defining specific criteria for establishing subcriticality [1]. Whenever computational methods are used in criticality safety analyses, the standard requires that the calculated application multiplication factor k_a must be less than or equal to the allowable multiplication factor.

The easiest way to think of this is to note that the largest, "worst case" value for k_a must be below the smallest, least conservative computed estimate. That is to say, the maximum expected application multiplication factor (*i.e.* $k_a + \Delta k_a$) must be less than the minimum expected computed value (*i.e.* $k_c - \Delta k_c$) for an applicable critical experiment, *i.e.* a real, measured system whose k_{eff} is unity, or an average computed k_{eff} for several such experiments. Mathematically, this can be written as

$$k_a + \Delta k_a \leq k_c - \Delta k_c \,, \tag{1.3}$$

or

$$k_a \leq k_c - \Delta k_a - \Delta k_c \,. \tag{1.4}$$

For added conservatism, the standard further requires that

$$k_a \leq k_c - \Delta k_a - \Delta k_c - \Delta k_m \,, \tag{1.5}$$

where the standard defines:

k_a is the calculated k_{eff} of the application system for all normal or credible abnormal conditions;

k_c is the average k_{eff} from the calculation of the benchmark criticality experiments. The experiments used should be neutronically similar to the application system. If the application system has parameters beyond the area of applicability established by the benchmark experiments, then the area of applicability may be extended using trends in the calculated values of k_c as functions of those parameters;

Δk_a is an allowance for

- statistical or convergence uncertainties in the computed k_a;
- material and fabrication tolerances;
- uncertainties due to geometry or material simplifications and approximations;

Δk_c is a margin for uncertainty in k_c that includes allowances for

- uncertainties in the critical experiments;
- statistical or convergence uncertainties in the computated k_c;
- uncertainties due to extrapolation of k_c outside the experimental data range;
- uncertainties due to geometry or material simplifications and approximations;

Δk_m is an arbitrary "administrative" margin to ensure the subcriticality of k_a.

Eq. 1.5 can be rewritten as

$$k_a + \Delta k_a + \Delta k_m - \beta + \Delta\beta \leq 1 \,, \tag{1.6}$$

where $\beta = k_c - 1$ is the bias and $\Delta\beta = \Delta k_c$ is the uncertainty in the bias. The definition of β is based on the fact that critical experiments, by their definition, have k_{eff} = 1, and hence the bias is just the difference between the computed value and unity. By convention, the bias is defined such that a *negative* β indicates an *underestimated* k_{eff}, which is undesirable.

To ensure subcriticality in the application system, an *upper subcritical limit* is defined as

$$USL = 1 - \Delta k_m + \beta - \Delta\beta\,, \tag{1.7}$$

and from Eq. 1.6, it is apparent that $k_a + \Delta k_a \leq USL$ [8]. Thus, the USL is the maximum value an application k_{eff} (plus any uncertainties) may have for which the application can, with a high degree of confidence, be considered subcritical. The value $1 - USL$ is the mathematical definition of the subcritical margin. In the event the bias β is positive, *i.e.* the computed value k_c overestimates k_{eff}, current practice is to set the bias to zero rather than reduce the subcritical margin.

Traditional Bias Determination

In the United States, biases and associated uncertainties and USL's have often been found through use of *trending analysis*. A suite of critical experiments is selected for use in a specific safety analysis based on the similarity of the experiments to the safety analysis system. Traditionally, this similarity is based on physical parameters that include the fissile material, hydrogen-to-fissile atom ratio (H/X), average neutron energy group causing fission, and energy of average lethargy causing fission (EALF), among others [5].

While various methods using trending analysis exist for determining the biases and USL's, one common approach, denoted USL_1 in the literature [5], is discussed here to provide at least some background of current practice. The following description is largely adapted from the technical report in which it was first developed [8].

The method computes $k_c(x)$ as a function of a trending parameter x using linear regression. The bias, $\beta(x)$, is just $k_c(x) - 1$. .

