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# ACM TEMPLATE

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# 1 Datastructure

## 1.1 KD tree

```

1 bool Div[MaxN];
2 void BuildKD(int deep,int l, int r, Point p[]) {
3     if (l > r) return;
4     int mid = l + r >> 1;
5     int minX, minY, maxX, maxY;
6     minX = min_element(p + l, p + r + 1, cmpX)->x;
7     minY = min_element(p + l, p + r + 1, cmpY)->y;
8     maxX = max_element(p + l, p + r + 1, cmpX)->x;
9     maxY = max_element(p + l, p + r + 1, cmpY)->y;
10    Div[mid] = (maxX - minX >= maxY - minY);
11    nth_element(p + l, p + mid, p + r + 1, Div[mid] ? cmpX : cmpY);
12    BuildKD(l, mid - 1, p);
13    BuildKD(mid + 1, r, p);
14 }
15 long long res;
16 void Find(int l, int r, Point a, Point p[]) {
17     if (l > r) return;
18     int mid = l + r >> 1;
19     long long dist = dist2(a, p[mid]);
20     if (dist > 0)//NOTICE
21         res = min(res, dist);
22     long long d = Div[mid] ? (a.x - p[mid].x) : (a.y - p[mid].y);
23     int l1, l2, r1, r2;
24     l1 = l, l2 = mid + 1;
25     r1 = mid - 1, r2 = r;
26     if (d > 0)
27         swap(l1, l2), swap(r1, r2);
28     Find(l1, r1, a, p);
29     if (d * d < res)
30         Find(l2, r2, a, p);
31 }

```

## 1.2 Binary indexed tree

```

1 int read(int k) {
2     int sum = 0;
3     for (; k; k^=k&-k) sum+=tree[k];
4     return sum;
5 }
6 void update(int k, int v) {
7     for (; k<=MaxN; k+=k&-k) tree[k]+=v;
8 }
9 int find_Kth(int k) {
10    int idx = 0;
11    for(int i=20; i>=0; i--) {
12        idx |= 1 << i;
13        if(idx <= MaxN && tree[idx] < k)
14            k -= tree[idx];
15        else idx ^= 1 << i;
16    }
17    return idx + 1;
18 }

```

## 1.3 Splay

```

1 //Node
2 struct Node {

```

```

3   int size, key;
4   Node *c[2], *p;
5 } mem[MaxN], *cur, *nil;
6 //Initialize functions without memory pool
7 Node *newNode(int v, Node *p) {
8     cur->c[0] = cur->c[1] = nil, cur->p = p;
9     cur->size = 1;
10    cur->key = v;
11    return cur++;
12 }
13 void Init() {
14     cur = mem;
15     nil = newNode(0, cur);
16     nil->size = 0;
17 }
18 //Splay tree
19 struct SplayTree {
20     Node *root;
21     void Init() {
22         root = nil;
23     }
24     void Pushup(Node *x) {
25         if (x == nil) return;
26         Pushdown(x);
27         Pushdown(x->c[0]);
28         Pushdown(x->c[1]);
29         x->size = x->c[0]->size + x->c[1]->size + 1;
30     }
31     void Pushdown(Node *x) {
32         if (x == nil) return;
33         //do something
34     }
35     void Rotate(Node *x, int f) {
36         if (x == nil) return;
37         Node *y = x->p;
38         y->c[f ^ 1] = x->c[f], x->p = y->p;
39         if (x->c[f] != nil)
40             x->c[f]->p = y;
41         if (y->p != nil)
42             y->p->c[y->p->c[1] == y] = x;
43         x->c[f] = y, y->p = x;
44         Pushup(y);
45     }
46     void Splay(Node *x, Node *f) {
47         static Node *stack[maxn];
48         int top = 0;
49         stack[top++] = x;
50         for (Node *y = x; y != f; y = y->p)
51             stack[top++] = y->p;
52         while (top)
53             Pushdown(stack[--top]);
54         while (x->p != f) {
55             Node *y = x->p;
56             if (y->p == f)
57                 Rotate(x, x == y->c[0]);
58             else {
59                 int fd = y->p->c[0] == y;
60                 if (y->c[fd] == x)
61                     Rotate(x, fd ^ 1), Rotate(x, fd);
62                 else
63                     Rotate(y, fd), Rotate(x, fd);
64             }
65         }
66         Pushup(x);

```

```

67     if (f == nil)
68         root = x;
69 }
70 void Select(int k, Node *f) {
71     Node *x = root;
72     Pushdown(x);
73     int tmp;
74     while ((tmp = x->c[0]->size) != k) {
75         if (k < tmp) x = x->c[0];
76         else
77             x = x->c[1], k -= tmp + 1;
78         Pushdown(x);
79     }
80     Splay(x, f);
81 }
82 void Select(int l, int r) {
83     Select(l, nil), Select(r + 2, root);
84 }
85 Node *Make_tree(int a[], int l, int r, Node *p) {
86     if (l > r) return nil;
87     int mid = l + r >> 1;
88     Node *x = newNode(a[mid], p);
89     x->c[0] = Make_tree(a, l, mid - 1, x);
90     x->c[1] = Make_tree(a, mid + 1, r, x);
91     Pushup(x);
92     return x;
93 }
94 void Insert(int pos, int a[], int n) {
95     Select(pos, nil), Select(pos + 1, root);
96     root->c[1]->c[0] = Make_tree(a, 0, n - 1, root->c[1]);
97     Splay(root->c[1]->c[0], nil);
98 }
99 void Insert(int v) {
100     Node *x = root, *y = nil;
101     //Need pushdown
102     while (x != nil) {
103         y = x;
104         y->size++;
105         x = x->c[v >= x->key];
106     }
107     y->c[v >= y->key] = x = newNode(v, y);
108     Splay(x, nil);
109 }
110 void Remove(int l, int r) {
111     Select(l, r);
112     //Recycle(root->c[1]->c[0]);
113     root->c[1]->c[0] = nil;
114     Splay(root->c[1], nil);
115 }
116 };

```

#### 1.4 Dynamic tree

```

1 struct SplayTree {
2     void Pushup(Node *x) {
3         if (x == nil) return;
4         Pushdown(x);
5         Pushdown(x->c[0]);
6         Pushdown(x->c[1]);
7         x->size = x->c[0]->size + x->c[1]->size + 1;
8     }
9     void Pushdown(Node *x) {
10        if (x == nil) return;

```

```

11     if (x->rev) {
12         x->rev = 0;
13         x->c[0]->rev ^= 1;
14         x->c[1]->rev ^= 1;
15         swap(x->c[0], x->c[1]);
16     }
17 }
18 bool isRoot(Node *x) {
19     return (x == nil) || (x->p->c[0] != x && x->p->c[1] != x);
20 }
21 void Rotate(Node *x, int f) {
22     if (isRoot(x)) return;
23     Node *y = x->p;
24     y->c[f ^ 1] = x->c[f], x->p = y->p;
25     if (x->c[f] != nil)
26         x->c[f]->p = y;
27     if (y != nil) {
28         if (y == y->p->c[1])
29             y->p->c[1] = x;
30         else if (y == y->p->c[0])
31             y->p->c[0] = x;
32     }
33     x->c[f] = y, y->p = x;
34     Pushup(y);
35 }
36 void Splay(Node *x) {
37     static Node *stack[MaxN];
38     int top = 0;
39     stack[top++] = x;
40     for (Node *y = x; !isRoot(y); y = y->p)
41         stack[top++] = y->p;
42     while (top)
43         Pushdown(stack[--top]);
44     while (!isRoot(x)) {
45         Node *y = x->p;
46         if (isRoot(y))
47             Rotate(x, x == y->c[0]);
48         else {
49             int fd = y->p->c[0] == y;
50             if (y->c[fd] == x)
51                 Rotate(x, fd ^ 1), Rotate(x, fd);
52             else
53                 Rotate(y, fd), Rotate(x, fd);
54         }
55     }
56     Pushup(x);
57 }
58 Node *Access(Node *u) {
59     Node *v = nil;
60     while (u != nil) {
61         Splay(u);
62         v->p = u;
63         u->c[1] = v;
64         Pushup(u);
65         u = (v = u)->p;
66         if (u == nil)
67             return v;
68     }
69 }
70 Node *LCA(Node *u, Node *v) {
71     Access(u);
72     return Access(v);
73 }
74 Node *Link(Node *u, Node *v) {

```



```

75     Access(u);
76     Splay(u);
77     u->rev = true;
78     u->p = v;
79 }
80 void ChangeRoot(Node *u) {
81     Access(u)->rev ^= 1;
82 }
83 Node *GetRoute(Node *u, Node *v) {
84     ChangeRoot(u);
85     return Access(v);
86 }
87 };

```

## 1.5 Partition tree

```

1  int n,m;
2  struct elem {
3      int v,index;
4  } a[120000];
5  int d[30][120000];
6  int s[30][120000];
7  bool cmp(elem a,elem b) {
8      if (a.v == b.v)
9          return a.index <= b.index;
10     return a.v < b.v;
11 }
12 void build(int depth,int l,int r) {
13     if (l == r)
14         return;
15     int mid = (l+r)/2;
16     int tl,tr;
17     tl = tr = 0;
18     for (int i = l; i <= r; i++) {
19         if (cmp(a[d[depth][i]],a[mid])) {
20             d[depth+1][l+tl] = d[depth][i];
21             tl++;
22         } else {
23             d[depth+1][mid+1+tr] = d[depth][i];
24             tr++;
25         }
26     }
27     s[depth][i] = tl;
28     build(depth+1,l,mid);
29     build(depth+1,mid+1,r);
30 }
31 int find(int depth,int dl,int dr,int fl,int fr,int k) {
32     if (fl == fr)
33         return a[d[depth][fl]].v;
34     int ls,rs;
35     int mid = (dl+dr)/2;
36     ls = (fl == dl)? 0 : s[depth][fl-1];
37     rs = s[depth][fr];
38     return (rs-ls < k)?
39         find(depth+1,mid+1,dr,mid+fl-dl-ls+1,mid+fr-dl-rs+1,k-(rs-ls))
40         : find(depth+1,dl,mid,dl+ls,dl+rs-1,k);
41 }
42 int main() {
43     while (scanf("%d%d",&n,&m) != EOF) {
44         for (int i = 1; i <= n; i++) {
45             scanf("%d",&a[i].v);
46             a[i].index = i;

```

```

47     }
48     sort(a+1,a+n+1,cmp);
49     for (int i = 1; i <= n; i++)
50         d[0][a[i].index] = i;
51     build(0,1,n);
52     int l,r,k;
53     for (int i = 1; i <= m; i++) {
54         scanf("%d%d%d",&l,&r,&k);
55         printf("%d\n",find(0,1,n,l,r,k));
56     }
57 }
58 return 0;
59 }

```

## 1.6 Treap

```

1 struct Treap {
2     Treap* ch[2];
3     int key, fix, size;
4
5     Treap(int x) : key(x) {
6         size = 1;
7         fix = rand();
8         ch[0] = ch[1] = nullptr;
9     }
10    int comp(int x) const {
11        if (x == key) return -1;
12        return x < key ? 0 : 1;
13    }
14    void maintain() {
15        size = 1;
16        if (ch[0] != nullptr) size += ch[0]->size;
17        if (ch[1] != nullptr) size += ch[1]->size;
18    }
19 };
20 bool fnd(Treap* o, int x) {
21     while (o != nullptr) {
22         int d = o->comp(x);
23         if (d == -1) return true;
24         o = o->ch[d];
25     }
26     return false;
27 }
28 void rotate(Treap* &o, int d) {
29     Treap* k = o->ch[d ^ 1];
30     o->ch[d ^ 1] = k->ch[d];
31     k->ch[d] = o;
32     o->maintain();
33     k->maintain();
34     o = k;
35 }
36 void insert(Treap* &o, int x) {;
37     if (o == nullptr) {
38         o = new Treap(x);
39     } else {
40         int d = o->comp(x);
41         // int d = (x < o->key ? 0 : 1);
42         insert(o->ch[d], x);
43         if (o->ch[d]->fix > o->fix) rotate(o, d ^ 1);
44     }
45     o->maintain();
46 }

```

```

47 void remove(Treap* &o, int x) {
48     int d = o->comp(x);
49     if (d == -1) {
50         Treap* u = o;
51         if (o->ch[0] != nullptr && o->ch[1] != nullptr) {
52             int dd = (o->ch[0]->fix > o->ch[1]->fix ? 1 : 0);
53             rotate(o, dd);
54             remove(o->ch[dd], x);
55         } else {
56             if (o->ch[0] == nullptr) o = o->ch[1];
57             else o = o->ch[0];
58             delete u;
59             u = nullptr;
60         }
61     } else {
62         remove(o->ch[d], x);
63     }
64     if (o != nullptr) o->maintain();
65 }
66 void clear(Treap* &o) {
67     if (o->ch[0] != nullptr) clear(o->ch[0]);
68     if (o->ch[1] != nullptr) clear(o->ch[1]);
69     delete o;
70     o = nullptr;
71 }
72 int Kth(Treap* o, int k) {
73     if (o == nullptr || k <= 0 || k > o->size) return -1;
74     int s = o->ch[0] == nullptr ? 0 : o->ch[0]->size;
75     if (s + 1 == k) return o->key;
76     else if (s >= k) return Kth(o->ch[0], k);
77     else return Kth(o->ch[1], k - s - 1);
78 }
79 int Rnk(Treap* o, int x) {
80     int r;
81     if (o == nullptr) return 0;
82     if (o->ch[0] == nullptr) r = 0;
83     else r = o->ch[0]->size;
84     if (x == o->key) return r + 1;
85     if (x < o->key) return Rnk(o->ch[0], x);
86     else return r + 1 + Rnk(o->ch[1], x);
87 }

```

## 2 Dynamic programming

### 2.1 RMQ

```

1 struct RMQ {
2     void init(const vector<int> &a) {
3         int n = a.size();
4         for (int i = 0; i < n; ++i) dp[i][0] = a[i];
5         for (int j = 1; (1 << j) <= n; ++j) {
6             for (int i = 0; i + (1 << j) - 1 < n; ++i) {
7                 dp[i][j] = min(dp[i][j - 1], dp[i + (1 << (j - 1))][j - 1]);
8             }
9         }
10    }
11
12    // [l, r]
13    int query(int l, int r) {
14        // // int k = 0;
15        // while ((1 << k + 1) <= r - l + 1) ++k;
16        int k = log(r - l + 1) / log(2.0);

```

```

17     return min(dp[l][k], dp[r - (1 << k) + 1][k]);
18 }
19 } rmq;

```

## 2.2 2D-LIS

```

1 #include<cstdio>
2 #include<map>
3 using namespace std;
4 map<int,int> mp[100001];
5 bool check(int idx,int x,int y) {
6     if (!idx) return 1;
7     if (mp[idx].begin()->first>=x) return 0;
8     map<int,int> ::iterator it=mp[idx].lower_bound(x);
9     it--;
10    if (it->second<y) return 1;
11    else return 0;
12 }
13 int main() {
14     int n;
15     scanf("%d",&n);
16     int l=0,r=0;
17     for (int i=0; i<n; i++) {
18         int x,y;
19         scanf("%d%d",&x,&y);
20         int tl=l,tr=r;
21         while (tl<tr) {
22             int mid=(tl+tr+1)/2;
23             if (check(mid,x,y))
24                 tl=mid;
25             else
26                 tr=mid-1;
27         }
28         if (tl==r) r++;
29         int idx=tl+1;
30         map<int,int> ::iterator itl=mp[idx].lower_bound(x),itr=itl;
31         while (itr!=mp[idx].end() && itr->second>y) itr++;
32         if (mp[idx].find(x)!=mp[idx].end())
33             y=min(y,mp[idx][x]);
34         if (itl!=itr) mp[idx].erase(itl,itr);
35         if (mp[idx].find(x)==mp[idx].end() || mp[idx][x]>y)
36             mp[idx][x]=y;
37     }
38     printf("%d\n",r);
39     return 0;
40 }

```

## 3 Geometry

### 3.1 2D

#### 3.1.1 Point

```

1 //Use cross product instead of atan2
2 bool cmp(const Point& a,const Point& b) {
3     if (a.y*b.y <= 0) {
4         if (a.y > 0 || b.y > 0) return a.y < b.y;
5         if (a.y == 0 && b.y == 0) return a.x < b.x;
6     }
7     return a*b > 0;
8 }

```

## 3.1.2 Line

```

1 Point operator &(const Line& b) const {
2     Point res = s;
3     double t = ((s - b.s) * (b.s - b.e)) / ((s - e) * (b.s - b.e));
4     res.x += (e.x - s.x) * t;
5     res.y += (e.y - s.y) * t;
6     return res;
7 }

```

## 3.1.3 Functions

```

1 Point nearestPointToLine(Point P, Line L) {
2     Point result;
3     double a, b, t;
4     a = L.e.x - L.s.x;
5     b = L.e.y - L.s.y;
6     t = ( (P.x - L.s.x) * a + (P.y - L.s.y) * b ) / (a * a + b * b);
7     if (t >= 0 && t <= 1) {
8         result.x = L.s.x + a * t;
9         result.y = L.s.y + b * t;
10    }
11    return result;
12 }
13 //Segment
14 bool inter(Line l1, Line l2) {
15     return
16         max(l1.s.x, l1.e.x) >= min(l2.s.x, l2.e.x) &&
17         max(l2.s.x, l2.e.x) >= min(l1.s.x, l1.e.x) &&
18         max(l1.s.y, l1.e.y) >= min(l2.s.y, l2.e.y) &&
19         max(l2.s.y, l2.e.y) >= min(l1.s.y, l1.e.y) &&
20         sgn((l2.s - l1.s) * (l1.e - l1.s)) * sgn((l2.e - l1.s) * (l1.e - l1.s)) <= 0 &&
21         sgn((l1.s - l2.s) * (l2.e - l2.s)) * sgn((l1.e - l2.s) * (l2.e - l2.s)) <= 0;
22 }
23 bool onSeg(Line a, Point b) {
24     return ((a.s - b) * (a.e - b) == 0 &&
25         (b.x - a.s.x) * (b.x - a.e.x) <= 0 &&
26         (b.y - a.s.y) * (b.y - a.e.y) <= 0);
27 }
28 int inPoly(Point p, Point poly[], int n) {
29     int i, count;
30     Line ray, side;
31     count = 0;
32     ray.s = p;
33     ray.e.y = p.y;
34     ray.e.x = -1; // -∞
35     for (i = 0; i < n; i++) {
36         side.s = poly[i];
37         side.e = poly[(i + 1) % n];
38         if (OnSeg(p, side))
39             return 1;
40         if (side.s.y == side.e.y)
41             continue;
42         if (OnSeg(side.s, ray)) {
43             if (side.s.y > side.e.y) count++;
44         } else if (OnSeg(side.e, ray)) {
45             if (side.e.y > side.s.y) count++;
46         } else if (inter(ray, side)) {
47             count++;
48         }
49     }
50     return ((count % 2 == 1) ? 0 : 2);
51 }

```

```

52 Point centerOfPolygon(Point poly[],int n) {
53     Point p, p0, p1, p2, p3;
54     double m, m0;
55     p1 = poly[0];
56     p2 = poly[1];
57     p.x = p.y = m = 0;
58     for (int i = 2; i < n; i++) {
59         p3 = poly[i];
60         p0.x = (p1.x + p2.x + p3.x) / 3.0;
61         p0.y = (p1.y + p2.y + p3.y) / 3.0;
62         m0 = p1.x*p2.y+p2.x*p3.y+p3.x*p1.y-p1.y*p2.x-p2.y*p3.x-p3.y*p1.x;
63         if (cmp(m + m0,0.0) == 0)
64             m0 += eps;
65         p.x = (m * p.x + m0 * p0.x) / (m + m0);
66         p.y = (m * p.y + m0 * p0.y) / (m + m0);
67         m = m + m0;
68         p2 = p3;
69     }
70     return p;
71 }

```

### 3.1.4 Half plane intersection

```

1  bool HPlcmp(Line a, Line b) {
2      if (fabs(a.k - b.k) > EPS) return a.k < b.k;
3      return ((a.s - b.s) * (b.e - b.s)) < 0;
4  }
5  Line Q[MAXN];
6  void HPI(Line line[], int n, Point res[], int &resn) {
7      int tot = n;
8      sort(line, line + n, HPlcmp);
9      tot = 1;
10     for (int i = 1; i < n; i++)
11         if (fabs(line[i].k - line[i - 1].k) > EPS)
12             line[tot++] = line[i];
13     int head = 0, tail = 1;
14     Q[0] = line[0];
15     Q[1] = line[1];
16     resn = 0;
17     for (int i = 2; i < tot; i++) {
18         if (fabs((Q[tail].e - Q[tail].s) * (Q[tail - 1].e - Q[tail - 1].s)) < EPS ||
19             fabs((Q[head].e - Q[head].s) * (Q[head + 1].e - Q[head + 1].s)) < EPS)
20             return;
21         while (head < tail && (((Q[tail] & Q[tail - 1]) - line[i].s) * (line[i].e - line[
22             i].s)) > EPS)
23             tail--;
24         while (head < tail && (((Q[head] & Q[head + 1]) - line[i].s) * (line[i].e - line[
25             i].s)) > EPS)
26             head++;
27         Q[++tail] = line[i];
28     }
29     while (head < tail && (((Q[tail] & Q[tail - 1]) - Q[head].s) * (Q[head].e - Q[head
30         ].s)) > EPS)
31         tail--;
32     while (head < tail && (((Q[head] & Q[head + 1]) - Q[tail].s) * (Q[tail].e - Q[tail
33         ].s)) > EPS)
34         head++;
35     if (tail <= head + 1) return;
36     for (int i = head; i < tail; i++)
37         res[resn++] = Q[i] & Q[i + 1];
38     if (head < tail + 1)
39         res[resn++] = Q[head] & Q[tail];
40 }

```

### 3.1.5 Convex hull

```

1 bool GScmp(Point a, Point b) {
2     if (fabs(a.x - b.x) < eps)
3         return a.y < b.y - eps;
4     return a.x < b.x - eps;
5 }
6 void GS(Point p[],int n,Point res[],int &resn) {
7     resn = 0;
8     int top = 0;
9     sort(p,p+n,GScmp);
10    if (conPoint(p,n)) {
11        res[resn++] = p[0];
12        return;
13    }
14    if (conLine(p,n)) {
15        res[resn++] = p[0];
16        res[resn++] = p[n-1];
17        return;
18    }
19    for (int i = 0; i < n; i++)
20        if (resn < 2 ||
21            (res[resn-1]-res[resn-2])*(p[i]-res[resn-1]) > 0)
22            res[resn++] = p[i++];
23        else
24            --resn;
25    top = resn-1;
26    for (int i = n-2; i >= 0; i--)
27        if (resn < top+2 ||
28            (res[resn-1]-res[resn-2])*(p[i]-res[resn-1]) > 0)
29            res[resn++] = p[i--];
30        else
31            --resn;
32    resn--;
33 }

```

### 3.1.6 Intersections of line and polygon

```

1 //Intersecting segment between [la,lb]
2 int Gao(int la,int lb,Line line) {
3     if (la > lb)
4         lb += n;
5     int l = la,r = lb,mid;
6     while (l < r) {
7         mid = l+r+1>>1;
8         if (cmp((line.e-line.s)*(p[la]-line.s),0)*cmp((line.e-line.s)*(p[mid]-line.s),0)
9             >= 0)
10             l = mid;
11         else
12             r = mid-1;
13     }
14     return l%n;
15 }
16 double theta[maxn];
17 void Gettheta() {
18     for (int i = 0; i < n; i++) {
19         Point v = p[(i+1)%n]-p[i];
20         theta[i] = atan2(v.y,v.x);
21     }
22     for (int i = 1; i < n; i++)
23         if (theta[i-1] > theta[i]+eps)
24             theta[i] += 2*pi;
25 }

```

```

25 void Calc(Line l) {
26     double tnow;
27     Point v = l.e-l.s;
28     tnow = atan2(v.y,v.x);
29     if (cmp(tnow,theta[0]) < 0) tnow += 2*pi;
30     int pl = lower_bound(theta,theta+n,tnow)-theta;
31     tnow = atan2(-v.y,-v.x);
32     if (cmp(tnow,theta[0]) < 0) tnow += 2*pi;
33     int pr = lower_bound(theta,theta+n,tnow)-theta;
34     //Farest points with l on polygon
35     pl = pl%n;
36     pr = pr%n;
37     if (cmp(v*(p[pl]-l.s),0)*cmp(v*(p[pr]-l.s),0) >= 0)
38         return 0.0;
39     int xa = Gao(pl,pr,l);
40     int xb = Gao(pr,pl,l);
41     if (xa > xb) swap(xa,xb);
42     //Intersecting with line  $P_{xa} \rightarrow P_{xa+1}$  and  $P_{xb} \rightarrow P_{xb+1}$ 
43     if (cmp(v*(p[xa+1]-p[xa]),0) == 0) return 0.0;
44     if (cmp(v*(p[xb+1]-p[xb]),0) == 0) return 0.0;
45     Point pa,pb;
46     //Intersections
47     pa = Line(p[xa],p[xa+1])&l;
48     pb = Line(p[xb],p[xb+1])&l;
49 }

```

## 3.2 3D

### 3.2.1 Point

```

1 Point3D operator *(const Point3D& b) const {
2     return Point3D(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);
3 }
4 //Rotate around V, notice that |V|=1
5 Point3D Trans(Point3D pa,Point3D V,double theta) {
6     double s = sin(theta);
7     double c = cos(theta);
8     double x,y,z;
9     x = V.x;
10    y = V.y;
11    z = V.z;
12    Point3D pp =
13        Point3D(
14            (x*x*(1-c)+c)*pa.x+(x*y*(1-c)-z*s)*pa.y+(x*z*(1-c)+y*s)*pa.z,
15            (y*x*(1-c)+z*s)*pa.x+(y*y*(1-c)+c)*pa.y+(y*z*(1-c)-x*s)*pa.z,
16            (x*z*(1-c)-y*s)*pa.x+(y*z*(1-c)+x*s)*pa.y+(z*z*(1-c)+c)*pa.z);
17    return pp;
18 }

```

### 3.2.2 Functions

```

1 bool lineIntersect(Line3D L1, Line3D L2) {
2     Point3D s = L1.s-L1.e;
3     Point3D e = L2.s-L2.e;
4     Point3D p = s*e;
5     if (ZERO(p)) return false; //Parallel
6     p = (L2.s-L1.e)*(L1.s-L1.e);
7     return ZERO(p&L2.e); //Common face
8 }
9 //Please check whether a,b,c,d on a plane first
10 bool segmentIntersect(Point a,Point b,Point c,Point d) {
11     Point ret = (a-b)*(c-d);

```



```

12 Point t1 = (b-a)*(c-a);
13 Point t2 = (b-a)*(d-a);
14 Point t3 = (d-c)*(a-c);
15 Point t4 = (d-c)*(b-c);
16 return sgn(t1&ret)*sgn(t2&ret) < 0 &&
17        sgn(t3&ret)*sgn(t4&ret) < 0;
18 }
19 //Distance from point  $p$  to line  $L$ 
20 double distance(Point3D p, Line3D L) {
21     return (Norm((p-L.s)*(L.e-L.s))/Norm(L.e-L.s));
22 }
23 //Angle between line  $L_1$  and  $L_2$ ,  $\theta \in [0, \pi]$ 
24 double calcTheta(Line3D L1, Line3D L2) {
25     Point3D u = L1.e - L1.s;
26     Point3D v = L2.e - L2.s;
27     return acos( (u & v) / (Norm(u)*Norm(v)) );
28 }

```

### 3.2.3 Convex hull

Don't forget **Randomshuffle**!

```

1 struct pt {
2     double x, y, z;
3     pt() {}
4     pt(double _x, double _y, double _z): x(_x), y(_y), z(_z) {}
5     pt operator - (const pt p1) {}
6     pt operator * (pt p) {}
7     double operator ^ (pt p) {}
8 };
9 struct _3DCH {
10     struct fac {
11         int a, b, c;
12         bool ok;
13     };
14     int n;
15     pt P[MAXV];
16     int cnt;
17     fac F[MAXV*8];
18     int to[MAXV][MAXV];
19     double vlen(pt a) {
20         return sqrt(a.x*a.x+a.y*a.y+a.z*a.z);
21     }
22     double area(pt a, pt b, pt c) {
23         return vlen((b-a)*(c-a));
24     }
25     double volume(pt a, pt b, pt c, pt d) {
26         return (b-a)*(c-a)^(d-a);
27     }
28     double ptof(pt &p, fac &f) {
29         pt m = P[f.b]-P[f.a], n = P[f.c]-P[f.a], t = p-P[f.a];
30         return (m * n) ^ t;
31     }
32     void deal(int p, int a, int b) {
33         int f = to[a][b];
34         fac add;
35         if (F[f].ok) {
36             if (ptof(P[p], F[f]) > eps)
37                 dfs(p, f);
38             else {
39                 add.a = b, add.b = a, add.c = p, add.ok = 1;
40                 to[p][b] = to[a][p] = to[b][a] = cnt;
41                 F[cnt++] = add;

```

```

42     }
43 }
44 }
45 void dfs(int p, int cur) {
46     F[cur].ok = 0;
47     deal(p, F[cur].b, F[cur].a);
48     deal(p, F[cur].c, F[cur].b);
49     deal(p, F[cur].a, F[cur].c);
50 }
51 bool same(int s, int t) {
52     pt &a = P[F[s].a], &b = P[F[s].b], &c = P[F[s].c];
53     return fabs(volume(a, b, c, P[F[t].a])) < eps && fabs(volume(a, b, c,
54         P[F[t].b])) < eps && fabs(volume(a, b, c, P[F[t].c])) < eps;
55 }
56 void construct() {
57     cnt = 0;
58     if (n < 4)
59         return;
60     bool sb = 1;
61     for (int i = 1; i < n; i++) {
62         if (vlen(P[0] - P[i]) > eps) {
63             swap(P[1], P[i]);
64             sb = 0;
65             break;
66         }
67     }
68     if (sb) return;
69     sb = 1;
70     for (int i = 2; i < n; i++) {
71         if (vlen((P[0] - P[1]) * (P[1] - P[i])) > eps) {
72             swap(P[2], P[i]);
73             sb = 0;
74             break;
75         }
76     }
77     if (sb) return;
78     sb = 1;
79     for (int i = 3; i < n; i++) {
80         if (fabs((P[0] - P[1]) * (P[1] - P[2]) ^ (P[0] - P[i])) > eps) {
81             swap(P[3], P[i]);
82             sb = 0;
83             break;
84         }
85     }
86     if (sb) return;
87     fac add;
88     for (int i = 0; i < 4; i++) {
89         add.a = (i+1)%4, add.b = (i+2)%4, add.c = (i+3)%4, add.ok = 1;
90         if (ptof(P[i], add) > 0)
91             swap(add.b, add.c);
92         to[add.a][add.b] = to[add.b][add.c] = to[add.c][add.a] = cnt;
93         F[cnt++] = add;
94     }
95     for (int i = 4; i < n; i++) {
96         for (int j = 0; j < cnt; j++) {
97             if (F[j].ok && ptof(P[i], F[j]) > eps) {
98                 dfs(i, j);
99                 break;
100             }
101         }
102     }
103     int tmp = cnt;
104     cnt = 0;
105     for (int i = 0; i < tmp; i++) {

```

```

106     if (F[i].ok) {
107         F[cnt++] = F[i];
108     }
109 }
110 }
111 double area() {
112     double ret = 0.0;
113     for (int i = 0; i < cnt; i++) {
114         ret += area(P[F[i].a], P[F[i].b], P[F[i].c]);
115     }
116     return ret / 2.0;
117 }
118 double volume() {
119     pt O(0, 0, 0);
120     double ret = 0.0;
121     for (int i = 0; i < cnt; i++) {
122         ret += volume(O, P[F[i].a], P[F[i].b], P[F[i].c]);
123     }
124     return fabs(ret / 6.0);
125 }
126 int facetCnt_tri() {
127     return cnt;
128 }
129 int facetCnt() {
130     int ans = 0;
131     for (int i = 0; i < cnt; i++) {
132         bool nb = 1;
133         for (int j = 0; j < i; j++) {
134             if (same(i, j)) {
135                 nb = 0;
136                 break;
137             }
138         }
139         ans += nb;
140     }
141     return ans;
142 }
143 pt Fc[MAXV*8];
144 double V[MAXV*8];
145 pt Center() {
146     pt O(0,0,0);
147     for (int i = 0; i < cnt; i++) {
148         Fc[i].x = (O.x+P[F[i].a].x+P[F[i].b].x+P[F[i].c].x)/4.0;
149         Fc[i].y = (O.y+P[F[i].a].y+P[F[i].b].y+P[F[i].c].y)/4.0;
150         Fc[i].z = (O.z+P[F[i].a].z+P[F[i].b].z+P[F[i].c].z)/4.0;
151         V[i] = volume(O,P[F[i].a],P[F[i].b],P[F[i].c]);
152     }
153     pt res = Fc[0],tmp;
154     double m = V[0];
155     for (int i = 1; i < cnt; i++) {
156         if (fabs(m+V[i]) < eps)
157             V[i] += eps;
158         tmp.x = (m*res.x+V[i]*Fc[i].x)/(m+V[i]);
159         tmp.y = (m*res.y+V[i]*Fc[i].y)/(m+V[i]);
160         tmp.z = (m*res.z+V[i]*Fc[i].z)/(m+V[i]);
161         m += V[i];
162         res = tmp;
163     }
164     return res;
165 }
166 };

```

### 3.3 Circle

#### 3.3.1 Functions

```

1 //Common area of two circle
2 double area(int x1,int y1,int x2,int y2,double r1,double r2) {
3     double s=dis(x2-x1,y2-y1);
4     if(r1+r2<s) return 0;
5     else if(r2-r1>s) return PI*r1*r1;
6     else if(r1-r2>s) return PI*r2*r2;
7     double q1=acos((r1*r1+s*s-r2*r2)/(2*r1*s));
8     double q2=acos((r2*r2+s*s-r1*r1)/(2*r2*s));
9     return (r1*r1*q1+r2*r2*q2-r1*s*sin(q1));
10 }

```

#### 3.3.2 Union

```

1 for (int i = 1; i <= n; i++)
2     ans[i] = 0.0;
3 for (int i = 0; i < n; i++) {
4     tote = 0;
5     e[tote++] = Event(-pi,1);
6     e[tote++] = Event(pi,-1);
7     for (int j = 0; j < n; j++)
8         if (j != i) {
9             lab = Point(c[j].c.x-c[i].c.x,c[j].c.y-c[i].c.y);
10            AB = lab.Length();
11            AC = c[i].r;
12            BC = c[j].r;
13            if (cmp(AB+AC,BC) <= 0) {
14                e[tote++] = Event(-pi,1);
15                e[tote++] = Event(pi,-1);
16                continue;
17            }
18            if (cmp(AB+BC,AC) <= 0) continue;
19            if (cmp(AB,AC+BC) > 0) continue;
20            theta = atan2(lab.y,lab.x);
21            fai = acos((AC*AC+AB*AB-BC*BC)/(2.0*AC*AB));
22            a0 = theta-fai;
23            if (cmp(a0,-pi) < 0) a0 += 2*pi;
24            a1 = theta+fai;
25            if (cmp(a1,pi) > 0) a1 -= 2*pi;
26            if (cmp(a0,a1) > 0) {
27                e[tote++] = Event(a0,1);
28                e[tote++] = Event(pi,-1);
29                e[tote++] = Event(-pi,1);
30                e[tote++] = Event(a1,-1);
31            } else {
32                e[tote++] = Event(a0,1);
33                e[tote++] = Event(a1,-1);
34            }
35        }
36    sort(e,e+tote,Eventcmp);
37    cur = 0;
38    for (int j = 0; j < tote; j++) {
39        if (cur != 0 && cmp(e[j].tim,pre[cur]) != 0) {
40            ans[cur] += Area(e[j].tim-pre[cur],c[i].r);
41            ans[cur] += xmult(Point(c[i].c.x+c[i].r*cos(pre[cur]),c[i].c.y+c[i].r*sin(pre[
42                cur])),
43                            Point(c[i].c.x+c[i].r*cos(e[j].tim),c[i].c.y+c[i].r*sin(e[j].
44                                tim)))/2.0;
45        }
46        cur += e[j].typ;

```

```

45     pre[cur] = e[j].tim;
46 }
47 }
48 for (int i = 1; i < n; i++)
49     ans[i] -= ans[i+1];

```

### 3.3.3 Area of intersection part with polygon

```

1 bool InCircle(Point a, double r) {
2     return cmp(a.x*a.x+a.y*a.y, r*r) <= 0;
3     //ε should big enough
4 }
5 double CalcArea(Point a, Point b, double r) {
6     Point p[4];
7     int tot = 0;
8     p[tot++] = a;
9     Point tv = Point(a, b);
10    Line tmp = Line(Point(0, 0), Point(tv.y, -tv.x));
11    Point near = LineToLine(Line(a, b), tmp);
12    if (cmp(near.x*near.x+near.y*near.y, r*r) <= 0) {
13        double A, B, C;
14        A = near.x*near.x+near.y*near.y;
15        C = r;
16        B = C*C-A;
17        double tvl = tv.x*tv.x+tv.y*tv.y;
18        double tmp = sqrt(B/tvl);
19        p[tot] = Point(near.x+tmp*tv.x, near.y+tmp*tv.y);
20        if (OnSeg(Line(a, b), p[tot]) == true) tot++;
21        p[tot] = Point(near.x-tmp*tv.x, near.y-tmp*tv.y);
22        if (OnSeg(Line(a, b), p[tot]) == true) tot++;
23    }
24    if (tot == 3) {
25        if (cmp(Point(p[0], p[1]).Length(), Point(p[0], p[2]).Length()) > 0)
26            swap(p[1], p[2]);
27    }
28    p[tot++] = b;
29    double res = 0.0, theta, a0, a1, sgn;
30    for (int i = 0; i < tot-1; i++) {
31        if (InCircle(p[i], r) == true && InCircle(p[i+1], r) == true) {
32            res += 0.5*xmult(p[i], p[i+1]);
33        } else {
34            a0 = atan2(p[i+1].y, p[i+1].x);
35            a1 = atan2(p[i].y, p[i].x);
36            if (a0 < a1) a0 += 2*pi;
37            theta = a0-a1;
38            if (cmp(theta, pi) >= 0) theta = 2*pi-theta;
39            sgn = xmult(p[i], p[i+1])/2.0;
40            if (cmp(sgn, 0) < 0) theta = -theta;
41            res += 0.5*r*r*theta;
42        }
43    }
44    return res;
45 }
46 area2 = 0.0;
47 for (int i = 0; i < resn; i++) //counterclockwise
48     area2 += CalcArea(p[i], p[(i+1)%resn], r);

```

### 3.4 Matrix

#### 3.4.1 基本矩阵

按向量  $\overrightarrow{(x, y, z)}$  平移：

$$\begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

按比例  $(x, y, z)$  缩放：

$$\begin{pmatrix} x & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

绕单位向量  $\overrightarrow{(x, y, z)}$  旋转  $angle$  角度：

$$\begin{pmatrix} x^2 \times (1 - c) + c & x \times y \times (1 - c) - z \times s & x \times z \times (1 - c) + y \times s & 0 \\ y \times x \times (1 - c) + z \times s & y^2 \times (1 - c) + c & y \times z \times (1 - c) - x \times s & 0 \\ x \times z \times (1 - c) - y \times s & y \times z \times (1 - c) + x \times s & z^2 \times (1 - c) + c & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{cases} s = \sin(angle) \\ c = \cos(angle) \end{cases}$$

以上矩阵变换都把点当作列向量，旋转角度的正负由右手定则决定

## 4 Graph

### 4.1 Dinic

```

1 struct Edge {int from, to, cap, flow;} ;
2 struct Dinic {
3     int n, m, s, t;
4     vector<Edge> edges;
5     vector<int> G[maxn];
6     bool vis[maxn];
7     int d[maxn];
8     int cur[maxn];
9
10    void init(int n) {
11        for (int i = 0; i <= n; ++i) G[i].clear();
12        edges.clear();
13    }
14
15    void add_edge(int from, int to, int cap) {
16        edges.push_back((Edge){from, to, cap, 0});
17        edges.push_back((Edge){to, from, 0, 0});
18        m = edges.size();
19        G[from].push_back(m - 2);
20        G[to].push_back(m - 1);
21    }
22
23    bool bfs() {
24        memset(vis, 0, sizeof(vis));
25        queue<int> Q;
26        Q.push(s);
27        d[s] = 0;

```

```

28     vis[s] = 1;
29     while (!Q.empty()) {
30         int x = Q.front(); Q.pop();
31         for (int i = 0; i < (int)G[x].size(); ++i) {
32             Edge& e = edges[G[x][i]];
33             if (!vis[e.to] && e.cap > e.flow) {
34                 vis[e.to] = 1;
35                 d[e.to] = d[x] + 1;
36                 Q.push(e.to);
37             }
38         }
39     }
40     return vis[t];
41 }
42
43 int dfs(int x, int a) {
44     if (x == t || a == 0) return a;
45     int flow = 0, f;
46     for (int& i = cur[x]; i < G[x].size(); ++i) {
47         Edge& e = edges[G[x][i]];
48         if (d[x] + 1 == d[e.to] && (f = dfs(e.to, min(a, e.cap - e.flow))) > 0) {
49             e.flow += f;
50             edges[G[x][i] ^ 1].flow -= f;
51             flow += f;
52             a -= f;
53             if (a == 0) break;
54         }
55     }
56     return flow;
57 }
58
59 int run(int s, int t) {
60     this->s = s; this->t = t;
61     int flow = 0;
62     while (bfs()) {
63         memset(cur, 0, sizeof(cur));
64         flow += dfs(s, inf);
65     }
66     return flow;
67 }
68 } max_flow;

```

## 4.2 ISAP

```

1 struct Edge {
2     int from, to, cap, flow;
3 };
4 struct ISAP {
5     int n, m, s, t; //结点数，边数（包括反向弧），源点编号，汇点编号
6     vector<Edge> edges;
7     vector<int> G[maxn];
8     bool vis[maxn];
9     int d[maxn];
10    int cur[maxn];
11    int p[maxn];
12    int num[maxn];
13    void init(int n) {
14        this->n = n;
15        for (int i = 0; i <= n; ++i) G[i].clear();
16        edges.clear();
17    }
18    void add_edge(int from, int to, int cap) {
19        edges.push_back((Edge){from, to, cap, 0});

```

```

20     edges.push_back((Edge){to, from, 0, 0});
21     m = edges.size();
22     G[from].push_back(m - 2);
23     G[to].push_back(m - 1);
24 }
25 bool RevBFS() {
26     memset(vis, 0, sizeof(vis));
27     queue<int> Q;
28     Q.push(t);
29     d[t] = 0;
30     vis[t] = 1;
31     while (!Q.empty()) {
32         int x = Q.front();
33         Q.pop();
34         for (int i = 0; i < G[x].size(); i++) {
35             Edge &e = edges[G[x][i] ^ 1];
36             if (!vis[e.from] && e.cap > e.flow) {
37                 vis[e.from] = 1;
38                 d[e.from] = d[x] + 1;
39                 Q.push(e.from);
40             }
41         }
42     }
43     return vis[s];
44 }
45 int Augment() {
46     int x = t, a = inf;
47     while (x != s) {
48         Edge &e = edges[p[x]];
49         a = min(a, e.cap - e.flow);
50         x = edges[p[x]].from;
51     }
52     x = t;
53     while (x != s) {
54         edges[p[x]].flow += a;
55         edges[p[x] ^ 1].flow -= a;
56         x = edges[p[x]].from;
57     }
58     return a;
59 }
60 int run(int s, int t) {
61     this->s = s; this->t = t;
62     int flow = 0;
63     RevBFS();
64     memset(num, 0, sizeof(num));
65     for (int i = 0; i < n; i++) {
66         num[d[i]]++;
67     }
68     int x = s;
69     memset(cur, 0, sizeof(cur));
70     while (d[s] < n) {
71         if (x == t) {
72             flow += Augment();
73             x = s;
74         }
75         int ok = 0;
76         for (int i = cur[x]; i < G[x].size(); i++) {
77             Edge &e = edges[G[x][i]];
78             if (e.cap > e.flow && d[x] == d[e.to] + 1) {
79                 ok = 1;
80                 p[e.to] = G[x][i];
81                 cur[x] = i;
82                 x = e.to;
83                 break;

```



```

84     }
85 }
86 if (!ok) {
87     int m = n - 1;
88     for (int i = 0; i < G[x].size(); i++) {
89         Edge &e = edges[G[x][i]];
90         if (e.cap > e.flow) m = min(m, d[e.to]);
91     }
92     if (--num[d[x]] == 0) break;
93     num[d[x] = m + 1]++;
94     cur[x] = 0;
95     if (x != s) x = edges[p[x]].from;
96 }
97 }
98 return flow;
99 }
100 } max_flow;

```

### 4.3 Minimal cost maximal flow

```

1 struct Edge {
2     int from, to, cap, flow, cost;
3 };
4 struct MCMF {
5     int n, m, s, t;
6     vector<Edge> edges;
7     vector<int> G[maxn];
8     int inq[maxn];
9     int d[maxn];
10    int p[maxn];
11    int a[maxn];
12
13    void init(int n) {
14        this->n = n;
15        for (int i = 0; i < n; ++i) G[i].clear();
16        edges.clear();
17    }
18
19    void add_edge(int from, int to, int cap, int cost) {
20        edges.push_back((Edge) {from, to, cap, 0, cost});
21        edges.push_back((Edge) {to, from, 0, 0, -cost});
22        m = edges.size();
23        G[from].push_back(m - 2);
24        G[to].push_back(m - 1);
25    }
26
27    bool SPFA(int s, int t, int& flow, int& cost) {
28        for (int i = 0; i < n; ++i) d[i] = inf;
29        memset(inq, 0, sizeof(inq));
30        d[s] = 0; inq[s] = 1; p[s] = 0; a[s] = inf;
31
32        queue<int> Q;
33        Q.push(s);
34        while (!Q.empty()) {
35            int u = Q.front(); Q.pop();
36            inq[u] = 0;
37            for (int i = 0; i < (int)G[u].size(); ++i) {
38                Edge& e = edges[G[u][i]];
39                if (e.cap > e.flow && d[e.to] > d[u] + e.cost) {
40                    d[e.to] = d[u] + e.cost;
41                    p[e.to] = G[u][i];
42                    a[e.to] = min(a[u], e.cap - e.flow);
43                    if (!inq[e.to]) { Q.push(e.to); inq[e.to] = 1; }