A lower confidence band $w(x)$ is statistically computed using current experiments and uncertainties as well as a specified confidence. This confidence band defines the value below which an additional computed k_{eff} value (*i.e.* not included in the analysis) must be for the additional system to be considered subcritical with a high degree of confidence. Equivalently, for a given value of the trending parameter x, $w(x)$ represents the value *above* which the additional computed value k_c will be *if* the system in question is critical—and this consequently implies any negative $\beta(x)$ is no worse than $w(x) - 1$. Hence, if our aim is to ensure the actual k_{eff} of a system is subcritical, then its computed value should be *less* than the appropriate confidence band value.

To simplify analyses, the limiting width, W, of the confidence band is used, which (like $w(x)$) takes into account all uncertainties associated with the experiments (*e.g.* experimental, stochastic, etc), and consequently, is a statistical measure of $\Delta\beta$. The width W of the confidence band is defined

$$W = \max\Big\{ w(x) | x_{\min}, x_{\max} \Big\}\,, \tag{1.8}$$

where

$$w(x) = t_{1-\gamma} s_p \left(1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \right)^{1/2}\,, \tag{1.9}$$

and n is the number of critical experiments included in the analysis, $t_{1-\gamma}$ is the student-t distribution statistic for $1-\gamma$ and $n-2$ degrees of freedom (γ is the desired confidence level), $\bar{x}$ is the mean value of x in the set of experiments, and s_p is the pooled standard deviation for the set of calculations.

The pooled standard deviation s_p is defined by

$$s_p^2 = s_{k(x)}^2 + s_w^2\,, \tag{1.10}$$

where $s_{k(x)}^2$ is the mean-square error of the linear regression, defined as

$$s_{k(x)}^2 = \frac{1}{n-2} \left(\sum_{i=1}^{n} (k_i - \bar{k})^2 - \frac{\Big(\sum_{i=1}^{n} (x_i - \bar{x})(k_i - \bar{k}) \Big)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \right)\,, \tag{1.11}$$

and s_w^2 is the variance of the data, defined as

$$s_w^2 = \frac{1}{n} \sum_{i=1}^{n} \sigma_i^2\,, \tag{1.12}$$

where σ_i is the uncertainty in the ith calculated value, accounting for method uncertainty (*e.g.* Monte Carlo statistics) and estimated uncertainty due to cross-section uncertainty.

The width of the confidence W is chosen to be the *maximum* value of w at the limits of the range of x corresponding to the experiments to be conservative, and moreover, it serves to simplify the definition of USL. Current guidance is to define W at the 95% confidence level, *i.e.* choose $\gamma = 0.95$. Below, we provide an illustrative example of the USL_1 methodology.

S/U-Techniques

Unfortunately, the proper selection of parameters over which to trend, and the experiments with which to trend, is largely based on expert judgement. As experiments continue to become more expensive, use of computational methods will grow. Furthermore, new applications will continue to extend beyond the areas of applicability of current experimental data, and consequently, methods to extend beyond these areas are needed.

For the past several years, ORNL has worked with the support of the NRC and the Department of Energy's (DOE) Nuclear Criticality Safety Program to develop "a rigorous physics-based approach for the determination of system similarity" for determining areas of applicability [5]. Additionally, their efforts aimed to develop the methodology and computational tools needed for "determination of biases and uncertainties due to experimental descriptions, computational methods, and nuclear data."

As an alternative to traditional trending analysis, work was done to develop S/U-based, easily-quantifiable parameters to measure the similarity of systems. It is beyond our scope to describe the methods in detail. Interested readers should see the exercises of Lecture **??** for some basic theory needed to derive the results, and Ref. [5] for greater detail.

Skipping the theory, what we end up with is the sensitivity of k_{eff} to the various underlying nuclear data,

$$S_{k,\sigma_x} = \frac{\sigma_x}{k}\frac{\partial k}{\partial \sigma_x} \tag{1.13}$$

where x denotes some reaction of interest. We express the sensitivity of k_{eff} to all nuclides in a system as the vector $\mathbf{S}_k$, which can implicitly represent dependence on multigroup data.

Sensitivity vectors can be used to compare both the qualitative and quantitative similarity between two systems with respect to single or several nuclide-reactions. Let the entire set of group-wise, nuclide-reaction cross-sections be denoted $\boldsymbol{\sigma} \equiv \sigma_i, \quad n = 1,\ 2, \ldots, N$, where N is the total number of nuclide-reactions multiplied by the number of energy groups. The $N \times N$ correlation (*i.e.* relative covariance) matrix of $\boldsymbol{\sigma}$ is defined

$$\mathbf{C}_{\sigma\sigma} \equiv \left[\frac{\text{COV}(\sigma_i, \sigma_j)}{\sigma_i \sigma_j}\right], \tag{1.14}$$

where i and j range from 1 to N, and

$$\text{COV} = \langle (\sigma_i - \bar{\sigma_i})(\sigma_j - \bar{\sigma_j}) \rangle = \langle \delta\sigma_i \delta\sigma_j \rangle\,, \tag{1.15}$$

where $\bar{\sigma_i}$ represents the expected value of the data σ_i.

Because the components of $\mathbf{C}_{\sigma\sigma}$ represent relative uncertainties, and because from above, we know the k_{eff} sensitivity represents the relative change in k_{eff} due to relative changes in some nuclide-reaction data, the relative uncertainty in k_{eff} is therefore

$$\frac{\delta k}{k} = \sqrt{\mathbf{S}_k \mathbf{C}_{\sigma\sigma} \mathbf{S}_k^T}\,, \tag{1.16}$$

where T denotes the matrix transpose. If several systems are of interest, then an $N \times I$ matrix of sensitivity vectors $\bar{\mathbf{S}}_k$ can be defined, where I is the number of systems. By folding $\bar{\mathbf{S}}_k$ with $\mathbf{C}_{\sigma\sigma}$, one obtains

$$\mathbf{C}_{kk} = \bar{\mathbf{S}}_k \mathbf{C}_{\sigma\sigma} \bar{\mathbf{S}}_k^T\,, \tag{1.17}$$

an $I \times I$ matrix consisting of each system's relative variance in k_{eff} (*i.e.* $(\Delta k/k)^2$) due to data uncertainties (the diagonal terms, α_{ii}) and the relative covariances in k_{eff} between systems (the off-diagonal terms, α_{ij}). These off-diagonal terms represent "shared variance" or "shared uncertainty" between systems.

The correlation coefficient between system i and j is defined

$$c_k = \frac{\alpha_{ij}^2}{\alpha_i \alpha_j}\,, \tag{1.18}$$

which, as is expected, takes on values between -1 (completely anti-correlated) and 1 (completely correlated). A value $c_k = 0$ indicates no correlation between systems.

The correlation of two systems is greatest if they share basic characteristics, *e.g.* fuel type, moderator, other materials. Two water-moderated thermal UO_2 systems would be expected to have a higher degree of correlation than would such a thermal system and a molten salt fast reactor. Use of c_k as a trending parameter in the USL_1 method is illustrated below.

Example Analyses

To illustrate the USL_1 method using both traditional parameters and c_k, 25 thermal LEU systems were chosen for example analyses. Two additional systems were selected as "applications" for which the β, $\Delta\beta$, and the USL are determined[§]. The experiments range from 2.35% to 5% enrichement, and have EALF values ranging from 0.017 to 2.24 eV. All experiments are included in the International Handbook of Evaluated Criticality Benchmark Experiments [4] and are low enriched uranium, thermal assemblies. The LCT-079 and LCT-050 are experiments of use to burnup credit, a topic discussed below; however, their use here is purely for example.

[§]The minimum recommended number of experiments for trending with the USL_1 method is 25; here, just the minimum was used for these example cases.

Figures 1.4-1.7 show the trending analysis for using EALF, enrichment, and c_k. For both EALF and enrichment, only one analysis was needed for the applications since neither parameter depends on the application. However, separate cases were run using c_k values specific to the given application. The experiments are the black dots, and the applications are red shapes. For all cases, the uncertainty is computed using Eq. 1.17, where the uncertainty for the system is its associated diagonal element of $\mathbf{C}_{kk}$, *i.e.* the uncertainty in cross-sections propagated to k_{eff} via use of sensitivity profiles. This is in line with previous analyses [5]. Note, for the USL, an administrative margin of $\Delta k_m = 0.05$ was used.