```

```

44     }
45     }
46 }
47
48 if (d[t] == inf) return false;
49 // if (d[t] >= 0) return false;
50
51 // add flow
52 flow += a[t];
53 cost += d[t] * a[t];
54 int u = t;
55 while (u != s) {
56     edges[p[u]].flow += a[t];
57     edges[p[u] ^ 1].flow -= a[t];
58     u = edges[p[u]].from;
59 }
60
61 return true;
62 }
63
64 int run(int s, int t) {
65     int flow = 0, cost = 0;
66     while (SPFA(s, t, flow, cost));
67     return cost;
68 }
69
70 } min_cost;

```

#### 4.4 Johnson Minimal cost flow

```

1 #include <cstdio>
2 #include <cstring>
3 #include <algorithm>
4 #include <queue>
5 #include <stack>
6 using namespace std;
7 const int MAXN = 2003;
8 const int MAXM = 2000 * 1999 / 2 + 2000 * 3;
9 int N, L;
10 int head[MAXN];
11 struct Edge {
12     int to, next, flow, cost;
13 } edge[MAXM * 2];
14 int h[MAXN], dis[MAXN], pre[MAXN];
15 struct Heap {
16     int value[MAXN + 1], id[MAXN + 1];
17     int pos[MAXN];
18     int size;
19     void init() {
20         size = 1;
21     }
22     void swap2(int p, int q) {
23         swap(value[p], value[q]);
24         swap(id[p], id[q]);
25         pos[id[p]] = p;
26         pos[id[q]] = q;
27     }
28     void push_up(int p) {
29         while (p > 1 && value[p / 2] > value[p]) {
30             swap2(p, p / 2);
31             p /= 2;
32         }
33     }

```

```

34 void push_down(int p) {
35     while (p * 2 < size) {
36         int best = p;
37         if (p * 2 < size && value[p] > value[p * 2])
38             best = p * 2;
39         if (p * 2 + 1 < size && value[best] > value[p * 2 + 1])
40             best = p * 2 + 1;
41         if (p == best)
42             break;
43         swap2(p, best);
44         p = best;
45     }
46 }
47 void push(int _value, int _id) {
48     value[size] = _value;
49     id[size] = _id;
50     pos[_id] = size;
51     push_up(size++);
52 }
53 int top() {
54     return id[1];
55 }
56 void pop() {
57     value[1] = value[size - 1];
58     id[1] = id[--size];
59     pos[id[1]] = 1;
60     push_down(1);
61 }
62 void update(int _value, int _id) {
63     int p = pos[_id];
64     value[p] = _value;
65     push_up(p);
66 }
67 } heap;
68 bool inque[MAXN];
69 void init(int n) {
70     N = n;
71     L = 0;
72     memset(head, -1, 4 * n);
73 }
74 void add_edge(int u, int v, int flow, int cost) {
75     edge[L].to = v;
76     edge[L].flow = flow;
77     edge[L].cost = cost;
78     edge[L].next = head[u];
79     head[u] = L++;
80     edge[L].to = u;
81     edge[L].flow = 0;
82     edge[L].cost = -cost;
83     edge[L].next = head[v];
84     head[v] = L++;
85 }
86 void spfa(int s) {
87     memset(dis, 63, 4 * N);
88     memset(inque, 0, N);
89     memset(pre, -1, 4 * N);
90     dis[s] = 0;
91     queue<int> que;
92     que.push(s);
93     while (!que.empty()) {
94         int u = que.front();
95         inque[u] = 0;
96         que.pop();
97         for (int i = head[u]; i != -1; i = edge[i].next)

```

```

98     if (edge[i].flow) {
99         int v = edge[i].to;
100         if (dis[v] > dis[u] + edge[i].cost) {
101             dis[v] = dis[u] + edge[i].cost;
102             pre[v] = i;
103             if (!inque[v]) {
104                 inque[v] = 1;
105                 que.push(v);
106             }
107         }
108     }
109 }
110 }
111 void dijkstra(int s) {
112     for (int i = 0; i < N; ++i)
113         h[i] += dis[i];
114     memset(dis, 63, 4 * N);
115     memset(pre, -1, 4 * N);
116     memset(inque, 0, N);
117     dis[s] = 0;
118     inque[s] = 1;
119     heap.init();
120     heap.push(0, s);
121     while (heap.size > 1) {
122         int u = heap.top();
123         heap.pop();
124         for (int i = head[u]; i != -1; i = edge[i].next)
125             if (edge[i].flow) {
126                 int v = edge[i].to;
127                 if (dis[v] > dis[u] + edge[i].cost + h[u] - h[v]) {
128                     dis[v] = dis[u] + edge[i].cost + h[u] - h[v];
129                     pre[v] = i;
130                     if (!inque[v]) {
131                         heap.push(dis[v], v);
132                         inque[v] = 1;
133                     } else
134                         heap.update(dis[v], v);
135                 }
136             }
137     }
138 }
139 int minimumCostFlow(int s, int t, int &cost) {
140     int flow = 0;
141     memset(h, 0, 4 * N);
142     for (spfa(s); pre[t] != -1; dijkstra(s)) {
143         int maxs = edge[pre[t]].flow;
144         for (int i = pre[t]; i != -1; i = pre[edge[i ^ 1].to])
145             maxs = min(maxs, edge[i].flow);
146         for (int i = pre[t]; i != -1; i = pre[edge[i ^ 1].to]) {
147             edge[i].flow -= maxs;
148             edge[i ^ 1].flow += maxs;
149             cost += edge[i].cost * maxs;
150         }
151         flow += maxs;
152     }
153     return flow;
154 }
155 int main() {
156     return 0;
157 }

```

## 4.5 Bi-connect

```

1 struct edges {
2     int to,next;
3     bool cut,visit;
4 } edge[MAXM<<1];
5 int head[MAXN],low[MAXN],dpt[MAXN],L;
6 bool visit[MAXN],cut[MAXN];
7 void init(int n) {
8     L=0;
9     memset(head,-1,4*n);
10    memset(visit,0,n);
11 }
12 void add_edge(int u,int v) {
13     edge[L].cut=edge[L].visit=0;
14     edge[L].to=v;
15     edge[L].next=head[u];
16     head[u]=L++;
17 }
18 int idx;
19 stack<int> st;
20 int bcc[MAXM];
21 void dfs(int u,int fu,int deg) {
22     cut[u]=0;
23     visit[u]=1;
24     low[u]=dpt[u]=deg;
25     int tot=0;
26     for (int i=head[u]; i!=-1; i=edge[i].next) {
27         int v=edge[i].to;
28         if (edge[i].visit)
29             continue;
30         st.push(i/2);
31         edge[i].visit=edge[i^1].visit=1;
32         if (visit[v]) {
33             low[u]=dpt[v]>low[u]?low[u]:dpt[v];
34             continue;
35         }
36         dfs(v,u,deg+1);
37         edge[i].cut=edge[i^1].cut=(low[v]>dpt[u] || edge[i].cut);
38         if (u!=fu) cut[u]=low[v]>=dpt[u]?1:cut[u];
39         if (low[v]>=dpt[u] || u==fu) {
40             while (st.top()!=i/2) {
41                 int x=st.top()*2,y=st.top()*2+1;
42                 bcc[st.top()]=idx;
43                 st.pop();
44             }
45             bcc[i/2]=idx++;
46             st.pop();
47         }
48         low[u]=low[v]>low[u]?low[u]:low[v];
49         tot++;
50     }
51     if (u==fu && tot>1) cut[u]=1;
52 }
53 int main() {
54     int n,m;
55     while (scanf("%d%d",&n,&m)!=EOF) {
56         init(n);
57         for (int i=0; i<m; i++) {
58             int u,v;
59             scanf("%d%d",&u,&v);
60             add_edge(u,v);
61             add_edge(v,u);
62         }
63         idx=0;

```

```

64     for (int i=0; i<n; i++)
65         if (!visit[i])
66             dfs(i,i,0);
67     }
68     return 0;
69 }

```

#### 4.6 Cut and bridge

```

1  vector<int> G[maxn];
2  int dfn[maxn], low[maxn], dfs_clock;
3  //割点答案
4  bool iscut[maxn];
5  //桥答案
6  vector<pair<int,int> > bridge;
7  void init()
8  {
9      dfs_clock = 1;
10     memset(dfn, 0, sizeof(dfn));
11     for (int i = 1; i <= n; i++)
12     {
13         G[i].clear();
14     }
15     memset(iscut, 0, sizeof(iscut));
16     bridge.clear();
17 }
18 void addedge(int u, int v)
19 {
20     G[u].push_back(v);
21     G[v].push_back(u);
22 }
23 void dfs(int u, int fa)
24 {
25     low[u] = dfn[u] = dfs_clock++;
26     int cnt = 0;
27     for (int v: G[u])
28     {
29         if (v != fa)
30         {
31             if (!dfn[v])
32             {
33                 dfs(v, u);
34                 cnt++;
35                 low[u] = min(low[u], low[v]);
36                 //判断割点 u?=1用于判断树根
37                 if (u == 1 && cnt > 1) iscut[u] = true;
38                 if (u != 1 && low[v] >= dfn[u]) iscut[u] = true;
39                 //判断桥
40                 if (low[v] > dfn[u]) bridge.push_back({u, v});
41             }
42             else
43             {
44                 low[u] = min(low[u], dfn[v]);
45             }
46         }
47     }
48     if (cnt <= 1 && u == 1) iscut[u] = false;
49 }

```

#### 4.7 Stoer-Wagner

```

1 int map[maxn][maxn];
2 int n;
3 void contract(int x,int y) {
4     int i,j;
5     for (i=0; i<n; i++)
6         if (i!=x) map[x][i]+=map[y][i],map[i][x]+=map[i][y];
7     for (i=y+1; i<n; i++) for (j=0; j<n; j++) {
8         map[i-1][j]=map[i][j];
9         map[j][i-1]=map[j][i];
10    }
11    n--;
12 }
13 int w[maxn],c[maxn];
14 int sx,tx;
15 int mincut() {
16     int i,j,k,t;
17     memset(c,0,sizeof(c));
18     c[0]=1;
19     for (i=0; i<n; i++) w[i]=map[0][i];
20     for (i=1; i+1<n; i++) {
21         t=k=-1;
22         for (j=0; j<n; j++) if (c[j]==0&&w[j]>k)
23             k=w[t=j];
24         c[sx=t]=1;
25         for (j=0; j<n; j++) w[j]+=map[t][j];
26     }
27     for (i=0; i<n; i++) if (c[i]==0) return w[tx=i];
28 }
29 int main() {
30     int i,j,k,m;
31     while (scanf("%d%d",&n,&m)!=EOF) {
32         memset(map,0,sizeof(map));
33         while (m--) {
34             scanf("%d%d%d",&i,&j,&k);
35             map[i][j]+=k;
36             map[j][i]+=k;
37         }
38         int mint=999999999;
39         while (n>1) {
40             k=mincut();
41             if (k<mint) mint=k;
42             contract(sx,tx);
43         }
44         printf("%d\n",mint);
45     }
46     return 0;
47 }

```

#### 4.8 Euler path

```

1 //Directed graph
2 void solve(int x) {
3     int i;
4     if (!match[x]) {
5         path[++l]=x;
6         return ;
7     }
8     for (i=1; i<=n; i++)
9         if (b[x][i]) {
10             b[x][i]--;
11             match[x]--;
12             solve(i);

```

```

13     }
14     path[++l]=x;
15 }
16 //Undirected graph
17 void solve(int x) {
18     int i;
19     if (!match[x]) {
20         path[++l]=x;
21         return ;
22     }
23     for (i=1; i<=n; i++)
24         if (b[x][i]) {
25             b[x][i]--;
26             b[i][x]--;
27             match[x]--;
28             match[i]--;
29             solve(i);
30         }
31     path[++l]=x;
32 }

```

#### 4.9 Strongly connected component

```

1 int dfsnum[2000];
2 int low[2000];
3 int stack[2000];
4 int top;
5 int ans;
6 int an;
7 int be[2000];
8 int flag[2000];
9 void dfs(int x) {
10     dfsnum[x] = low[x] = ans++;
11     stack[++top] = x;
12     flag[x] = 1;
13     for (int i = head[x]; i != -1; i = edge[i].next) {
14         int y = edge[i].to;
15         if (dfsnum[y] == -1) {
16             dfs(y);
17             low[x] = min(low[x], low[y]);
18         } else if (flag[y] == 1)
19             low[x] = min(low[x], dfsnum[y]);
20     }
21     if (dfsnum[x] == low[x]) {
22         while (stack[top] != x) {
23             flag[stack[top]] = 0;
24             be[stack[top]] = an;
25             top--;
26         }
27         flag[x] = 0;
28         be[x] = an++;
29         top--;
30     }
31 }
32 void SC() {
33     memset(dfsnum, -1, sizeof(dfsnum));
34     memset(flag, 0, sizeof(flag));
35     top = 0;
36     an = 0;
37     ans = 0;
38     for (int i = 0; i < n; i++)
39         if (dfsnum[i] == -1)
40             dfs(i);

```



41 | }

## 4.10 Match

### 4.10.1 Bipartite graph

```

1  bool check(int u) {
2      for (int i=head[u]; i!=-1; i=edge[i].next) {
3          int v=edge[i].to;
4          if (!use[v]) {
5              use[v]=1;
6              if (pre[v]==-1 || check(pre[v])) {
7                  pre[v]=u;
8                  return 1;
9              }
10         }
11     }
12     return 0;
13 }
14 int match() {
15     int ret=0;
16     memset(pre,-1,sizeof(pre));
17     for (int u=1; u<=N; u++) {
18         memset(use,0,sizeof(use));
19         if (check(u))
20             ret++;
21     }
22     return ret;
23 }
```

### 4.10.2 Edmonds

```

1  int N;
2  bool Graph[MaxN+1][MaxN+1];
3  int Match[MaxN+1];
4  bool InQueue[MaxN+1], InPath[MaxN+1], InBlossom[MaxN+1];
5  int Head, Tail;
6  int Queue[MaxN+1];
7  int Start, Finish;
8  int NewBase;
9  int Father[MaxN+1], Base[MaxN+1];
10 int Count;
11 void CreateGraph() {}
12 void Push(int u) {
13     Queue[Tail] = u;
14     Tail++;
15     InQueue[u] = true;
16 }
17 int Pop() {
18     int res = Queue[Head];
19     Head++;
20     return res;
21 }
22 int FindCommonAncestor(int u, int v) {
23     memset(InPath, false, sizeof(InPath));
24     while (true) {
25         u = Base[u];
26         InPath[u] = true;
27         if (u == Start) break;
28         u = Father[Match[u]];
29     }
30     while (true) {
```

```

31     v = Base[v];
32     if (InPath[v]) break;
33     v = Father[Match[v]];
34 }
35 return v;
36 }
37 void ResetTrace(int u) {
38     int v;
39     while (Base[u] != NewBase) {
40         v = Match[u];
41         InBlossom[Base[u]] = InBlossom[Base[v]] = true;
42         u = Father[v];
43         if (Base[u] != NewBase) Father[u] = v;
44     }
45 }
46 void BlossomContract(int u, int v) {
47     NewBase = FindCommonAncestor(u, v);
48     memset(InBlossom, false, sizeof(InBlossom));
49     ResetTrace(u);
50     ResetTrace(v);
51     if (Base[u] != NewBase) Father[u] = v;
52     if (Base[v] != NewBase) Father[v] = u;
53     for (int tu = 1; tu <= N; tu++)
54         if (InBlossom[Base[tu]]) {
55             Base[tu] = NewBase;
56             if (!InQueue[tu]) Push(tu);
57         }
58 }
59 void FindAugmentingPath() {
60     memset(InQueue, false, sizeof(InQueue));
61     memset(Father, 0, sizeof(Father));
62     for (int i = 1; i <= N; i++)
63         Base[i] = i;
64     Head = Tail = 1;
65     Push(Start);
66     Finish = 0;
67     while (Head < Tail) {
68         int u = Pop();
69         for (int v = 1; v <= N; v++)
70             if (Graph[u][v] && (Base[u] != Base[v]) && (Match[u] != v)) {
71                 if ((v == Start) ||
72                     ((Match[v] > 0) && (Father[Match[v]] > 0)))
73                     BlossomContract(u, v);
74                 else if (Father[v] == 0) {
75                     Father[v] = u;
76                     if (Match[v] > 0)
77                         Push(Match[v]);
78                     else {
79                         Finish = v;
80                         return;
81                     }
82                 }
83             }
84     }
85 }
86 void AugmentPath() {
87     int u, v, w;
88     u = Finish;
89     while (u > 0) {
90         v = Father[u];
91         w = Match[v];
92         Match[v] = u;
93         Match[u] = v;
94         u = w;

```

```

95     }
96 }
97 void Edmonds() {
98     memset(Match,0,sizeof(Match));
99     for (int u = 1; u <= N; u++)
100         if (Match[u] == 0) {
101             Start = u;
102             FindAugmentingPath();
103             if (Finish > 0) AugmentPath();
104         }
105 }
106 void PrintMatch() {}
107 int main() {
108     CreateGraph();
109     Edmonds();
110     PrintMatch();
111 }

```

#### 4.10.3 KM

```

1  bool visx[N],visy[N];
2  int lx[N],ly[N];
3  int matchy[N];
4  int map[N][N];
5  bool find(int x) {
6      visx[x]=true;
7      int t;
8      for (int y=0; y<ycnt; y++) {
9          if (!visy[y]) {
10             t=lx[x]+ly[y]-map[x][y];
11             if (t==0) {
12                 visy[y]=true;
13                 if (matchy[y]==-1 || find(matchy[y])) {
14                     matchy[y]=x;
15                     return true;
16                 }
17             } else if (lack>t) lack=t;
18         }
19     }
20     return false;
21 }
22 void KM() {
23     memset(lx,0,sizeof(lx));
24     memset(ly,0,sizeof(ly));
25     memset(matchy,-1,sizeof(matchy));
26     for (int i=0; i<xcnt; i++)
27         for (int j=0; j<ycnt; j++)
28             if (map[i][j]>lx[i])
29                 lx[i]=map[i][j];
30     for (int x=0; x<xcnt; x++) {
31         while (true) {
32             memset(visx,false,sizeof(visx));
33             memset(visy,false,sizeof(visy));
34             lack=INFI;
35             if (find(x)) break;
36             for (int i=0; i<xcnt; i++) {
37                 if (visx[i]) lx[i]-=lack;
38                 if (visy[i]) ly[i]+=lack;
39             }
40         }
41     }
42     int cost=0;
43     for (int i=0; i<ycnt; i++)

```

```

44 |     cost+=map[matchy[i]][i];
45 | }

```

#### 4.11 Clique

```

1 | bool am[100][100];
2 | int ans;
3 | int c[100];
4 | int U[100][100];
5 | int n;
6 | bool dfs(int rest,int num) {
7 |     if (!rest) {
8 |         if (num>=ans)
9 |             return 1;
10 |        else
11 |            return 0;
12 |    }
13 |    int pre=-1;
14 |    for (int i=0; i<rest && rest-i+num>=ans; i++) {
15 |        int idx=U[num][i];
16 |        if (num+c[idx]<ans)
17 |            return 0;
18 |        int nrest=0;
19 |        for (int j=i+1; j<rest; j++)
20 |            if (am[idx][U[num][j]])
21 |                U[num+1][nrest++]=U[num][j];
22 |        if (dfs(nrest,num+1))
23 |            return 1;
24 |    }
25 |    return 0;
26 | }
27 | int main() {
28 |     while (scanf("%d",&n),n) {
29 |         for (int i=0; i<n; i++)
30 |             for (int j=0; j<n; j++)
31 |                 scanf("%d",&am[i][j]);
32 |         ans=0;
33 |         for (int i=n-1; i>=0; i--) {
34 |             int rest=0;
35 |             for (int j=i+1; j<n; j++)
36 |                 if (am[i][j])
37 |                     U[0][rest++]=j;
38 |             ans+=dfs(rest,0);
39 |             c[i]=ans;
40 |         }
41 |         printf("%d\n",ans);
42 |     }
43 |     return 0;
44 | }

```

最大团的压位做法 by Claris

```

1 | typedef unsigned long long U;
2 | typedef long long ll;
3 | const int N=45;
4 | //0为有边,1为无边
5 | int n,K,x,i,j,ans;bool flag;U g[N];double res;
6 | inline int ctz(U s){return s?__builtin_ctzll(s):64;}
7 | void BornKerbosch(U cur,U allow,U forbid){
8 |     if (!allow&&!forbid){
9 |         ans=max(ans,__builtin_popcountll(cur));
10 |        return;

```

```

11 }
12 if(!allow)return;
13 int pivot=ctz(allow|forbid);
14 U z=allow&~g[pivot];
15 for(int u=ctz(z);u<n;u+=ctz(z>>(u+1))+1){
16     BornKerbosch(cur|(1ULL<<u),allow&g[u],forbid&g[u]);
17     allow^=1ULL<<u,forbid|=1ULL<<u;
18 }
19 }
20 int main(){
21     scanf("%d",&n);
22     for(i=0;i<n;i++)g[i]=(1ULL<<n)-1-(1ULL<<i);
23     for(i=0;i<n;i++)for(j=0;j<n;j++){
24         scanf("%d",&x);
25         //0为有边,1为无边
26         if(!x&&i!=j)g[i]^=1ULL<<j;
27     }
28     BornKerbosch(0,(1ULL<<n)-1,0);
29     //ans为最大团大小
30     printf("%d",ans);
31 }

```

## 4.12 Spanning tree

### 4.12.1 Count the number of spanning tree

```

1 Matrix laplacian;
2 laplacian.clear();
3 for (int i = 0; i < n; i++)
4     for (int j = 0; j < n; j++)
5         if (i != j && G[i][j]) {
6             laplacian.a[i][j] = -1;
7             laplacian.a[i][i]++;
8         }
9 printf("%d\n",laplacian.det(n-1));

```

### 4.12.2 Spanning tree on directed graph

```

1 struct Edge {
2     int u,v,cost;
3 };
4 Edge e[1001*1001];
5 int pre[1001],id[1001],visit[1001],in[1001];
6 int zhuliu(int root,int n,int m,Edge e[]) {
7     int res = 0,u,v;
8     while (true) {
9         for (int i = 0; i < n; i++)
10             in[i] = inf;
11         for (int i = 0; i < m; i++)
12             if (e[i].u != e[i].v && e[i].cost < in[e[i].v]) {
13                 pre[e[i].v] = e[i].u;
14                 in[e[i].v] = e[i].cost;
15             }
16         for (int i = 0; i < n; i++)
17             if (i != root)
18                 if (in[i] == inf) return -1;
19         int tn = 0;
20         memset(id,-1,sizeof(id));
21         memset(visit,-1,sizeof(visit));
22         in[root] = 0;
23         for (int i = 0; i < n; i++) {