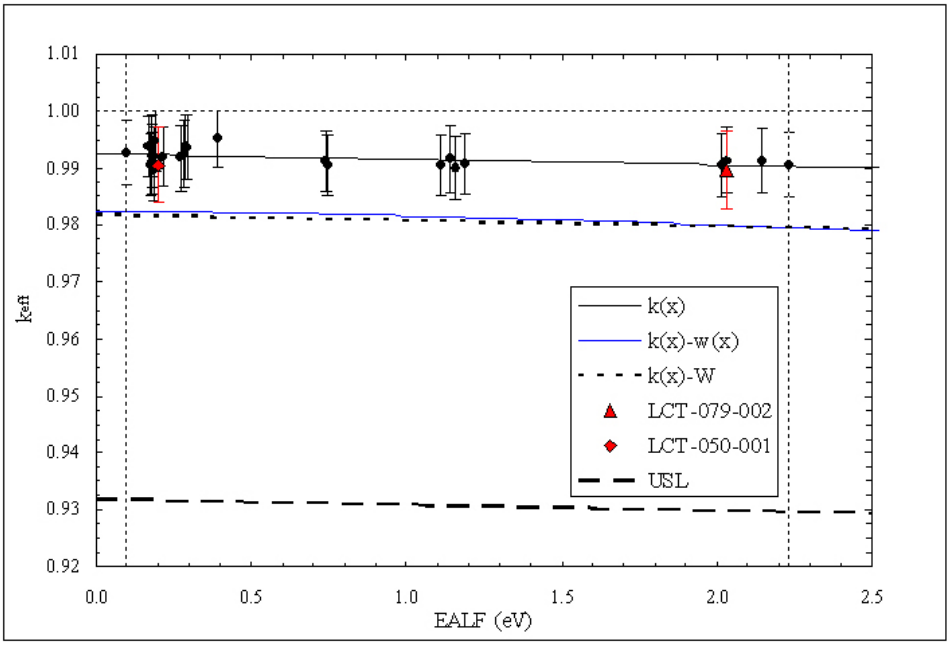

Figure 1.4: Example trending analysis using EALF.

From the trends, the bias $k(x) - 1$ (where x is the parameter value for the application) and associated bias uncertainty (*i.e.* W from Eq. 1.8) can be computed in addition to the USL. Since the "applications" are known critical experiments, it is useful to compare the observed bias (*i.e.* $k_{\text{calc}} - 1$) to the bias as predicted via trending. Table 1.3 provides the observed and predicted biases (with uncertainty) and the USL for both applications. For each application, all three methods yield very similar USL's, and all the biases are slightly underpredicted but well within one standard deviation of the observed biases.

With such a large Δk_m, it is easy to wonder why we care about $\Delta\beta$. However,

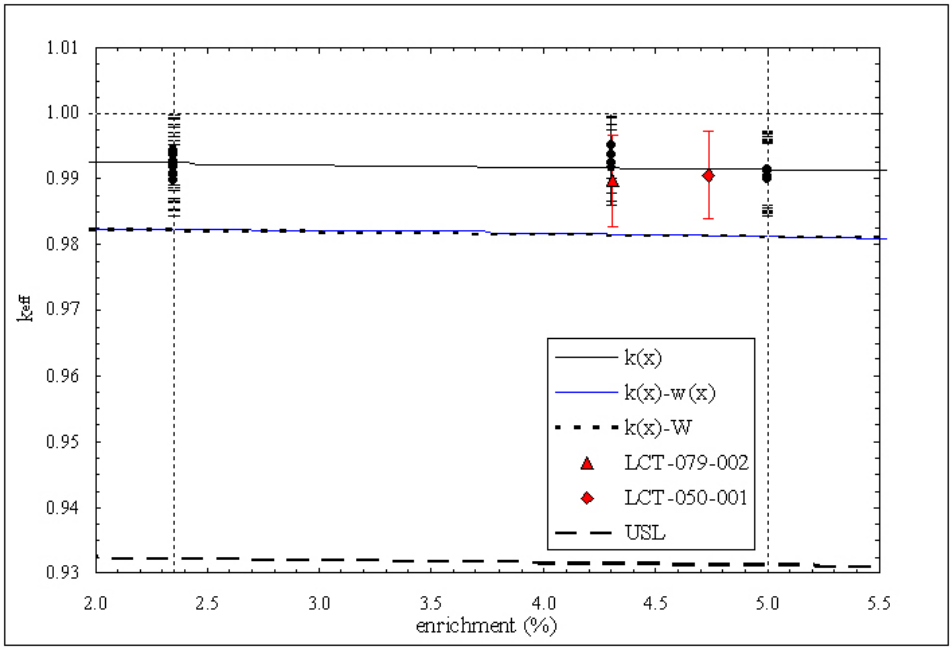

Figure 1.5: Example trending analysis using enrichment.

if we interpret subtracting Δk_m from k_{eff} as simply shifting the definition of critical, then it becomes clearer that $\Delta\beta$ is still important.