```

```

24     res += in[i];
25     v = i;
26     while (visit[v] != i && id[v] == -1 && v != root) {
27         visit[v] = i;
28         v = pre[v];
29     }
30     if (v != root && id[v] == -1) {
31         for (int u = pre[v] ; u != v ; u = pre[u])
32             id[u] = tn;
33         id[v] = tn++;
34     }
35 }
36 if (tn == 0) break;
37 for (int i = 0; i < n; i++)
38     if (id[i] == -1)
39         id[i] = tn++;
40 for (int i = 0; i < m; i++) {
41     int v = e[i].v;
42     e[i].u = id[e[i].u];
43     e[i].v = id[e[i].v];
44     if (e[i].u != e[i].v)
45         e[i++].cost -= in[v];
46     else
47         swap(e[i], e[--m]);
48 }
49 n = tn;
50 root = id[root];
51 }
52 return res;
53 }

```

#### 4.13 Kth shortest path

```

1  #include<cstdio>
2  #include<cstring>
3  #include<queue>
4  using namespace std;
5  int K;
6  class states {
7  public:
8      int cost,id;
9  };
10 int dist[1000];
11 class cmp {
12 public:
13     bool operator ()(const states &i,const states &j) {
14         return i.cost>j.cost;
15     }
16 };
17 class cmp2 {
18 public:
19     bool operator ()(const states &i,const states &j) {
20         return i.cost+dist[i.id]>j.cost+dist[j.id];
21     }
22 };
23 struct edges {
24     int to,next,cost;
25 } edger[100000],edge[100000];
26 int headr[1000],head[1000],Lr,L;
27 void dijkstra(int s) {
28     states u;
29     u.id=s;
30     u.cost=0;

```

```

31 dist[s]=0;
32 priority_queue<states,vector<states>,cmp> q;
33 q.push(u);
34 while (!q.empty()) {
35     u=q.top();
36     q.pop();
37     if (u.cost!=dist[u.id]) continue;
38     for (int i=headr[u.id]; i!=-1; i=edger[i].next) {
39         states v=u;
40         v.id=edger[i].to;
41         if (dist[v.id]>dist[u.id]+edger[i].cost) {
42             v.cost=dist[v.id]=dist[u.id]+edger[i].cost;
43             q.push(v);
44         }
45     }
46 }
47 }
48 int num[1000];
49 void init(int n) {
50     Lr=L=0;
51     memset(head,-1,4*n);
52     memset(headr,-1,4*n);
53     memset(dist,63,4*n);
54     memset(num,0,4*n);
55 }
56 void add_edge(int u,int v,int x) {
57     edge[L].to=v;
58     edge[L].cost=x;
59     edge[L].next=head[u];
60     head[u]=L++;
61     edger[Lr].to=u;
62     edger[Lr].cost=x;
63     edger[Lr].next=headr[v];
64     headr[v]=Lr++;
65 }
66 int a_star(int s,int t) {
67     if (dist[s]==0x3f3f3f3f)
68         return -1;
69     priority_queue<states,vector<states>,cmp2> q;
70     states tmp;
71     tmp.id=s;
72     tmp.cost=0;
73     q.push(tmp);
74     while (!q.empty()) {
75         states u=q.top();
76         q.pop();
77         num[u.id]++;
78         if (num[t]==K)
79             return u.cost;
80         for (int i=head[u.id]; i!=-1; i=edge[i].next) {
81             int v=edge[i].to;
82             tmp.id=v;
83             tmp.cost=u.cost+edge[i].cost;
84             q.push(tmp);
85         }
86     }
87     return -1;
88 }
89 int main() {
90     int n,m;
91     scanf("%d%d",&n,&m);
92     init(n);
93     for (int i=0; i<m; i++) {

```

```

94     int u,v,x;
95     scanf("%d%d%d",&u,&v,&x);
96     add_edge(u-1,v-1,x);
97 }
98 int s,t;
99 scanf("%d%d%d",&s,&t,&K);
100 if (s==t)
101     K++;
102     dijkstra(t-1);
103     printf("%d\n",a_star(s-1,t-1));
104 }

```

#### 4.14 LCA

```

1  typedef long long ll;
2  const int maxn = 100000 + 100;
3  const int maxk = 20;
4  struct edge
5  {
6      int v, w;
7  } es[maxn * 2];
8  int tot;
9  vector<int> G[maxn];
10 ll dis[maxn];
11 int fa[maxn][maxk];
12 int depth[maxn];
13 int n, m;
14 void init()
15 {
16     for (int i = 1; i <= n; i++)
17     {
18         G[i].clear();
19     }
20     tot = 0;
21 }
22 void addedge(int u, int v, int w)
23 {
24     G[u].push_back(tot);
25     es[tot++] = {v, w};
26     G[v].push_back(tot);
27     es[tot++] = {u, w};
28 }
29 int lca(int x, int y)
30 {
31     if (depth[x] > depth[y]) swap(x, y);
32     for (int k = maxk - 1; k >= 0; k--)
33     {
34         if (depth[fa[y][k]] >= depth[x])
35         {
36             y = fa[y][k];
37         }
38     }
39     if (x == y) return x;
40     for (int k = maxk - 1; k >= 0; k--)
41     {
42         if (fa[x][k] != fa[y][k])
43         {
44             x = fa[x][k];
45             y = fa[y][k];
46         }
47     }
48     return fa[x][0];

```



```

49 }
50 void dfs(int cur, int parent)
51 {
52     fa[cur][0] = parent;
53     for (int k = 1; k < maxk; k++)
54     {
55         fa[cur][k] = fa[fa[cur][k - 1]][k - 1];
56     }
57     for (int eno: G[cur])
58     {
59         edge & e = es[eno];
60         if (e.v != parent)
61         {
62             dis[e.v] = e.w + dis[cur];
63             depth[e.v] = 1 + depth[cur];
64             dfs(e.v, cur);
65         }
66     }
67 }
68 ll dist(int u, int v)
69 {
70     int _lca = lca(u, v);
71     return dis[u] + dis[v] - 2 * dis[_lca];
72 }

```

#### 4.15 VirtualTree

dfs 部分参照 lca 部分自己写，碰上部分点问题优先想 dfs 序

```

1  typedef long long ll;
2  const int inf = 0x3f3f3f3f;
3  const int maxn = 100000;
4  const int maxk = 21;
5  int dfn[maxn], dfs_clock;
6  inline bool cmp(const int & i, const int & j)
7  {
8      return dfn[i] < dfn[j];
9  }
10 int fa[maxn][maxk], depth[maxn];
11 struct edge
12 {
13     int v;
14     ll c;
15 } es[maxn * 2];
16 vector<int> G[maxn], H[maxn];
17 void addH(int u, int v)
18 {
19     H[u].push_back(v);
20 }
21 int lca(int x, int y)
22 {
23     if (depth[x] < depth[y]) swap(x, y);
24     for (int k = maxk - 1; k >= 0; k--)
25     {
26         if (depth[fa[x][k]] >= depth[y])
27         {
28             x = fa[x][k];
29         }
30     }
31     if (x == y) return x;
32     for (int k = maxk - 1; k >= 0; k--)
33     {
34         if (fa[x][k] != fa[y][k])

```

```

35     {
36         x = fa[x][k];
37         y = fa[y][k];
38     }
39 }
40 return fa[x][0];
41 }
42 //vertices need to be arrange
43 int key_node[maxn], kcnt;
44 void build()
45 {
46     static int stk[maxn];
47     sort(key_node, key_node + kcnt, cmp);
48     //注释部分可用于去除关键点子树中的多余关键点
49     // int p = 0;
50     // for (int i = 1; i < kcnt; i++)
51     // {
52     //     if (lca(key_node[i], key_node[p]) != key_node[p])
53     //     {
54     //         key_node[++p] = key_node[i];
55     //     }
56     // }
57     // kcnt = p + 1;
58     int sz = 0;
59     stk[sz++] = 0;
60     for (int i = 0; i < kcnt; i++)
61     {
62         int f = lca(stk[sz - 1], key_node[i]);
63         if (f == stk[sz - 1])
64         {
65             stk[sz++] = key_node[i];
66         }
67         else
68         {
69             while(sz - 2 >= 0 && depth[stk[sz - 2]] >= depth[f])
70             {
71                 addH(stk[sz - 2], stk[sz - 1]);
72                 sz--;
73             }
74             if (stk[sz - 1] != f)
75             {
76                 addH(f, stk[--sz]);
77                 stk[sz++] = f;
78             }
79             stk[sz++] = key_node[i];
80         }
81     }
82     for (int i = 1; i < sz; i++)
83     {
84         addH(stk[i - 1], stk[i]);
85     }
86 }

```

#### 4.16 Stable marriage problem

假定有  $n$  个男生和  $n$  个女生，理想的拍拖状态就是对于每对情侣  $(a, b)$ ，找不到另一对情侣  $(c, d)$  使得  $c$  更喜欢  $b$ ， $b$  也更喜欢  $c$ ，同理，对  $a$  来说也没有  $(e, f)$  使得  $a$  更喜欢  $e$  而  $e$  更喜欢  $a$ ，当然最后会有一些人落单。这样子一个状态可以称为理想拍拖状态，它也有一个专业的名词叫稳定婚姻。

求解这个问题可以用一个专有的算法，延迟认可算法，其核心就是让每个男生按自己喜欢的顺序逐个向女生表白，例如 leokan 向一个女生求爱，这个过程中，若这个女生没有男朋友，那么这个

女生就暂时成为 leokan 的女朋友，或这个女生喜欢她现有男朋友的程度没有喜欢 leokan 高，这个女生也暂时成为 leokan 的女朋友，而她原有的男朋友则再将就找下一个次喜欢的女生来当女朋友。

```

1 #include<string.h>
2 #include<stdio.h>
3 #define N 1050
4 int boy[N][N];
5 int girl[N][N];
6 int ans[N];
7 int cur[N];
8 int n;
9 void getMarry(int g) {
10     for (int i=ans[g]+1; i<n; i++) {
11         int b=girl[g][i]-1;
12         if (cur[b]<0) {
13             ans[g]=i;
14             cur[b]=g;
15             return;
16         }
17         int og=cur[b];
18         if (boy[b][og] > boy[b][g]) {
19             cur[b]=g;
20             ans[g]=i;
21             getMarry(og);
22             return;
23         }
24     }
25 };
26 int main() {
27     int t,a;
28     scanf("%d",&t);
29     while(t-->0) {
30         memset(girl,0,sizeof(girl));
31         memset(boy,0,sizeof(boy));
32         scanf("%d",&n);
33         for (int i=0; i<n; i++)
34             for (int j=0; j<n; j++)
35                 scanf("%d",&girl[i][j]);
36         for (int i=0; i<n; i++)
37             for (int j=0; j<n; j++) {
38                 scanf("%d",&a);
39                 boy[i][a-1]=j;
40             }
41         memset(cur,0xff,sizeof(cur));
42         memset(ans,0xff,sizeof(ans));
43         for (int i=0; i<n; i++)
44             getMarry(i);
45         for (int i=0; i<n; i++)
46             printf("%d\n",girl[i][ans[i]]);
47     }
48     return 0;
49 }

```

## 5 Math

### 5.1 Hill climbing

Hill climbing is an useful function to get the maximum value if you don't know what to do! just make a function and follow the instruction below:

```

1 for (s = 1; s > 1e-6; f = 0)
2 {

```

```

3   if (F(dx, dy) > F(dx + s, dy)) dx += s, f = 1;
4   else if (F(dx, dy) > F(dx - s, dy)) dx -= s, f = 1;
5   else if (F(dx, dy) > F(dx, dy + s)) dy += s, f = 1;
6   else if (F(dx, dy) > F(dx, dy - s)) dy -= s, f = 1;
7   if (!f) s *= 0.7;
8 }

```

## 5.2 Linear Seq

杜教的递推板子，目测大概需要暴力递推阵的大小的两倍。

```

1  const ll moder = 998244353;
2  typedef vector<int> VI;
3  ll p_m(ll base, ll index)
4  {
5      ll ret = 1;
6      while(index)
7      {
8          if (index & 1) ret = ret * base % moder;
9          base = base * base % moder;
10         index >>= 1;
11     }
12     return ret;
13 }
14 int n;
15 namespace linear_seq
16 {
17     const int N = 10000 + 10;
18     ll res[N], base[N], _c[N], _md[N];
19     vector<int> Md;
20     void mul(ll *a, ll *b, int k)
21     {
22         for (int i = 0; i < k+k; i++) _c[i] = 0;
23         for (int i = 0; i < k; i++)
24             if (a[i])
25                 for (int j = 0; j < k; j++)
26                     _c[i+j] = (_c[i+j] + a[i] * b[j]) % moder;
27         for (int i = k + k - 1; i >= k; i--)
28             if (_c[i])
29                 for (int j = 0; j < Md.size(); j++)
30                     _c[i-k+Md[j]] = (_c[i-k+Md[j]] - _c[i] * _md[Md[j]]) % moder;
31         for (int i = 0; i < k; i++) a[i] = _c[i];
32     }
33     int solve(ll n, VI a, VI b)
34     {
35         ll ans = 0, pnt = 0;
36         int k = a.size();
37         for (int i = 0; i < k; i++) _md[k-1-i] = -a[i];
38         _md[k] = 1;
39         Md.clear();
40         for (int i = 0; i < k; i++) if (_md[i] != 0) Md.push_back(i);
41         for (int i = 0; i < k; i++) res[i] = base[i] = 0;
42         res[0] = 1;
43         while((1LL << pnt) <= n) pnt++;
44         for (int p = pnt; p >= 0; p--)
45         {
46             mul(res, res, k);
47             if ((n>>p) & 1)
48             {
49                 for (int i = k - 1; i >= 0; i--) res[i+1] = res[i];
50                 res[0] = 0;
51                 for (int j = 0; j < Md.size(); j++) res[Md[j]] = (res[Md[j]] - res[k] * _md[

```

```

52     }
53 }
54 for (int i = 0; i < k; i++) ans = (ans + res[i] * b[i]) % moder;
55 if (ans < 0) ans += moder;
56 return ans;
57 }
58 VI BM(VI s)
59 {
60     VI C(1,1), B(1,1);
61     int L = 0, m = 1, b = 1;
62     for (int n = 0; n < s.size(); n++)
63     {
64         ll d = 0;
65         for (int i = 0; i <= L; i++) d = (d + (ll)C[i]*s[n-i])%moder;
66         if (d == 0) ++m;
67         else if (2 * L <= n)
68         {
69             VI T = C;
70             ll c = moder - d * p_m(b, moder - 2) % moder;
71             while(C.size() < B.size() + m) C.push_back(0);
72             for (int i = 0; i < B.size(); i++) C[i+m] = (C[i+m] + c * B[i]) % moder;
73             L = n + 1 - L; B = T; b = d; m = 1;
74         }
75         else
76         {
77             ll c = moder - d * p_m(b, moder - 2) % moder;
78             while(C.size() < B.size() + m) C.push_back(0);
79             for (int i = 0; i < B.size(); i++) C[i+m] = (C[i+m] + c * B[i]) % moder;
80             ++m;
81         }
82     }
83     return C;
84 }
85 int gao(VI a, ll n)
86 {
87     VI c = BM(a);
88     c.erase(c.begin());
89     for (int i = 0; i < c.size(); i++) c[i] = (moder - c[i]) % moder;
90     return solve(n, c, VI(a.begin(), a.begin() + c.size()));
91 }
92 }
93 int main() {
94     while(~scanf("%d", &n))
95         printf("%d\n", linear_seq::gao(VI{1, 4, 12, 33, 88, 232, 609}, n));
96 }

```

## 5.3 FFT

### 5.3.1 Bit operation

$tf(X1, X2) = (tf(X1) - tf(X2), tf(X1) + tf(X2))$

异或:  $tf(X1, X2) = (tf(X1) - tf(X2), tf(X1) + tf(X2))$

与:  $tf(x1, x2) = (tf(x1) + tf(x2), tf(x1))$

```

1 // Transforms the interval [x, y) in a.
2 void transform(int x, int y) {
3     if (x == y - 1) {
4         return;
5     }
6     int l2 = (y - x) / 2;
7     int z = x + l2;
8     transform(x, z);
9     transform(z, y);

```

```

10  for (int i=x; i<z; i++) {
11      int x1 = a[i];
12      int x2 = a[i+l2];
13      a[i] = (x1 - x2 + MOD) % MOD;
14      a[i+l2] = (x1 + x2) % MOD;
15  }
16  }
17  // Reverses the transform in
18  // the interval [x, y) in a.
19  void untransform(int x, int y) {
20      if ( x == y - 1) {
21          return;
22      }
23      int l2 = ( y - x ) / 2;
24      int z = x + l2;
25      for (int i=x; i<z; i++) {
26          long long y1 = a[i];
27          long long y2 = a[i+l2];
28          // x1 - x2 = y1
29          // x1 + x2 = y2
30          // 2 * x1 = y1 + y2
31          // 2 * x2 = y2 - y1
32
33          // In order to solve those equations, we need to divide by 2
34          // But we are performing operations modulo 1000000007
35          // that needs us to find the modular multiplicative inverse of 2.
36          // That is saved in the INV2 variable.
37
38          a[i] = (int)( ((y1 + y2)*INV2) % MOD );
39          a[i+l2] = (int)( ((y2 - y1 + MOD)*INV2) % MOD );
40      }
41      untransform(x, z);
42      untransform(z, y);
43  }

```

### 5.3.2 Standard

```

1  struct vir {
2      long double re, im;
3      vir(long double a = 0, long double b = 0) {
4          re = a;
5          im = b;
6      }
7      vir operator +(const vir& b) const {
8          return vir(re + b.re, im + b.im);
9      }
10     vir operator -(const vir& b) const {
11         return vir(re - b.re, im - b.im);
12     }
13     vir operator *(const vir& b) const {
14         return vir(re * b.re - im * b.im, re * b.im + im * b.re);
15     };
16 };
17 void change(vir *x, int len, int loglen) {
18     int i, j, k, t;
19     for (i = 0; i < len; i++) {
20         t = i;
21         for (j = k = 0; j < loglen; j++, t >= 1)
22             k = (k << 1) | (t & 1);
23         if (k < i) {
24             vir wt = x[k];
25             x[k] = x[i];
26             x[i] = wt;

```

```

27     }
28 }
29 }
30 void fft(vir *x, int len, int loglen) {
31     int i, j, t, s, e;
32     change(x, len, loglen);
33     t = 1;
34     for (i = 0; i < loglen; i++, t <= 1) {
35         s = 0;
36         e = s + t;
37         while (s < len) {
38             vir a, b, wo(cos(PI / t), sin(PI / t)), wn(1, 0);
39             for (j = s; j < s + t; j++) {
40                 a = x[j];
41                 b = x[j + t] * wn;
42                 x[j] = a + b;
43                 x[j + t] = a - b;
44                 wn = wn * wo;
45             }
46             s = e + t;
47             e = s + t;
48         }
49     }
50 }
51 void dit_fft(vir *x, int len, int loglen) {
52     int i, j, s, e, t = 1 << loglen;
53     for (i = 0; i < loglen; i++) {
54         t >>= 1;
55         s = 0;
56         e = s + t;
57         while (s < len) {
58             vir a, b, wn(1, 0), wo(cos(PI / t), -sin(PI / t));
59             for (j = s; j < s + t; j++) {
60                 a = x[j] + x[j + t];
61                 b = (x[j] - x[j + t]) * wn;
62                 x[j] = a;
63                 x[j + t] = b;
64                 wn = wn * wo;
65             }
66             s = e + t;
67             e = s + t;
68         }
69     }
70     change(x, len, loglen);
71     for (i = 0; i < len; i++)
72         x[i].re /= len;
73 }

```

### 5.3.3 Usage

```

1 vir x1[MAXN], x2[MAXN];
2 void solve(long long *a, int lena, long long *b, int lenb, long long *ret, int& len)
3 {
4     int len1 = lena << 1;
5     int len2 = lenb << 1;
6     len = 1;
7     int loglen = 0;
8     while (len < len1 || len < len2) {
9         len <= 1;
10        loglen++;
11    }
12    for (int i = 0; i < lena; i++)
13        x1[i] = vir(a[i], 0);

```

```

13  for (int i = lena; i < len; i++)
14      x1[i] = vir(0, 0);
15  for (int i = 0; i < lenb; i++)
16      x2[i] = vir(b[i], 0);
17  for (int i = lenb; i < len; i++)
18      x2[i] = vir(0, 0);
19  fft(x1, len, loglen);
20  fft(x2, len, loglen);
21  for (int i = 0; i < len; i++)
22      x1[i] = x1[i] * x2[i];
23  dit_fft(x1, len, loglen);
24  for (int i = 0; i < len; i++)
25      ret[i] = (long long)(x1[i].re + 0.5);
26  }

```

## 5.4 Euler function

```

1  int getEuler(int x) {
2      getFactor(x);
3      int ret=x;
4      for (int i=0; i<N; i++)
5          ret = ret/fac[i]*(fac[i]-1);
6      return ret;
7  }
8  void getEuler2() {
9      memset(euler,0,sizeof(euler));
10     euler[1] = 1;
11     for (int i = 2; i <= 3000000; i++) {
12         if (!euler[i]) {
13             for (int j = i; j <= 3000000; j += i) {
14                 if (!euler[j])
15                     euler[j] = j;
16                 euler[j] = euler[j]/i*(i-1);
17             }
18         }
19     }
20 }

```

## 5.5 Ex-GCD

```

1  //Find one solution (x,y) of ax + by = gcd(a,b)
2  long long ex_gcd(long long a,long long b,long long &x,long long &y) {
3      if (b) {
4          long long ret = ex_gcd(b,a%b,x,y),tmp = x;
5          x = y;
6          y = tmp-(a/b)*y;
7          return ret;
8      } else {
9          x = 1;
10         y = 0;
11         return a;
12     }
13 }

```

## 5.6 Möbius

两个公式:

$$F(n) = \sum_{d|n} f(d) \implies f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right) \quad (1)$$



$$F(n) = \sum_{n|d} f(d) \implies f(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) F(d) \quad (2)$$

$$\mu(n) = \begin{cases} 1 & n = 1 \\ (-1)^k & n = p_1 p_2 \dots p_k \\ 0 & \text{其余情况} \end{cases}$$

### 5.6.1 Möbius 用于容斥

容斥原理: 在集合  $S$  中至少具有  $P_1 P_2 \dots P_m$  中一个元素的个数是:

$$|S_1 \cup S_2 \cup S_3 \dots \cup S_n| = \sum |S_i| - \sum |S_i \cup S_j| + \dots + \sum (-1)^{m+1} |S_1 \cup S_2 \dots \cup S_m|$$

常用转化式:

$$\sum_{i=1}^n \left\lfloor \frac{n}{i} \right\rfloor = \sum_{i=1}^n d(i), d(n) \text{ 是 } n \text{ 的正因子数目 (埃筛)}$$

$$[x = 1] = \sum_{d|x} \mu(d)$$

```

1  const int maxn = 1000000 + 100;
2  int primes[maxn], ptot;
3  int mu[maxn];
4  bool nprime[maxn];
5  void init()
6  {
7      nprime[1] = true;
8      mu[1] = 1;
9      for (int i = 2; i < maxn; i++)
10     {
11         if (!nprime[i])
12         {
13             primes[ptot++] = i;
14             mu[i] = -1;
15         }
16         for (int j = 0; j < ptot && i * primes[j] < maxn; j++)
17         {
18             nprime[i * primes[j]] = true;
19             if (i % primes[j] == 0)
20             {
21                 mu[i * primes[j]] = -mu[i];
22                 break;
23             }
24         }
25     }
26 }
```