Table 1.3: Observed and predicted biases (β_{ob} and β), $\Delta\beta$, all in percent, and USL using each trending technique.

	LCT-079-002				LCT-050-001			
	β_{ob}	β	$\Delta\beta$	USL	β_{ob}	β	$\Delta\beta$	USL
EALF	-1.08	-0.95	1.08	0.9298	-0.94	-0.77	1.08	0.9316
Enrich.	-1.08	-0.84	1.02	0.9314	-0.94	-0.85	1.02	0.9313
c_k	-1.08	-0.74	1.16	0.9306	-0.94	-0.71	1.07	0.9322

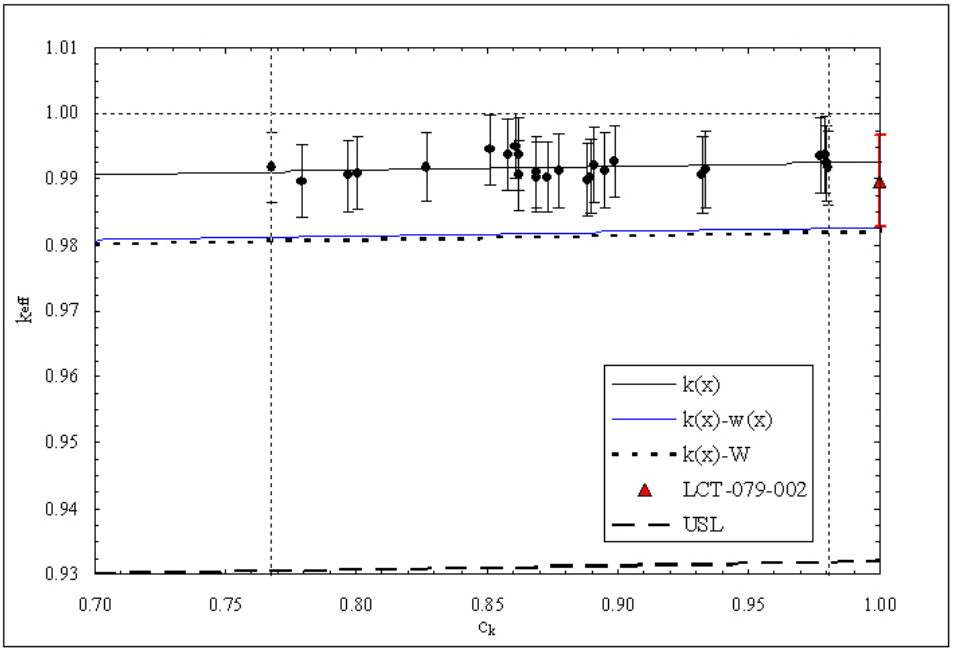

Figure 1.6: Example trending analysis using c_k (for LCT-079-002).

Case Study: Burnup Credit

Overview and Motivation

The transportation and storage of used nuclear fuel is an integral component of any nuclear fuel cycle. During handling of used nuclear fuel, strict attention must be paid to criticality safety. The chief concern of criticality safety is to ensure the effective multiplication factor k_{eff} of the system in question is below unity at all times, i.e. to ensure subcriticality. For the case of used nuclear fuel, several factors affect the subcritical margin, i.e. by how much a system is subcritical. Typical, unirradiated light water reactor (LWR) fuel consists of uranium-oxide (UO2) enriched to 3-5% ^{235}U. During its time in the reactor, nuclear fuel undergoes significant compositional changes, a process referred to as burnup. Most importantly, the fissile isotope 235 U is depleted, generating various fission products (FP's), many of which are parasitic neutron absorbers. Simultaneously, ^{235}U breeds ^{239}Pu which also undergoes fission and produces FP's. The net effect of these compositional changes is to decrease the k_{eff} of the fuel with increased burnup. Accounting for this decrease in k_{eff} (or equivalently, a decrease in reactivity) in subcritical margins is often referred to as

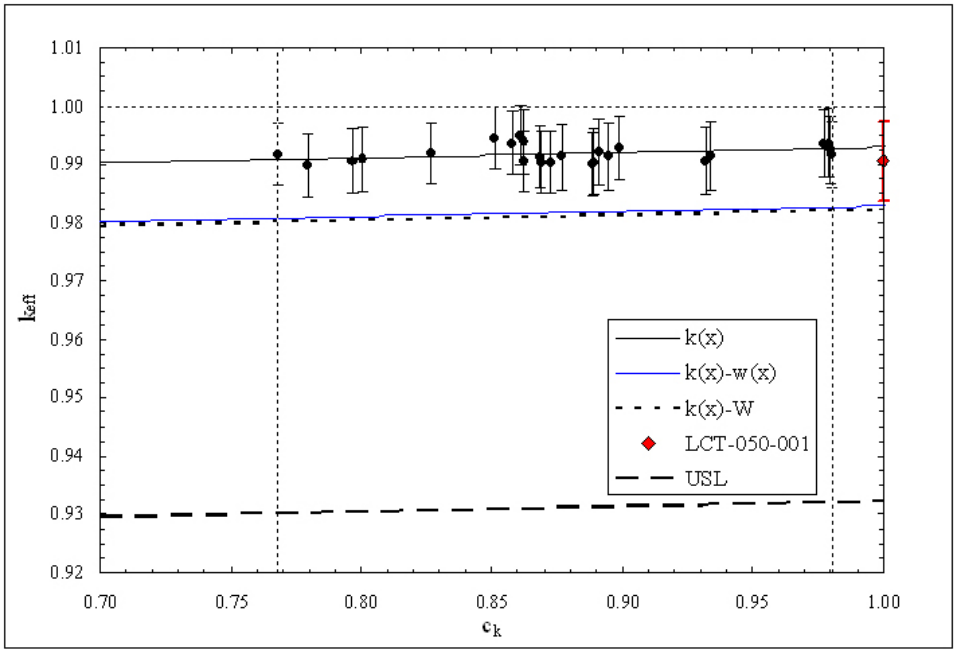

Figure 1.7: Example trending analysis using c_k (for LCT-050-001).

burnup credit.

Historically, the so-called *fresh fuel assumption* was used as a conservative bound in criticality safety analysis of used nuclear fuel [1]. More recently, the Nuclear Regulatory Commission (NRC) offered guidance for crediting the major (fissile) actinides in such analyses in its Interim Staff Guidance - 8, revision 2 (ISG-8R2) report [2]. However, even this results in a very conservative estimate of the subcritical margin of used nuclear fuel.

Changes in the major actinides account for about 66-75% of the net reduction in reactivity; FP's account for the remainder, the percentages depending on burnup. The FP's relevant to burnup credit, roughly in order of importance, include SM.

Why do we care? Naturally, assuming a canister of some number of burned assemblies contains only fresh fuel is quite conservative. Figure 1.8 shows loading curves for a generic used fuel canister with 32 WH 17x17 assemblies of varying initial enrichments and burnups. Configurations below a given line do not meet subcritical limits under the given assumptions. The reference case assumes fresh fuel, and each subsequence curve relaxes the assumptions.

Considering that much of the current U.S. inventory of used fuel lies below the reference curve, a 32-assembly canister could not be widely used (and instead, canis-

ters such as the 21-assembly Transportation, Aging, and Disposal (TAD) canister [3] intended for ultimate disposition in Yucca Mountain would be required). The cost of producing, loading, shipping, and disposing canisters is not negligible, and a study by Parks and Wagner suggest that crediting the FP's in criticality safety analysis, thus allowing the high capacity canisters, would potentially save upwards of $200 million dollars in total disposal costs [9].