## 5.7 Prime

### 5.7.1 Get primes

```

1  int N;
2  bool isPrime[10001];
3  int prime[10000];
4  void getPrime(int n) {
5      memset(isPrime, 1, ++n);
6      N=0;
7      isPrime[0]=isPrime[1]=0;
```

```

8   for (int i=2; i<n; i++) {
9       if (isPrime[i])
10          prime[N++]=i;
11       for (int j=0; j<N && prime[j]*i<n; j++) {
12          isPrime[i*prime[j]]=0;
13          if (i%prime[j]==0)
14             break;
15       }
16   }
17 }

```

### 5.7.2 Get factors

```

1  const int TIME = 8;
2  int factor[100], fac_top = -1;
3  //GCD of bint
4  bint gcd(bint small, bint big) {
5      while(small) {
6          swap(small, big);
7          small%=big;
8      }
9      return abs(big);
10 }
11 //ret = (a*b)%n (n<2^62)
12 bint muti_mod(bint a, bint b, bint n) {
13     bint exp = a%n, res = 0;
14     while(b) {
15         if(b&1) {
16             res += exp;
17             if(res>n) res -= n;
18         }
19         exp <<= 1;
20         if (exp>n) exp -= n;
21         b>>=1;
22     }
23     return res;
24 }
25 // ret = (a^b)%n
26 bint mod_exp(bint a, bint p, bint m) {
27     bint exp=a%m, res=1;
28     while(p>1) {
29         if(p&1)
30             res=muti_mod(res, exp, m);
31         exp = muti_mod(exp, exp, m);
32         p>>=1;
33     }
34     return muti_mod(res, exp, m);
35 }
36 //miller-rabin
37 bool miller_rabin(bint n, int times) {
38     if(n==2) return 1;
39     if(n<2 || !(n&1)) return 0;
40     bint a, u=n-1, x, y;
41     int t=0;
42     while(u%2==0) {
43         t++;
44         u/=2;
45     }
46     srand(time(0));
47     for(int i=0; i<times; i++) {
48         a = rand() % (n-1) + 1;
49         x = mod_exp(a, u, n);
50         for(int j=0; j<t; j++) {

```

```

51     y = muti_mod(x, x, n);
52     if ( y == 1 && x != 1 && x != n-1 )
53         return false; //must not
54     x = y;
55 }
56 if( y!=1) return false;
57 }
58 return true;
59 }
60 bint pollard_rho(bint n,int c) {
61     bint x,y,d,i = 1,k = 2;
62     srand(time(0));
63     x = rand()%(n-1)+1;
64     y = x;
65     while(true) {
66         i++;
67         x = (muti_mod(x,x,n) + c) % n;
68         d = gcd(y-x, n);
69         if (1 < d && d < n) return d;
70         if( y == x) return n;
71         if(i == k) {
72             y = x;
73             k <<= 1;
74         }
75     }
76 }
77 void findFactor(bint n,int k) {
78     if(n==1)return;
79     if(miller_rabin(n, TIME)) {
80         factor[++fac_top] = n;
81         return;
82     }
83     bint p = n;
84     while(p >= n)
85         p = pollard_rho(p,k--);
86     findFactor(p,k);
87     findFactor(n/p,k);
88 }

```

### 5.7.3 区间筛

```

1  const int maxn = 1000000 + 10;
2  int primes[maxn];
3  int ptot;
4  bool nprime[maxn];
5  void intervalprime(int L, int U)
6  {
7      int i, j;
8      int SU = sqrt((double) U);
9      int d = U - L + 1;
10     for (int i = 0; i < d; i++) nprime[i] = 0;
11     //去偶数，可删(改下面起始点为2，步长为1)
12     for (int i = (L % 2 != 0); i < d; i+= 2) nprime[i] = 1;
13
14     for (int i = 3; i <= SU; i += 2)
15     {
16         if (i > L && nprime[i - L]) continue;
17         j = (L / i) * i;
18         if (j < L) j += i;
19         j = j - L;
20         for (; j < d; j += i) nprime[j] = 1;
21     }
22     if (L <= 1) nprime[1 - L] = 1;

```

```

23 |   if (L <= 2) nprime[2 - L] = 0;
24 |   ptot = 0;
25 |   for (int i = 0; i < d; i++) if (!nprime[i]) primes[ptot++] = i + L;
26 | }

```

## 5.8 Simpson

```

1 | double Simp(double l, double r) {
2 |     double h = (r-l)/2.0;
3 |     return h*(calc(l)+4*calc((l+r)/2.0)+calc(r))/3.0;
4 | }
5 | double rSimp(double l, double r) {
6 |     double mid = (l+r)/2.0;
7 |     if (abs((Simp(l, r)-Simp(l, mid)-Simp(mid, r)))/15 < eps)
8 |         return Simp(l, r);
9 |     else
10 |         return rSimp(l, mid)+rSimp(mid, r);
11 | }

```

## 5.9 Chinese remainder theorem

```

1 | int m[10], a[10]; //  $x \bmod m_i = a_i$ 
2 | bool solve(int &m0, int &a0, int m, int a) {
3 |     int y, x;
4 |     int g = ex_gcd(m0, m, x, y);
5 |     if (abs(a-a0)%g) return 0;
6 |     x*=(a-a0)/g;
7 |     x%=m/g;
8 |     a0=(x*m0+a0);
9 |     m0*=m/g;
10 |    a0%=m0;
11 |    if (a0<0) a0+=m0;
12 |    return 1;
13 | }
14 | int MLES() {
15 |     bool flag=1;
16 |     int m0=1, a0=0;
17 |     for (int i=0; i<n; i++)
18 |         if (!solve(m0, a0, m[i], a[i])) {
19 |             flag=0;
20 |             break;
21 |         }
22 |     if (flag)
23 |         return a0;
24 |     else
25 |         return -1;
26 | }

```

## 5.10 Lucas

```

1 | // num[i] = i!
2 | int comLucas(int n, int m, int p) {
3 |     int ans=1;
4 |     for (; n && m && ans; n/=p, m/=p) {
5 |         if (n%p>=m%p)
6 |             ans = ans*num[n%p]%p*getInv(num[n%p]%p)%p
7 |                 *getInv(num[m%p-n%p])%p;
8 |         else
9 |             ans=0;
10 |    }

```

```

11 |   return ans;
12 | }

```

### 5.11 Primitive root

```

1 | int getPriRoot(int p) {
2 |     if (p==2) return 1;
3 |     int phi = p - 1;
4 |     getFactor(phi);
5 |     for (int g = 2; g < p; ++g) {
6 |         bool flag=1;
7 |         for (int i = 0; flag && i < N; ++i)
8 |             if (power(g, phi/fac[i], p) == 1)
9 |                 flag=0;
10 |        if (flag)
11 |            return g;
12 |    }
13 | }

```

### 5.12 Inverse element

```

1 | void getInv2(int x) {
2 |     inv[1]=1;
3 |     for (int i=2; i<=x; i++)
4 |         inv[i]=(mod-(mod/i)*inv[mod%i]%mod)%mod;
5 | }

```

### 5.13 Calculator

注意灵活运用。

双目运算符在 calc() 中，左结合单目运算符在 P() 中，右结合单目运算符在 calc\_exp 中。（但是还没遇到过。。）

```

1 | #include <iostream>
2 | #include <cstdio>
3 | #include <cstring>
4 | #include <algorithm>
5 | #include <string>
6 | using namespace std;
7 |
8 | char s[100000];
9 | int n,cur;
10 | const string OP = "+-*/";
11 |
12 | char next_char() {
13 |     if (cur >= n) return EOF;
14 |     return s[cur];
15 | }
16 |
17 | int get_priority(char ch) {
18 |     if (ch == '*' ) return 2;
19 |     return 1;
20 | }
21 |
22 | int P();
23 |
24 | int calc(int a,char op,int b) {
25 |     if (op == '+' )
26 |         return a+b;

```

```

27     if (op == ' - ')
28         return a-b;
29     if (op == ' * ')
30         return a*b;
31 }
32
33 int calc_exp(int p) {
34     int a = P();
35     while ((OP.find(next_char()) != OP.npos) &&
36            (get_priority(next_char()) >= p)) {
37         char op = next_char();
38         cur++;
39         a = calc(a,op,calc_exp(get_priority(op)+1));
40     }
41     return a;
42 }
43
44 int totvar,m,var[26],varid[26];
45
46 int P() {
47     if (next_char() == ' - ') {
48         cur++;
49         return -P();
50     } else if (next_char() == ' + ') {
51         cur++;
52         return P();
53     } else if (next_char() == ' ( ') {
54         cur++;
55         int res = calc_exp(0);
56         cur++;
57         return res;
58     } else {
59         cur++;
60         return var[varid[s[cur-1]-' a ']];
61     }
62 }
63
64 int id[26],minid;
65
66 int main() {
67     while (true) {
68         scanf("%d%d",&totvar,&var[0]);
69         if (totvar == 0 && var[0] == 0) break;
70         for (int i = 1; i < totvar; i++)
71             scanf("%d",&var[i]);
72         scanf("%d",&m);
73         scanf("%s",s);
74         for (int i = 0; i < 26; i++)
75             id[i] = -1;
76         minid = 0;
77         n = strlen(s);
78         for (int i = 0; i < n; i++)
79             if (s[i] >= ' a ' && s[i] <= ' z ') {
80                 if (id[s[i]-' a '] == -1) {
81                     id[s[i]-' a '] = minid;
82                     minid++;
83                 }
84                 s[i] = ' a ' + id[s[i]-' a '];
85             }
86         for (int i = 0; i < totvar; i++)
87             varid[i] = i;
88         int res = 0;

```

```

89     do {
90         cur = 0;
91         int tmp = calc_exp(0);
92         if (tmp == m) {
93             res++;
94             break;
95         }
96     } while (next_permutation(varid, varid+totvar));
97     //puts(s);
98     if (res > 0)
99         puts(" YES" );
100     else
101         puts(" NO" );
102 }
103 return 0;
104 }

```

### 5.14 Linear programming

```

1  #define MAXM 20 //max num of basic variables
2  #define INF 1E200
3
4  double A[MAXM+5][MAXN+MAXM+5];
5  double b[MAXM+5], c[MAXN+MAXM+5];
6  int N[MAXN+5], B[MAXM+5];
7  double X[MAXN+MAXM+5], V;
8  int n, m, R, C, nCnt, bCnt;
9  int v1[MAXN], v2[MAXN];
10
11 int fcmp(double a, double b) {
12     if(fabs(a-b)<1E-7) return 0;
13     if(a>b) return 1;
14     return -1;
15 }
16
17 void Pivot(int l, int e) {
18     double t=A[l][e], p=c[e];
19     b[l]=b[l]/t;
20     for(int i=1; i<=C; i++)
21         A[l][i]/=t;
22     V=V-c[e]*b[l];
23     for(int i=1; i<=R; i++) {
24         if(i==l || fcmp(A[i][e], 0.0)==0)
25             continue;
26         t=A[i][e];
27         b[i]=b[i]-t*b[l];
28         for(int j=1; j<=C; j++)
29             A[i][j]=A[i][j]-t*A[l][j];
30     }
31     for(int i=1; i<=C; i++)
32         c[i]=c[i]-p*A[l][i];
33     for(int i=1; i<=nCnt; i++) {
34         if(N[i]==e) {
35             N[i]=B[l];
36             break;
37         }
38     }
39     B[l]=e;
40 }
41
42 bool Process(double P[]) {
43     while(true) {

```

```

44     int e=-1;
45     double mV=-INF;
46     for(int i=1; i<=nCnt; i++)
47         if (fcmp(P[N[i]],mV)==1)
48             mV=P[N[i]],e=N[i];
49
50     if (fcmp(mV,0.0)<=0) break;
51     int l=-1;
52     mV=INF;
53     for(int i=1; i<=bCnt; i++) {
54         if (fcmp(A[i][e],0.0)==1) {
55             double t=b[i]/A[i][e];
56             if (fcmp(mV,t)==1|| (fcmp(mV,t)==0&&(l==-1||B[l]>B[i])))
57                 mV=t,l=i;
58         }
59     }
60     if (l==-1) return false;
61     Pivot(l,e);
62 }
63 return true;
64 }
65
66 bool initSimplex() {
67     nCnt=bCnt=0;
68     for(int i=1; i<=n; i++)
69         N[++nCnt]=i;
70     for(int i=1; i<=m; i++)
71         B[++bCnt]=i+n,A[i][n+i]=1.0;
72     R=bCnt,C=bCnt+nCnt;
73     double minV=INF;
74     int p=-1;
75     for(int i=1; i<=m; i++)
76         if (fcmp(minV,b[i])==1)
77             minV=b[i],p=i;
78     if (fcmp(minV,0.0)>=0)
79         return true;
80     N[++nCnt]=n+m+1;
81     R++,C++;
82     for(int i=0; i<=C; i++)
83         A[R][i]=0.0;
84     for(int i=1; i<=R; i++)
85         A[i][n+m+1]=-1.0;
86     Pivot(p,n+m+1);
87     if (!Process(A[R])) return false;
88     if (fcmp(b[R],0.0)!=0)
89         return false;
90     p=-1;
91     for(int i=1; i<=bCnt&& p==-1; i++)
92         if (B[i]==n+m+1) p=i;
93     if (p!=-1) {
94         for(int i=1; i<=nCnt; i++) {
95             if (fcmp(A[p][N[i]],0.0)!=0) {
96                 Pivot(p,N[i]);
97                 break;
98             }
99         }
100     }
101     bool f=false;
102     for(int i=1; i<=nCnt; i++) {
103         if (N[i]==n+m+1) f=true;
104         if (f&& i+1<=nCnt)
105             N[i]=N[i+1];
106     }
107     nCnt--;

```



```

108 R--,C--;
109 return true;
110 }
111
112 // -1: no solution 1: no bound 0: has a solution -V
113 int Simplex() {
114     if(!initSimplex())
115         return -1;
116     if(!Process(c))
117         return 1;
118     for(int i=1; i<=nCnt; i++)
119         X[N[i]]=0.0;
120     for(int i=1; i<=bCnt; i++)
121         X[B[i]]=b[i];
122     return 0;
123 }
124
125 int main() {
126     //n = 1;m=1;
127     //V= 0.0;
128     //c[1] = 1.0;
129     //A[1][1] = 1.0;
130     //b[1] = 5.0;
131     //Simplex();
132     //printf(" V = %.3f\n",V);
133
134     while(scanf("%d",&v1[1]) == 1) {
135         for(int i = 2; i<=6; i++)
136             scanf("%d",&v1[i]);
137         n = 4;
138         m = 6;
139         for(int i = 0 ; i<=m+1; i++)
140             for(int j=0; j<=n+m+2; j++)
141                 A[i][j] = c[j] = 0;
142         memset(b,0,sizeof(b));
143         V = 0.0;
144         /*
145         n 为未知数个数
146         m 为约束个数
147         目标：siana(c[i]*xi)
148         约束：sigma(A[i][j]*xj) <=b[i]; j = 1 ... n
149         解存在X里面
150         */
151         b[1] = v1[1] ;
152         A[1][1] = 1;
153         A[1][4] = 1;
154         b[2] = v1[2] ;
155         A[2][1] = 1;
156         A[2][3] = 1;
157         b[3] = v1[3] ;
158         A[3][3] = 1;
159         A[3][4] = 1;
160         b[4] = v1[4] ;
161         A[4][2] = 1;
162         A[4][3] = 1;
163         b[5] = v1[5] ;
164         A[5][2] = 1;
165         A[5][4] = 1;
166         b[6] = v1[6] ;
167         A[6][1] = 1;
168         A[6][2] = 1;
169         c[1] = 1;
170         c[2] = 1;

```

```

171     c[3] = 1;
172     c[4] = 1;
173     Simplex();
174     //printf(" V = %.3f\n",V);
175     printf(" %.3f_%.3f_%.3f_%.3f\n",X[1],X[2],X[3],X[4]);
176
177 }
178 return 0;
179 }

```

### 5.15 Factorization prime number $p$ into $x^2 + y^2$

```

1  #include <stdio.h>
2  #include <string.h>
3  #include <stdlib.h>
4  int p,expp,A,B,aa,ans,tt;
5  long long M;
6  long long exp(int a,int b,long long mod) {
7      long long ans=1,num=a;
8      while (b!=0) {
9          if (b&1) {
10             ans=((ans%mod)*(num%mod))%mod;
11         }
12         num=((num%mod)*(num%mod))%mod;
13         b>>=1;
14     }
15     return ans;
16 }
17 int calcu(int p,int &x,int &y) {
18     if (p%4!=1) return -1;
19     else {
20         expp=(p-1)/4;
21         A,B;
22         while (1) {
23             aa=rand()%p;
24             if (aa==0) continue;
25             A=exp(aa,expp,p);
26             ans=((long long)A%p)*((long long)A%p)%p;
27             if (ans==p-1) break;
28         }
29         B=1;
30         M=((long long)A*(long long)A+(long long)B*(long long)B)/p;
31         if (M!=1) B=p;
32         while (M!=1) {
33             if (B>A) {
34                 tt=A;
35                 A=B;
36                 B=tt;
37             }
38             tt=A;
39             A=B;
40             B=tt%B;
41             M=((long long)A*(long long)A
42                +(long long)B*(long long)B)/p;
43         }
44         if (B<=A) {
45             x=B;
46             y=A;
47         } else {
48             x=A;
49             y=B;
50         }

```

```

51 | }
52 | }
53 | int main() {
54 |     while (scanf("%d", &p) != EOF) {
55 |         int x, y;
56 |         if (calcu(p, x, y) != -1)
57 |             ;
58 |         return 0;
59 |     }

```

### 5.16 Partition ways of an integer

$O(n\sqrt{n})$

```

1 | #include <cstdio>
2 | #include <cmath>
3 | #include <cstring>
4 | #include <map>
5 | #include <algorithm>
6 | using namespace std;
7 | bool check(int x) {
8 |     for (int i=2; i*i<=x; i++)
9 |         if (x%i==0)
10 |             return 0;
11 |     return 1;
12 | }
13 | int p[1000000];
14 | inline int calc(int x) {
15 |     return x*(x*3-1)/2;
16 | }
17 | int main() {
18 |     p[0]=1;
19 |     for (int i=1; i<1000000; i++) {
20 |         for (int j=1, k=1; calc(j)<=i; j++, k*=-1) {
21 |             p[i]+=k*p[i-calc(j)];
22 |             if (p[i]<0)
23 |                 p[i]+=1000000;
24 |             if (p[i]>=1000000)
25 |                 p[i]-=1000000;
26 |             if (calc(-j)<=i)
27 |                 p[i]+=k*p[i-calc(-j)];
28 |             if (p[i]<0)
29 |                 p[i]+=1000000;
30 |             if (p[i]>=1000000)
31 |                 p[i]-=1000000;
32 |         }
33 |         if (!p[i])
34 |             printf("%d\n", i);
35 |     }
36 |     return 0;
37 | }

```

### 5.17 Pell's equation

```

1 | import java.math.BigInteger;
2 | import java.util.*;
3 | public class Main {
4 |     public static class Fraction {
5 |         public BigInteger num, den;
6 |         public Fraction() {
7 |             num=BigInteger.ZERO;
8 |             den=BigInteger.ONE;

```

```

9      }
10     public Fraction(int _num,int _den) {
11         num=BigInteger.valueOf(_num);
12         den=BigInteger.valueOf(_den);
13     }
14     public Fraction(BigInteger _num,BigInteger _den) {
15         num=_num;
16         den=_den;
17     }
18     public Fraction gen() {
19         BigInteger g=num.gcd(den);
20         return new Fraction(num.divide(g),den.divide(g));
21     }
22     public Fraction add(Fraction x) {
23         return new Fraction(x.num.multiply(den).add(num.multiply(x.den)),x.den.multiply
            (den)).gen();
24     }
25     public Fraction reciprocal() {
26         return new Fraction(den,num);
27     }
28     public void out() {
29         System.out.println(num+" / " +den);
30     }
31 }
32 public static BigInteger sqrt(BigInteger a) {
33     BigInteger b=a;
34     while (a.compareTo(b.multiply(b))<0)
35         b=b.multiply(b).add(a).divide(b.multiply(BigInteger.valueOf(2)));
36     return b;
37 }
38 public static boolean check(Fraction x,int n) {
39     return x.num.multiply(x.num).add(x.den.multiply(x.den.multiply(BigInteger.valueOf
        (n))))).negate().compareTo(BigInteger.ONE)==0;
40 }
41 static int p[]=new int[1000];
42 static int l;
43 public static void main(String[] args) {
44     BigInteger ans=BigInteger.ZERO;
45     int idx=0;
46     for (int n=2,r=2; n<=1000; n++) {
47         if (n==r*r) {
48             r++;
49             continue;
50         }
51         int tmp=calc(n,0,1),a=tmp,b=n-tmp*tmp;
52         p[0]=tmp;
53         l=1;
54         while (true) {
55             tmp=calc(n,a,b);
56             p[l++]=tmp;
57             a=a-tmp*b;
58             Fraction x=getFrac();
59             if (check(x,n)) {
60                 if (ans.compareTo(x.num)<0) {
61                     ans=x.num;
62                     idx=n;
63                 }
64                 break;
65             }
66             a=-a;
67             b=(n-a*a)/b;
68         }
69     }

```

```

70     System.out.println(idx);
71 }
72 private static Fraction getFrac() {
73     Fraction ret=new Fraction(p[l-1],1);
74     for (int i=l-2; i>=0; i--)
75         ret=new Fraction(p[i],1).add(ret.reciprocal());
76     return ret;
77 }
78 private static int calc(int n, int a, int b) {
79     for (long i=2;; i++)
80         if ((i*b-a)*(i*b-a)>n)
81             return (int)i-1;
82 }
83 }

```

### 5.18 Polya

设  $G$  是  $p$  个对象的一个置换群，用  $k$  种颜色去染这  $p$  个对象，若一种染色方案在群  $G$  的作用下变为另一种方案，则这两个方案当作是同一种方案，这样的不同染色方案数为：

$$L = \frac{1}{|G|} \times \sum (k^{C(f)}), f \in G$$

$C(f)$  为循环节， $|G|$  表示群的置换方法数

对于有  $n$  个位置的手镯，有  $n$  种旋转置换和  $n$  种翻转置换

对于旋转置换：

$$C(f_i) = \gcd(n, i), i \text{ 表示一次转过 } i \text{ 颗宝石}, i = 0 \text{ 时 } c = n;$$

对于翻转置换：

如果  $n$  为偶数：则有  $\frac{n}{2}$  个置换  $C(f) = \frac{n}{2}$ ，有  $\frac{n}{2}$  个置换  $C(f) = \frac{n}{2} + 1$

如果  $n$  为奇数： $C(f) = \frac{n}{2} + 1$

### 5.19 拉格朗日插值法

已知  $y = a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1}$  曲线上的  $n$  个点  $(x_1, y_1), (x_2, y_2), (x_3, y_3) \cdots (x_n, y_n)$  用拉格朗日插值法可以不求系数可知任意  $x$  对应的  $y$  值。

$$\begin{aligned}
 y = & y_1 \frac{(x-x_2)(x-x_3) \cdots (x-x_n)}{(x_1-x_2)(x_1-x_3) \cdots (x_1-x_n)} \\
 & + y_2 \frac{(x-x_1)(x-x_3) \cdots (x-x_n)}{(x_2-x_1)(x_2-x_3) \cdots (x_2-x_n)} \\
 & + \cdots \\
 & + y_n \frac{(x-x_1)(x-x_2) \cdots (x-x_{n-1})}{(x_n-x_1)(x_n-x_2) \cdots (x_n-x_{n-1})}
 \end{aligned}$$

特别的，如果  $x_1 \sim x_n$  为连续自然数，那么对于下一个自然数对应的  $y$  值为：

$$y_{n+1} = (-1)^{n-1} C_n^0 y_1 + (-1)^{n-2} C_n^1 y_2 + \cdots + (-1)^0 C_n^{n-1} y_n$$

这个组合系数可以通过高斯消元暴出来，前提是要猜到它满足递推关系。

## 5.20 正多面体顶点着色

$$\text{正四面体: } N = \frac{(n^4 + 11 \times n^2)}{12}$$

$$\text{正六面体: } N = \frac{(n^8 + 17 \times n^4 + 6 \times n^2)}{24}$$

$$\text{正八面体: } N = \frac{(n^6 + 3 \times n^4 + 12 \times n^3 + 8 \times n^2)}{24}$$

$$\text{正十二面体: } N = \frac{(n^{20} + 15 \times n^{10} + 20 \times n^8 + 24 \times n^4)}{60}$$

$$\text{正二十面体: } N = \frac{(n^{12} + 15 \times n^6 + 44 \times n^4)}{60}$$

## 5.21 求和公式

$$\sum k = \frac{n \times (n+1)}{2}$$

$$\sum 2k - 1 = n^2$$

$$\sum k^2 = \frac{n \times (n+1) \times (2n+1)}{6}$$

$$\sum (2k - 1)^2 = \frac{n \times (4n^2 - 1)}{3}$$

$$\sum k^3 = \left( \frac{n \times (n+1)}{2} \right)^2$$

$$\sum (2k - 1)^3 = n^2 \times (2n^2 - 1)$$

$$\sum k^4 = \frac{n \times (n+1) \times (2n+1) \times (3n^2 + 3n - 1)}{30}$$

$$\sum k^5 = \frac{n^2 \times (n+1)^2 \times (2n^2 + 2n - 1)}{12}$$

$$\sum k \times (k+1) = \frac{n \times (n+1) \times (n+2)}{3}$$

$$\sum k \times (k+1) \times (k+2) = \frac{n \times (n+1) \times (n+2) \times (n+3)}{4}$$

$$\sum k \times (k+1) \times (k+2) \times (k+3) = \frac{n \times (n+1) \times (n+2) \times (n+3) \times (n+4)}{5}$$

$$\sum i \times \binom{n}{i} = n \times 2^{n-1} \quad (\text{子集长度和})$$

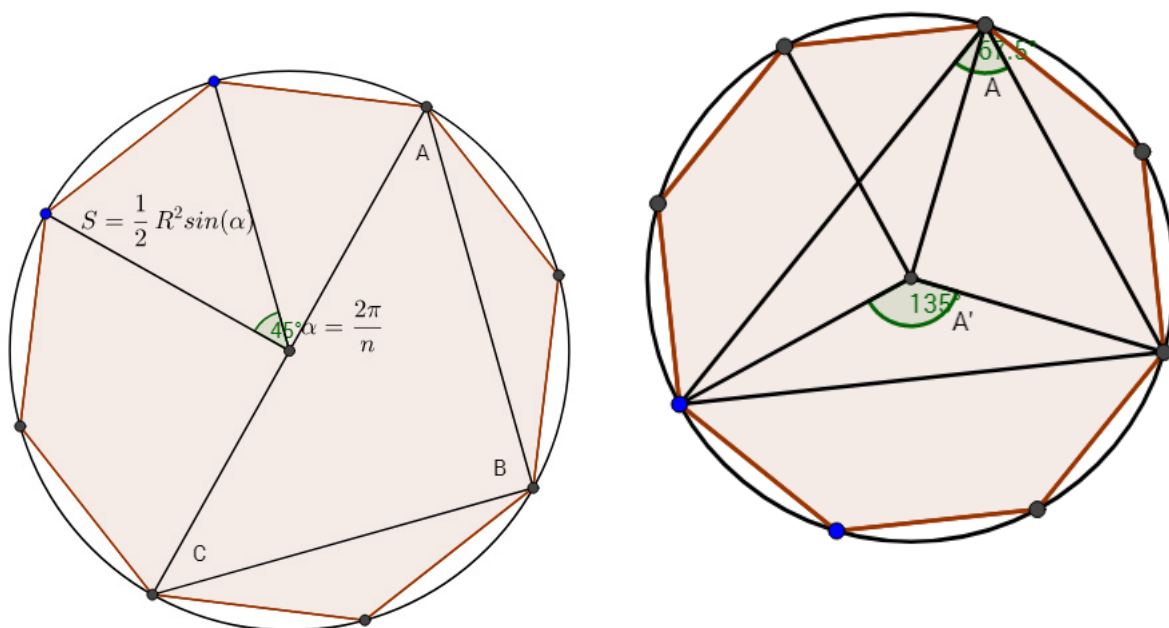
$$\sum_{i=1}^n \left\lfloor \frac{n}{i} \right\rfloor = \sum_{i=1}^n d(i), d(n) \text{ 是 } n \text{ 的正因子数目 (埃筛)}$$

$$[x=1] = \sum_{d|x} \mu(d)$$

## 5.22 几何公式

$$n\text{-Polygon: } \frac{n}{2} * R^2 * \sin\left(\frac{2\pi}{n}\right)$$

已知任意三点求边数最少的 (同时也是面积最小) 正多边形: 由于  $A = \frac{A'}{2}$  所以有  $A = k \frac{\alpha}{2}$ , 结论:  $\alpha = 2gcd(A, B, C)$  其中 A, B, C 为三角形内角



球扇形:

全面积:  $T = \pi r(2h + r_0)$ ,  $h$  为球冠高,  $r_0$  为球冠底面半径

体积： $V = \frac{2\pi r^2 h}{3}$

### 5.23 小公式

Pick 公式： $A = E \times 0.5 + I - 1$  ( $A$  是多边形面积， $E$  是边界上的整点， $I$  是多边形内部的整点)

海伦公式： $S = \sqrt{p(p-a)(p-b)(p-c)}$ ，其中  $p = \frac{(a+b+c)}{2}$ ， $abc$  为三角形的三条边长

求  $\binom{n}{k}$  中素因子  $P$  的个数：

1. 把  $n$  转化为  $P$  进制，并记它每个位上的和为  $S1$

2. 把  $n - k$ ， $k$  做同样的处理，得到  $S2$ ， $S3$

则  $\binom{n}{k}$  中素因子  $P$  的个数： $\frac{S2+S3-S1}{P-1}$

部分错排公式：

$n + m$  个数中  $m$  个数必须错排求排列数

```
1 dp[i] = n*dp[i-1]+(i-1)*(dp[i-1]+dp[i-2]);
2 dp[0] = n!;
3 dp[1] = n*n!;
```

$dp[m]$  为所求解

## 6 Search

### 6.1 Dancing links

```
1 struct DLX {
2     int h,n,m,tot;
3     int U[MaxN*MaxM],D[MaxN*MaxM],L[MaxN*MaxM],R[MaxN*MaxM],Row[MaxN*MaxM],Col[MaxN*
4         MaxM];
5     int S[MaxM],O[MaxN];
6     bool hasans;
7     void init() {
8         h = 0;
9         hasans = false;
10        tot = m+n;
11        for (int i = 0; i <= m; i++) {
12            D[i] = U[i] = Col[i] = i;
13            Row[i] = S[i] = 0;
14            L[i] = (i+m)%(m+1);
15            R[i] = (i+1)%(m+1);
16        }
17        for (int i = 1; i <= n; i++) {
18            R[i+m] = L[i+m] = i+m;
19            Row[i+m] = i;
20            Col[i+m] = 0;
21        }
22    }
23    void insert(int x,int y) {
24        tot++;
25        Row[tot] = x;
26        Col[tot] = y;
27        S[y]++;
28        int colPos,rowPos;
```

```

29     while (true) {
30         colPos = D[colPos];
31         if (colPos == y || Row[colPos] > x) break;
32     }
33     colPos = U[colPos];
34     if (Row[colPos] == x) return;
35     U[tot] = colPos;
36     D[tot] = D[colPos];
37     U[D[tot]] = D[U[tot]] = tot;
38     rowPos = x+m;
39     while (true) {
40         rowPos = R[rowPos];
41         if (rowPos == x+m || Col[rowPos] > y) break;
42     }
43     rowPos = L[rowPos];
44     if (Col[rowPos] == y) return;
45     L[tot] = rowPos;
46     R[tot] = R[rowPos];
47     L[R[tot]] = R[L[tot]] = tot;
48 }
49 void print(int deep) {
50     for (int i = 0; i < deep; i++)
51         printf("%d_", O[i]);
52     printf("\n");
53 }
54 void cover(int col) {
55     L[R[col]] = L[col];
56     R[L[col]] = R[col];
57     for (int i = D[col]; i != col; i = D[i])
58         for (int j = R[i]; j != i; j = R[j])
59             if (Col[j] != col) {
60                 U[D[j]] = U[j];
61                 D[U[j]] = D[j];
62                 S[Col[j]]--;
63             }
64 }
65 void resume(int col) {
66     for (int i = U[col]; i != col; i = U[i])
67         for (int j = L[i]; j != i; j = L[j])
68             if (Col[j] != col) {
69                 S[Col[j]]++;
70                 U[D[j]] = j;
71                 D[U[j]] = j;
72             }
73     L[R[col]] = col;
74     R[L[col]] = col;
75 }
76 void initDFS() {
77     for (int i = 1; i <= n; i++) {
78         L[R[i+m]] = L[i+m];
79         R[L[i+m]] = R[i+m];
80     }
81 }
82 void DFS(int deep) {
83     if (hasans == true) return;
84     if (R[0] == 0) {
85         hasans = true;
86         print(deep);
87         return;
88     };
89     int tc = R[0];
90     for (int i = R[0]; i != 0; i = R[i])
91         if (S[i] < S[tc]) tc = i;

```



```

92     cover(tc);
93     for (int i = D[tc]; i != tc; i = D[i]) {
94         int temp = O[deep];
95         O[deep] = Row[i];
96         for (int j = R[i]; j != i; j = R[j])
97             cover(Col[j]);
98         DFS(deep+1);
99         for (int j = L[i]; j != i; j = L[j])
100             resume(Col[j]);
101         O[deep] = temp;
102     }
103     resume(tc);
104 }
105 }

```

### 6.1.1 Usage

```

1 DLX g;
2 g.n = ROW_SIZE;
3 g.m = COL_SIZE;
4 g.init();
5 g.insert(ROW, COL);
6 g.initDFS();
7 g.DFS(0);

```

## 6.2 Dancing links (A-star)

```

1 namespace DLX {
2     const int MAXN = 1000;
3     const int MAXM = 400;
4     const int INF = 0x3f3f3f3f;
5     int D[MAXN * MAXM], U[MAXN * MAXM], L[MAXN * MAXM], R[MAXN * MAXM], COL[MAXN * MAXM],
        ROW[MAXN * MAXM];
6     int CNT, BEG[MAXN * MAXM], END[MAXN * MAXM], ANS, USE[MAXM], _USE[MAXM];
7     int SUM[MAXM];
8     bool vis[MAXM];
9     void init(int n) {
10         memset(BEG, 0xff, sizeof(BEG));
11         for (int i = 1; i <= n; i++)
12             SUM[L[i + 1] = R[i - 1] = D[i] = U[i] = i] = 0;
13         L[L[1] = R[n] = 0] = n, CNT = n + 1;
14         ANS = n + 1;
15     }
16     void link(int r, int c) {
17         D[CNT] = D[c], U[CNT] = c, U[D[c]] = CNT, D[c] = CNT, COL[CNT] = c, ROW[CNT] = r,
            SUM[c]++;
18         if (BEG[r] == -1) BEG[r] = END[r] = CNT;
19         R[END[r]] = CNT, L[CNT] = END[r], R[CNT] = BEG[r], L[BEG[r]] = CNT, END[r] = CNT++;
20     }
21     void DLX_Remove_Repeat(int c) {
22         for (int i = D[c]; i != c; i = D[i])
23             L[R[i]] = L[i], R[L[i]] = R[i], SUM[COL[i]]--;
24     }
25     void DLX_Resume_Repeat(int c) {
26         for (int i = U[c]; i != c; i = U[i])
27             L[R[i]] = i, R[L[i]] = i, SUM[COL[i]]++;
28     }
29     int Heuristics() {
30         memset(vis, true, sizeof(vis));
31         int c, i, j, cnt=0;
32         for (c=R[0]; c; c=R[c])

```

```

33     if(vis[c])
34         for(cnt++, vis[c] = false, i = D[c]; i != c; i = D[i])
35             for(j = R[i]; j != i; j = R[j])
36                 vis[COL[j]] = false;
37     return cnt;
38 }
39 void DLX_Dfs(int n) {
40     if (Heuristics() + n >= ANS) return;
41     if (R[0] == 0) {
42         ANS = n;
43         for (int i = 0; i < n; i++)
44             USE[i] = _USE[i];
45         return ;
46     }
47     int i,now = INF,c;
48     for (i = R[0]; i; i = R[i])
49         if (now > SUM[i])
50             now = SUM[c = i];
51     for(i = D[c]; i != c; i = D[i]) {
52         DLX_Remove_Repeat(i);
53         for(int j = R[i]; j != i; j = R[j])
54             DLX_Remove_Repeat(j);
55         _USE[n] = ROW[i];
56         DLX_Dfs(n + 1);
57         for(int j = L[i]; j != i; j = L[j])
58             DLX_Resume_Repeat(j);
59         DLX_Resume_Repeat(i);
60     }
61 }
62 void solve() {
63     //ANS = m
64     DLX_Dfs(0);
65 }
66 };

```

## 7 String

### 7.1 Aho-Corasick automation

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int charsize = 26;
4  const int maxn = 500000;
5  struct Node { // you need to modify a lot ... before using it
6      int tot;
7      int root;
8      int next[maxn][charsize];
9      int fail[maxn];
10     int end[maxn]; // the cont of the word
11
12     inline int getid(const char& c) {}
13     int newnode() {
14         for (int i = 0; i < charsize; ++i) {
15             next[tot][i] = -1;
16         }
17         end[tot++] = 0;
18         return tot - 1;
19     }
20
21     void init() {
22         tot = 0;
23         root = newnode();

```

```

24 | }
25 |
26 | void insert(const char* str) {
27 |     int now = root;
28 |     while(*str) {
29 |         int charid = getid(*str);
30 |         if (next[now][charid] == -1) next[now][charid] = newnode();
31 |         now = next[now][charid];
32 |         ++str;
33 |     }
34 |     ++end[now];
35 | }
36 |
37 | void build() {
38 |     queue<int> q;
39 |     fail[root] = root;
40 |     for (int i = 0; i < charsize; ++i) {
41 |         if (next[root][i] == -1) {
42 |             next[root][i] = root;
43 |         } else {
44 |             fail[next[root][i]] = root;
45 |             q.push(next[root][i]);
46 |         }
47 |     }
48 |
49 |     while (!q.empty()) {
50 |         int now = q.front(); q.pop();
51 |         for (int i = 0; i < charsize; ++i) {
52 |             if (next[now][i] == -1) {
53 |                 next[now][i] = next[fail[now]][i];
54 |             } else {
55 |                 fail[next[now][i]] = next[fail[now]][i];
56 |                 q.push(next[now][i]);
57 |             }
58 |         }
59 |     }
60 | }
61 |
62 | int solve(const char* str) {
63 |     int ret = 0, k = 0;
64 |     while (*str) {
65 |         int charid = getid(*str);
66 |         k = next[k][charid];
67 |         int j = k;
68 |         while (j) {
69 |             ret += end[j];
70 |             end[j] = 0;
71 |             j = fail[j];
72 |         }
73 |         ++str;
74 |     }
75 |     return ret;
76 | }
77 |
78 | } AC;

```

## 7.2 KMP

Match the suffix of  $A[\cdots i]$  and the prefix of  $B$

```

1 | int fail[maxn];
2 | void get_fail(const string& t) {
3 |     fail[0] = -1;

```

```

4   int n = t.size();
5   int j = 0;
6   int k = -1;
7   while (j < n) {
8       if (k == -1 || t[j] == t[k]) {
9           fail[++j] = ++k;
10      } else {
11          k = fail[k];
12      }
13  }
14  }
15  int KMP(const string& s, const string& t) {
16      get_fail(t);
17      int sn = s.size();
18      int tn = t.size();
19      int i = 0;
20      int j = 0;
21      while (i < sn && j < tn) {
22          if (j == -1 || s[i] == t[j]) {
23              ++i; ++j;
24          } else {
25              j = fail[j];
26          }
27      }
28      if (j == tn) return i - tn + 1;
29      else return -1;
30  }

```

### 7.3 Extend-KMP

Common prefix of  $A[i \dots]$  and  $B$

```

1  //Self match
2  int j = 0;
3  while (j < lb && b[j] == b[j + 1])
4      j++;
5  p[0] = lb, p[1] = j;
6  int k = 1;
7  for (int i = 2; i < lb; i++) {
8      int Len = k + p[k] - 1, L = p[i - k];
9      if (L < Len - i + 1)
10         p[i] = L;
11     else {
12         j = max(0, Len - i + 1);
13         while (i + j < lb && b[i + j] == b[j])
14             j++;
15         p[i] = j, k = i;
16     }
17 }
18 //Match
19 j = 0;
20 while (j < la && j < lb && a[j] == b[j])
21     j++;
22 eKMP[0] = j;
23 k = 0;
24 for (int i = 1; i < la; i++) {
25     int Len = k + eKMP[k] - 1, L = p[i - k];
26     if (L < Len - i + 1)
27         eKMP[i] = L;
28     else {
29         j = max(0, Len - i + 1);
30         while (i + j < la && j < lb && a[i + j] == b[j])
31             j++;

```

```

32 |     eKMP[i] = j, k = i;
33 | }
34 | }

```

## 7.4 Manacher

```

1 | const int maxn = 110000;
2 |
3 | char Ma[maxn*2];
4 | int Mp[maxn*2];
5 | void Manacher(char s[],int len) {
6 |     int l = 0;
7 |     Ma[l++] = ' ' ;
8 |     Ma[l++] = ' ' ;
9 |     for (int i = 0; i < len; i++) {
10 |         Ma[l++] = s[i];
11 |         Ma[l++] = ' ' ;
12 |     }
13 |     Ma[l] = 0;
14 |     int pnow = 0,pid = 0;
15 |     for (int i = 1; i < l; i++) {
16 |         if (pnow > i)
17 |             Mp[i] = min(Mp[2*pid-i],pnow-i);
18 |         else
19 |             Mp[i] = 1;
20 |         for (; Ma[i-Mp[i]] == Ma[i+Mp[i]]; Mp[i]++);
21 |         if (i+Mp[i] > pnow) {
22 |             pnow = i+Mp[i];
23 |             pid = i;
24 |         }
25 |     }
26 | }
27 | /*
28 | abaaba
29 | .   ,   a   ,   b   ,   a   ,   a   ,   b   ,   a   ,
30 | 0   1   2   3   4   5   6   7   8   9   10  11  12  13
31 | */

```

## 7.5 Suffix array

```

1 | const int maxn = 200010;
2 | int wx[maxn],wy[maxn],*x,*y,wss[maxn],wv[maxn];
3 |
4 | bool cmp(int *r,int n,int a,int b,int l) {
5 |     return a+l<n && b+l<n && r[a]==r[b]&&r[a+l]==r[b+l];
6 | }
7 | void da(int str[],int sa[],int rank[],int height[],int n,int m) {
8 |     int *s = str;
9 |     int *x=wx,*y=wy,*t,p;
10 |    int i,j;
11 |    for(i=0; i<m; i++)wss[i]=0;
12 |    for(i=0; i<n; i++)wss[x[i]=s[i]]++;
13 |    for(i=1; i<m; i++)wss[i]=wss[i-1];
14 |    for(i=n-1; i>=0; i--)sa[--wss[x[i]]]=i;
15 |    for(j=1,p=1; p<n && j<n; j*=2,m=p) {
16 |        for(i=n-j,p=0; i<n; i++)y[p++]=i;
17 |        for(i=0; i<n; i++)if(sa[i]-j>=0)y[p++]=sa[i]-j;
18 |        for(i=0; i<n; i++)wv[i]=x[y[i]];
19 |        for(i=0; i<m; i++)wss[i]=0;
20 |        for(i=0; i<n; i++)wss[wv[i]]++;
21 |        for(i=1; i<m; i++)wss[i]=wss[i-1];

```

```

22     for(i=n-1; i>=0; i--)sa[--wss[wv[i]]]=y[i];
23     for(t=x,x=y,y=t,p=1,i=1,x[sa[0]]=0; i<n; i++)
24         x[sa[i]]=cmp(y,n,sa[i-1],sa[i],j)?p-1:p++;
25 }
26 for(int i=0; i<n; i++) rank[sa[i]]=i;
27 for(int i=0,j=0,k=0; i<n; height[rank[i++]]=k)
28     if(rank[i]>0)
29         for(k?k--:0,j=sa[rank[i]-1];
30             i+k < n && j+k < n && str[i+k]==str[j+k];
31             k++);
32 }

```

### 7.5.1 Longest common prefix

```

1 int lcp(int x,int y) {
2     if (x > y) swap(x,y);
3     if (x == y)
4         return len-sa[x]; //NOTICE!
5     x++;
6     int k = lent[y-x+1];
7     return min(f[x][k],f[y-(1<<k)+1][k]);
8 }
9 //Interval
10 void getinterval(int pos,int comlen,int& pl,int& pr) {
11     int l,r,mid,cp;
12     l = 0;
13     r = pos;
14     while (l < r) {
15         mid = l+r>>1;
16         cp = lcp(mid,pos);
17         if (cp < comlen)
18             l = mid+1;
19         else
20             r = mid;
21     }
22     pl = l;
23     l = pos;
24     r = len-1;
25     while (l < r) {
26         mid = l+r+1>>1;
27         cp = lcp(pos,mid);
28         if (cp < comlen)
29             r = mid-1;
30         else
31             l = mid;
32     }
33     pr = l;
34 }

```

## 7.6 Smallest representation

```

1 int Gao(char a[],int len) {
2     int i = 0,j = 1,k = 0;
3     while (i < len && j < len && k < len) {
4         int cmp = a[(j+k)%len]-a[(i+k)%len];
5         if (cmp == 0)
6             k++;
7         else {
8             if (cmp > 0)
9                 j += k+1;
10            else

```

```

11     i += k+1;
12     if (i == j) j++;
13     k = 0;
14 }
15 }
16 return min(i,j);
17 }

```

## 7.7 Hash