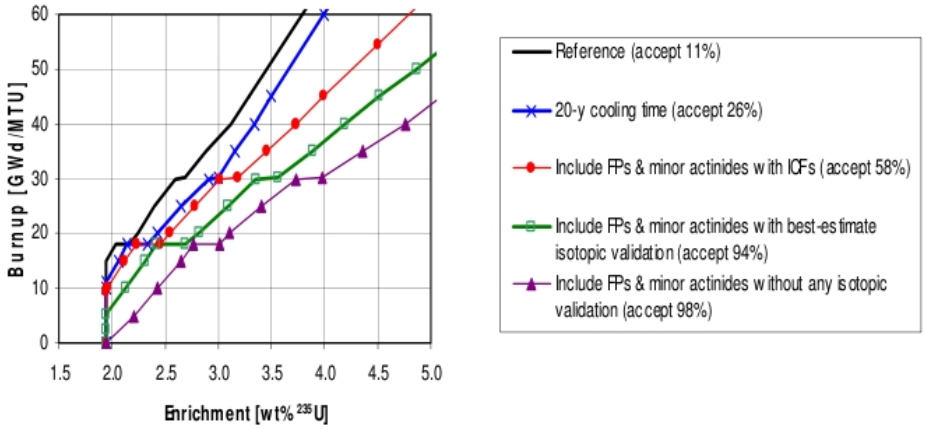

Figure 1.8: Effect of calculational assumptions on loading curves for the GBC-32 and WH 17x17 assemblies.

Current Research

To take into account the reduction in reactivity due to fission products, called *full burnup credit*, requires adequate knowledge of the isotopic content of the spent fuel (via validated depletion methods), and, our focus here, *methods* to determine criticality and *experiments* to validate those methods.

Current work has expanded on the S/U-based techniques outlined above to develop methods for determinining biases of individual nuclides. While the details are beyond our scope, we give a brief description and several references for interested readers.

The basis of the work is the generalized linear least-square method (GLLSM). GLLSM takes as input a set of nuclear data and covariances, the sensitivities of experiment models to nuclear data, the model k_{eff} and uncertainty, the experiment k_{eff} and uncertainty (and any experiment correlation), and *adjusts the nuclear data* so

that the the experiment and model k_{eff} match. It does so in a way that minimizes the variation of all parameters (data and k_{eff}) in terms of the standard deviations of those parameters. In other words, for all parameters p, the method forces the experiment and model k_{eff} to match while minimizing $\sum_i (\Delta p_i / \sigma_{p_i})^2$. The adjustments can be propogated to an application's k_{eff} via its model sensitivities, and the resulting difference is the bias.

To find biases of individual nuclides, the GLLSM method can be applied to so-called "replacement experiments". These experiments consists of a single reference case, and one or more cases that introduce some small perturbation to the system, such as small foils of a fission product between fuel pellets or small concentrations of a fission product in a bulk moderator. GLLSM can then be applied to the reactivity difference between a reference-perturbation pair of models and experiements (rather than on the eigenvalue), since the sensitivities of the reactivity difference should be small for all but the perturbation material. Consequently, the corresponding adjustment to the data should primarily be due to the test material, and as above, the adjustment can be propagated to the application to define a nuclide-specific bias.

Limitations

Two significant limitations apply to the methods under development. First, the methods require that relevant experiments are available. However, only a handful of experiments have been performed relevant to burnup credit, and of those, the difference between reference and perturbation cases may be too high to extract partial biases. Moreover, little data is available for the correlation between experiments. For the the reactivity difference method described above, the resulting biases are extremely sensitive to the correlation between experiments. This suggests that future experiments should be designed with the various S/U methods as guidance.

A second, perhaps more significant issue is the general lack of reliable nuclear covariance data. Only for the most important isotopes does credible data exist (such as uranium isotopes). For isotopes of generally less importance (like many fission products), little if any covariance data exists. Until reliable covariance data exists, the methods described above are of limited value.

1.1 Further Reading

Much of the content in this lecture was inspired by Knief's *Nuclear Criticality Safety: Theory and Practice* [6]. Of course, in one lecture, all that material cannot be covered, and the student is directed to that book for more info most of the topics addressed here. A more succinct overview of some of the topics is given by Pevey in a chapter of the *Nuclear Engineering Handbook* [7].