```

1  typedef long long ll;
2  typedef unsigned long long ull;
3  const int MAXN = 1000000 + 100;
4  //ll gao[MAXN];
5  ll BL, BR, ML, MR;
6  ull psl[MAXN],psr[MAXN];
7  //call this before everything
8  void init(){
9      int maxx = 1e9;
10     srand(time(0));
11     BL = (ll)maxx + rand() % maxx;
12     BR = (ll)maxx + rand() % maxx;
13     ML = (ll)maxx + rand() % maxx;
14     MR = (ll)maxx + rand() % maxx;
15 }
16 //n is the max length you need
17 void Hash2(int n){
18     for(int i = 0; i <= n ; i ++){
19         psl[i] = (i == 0 ? 1 : psl[i - 1] * BL) % ML;
20     }
21     for(int i = 0 ; i <= n; i ++){
22         psr[i] = (i == 0 ? 1 : psr[i - 1] * BR) % MR;
23     }
24 }
25 struct _hash{
26     //read your string in here.
27     char str[MAXN];
28     ull hs[MAXN];
29     void build(){
30         int n = strlen(str);
31         ll l = 0, r = 0;
32         for(int i = 0 ; i < n ; i ++){
33             l = (l * BL + str[i]) % ML;
34             r = (r * BR + str[i]) % MR;
35             if(l < 0) l+= ML;
36             if(r < 0) r += MR;
37             hs[i + 1] = (l << 32) | r;
38         }
39     }
40     //hash(str[b: e])
41     ll get(int b,int e){
42         ull el = (hs[e] >>32);
43         ull er = (hs[e] & 0xffffffffULL);
44         ull bl = (hs[b] >> 32);
45         ull br = (hs[b] & 0xffffffffULL);
46         ll l = el - bl * psl[e-b] % ML;
47         ll r = er - br *psr[e-b] % MR;
48         if(l < 0) l += ML;
49         if(r < 0) r += MR;
50         return (l << 32) | r;
51     }
52 }Hash;

```

## 8 Tool

### 8.1 Fast IO

```

1 // 加强版
2 const int MAXBUF = 10000;
3 char buf[MAXBUF], *ps = buf, *pe = buf + 1;
4 inline void rnext() {
5     if (++ps == pe)
6         pe = (ps = buf) +
7             fread(buf, sizeof(char), sizeof(buf) / sizeof(char), stdin);
8 }
9
10 template <class T>
11 inline bool in(T &ans) {
12     ans = 0;
13     T f = 1;
14     if (ps == pe) return false; // EOF
15     do {
16         rnext();
17         if ( '-' == *ps) f = -1;
18     } while (!isdigit(*ps) && ps != pe);
19     if (ps == pe) return false; // EOF
20     do {
21         ans = (ans << 1) + (ans << 3) + *ps - 48;
22         rnext();
23     } while (isdigit(*ps) && ps != pe);
24     ans *= f;
25     return true;
26 }
27
28 const int MAXOUT = 10000;
29 char bufout[MAXOUT], outtmp[50], *pout = bufout, *pend = bufout + MAXOUT;
30 inline void write() {
31     fwrite(bufout, sizeof(char), pout - bufout, stdout);
32     pout = bufout;
33 }
34 inline void out_char(char c) {
35     *(pout++) = c;
36     if (pout == pend) write();
37 }
38 inline void out_str(const char *s) {
39     while (*s) {
40         *(pout++) = *(s++);
41         if (pout == pend) write();
42     }
43 }
44 template <class T>
45 inline void out_int(T x) {
46     if (!x) {
47         out_char('0');
48         return;
49     }
50     if (x < 0) x = -x, out_char('-');
51     int len = 0;
52     while (x) {
53         outtmp[len++] = x % 10 + 48;
54         x /= 10;
55     }
56     outtmp[len] = 0;
57     for (int i = 0, j = len - 1; i < j; i++, j--) swap(outtmp[i], outtmp[j]);
58     out_str(outtmp);
59 }

```



```

60 // how to use
61 int main() {
62     int t, ca = 1;
63     in(t);
64     while (t--) {
65         int n;
66         in(n);
67
68         out_str(" Case_#" );
69         out_int(ca++);
70         out_str(" :_" );
71         out_int(n), out_char( '\n' );
72     }
73     write(); // 一定要记得 write() 啊!!!!
74     return 0;
75 }

```

## 8.2 日期函数

```

1 int days[12]={31,28,31,30,31,30,31,31,30,31,30,31};
2 struct date{
3     int year,month,day;
4 };
5 // 是否闰年
6 inline bool isleap(int year)
7 {
8     return (year%4==0&&year%100!=0)||year%400==0;
9 }
10 // 判合法性
11 inline bool islegal(date a){
12     if (a.month<0||a.month>12) return 0;
13     if (a.month==2)
14         return a.day>0&&a.day<=28+isleap(a.year);
15     return a.day>0&&a.day<=days[a.month-1];
16 }
17 // 比较日期大小, 正/负表示大于/小于, 0表示相等
18 // 如果用于sort等, 请把-改成<
19 inline int datecmp(date a,date b){
20     if (a.year!=b.year)
21         return a.year-b.year;
22     if (a.month!=b.month)
23         return a.month-b.month;
24     return a.day-b.day;
25 }
26 // 日期转天数偏移 ({0,1,1} 第1天)
27 int date2int(date a){
28     int ret=a.year*365+(a.year-1)/4-(a.year-1)/100+(a.year-1)/400,i;
29     days[1]+=isleap(a.year);
30     for (i=0;i<a.month-1;ret+=days[i++]);
31     days[1]=28;
32     return ret+a.day;
33 }
34 // 天数偏移转日期
35 date int2date(int a){
36     date ret;
37     ret.year=a/146097*400;
38     for (a%=146097;a>=365+isleap(ret.year);a-=365+isleap(ret.year),ret.year++);
39     days[1]+=isleap(ret.year);
40     for (ret.month=1;a>=days[ret.month-1];a-=days[ret.month-1],ret.month++);
41     days[1]=28;
42     ret.day=a+1;
43     return ret;

```

44 | }

### 8.3 Bit compression

```

1 | int bit[5];
2 | inline int getbit26(int sta, int pos) {
3 |     return sta / bit[pos] % bit[1];
4 | }
5 | inline int setbit26(int sta, int pos, int val) {
6 |     return sta / bit[pos + 1] * bit[pos + 1] + val * bit[pos] + sta % bit[pos];
7 | }
8 | //bin
9 | inline int getbit(int sta, int pos) {
10 |     return (sta >> pos) & 1;
11 | }
12 | inline int setbit(int sta, int pos, int val) {
13 |     return ((sta >> (pos + 1)) << (pos + 1)) | (val << pos) | (sta & ((1 << pos) - 1));
14 | }

```

### 8.4 Random

```

1 | // std::mt19937 rng_engine{randutils::auto_seed_128{}.base()};
2 | std::mt19937 rng_engine{(size_t)(new char)};
3 | std::uniform_int_distribution<int> dist{1, 1000}; //1-1000 inclusive
4 | int rand_integer = dist(rng_engine);

```

### 8.5 Hash map

```

1 | struct hash_map {
2 |     int head[MOD];
3 |     struct hash_tables {
4 |         int key1, key2;
5 |         long long val;
6 |         int next;
7 |     } ele[ELE];
8 |     int N;
9 |     int getHash(int key1, int key2) {
10 |         return (key1 * 1000000 + key2) % MOD;
11 |     }
12 |     void init() {
13 |         memset(head, -1, sizeof(head));
14 |         N = 0;
15 |     }
16 |     void clear() {
17 |         for (int i = 0; i < N; i++)
18 |             head[getHash(ele[i].key1, ele[i].key2)] = -1;
19 |         N = 0;
20 |     }
21 |     int fint(int key1, int key2) {
22 |         for (int i = head[getHash(key1, key2)]; i != -1; i = ele[i].next) {
23 |             if (ele[i].key1 == key1 && ele[i].key2 == key2)
24 |                 return i;
25 |         }
26 |         return -1;
27 |     }
28 |     void insert(int key1, int key2) {
29 |         int tmp = getHash(key1, key2);
30 |         ele[N].key1 = key1;
31 |         ele[N].key2 = key2;
32 |         ele[N].val = 0;

```

```

33     ele[N].next = head[tmp];
34     head[tmp] = N++;
35 }
36 long long get(int key1, int key2) {
37     int tmp = fint(key1, key2);
38     if (tmp == -1) {
39         insert(key1, key2);
40         return ele[N - 1].val;
41     } else
42         return ele[tmp].val;
43 }
44 void set(int key1, int key2, long long val) {
45     int tmp = fint(key1, key2);
46     if (tmp == -1) {
47         insert(key1, key2);
48         ele[N - 1].val = val;
49     } else
50         ele[tmp].val = val;
51 }
52 void add(int key1, int key2, long long val) {
53     int tmp = fint(key1, key2);
54     if (tmp == -1) {
55         insert(key1, key2);
56         ele[N - 1].val += val;
57     } else
58         ele[tmp].val += val;
59 }
60 };

```

## 8.6 树链剖分

```

1 // with Segtree or splay...
2 void dfs(int u, int p, int d) {
3     sz[u] = 1;
4     fa[u] = p;
5     deep[u] = d;
6     for (int i = head[u]; i != -1; i = edge[i].next) {
7         int v = edge[i].v;
8         if (v == p) continue;
9         dfs(v, u, d + 1);
10        sz[u] += sz[v];
11        if (son[u] == -1 || sz[v] > sz[son[u]]) son[u] = v;
12    }
13 }
14
15 void link(int u, int first) {
16     top[u] = first;
17     tid[u] = ++tot;
18     arc[tid[u]] = u;
19     if (son[u] != -1) link(son[u], first);
20     for (int i = head[u]; i != -1; i = edge[i].next) {
21         int v = edge[i].v;
22         if (v == fa[u] || v == son[u]) continue;
23         link(v, v);
24     }
25 }
26 int solve(int u, int v) {
27     int ret = 0;
28     while (top[u] != top[v]) {
29         if (deep[top[u]] < deep[top[v]]) swap(u, v);
30         ret += st.query(id[top[u]], id[u] + 1);
31         u = fa[top[u]];
32     }

```

```

33 | if (u == v) return ret; // 取决是否边权下放
34 | if (deep[u] > deep[v]) swap(u, v);
35 | ret += st.query(id[son[u]], id[v] + 1);
36 | return ret;
37 | }

```

## 8.7 单调栈

```

1 | for (int i = 0; i < n; i++) l[i] = r[i] = i;
2 | for (int i = 1; i < n; i++) {
3 |     int now = i;
4 |     while (now >= 1 && a[i] <= a[now - 1]) now = l[now - 1];
5 |     l[i] = now;
6 | }
7 | for (int i = n - 2; i >= 0; i--) {
8 |     int now = i;
9 |     while (now < n - 1 && a[i] <= a[now + 1]) now = r[now + 1];
10 |    r[i] = now;
11 | }

```

## 8.8 单调队列

```

1 | struct Deque {
2 |     int val, idx;
3 |     Deque(int v = 0, int x = 0) : val(v), idx(x) {}
4 | } Q[maxn];
5 | int head, tail;
6 | vector<int> Max, Min;
7 | int n, k;
8 | void solve_min() {
9 |     Min.clear();
10 |    head = 1;
11 |    tail = 0;
12 |    for (int i = 1; i < k; ++i) {
13 |        while (head <= tail && Q[tail].val >= a[i]) {
14 |            --tail;
15 |        }
16 |        ++tail;
17 |        Q[tail].val = a[i];
18 |        Q[tail].idx = i;
19 |    }
20 |    for (int i = k; i <= n; ++i) {
21 |        while (head <= tail && Q[tail].val >= a[i]) {
22 |            --tail;
23 |        }
24 |        ++tail;
25 |        Q[tail].val = a[i];
26 |        Q[tail].idx = i;
27 |        while (head <= tail && Q[head].idx < i - k + 1) {
28 |            ++head;
29 |        }
30 |        Min.push_back(Q[head].val);
31 |    }
32 | }
33 | void solve_max() {
34 |     Max.clear();
35 |     head = 1;
36 |     tail = 0;
37 |     for (int i = 1; i < k; ++i) {
38 |         while (head <= tail && Q[tail].val <= a[i]) {
39 |             --tail;

```

```

40     }
41     ++tail;
42     Q[tail].val = a[i];
43     Q[tail].idx = i;
44 }
45 for (int i = k; i <= n; ++i) {
46     while (head <= tail && Q[tail].val <= a[i]) {
47         --tail;
48     }
49     ++tail;
50     Q[tail].val = a[i];
51     Q[tail].idx = i;
52     while (head <= tail && Q[head].idx < i - k + 1) {
53         ++head;
54     }
55     Max.push_back(Q[head].val);
56 }
57 }

```

## 8.9 Big-integer

```

1  #include <algorithm>
2  #include <cstdio>
3  #include <cstdlib>
4  #include <cstring>
5  #include <iostream>
6  #include <string>
7  using namespace std;
8
9  const int MAXN = 410;
10
11 struct bign {
12     int len, s[MAXN];
13     bign() {
14         memset(s, 0, sizeof(s));
15         len = 1;
16     }
17     bign(int num) { *this = num; }
18     bign(const char *num) { *this = num; }
19     bign operator=(const int num) {
20         char s[MAXN];
21         sprintf(s, "%d", num);
22         *this = s;
23         return *this;
24     }
25     bign operator=(const char *num) {
26         for (int i = 0; num[i] != '\0'; num++)
27             ; // ÆÇ°µ¼0
28         len = strlen(num);
29         for (int i = 0; i < len; i++) s[i] = num[len - i - 1] - '0';
30         return *this;
31     }
32     bign operator+(const bign &b) const //+
33     {
34         bign c;
35         c.len = 0;
36         for (int i = 0, g = 0; g || i < max(len, b.len); i++) {
37             int x = g;
38             if (i < len) x += s[i];
39             if (i < b.len) x += b.s[i];
40             c.s[c.len++] = x % 10;
41             g = x / 10;

```

```

42     }
43     return c;
44 }
45 bign operator+=(const bign &b) {
46     *this = *this + b;
47     return *this;
48 }
49 void clean() {
50     while (len > 1 && !s[len - 1]) len--;
51 }
52 bign operator*(const bign &b) // *
53 {
54     bign c;
55     c.len = len + b.len;
56     for (int i = 0; i < len; i++) {
57         for (int j = 0; j < b.len; j++) {
58             c.s[i + j] += s[i] * b.s[j];
59         }
60     }
61     for (int i = 0; i < c.len; i++) {
62         c.s[i + 1] += c.s[i] / 10;
63         c.s[i] %= 10;
64     }
65     c.clean();
66     return c;
67 }
68 bign operator*=(const bign &b) {
69     *this = *this * b;
70     return *this;
71 }
72 bign operator-(const bign &b) {
73     bign c;
74     c.len = 0;
75     for (int i = 0, g = 0; i < len; i++) {
76         int x = s[i] - g;
77         if (i < b.len) x -= b.s[i];
78         if (x >= 0)
79             g = 0;
80         else {
81             g = 1;
82             x += 10;
83         }
84         c.s[c.len++] = x;
85     }
86     c.clean();
87     return c;
88 }
89 bign operator-=(const bign &b) {
90     *this = *this - b;
91     return *this;
92 }
93 bign operator/(const bign &b) {
94     bign c, f = 0;
95     for (int i = len - 1; i >= 0; i--) {
96         f = f * 10;
97         f.s[0] = s[i];
98         while (f >= b) {
99             f -= b;
100             c.s[i]++;
101         }
102     }
103     c.len = len;
104     c.clean();
105     return c;

```

```

106 }
107 bign operator/=(const bign &b) {
108     *this = *this / b;
109     return *this;
110 }
111 bign operator%(const bign &b) {
112     bign r = *this / b;
113     r = *this - r * b;
114     return r;
115 }
116 bign operator%=(const bign &b) {
117     *this = *this % b;
118     return *this;
119 }
120 bool operator<(const bign &b) {
121     if (len != b.len) return len < b.len;
122     for (int i = len - 1; i >= 0; i--) {
123         if (s[i] != b.s[i]) return s[i] < b.s[i];
124     }
125     return false;
126 }
127 bool operator>(const bign &b) {
128     if (len != b.len) return len > b.len;
129     for (int i = len - 1; i >= 0; i--) {
130         if (s[i] != b.s[i]) return s[i] > b.s[i];
131     }
132     return false;
133 }
134 bool operator==(const bign &b) { return !(*this > b) && !(*this < b); }
135 bool operator!=(const bign &b) { return !(*this == b); }
136 bool operator<=(const bign &b) { return *this < b || *this == b; }
137 bool operator>=(const bign &b) { return *this > b || *this == b; }
138 string str() const {
139     string res = "";
140     for (int i = 0; i < len; i++) res = char(s[i] + '0') + res;
141     return res;
142 }
143 };
144
145 istream &operator>>(istream &in, bign &x) {
146     string s;
147     in >> s;
148     x = s.c_str();
149     return in;
150 }
151
152 ostream &operator<<(ostream &out, const bign &x) {
153     out << x.str();
154     return out;
155 }

```

## 8.10 Code::blocks settings

```
1 |gnome-terminal -t $TITLE -x
```

## 8.11 Bit operation

### 8.11.1 基本操作

注意括号

功能	示例	位运算
返回 lsb 之后的 0 的个数	(1100010) $\rightarrow$ 1D	__builtin_ctz(x)[x==0 时 UB]
统计二进制 1 的个数	(1100110 $\rightarrow$ 4D)	__builtin_popcount(x)
取最后一个 1 的 pos+1(ffs)	(1000010 $\rightarrow$ 2D)	__builtin_ffs(x)
取最后一个 1 的 mask(lsb)	(1000010 $\rightarrow$ 10)	(x & (-x))
去掉最后一位	(101101 $\rightarrow$ 10110)	x shr 1
在最后加一个 0	(101101 $\rightarrow$ 1011010)	x shl 1
在最后加一个 1	(101101 $\rightarrow$ 1011011)	x shl 1+1
把最后一位变成 1	(101100 $\rightarrow$ 101101)	x or 1
把最后一位变成 0	(101101 $\rightarrow$ 101100)	x or 1-1
最后一位取反	(101101 $\rightarrow$ 101100)	x xor 1
把右数第 $k$ 位变成 1	(101001 $\rightarrow$ 101101, $k=3$ )	x or (1 shl (k-1))
把右数第 $k$ 位变成 0	(101101 $\rightarrow$ 101001, $k=3$ )	x and not (1 shl (k-1))
右数第 $k$ 位取反	(101001 $\rightarrow$ 101101, $k=3$ )	x xor (1 shl (k-1))
取末三位	(1101101 $\rightarrow$ 101)	x and 7
取末 $k$ 位	(1101101 $\rightarrow$ 1101, $k=5$ )	x and (1 shl k-1)
取右数第 $k$ 位	(1101101 $\rightarrow$ 1, $k=4$ )	x shr (k-1) and 1
把末 $k$ 位变成 1	(101001 $\rightarrow$ 101111, $k=4$ )	x or (1 shl k-1)
末 $k$ 位取反	(101001 $\rightarrow$ 100110, $k=4$ )	x xor (1 shl k-1)
把右边连续的 1 变成 0	(100101111 $\rightarrow$ 100100000)	x and (x+1)
把右起第一个 0 变成 1	(100101111 $\rightarrow$ 100111111)	x or (x+1)
把右边连续的 0 变成 1	(11011000 $\rightarrow$ 11011111)	x or (x-1)
取右边连续的 1	(100101111 $\rightarrow$ 1111)	(x xor (x+1)) shr 1
去掉右起第一个 1 的左边	(100101000 $\rightarrow$ 1000)	x and (x xor (x-1))

### 8.11.2 枚举长为 $n$ 含 $k$ 个 1 的 01 串

```

1 | int n = 5, k = 3;
2 | for (int s = (1 << k) - 1, u = 1 << n; s < u;) {
3 |     for (int i = 0; i < n; i++)
4 |         printf("%d", (((s >> (n-1-i)) & 1) == 1));
5 |     printf("\n");
6 |
7 |     int b = s & -s;
8 |     s = (s+b) | (((s^(s+b)) >> 2)/b);
9 | }
```

### 8.12 Profile of Vim

```

1 | set nu
2 | set history=1000
3 |
4 | set tabstop
5 | set shiftwidth=4
6 | set smarttab
7 |
8 | set cindent
9 |
10 | colo evening
11 |
12 | set nobackup
13 | set noswapfile
14 |
15 | set mouse=a
16 |
```



```

17 map <F5> : call CompileRun()<CR>
18 func! CompileRun()
19     exec "w"
20     exec " !g++ %_Wall_std=c++11_o_%>"
21     exec " !./%<"
22 endfunc

```

## 9 Appendix

### 9.1 Template by elfness

#### 9.1.1 AC machine

```

1  #include<cstdio>
2  #include<cstring>
3  #include<cstdlib>
4  #include<cmath>
5  #include<algorithm>
6  #include<iostream>
7  using namespace std;
8  typedef long long LL;
9  struct tree {
10     tree *ne[26],*fail;
11     int ct;
12 } tr[500100],VD,*root,*Q[500100];
13 int tn;
14 void init() {
15     tr[tn=0]=VD;
16     root=tr+(tn++);
17 }
18 char s[1000100];
19 void build() {
20     tree *p=root;
21     for(int i=0; s[i]; i++) {
22         if(p->ne[s[i]-'a']==NULL) {
23             tr[tn]=VD;
24             p->ne[s[i]-'a']=tr+(tn++);
25         }
26         p=p->ne[s[i]-'a'];
27     }
28     p->ct++;
29 }
30 void pre() {
31     int i,top,tail;
32     tree *p,*q;
33     top=0;
34     tail=0;
35     for(i=0; i<26; i++)
36         if(root->ne[i]!=NULL) {
37             Q[++tail]=root->ne[i];
38             root->ne[i]->fail=root;
39         } else root->ne[i]=root;
40     while(top<tail) {
41         p=Q[++top];
42         for(i=0; i<26; i++)
43             if(p->ne[i]!=NULL) {
44                 q=p->ne[i];
45                 Q[++tail]=q;
46                 q->fail=p->fail->ne[i];
47                 if(q->fail==NULL)q->fail=root;
48             } else p->ne[i]=p->fail->ne[i];
49     }

```

```

50 }
51 int doit() {
52     int ret=0;
53     tree *p=root,*q;
54     for(int i=0; s[i]; i++) {
55         p=p->ne[s[i]-'a'];
56         q=p;
57         while(root!=q&&q->ct!=-1) {
58             ret+=q->ct;
59             q->ct=-1;
60             q=q->fail;
61         }
62     }
63     return ret;
64 }
65 int i,n,_;
66 int main() {
67     for(i=0; i<26; i++)VD.ne[i]=NULL;
68     VD.ct=0;
69     scanf("%d",&_);
70     while(_--) {
71         scanf("%d",&n);
72         init();
73         for(i=0; i<n; i++) {
74             scanf("%s",s);
75             build();
76         }
77         pre();
78         scanf("%s",s);
79         printf("%d\n",doit());
80     }
81 }

```

### 9.1.2 E-KMP