For an overview of the application of sensitivity and uncertainty analysis to criticality safety, see Broadhead et. al [5], and for the latest work on adjustment techniques applicable to full burnup credit, see Rearden et al. [10].

Part I

Bibliography and Appendices

Bibliography

[1] "'American National Standard for Criticality Safety Criteria for the Handling, Storage, and Transportation of LWR Fuel Outside Reactors," ANSI/ANS-8.17-2004, American Nuclear Society (2004). (ANSI/ANS-8.17-2004).

[2] "American National Standard for Nuclear Criticality Safety in Operations with Fissionable Material Outside Reactors," ANSI/ANS-8.1-1998, American Nuclear Society (1998). (ANSI/ANS-8.1-1998).

[3] *Transportation, Aging, and Disposal Canister System Performance Specification*. U.S. Dept. of Energy, Office of Civilian Radioactive Waste Management.

[4] *International Handbook of Evaluated Criticality Safety Benchmark Experiments*. NEA/NSC/DOC(95)03/I-VIII, OECD-NEA, September 2007.

[5] B.L. Broadhead, B.T. Rearden, C.M. Hopper, J.J. Wagschal, and C.V. Parks. Sensitivity-and Uncertainty-Based Criticality Safety Validation Techniques. *Nucl. Sci. Eng*, 146(3):340–366, 2004.

[6] R.A. Knief. *Nuclear Criticality Safety*. American Nuclear Society, 1991.

[7] R.E. Knott. Neutronics. In K.D. Kok, editor, *Nuclear Engineering Handbook*, pages 575–608. CRC, 2009.

[8] J.J. Lichtenwalter, S.M. Bowman, M.D. DeHart, and C.M. Hopper. Criticality Benchmark Guide for Light-Water-Reactor Fuel in Transportation and Storage Packages. Technical report, NUREG/CR-6361 (ORNL/TM-13211), US Nuclear Regulatory Commission, Oak Ridge National Laboratory, 1997.

[9] C.V. Parks and J.C. Wagner. Current Status and Potential Benefits of Burnup Credit for Spent Fuel Transportation. In *14th Pacific Basin Nuclear Conference, Honolulu, HI, Mar. 21-25, 2004*, 1997.

[10] B.T. Rearden, M.L. Williams, M.A. Jessee, D.E. Mueller, and D.A. Wiarda. Sensitivity and uncertainty analysis capabilities and data in scale. *Nucl. Tech.*, 174(2):236–288, 2011.

Appendix A

To facilitate understanding of the various terms used throughout the lecture materials, we provide here a list of variables, short definitions, and common units where applicable. In very few cases, symbols are used more than once due to convention (e.g. ϕ for both flux and azimuthal angle). Bold symbols indicate a vector quantity.

Table A.1: Fundamental quantities

Symbol	Description	Units
$\psi(\vec{r}, \hat{\Omega}, E, t)$	angular flux	$\frac{\mathrm{n}}{\mathrm{cm}^2-\mathrm{s}-\mathrm{eV}-\mathrm{ster}}$
$\psi^+(\vec{r}, \hat{\Omega}, E, t)$	adjoint angular flux	$\frac{\mathrm{n}}{\mathrm{cm}^2-\mathrm{s}-\mathrm{eV}-\mathrm{ster}}$
$\phi(\vec{r}, E, t)$	scalar flux	$\frac{\mathrm{n}}{\mathrm{cm}^2-\mathrm{s}-\mathrm{eV}}$
$\phi^+(\vec{r}, E, t)$	adjoint scalar flux	$\frac{\mathrm{n}}{\mathrm{cm}^2-\mathrm{s}-\mathrm{eV}}$
$\mathbf{j}(\vec{r}, \hat{\Omega}, E, t)$	angular current density	$\frac{\mathrm{n}}{\mathrm{cm}^2-\mathrm{s}-\mathrm{eV}-\mathrm{ster}}$
$\mathbf{J}(\vec{r}, E, t)$	current density	$\frac{\mathrm{n}}{\mathrm{cm}^2-\mathrm{s}-\mathrm{eV}}$
$J_{\pm}(\vec{r}, E, t)$	partial current density	$\frac{\mathrm{n}}{\mathrm{cm}^2-\mathrm{s}-\mathrm{eV}}$