```

1  #include<cstdio>
2  #include<cstring>
3  #include<cstdlib>
4  #include<cmath>
5  #include<algorithm>
6  #include<iostream>
7  using namespace std;
8  typedef long long LL;
9  void e_kmp(char *s,char *t,int *has,int *e_has) {
10     int sp,p,mx,tn;
11     for(sp=p=mx=0; s[p]>0; p++) {
12         if(mx==p||p+e_has[p-sp]>=mx) {
13             for(tn=mx-p; s[mx]==t[tn]; tn++)mx++;
14             has[sp=p]=mx-p;
15             if(mx==p)sp=++mx;
16         } else has[p]=e_has[p-sp];
17     }
18 }
19 const int V=1001000;
20 char t[V],s[V];
21 int e_has[V],has[V],tn;
22 int main() {
23     scanf("%s%s",s,t);
24     tn=strlen(t);
25     t[tn]=-1;
26     e_has[0] = tn;
27     e_kmp(t+1,t,e_has+1,e_has);

```

```

28 | e_kmp(s,t,has,e_has);
29 | }

```

### 9.1.3 KM (list)

```

1 | #include<cstdio>
2 | #include<cstring>
3 | #include<cstdlib>
4 | #include<cmath>
5 | #include<algorithm>
6 | using namespace std;
7 | const int V=1200;
8 | const int En=21000;
9 | const int oo=1000000000;
10 | struct Edge {
11 |     int num,ne,w;
12 | } e[En];
13 | int p[V],K;
14 | void add(int x,int y,int z) {
15 |     e[K].num=y;
16 |     e[K].w=z;
17 |     e[K].ne=p[x];
18 |     p[x]=K++;
19 | }
20 | bool sx[V],sy[V];
21 | int lx[V],ly[V],mat[V];
22 | bool path(int u) {
23 |     sx[u]=true;
24 |     for(int i=p[u]; i!=-1; i=e[i].ne) {
25 |         int v=e[i].num;
26 |         if(!sy[v]&&lx[u]+ly[v]==e[i].w) {
27 |             sy[v]=true;
28 |             if(mat[v]==-1||path(mat[v])) {
29 |                 mat[v]=u;
30 |                 return true;
31 |             }
32 |         }
33 |     }
34 |     return false;
35 | }
36 | int N;
37 | int KM() {
38 |     int i,j;
39 |     for(i=0; i<N; i++) {
40 |         lx[i]=-oo;
41 |         for(j=p[i]; j!=-1; j=e[j].ne)
42 |             lx[i]=max(lx[i],e[j].w);
43 |     }
44 |     for(i=0; i<N; i++) ly[i]=0,mat[i]=-1;
45 |     for(int u=0; u<N; u++)
46 |         while(1) {
47 |             for(i=0; i<N; i++) sx[i]=0,sy[i]=0;
48 |             if(path(u)) break;
49 |             int dx=oo;
50 |             for(i=0; i<N; i++) if(sx[i])
51 |                 for(j=p[i]; j!=-1; j=e[j].ne)
52 |                     if(!sy[e[j].num])
53 |                         dx=min(dx,lx[i]+ly[e[j].num]-e[j].w);
54 |             if(dx==oo) return -1;
55 |             for(i=0; i<N; i++) if(sx[i]) lx[i]-=dx;
56 |             for(i=0; i<N; i++) if(sy[i]) ly[i]+=dx;
57 |         }
58 |     int ret=0;

```

```

59     for(i=0; i<N; i++)ret+=lx[i]+ly[i];
60     return -ret;
61 }
62 int _,ca,n,m,i,x,y,z,te;
63 int main() {
64     scanf("%d",&_);
65     ca=0;
66     while(_--) {
67         ca++;
68         scanf("%d%d",&n,&m);
69         N=n;
70         for(i=0; i<n; i++)p[i]=-1;
71         K=0;
72         for(i=0; i<m; i++) {
73             scanf("%d%d%d",&x,&y,&z);
74             x--;
75             y--;
76             add(x,y,-z);
77             add(y,x,-z);
78         }
79         te=KM();
80         printf(" Case_%d:_",ca);
81         if(te==-1)puts(" NO ");
82         else printf("%d\n",te);
83     }
84 }

```

#### 9.1.4 Nearest point pair

```

1  /*
2  * nearestPointPair.cpp
3  *
4  * Created on: 2011-10-10
5  * Author: Fish
6  */
7
8  #include <cstdio>
9  #include <cstring>
10 #include <cstdlib>
11 #include <cmath>
12 #include <algorithm>
13
14 using namespace std;
15
16 const int MaxN = 120000;
17 const int Log = 20;
18
19 struct Point {
20     double x, y;
21     Point() {
22     }
23     Point(double x, double y) :
24         x(x), y(y) {
25     }
26     Point operator-(const Point& p) const {
27         return Point(x - p.x, y - p.y);
28     }
29     double norm() const {
30         return hypot(x, y);
31     }
32     void init() {
33         scanf("%lf%lf", &x, &y);

```

```

34     }
35 } p[MaxN];
36 int x[MaxN], y[Log][MaxN], tmp[MaxN], n;
37 bool vst[MaxN];
38
39 bool comp_x(const int& i, const int& j) {
40     return p[i].x < p[j].x;
41 }
42
43 bool comp_y(const int& i, const int& j) {
44     return p[i].y < p[j].y;
45 }
46
47 double dfs(int k, int l, int r) {
48     double ret = 1e100;
49     if (r - l <= 2) {
50         for (int i = l; i < r; i++)
51             for (int j = i + 1; j <= r; j++)
52                 ret = min(ret, (p[x[i]] - p[x[j]]).norm());
53     }
54     return ret;
55 }
56
57 int mid = (l + r) >> 1;
58 int lp = l, rp = mid + 1;
59 for (int i = l; i <= r; i++)
60     vst[x[i]] = i <= mid;
61 for (int i = l; i <= r; i++)
62     if (vst[y[k][i]])
63         y[k + 1][lp++] = y[k][i];
64     else
65         y[k + 1][rp++] = y[k][i];
66 double lhs = dfs(k + 1, l, mid);
67 double rhs = dfs(k + 1, mid + 1, r);
68 double mx = (p[x[mid + 1]].x + p[x[mid]].x) / 2.0;
69 ret = min(lhs, rhs);
70
71 lp = 0;
72 for (int i = l; i <= r; i++)
73     if (fabs(mx - p[y[k][i]].x) < ret)
74         tmp[lp++] = y[k][i];
75
76 for (int i = 0; i < lp; i++)
77     for (int j = 1; j < 8 && i + j < lp && (p[tmp[i + j]].y - p[tmp[i]].y) < ret; j++)
78         ret = min(ret, (p[tmp[i]] - p[tmp[i + j]]).norm());
79
80 return ret;
81 }
82
83 int main() {
84 #ifdef __FISH__
85     freopen("data.in", "r", stdin);
86     freopen("nlogn.out", "w", stdout);
87 #endif
88 while (scanf("%d", &n) == 1 && n) {
89     for (int i = 0; i < n; i++) {
90         p[i].init();
91         x[i] = y[0][i] = i;
92     }
93     sort(x, x + n, comp_x);
94     sort(y[0], y[0] + n, comp_y);
95     printf("%.2f\n", dfs(0, 0, n - 1) / 2.0);
96     // printf("%.6f\n", dfs(0, 0, n - 1));

```

```

96     }
97
98     return 0;
99 }

```

### 9.1.5 SA

```

1  #include<cstdio>
2  #include<cstring>
3  #include<cstdlib>
4  #include<cmath>
5  #include<algorithm>
6  #include<iostream>
7  #include<vector>
8  #include<string>
9  using namespace std;
10 typedef long long LL;
11 const int N=100100;
12 char s[N];    /// 长度+1，对于非字符串，加一个小于最小值的元素，
13 int sa[N];    /// 倍增算法，结果 下标 1-n，第 i 大的是 sa[i]
14 int rk[N];    /// 第 i 位开始的后缀，的排名为 rk[i]
15 int wa[N],wb[N],wv[N],rmq[20][N];
16 int sn,to[N];
17 bool cmp(int *y,int a,int b,int L) {
18     return y[a]==y[b]&&y[a+L]==y[b+L];
19 }
20 void da(char *s,int *sa,int len,int dn) {
21     int i,j,p;
22     int *x,*y,*t;
23     x=wa;
24     y=wb;
25     for(i=0; i<dn; i++)rk[i]= 0;
26     for(i=0; i<len; i++)rk[x[i]=s[i]]++;
27     for(i=0; i<dn; i++)rk[i+1]+=rk[i];
28     for(i=len-1; i>=0; i--)sa[--rk[x[i]]]=i;
29     for(j=1,p=1; p<len; j*=2,dn=p) {
30         for(p=0; p<j; p++)y[p]=len-j+p;
31         for(i=0; i<len; i++)if(sa[i]>=j)y[p++]=sa[i]-j;
32         for(i=0; i<len; i++)wv[i]=x[y[i]];
33         for(i=0; i<dn; i++)rk[i]=0;
34         for(i=0; i<len; i++)rk[wv[i]]++;
35         for(i=0; i<dn; i++)rk[i+1]+=rk[i];
36         for(i=len-1; i>=0; i--)sa[--rk[wv[i]]]=y[i];
37         swap(x,y);
38         x[sa[0]]=0;
39         for(p=i=1; i<len; i++) {
40             p+=!cmp(y,sa[i],sa[i-1],j);
41             x[sa[i]]=p-1;
42         }
43     }
44 }
45 void find_height(char *s,int *sa,int len) {
46     int *h=rmq[0];
47     int i,j,k=0;
48     for(i=1; i<=len; i++)
49         rk[sa[i]] = i;
50     for(i=0; i<len; i++) {
51         if(k>0)k--;
52         j=sa[rk[i]-1];
53         while(s[i+k]==s[j+k])k++;
54         h[rk[i]]=k;
55     }
56 }

```

```

57 void RMQ(int n) {
58     int i,j;
59     int rn=(int)floor(log(n*2.0)/log(2.0));
60     for(i=1; i<rn; i++)
61         for(j=0; j<n+2-(1<<(i-1)); j++)
62             rmq[i][j]=min(rmq[i-1][j],rmq[i-1][j+(1<<(i-1))]);
63 }
64 int askRMQ(int a,int b) { /// [a,b] 闭区间
65     int rq=to[b-a];
66     return min(rmq[rq][a],rmq[rq][b+1-(1<<rq)]);
67 }
68 void PT(char *s,int *sa) {
69     int i,sn;
70     sn=strlen(s);
71     for(i=0; i<sn; i++)
72         puts(s+sa[i+1]);
73     puts("");
74     for(i=0; i<sn; i++)
75         printf("rank_%d=%d\n",i,rk[i]);
76 }
77 int lcp(int a,int b,int len) {
78     if(a==b)
79         return len-a;
80     a=rk[a];
81     b=rk[b];
82     if(a>b)swap(a,b);
83     return askRMQ(a+1,b);
84 }
85 void pre_log() {
86     int i;
87     to[0]=to[1]=0;
88     for(i=1; i*2<N; i++)
89         to[i*2]=to[i*2+1]=to[i]+1;
90 }
91 int main() {
92     int T,_=0;
93     pre_log();
94     while(~scanf("%s",s)) {
95         sn=strlen(s);
96         da(s,sa,sn+1,128);
97         find_height(s,sa,sn);
98         RMQ(sn);
99         PT(s,sa);
100         scanf("%d",&T);
101         while(T--) {
102             int a,b;
103             scanf("%d%d",&a,&b);
104             a--,b--;/// 求原串的 a b 开始的后缀的公共前缀
105             printf("lcp_=%d\n",lcp(a,b,sn));
106         }
107     }
108     return 0;
109 }

```

### 9.1.6 SAP

```

1 #include<cstdio>
2 #include<cstring>
3 #include<cstdlib>
4 #include<cmath>
5 #include<algorithm>
6 using namespace std;

```

```

7  const int V=220;
8  const int En=200000;
9  const int oo=0x3f3f3f3f;
10 struct Edge {
11     int num,ne,c;
12 } e[En];
13 int d[V],p[V],pre[V],low[V];
14 int gap[V],cur[V];
15 int N,K,st,ed;
16 void add(int x,int y,int c) {
17     e[K].num=y;
18     e[K].c=c;
19     e[K].ne=p[x];
20     p[x]=K++;
21     e[K].num=x;
22     e[K].c=0;
23     e[K].ne=p[y];
24     p[y]=K++;
25 }
26 int sap() {
27     int ret=0;
28     bool fail;
29     for(int i=0; i<=N; i++) {
30         low[i]=gap[i]=d[i]=0;
31         cur[i]=p[i];
32     }
33     low[st]=oo;
34     gap[0]=N;
35     int u=st;
36     while(d[st]<N) {
37         fail=true;
38         for(int i=cur[u]; i!=-1; i=e[i].ne) {
39             int v=e[i].num;
40             cur[u]=i;
41             if(e[i].c&&d[u]==d[v]+1) {
42                 pre[v]=i;
43                 low[v]=min(low[u],e[i].c);
44                 u=v;
45                 if(u==ed) {
46                     do {
47                         e[pre[u]].c-=low[ed];
48                         e[pre[u]^1].c+=low[ed];
49                         u=e[pre[u]^1].num;
50                     } while(u!=st);
51                     ret+=low[ed];
52                 }
53                 fail=false;
54                 break;
55             }
56         }
57         if(fail) {
58             gap[d[u]]--;
59             if(!gap[d[u]]) return ret;
60             d[u]=N;
61             for(int i=p[u]; i!=-1; i=e[i].ne)
62                 if(e[i].c)d[u]=min(d[u],d[e[i].num]+1);
63             gap[d[u]]++;
64             cur[u]=p[u];
65             if(u!=st)u=e[pre[u]^1].num;
66         }
67     }
68     return ret;
69 }

```



## 9.1.7 一般图最大匹配

```

1  #include <stdio.h>
2  #include <string.h>
3  #include <algorithm>
4  #include <vector>
5  #define maxn 300
6  #define maxm 90010
7
8  using namespace std;
9
10 int match[maxn];           // 标记是否匹配
11 int st[maxn], aim[maxm], nxt[maxm], ln; // 边表
12 int q[maxn];               // bfs队列
13 int level[maxn];           // 离根深度的奇偶性
14 vector<int> ar[maxn];       // 存每个点到根的路径
15 vector<int> a;              // 找到的一条增广路
16 int n;
17 void init() {
18     for(int i=0; i<n; i++) st[i]=-1;
19     ln=0;
20 }
21 void in_edge(int x, int y) {
22     aim[ln]=y;
23     nxt[ln]=st[x];
24     st[x]=ln++;
25 }
26 int lca(int p, int q) {      // 求p和q的最近公共祖先
27     int ret=0;
28     while (ret<ar[p].size() && ret<ar[q].size() && ar[p][ret]==ar[q][ret]) ret++;
29     return ret-1;
30 }
31 int FindAlterRoad(int sp) {
32     int qn=1;
33     memset(level, -1, sizeof(level));
34     level[q[0]=sp]=1;
35     ar[sp].clear();
36     ar[sp].push_back(sp);
37     for (int p=0; p<qn; p++) {
38         int x=q[p];
39         for (int i=st[x]; i!=-1; i=nxt[i]) {
40             int u=aim[i];
41             if (match[u]==u) continue;
42             if (level[u]==-1) { // u是未访问的点
43                 if (match[u]==-1) { // u是未匹配的, 找到增广路
44                     a=ar[x];
45                     a.push_back(u);
46                     return 1;
47                 } else { // u是已匹配的点
48                     int v=match[u];
49                     if (level[v]!=-1) continue;
50                     ar[v]=ar[x];
51                     ar[v].push_back(u);
52                     ar[v].push_back(v);
53                     level[u]=0;
54                     level[v]=1;
55                     q[qn++]=v;
56                 }
57             } else if (level[u]==1) { // u和x同为偶点. 形成花
58                 int root=lca(u, x);
59                 vector<int> tmp=ar[x];
60                 for (int i=ar[u].size()-1; i>root; i--) {
61                     int y=ar[u][i];

```

```

62         tmp.push_back(y);
63         if (level[y]==0) {
64             level[y]=1;
65             ar[y]=tmp;
66             level[y]=1;
67             q[qn++]=y;
68         }
69     }
70     tmp=ar[u];
71     for (int i=ar[x].size()-1; i>root; i--) {
72         int y=ar[x][i];
73         tmp.push_back(y);
74         if (level[y]==0) {
75             level[y]=1;
76             ar[y]=tmp;
77             level[y]=1;
78             q[qn++]=y;
79         }
80     }
81 }
82 }
83 }
84 return 0;
85 }
86 int MaximumMatch() {
87     int ret=0; //最大匹配数
88     memset(match,-1,sizeof(match));
89     for (int i=0; i<n; i++)
90         if (match[i]==-1)
91             if (FindAlterRoad(i)) {
92                 for (int i=0; i<a.size(); i+=2) {
93                     int u=a[i],v=a[i+1];
94                     match[u]=v;
95                     match[v]=u;
96                 }
97                 ret++;
98             } else match[i]=i;
99     return ret;
100 }

```

### 9.1.8 上下界最大流

```

1  /*
2  Author: elfness@UESTC
3  */
4  #include<cstdio>
5  #include<cstring>
6  #include<cstdlib>
7  #include<cmath>
8  #include<algorithm>
9  #include<iostream>
10 #include<vector>
11 #include<string>
12 using namespace std;
13 typedef long long LL;
14 const int V=1500;
15 const int En=900000;
16 const int inf=0x3f3f3f3f;
17 struct Edge {
18     int num,ne;
19     int c;
20 } e[En];
21 int p[V],K;

```

```

22 void add(int x,int y,int c) {
23     e[K].num=y;
24     e[K].c=c;
25     e[K].ne=p[x];
26     p[x]=K++;
27     e[K].num=x;
28     e[K].c=0;
29     e[K].ne=p[y];
30     p[y]=K++;
31 }
32 int d[V],pre[V],pree[V],gap[V],cur[V];
33 int N,st,ed;
34 int low[V];
35 int sap() {
36     int ret=0;
37     bool fail;
38     for(int i=0; i<=N; i++) {
39         d[i]=0;
40         gap[i]=0;
41         cur[i]=p[i];
42         low[i]=0;
43     }
44     low[st]=inf;
45     gap[0]=N;
46     int u=st;
47     while(d[st]<N) {
48         fail=true;
49         for(int i=cur[u]; i!=-1; i=e[i].ne) {
50             int v=e[i].num;
51             cur[u]=i;
52             if(e[i].c&& d[u]==d[v]+1) {
53                 pre[v]=u;
54                 pree[v]=i;
55                 low[v]=min(low[u],e[i].c);
56                 u=v;
57                 if(u==ed) {
58                     do {
59                         e[pree[u]].c-=low[ed];
60                         e[pree[u]^1].c+=low[ed];
61                         u=pree[u];
62                     } while(u!=st);
63                     ret+=low[ed];
64                 }
65                 fail=false;
66                 break;
67             }
68         }
69         if(fail) {
70             gap[d[u]]--;
71             if(!gap[d[u]]) return ret;
72             d[u]=N;
73             for(int i=p[u]; i!=-1; i=e[i].ne)
74                 if(e[i].c d[u]=min(d[u],d[e[i].num]+1);
75             gap[d[u]]++;
76             cur[u]=p[u];
77             if(u!=st) u=pree[u];
78         }
79     }
80     return ret;
81 }
82 int n,m,s,t;
83 struct Elf {
84     int u,v,lo,up;
85 } b[12000];

```

```

86 int lb[12000];
87 int doit() {
88     int i;
89     N=n+2;
90     st=n;
91     ed=n+1;
92     for(i=0; i<N; i++)p[i]=-1;
93     K=0;
94     for(i=0; i<n; i++)lb[i]=0;
95     for(i=0; i<m; i++) {
96         lb[b[i].u]-=b[i].lo;
97         lb[b[i].v]+=b[i].lo;
98         add(b[i].u,b[i].v,b[i].up-b[i].lo);
99     }
100    for(i=0; i<n; i++) {
101        if(lb[i]>0)add(st,i,lb[i]);
102        else add(i,ed,-lb[i]);
103    }
104    add(t,s,inf);
105    int te=sap();
106    for(i=p[st]; i!=-1; i=e[i].ne)
107        if(e[i].c!=0)return -1;
108    st=s;
109    ed=t;
110    te=sap();
111    return te;
112 }

```

### 9.1.9 上下界最小流

```

1  #include<cstdio>
2  #include<cstdlib>
3  #include<cstring>
4  #include<cmath>
5  #include<algorithm>
6  using namespace std;
7  const int V=600;
8  const int En=50000;
9  const int oo=0x3f3f3f3f;
10 struct Edge {
11     int num,ne,c;
12 } e[En];
13 int p[V],K;
14 void add(int x,int y,int c) {
15     e[K].num=y;
16     e[K].c=c;
17     e[K].ne=p[x];
18     p[x]=K++;
19     e[K].num=x;
20     e[K].c=0;
21     e[K].ne=p[y];
22     p[y]=K++;
23 }
24 int d[V],cur[V],low[V],pre[V],gap[V],pree[V];
25 int st,ed,N;
26 int sap() {
27     int ret=0;
28     bool fail;
29     memset(gap,0,sizeof(gap));
30     memset(low,0,sizeof(low));
31     memset(d,0,sizeof(d));
32     for(int i=0; i<N; i++)cur[i]=p[i];
33     gap[0]=N;

```

```

34 low[st]=oo;
35 int u=st;
36 while(d[st]<N) {
37     fail=true;
38     for(int i=cur[u]; i!=-1; i=e[i].ne) {
39         int v=e[i].num;
40         cur[u]=i;
41         if(e[i].c&&d[u]==d[v]+1) {
42             pre[v]=u;
43             pree[v]=i;
44             low[v]=min(low[u],e[i].c);
45             u=v;
46             if(u==ed) {
47                 do {
48                     e[pree[u]].c-=low[ed];
49                     e[pree[u]^1].c+=low[ed];
50                     u=pree[u];
51                 } while(u!=st);
52                 ret+=low[ed];
53             }
54             fail=false;
55             break;
56         }
57     }
58     if(fail) {
59         gap[d[u]]--;
60         if(!gap[d[u]]) return ret;
61         d[u]=N;
62         for(int i=p[u]; i!=-1; i=e[i].ne)
63             if(e[i].c)d[u]=min(d[u],d[e[i].num]+1);
64         gap[d[u]]++;
65         cur[u]=p[u];
66         if(u!=st)u=pree[u];
67     }
68 }
69 return ret;
70 }
71 struct ELF {
72     int u,v,lo;
73 } b[En];
74 int n,m,lb[V],ts,tt;
75 void solve() {
76     N=n+4;
77     ts=0;
78     tt=n+1;
79     st=n+2;
80     ed=n+3;
81     memset(lb,0,sizeof(lb));
82     int i,u,v;
83     for(i=0; i<N; i++)p[i]=-1;
84     K=0;
85     for(i=0; i<m; i++) {
86         u=b[i].u;
87         v=b[i].v;
88         lb[v]+=b[i].lo;
89         lb[u]-=b[i].lo;
90         add(u,v,oo-b[i].lo);
91     }
92     for(i=1; i<=n; i++) {
93         add(ts,i,oo);
94         add(i,tt,oo);
95     }
96     for(i=0; i<n+2; i++) {
97         if(lb[i]>0)add(st,i,lb[i]);

```

```

98     else add(i,ed,-lb[i]);
99 }
100 int ans=sap();
101 add(tt,ts,oo);
102 printf("%d\n",sap());
103 }
104 int _,ca,i;
105 int main() {
106     scanf("%d",&_);
107     ca=0;
108     while(_--) {
109         ca++;
110         scanf("%d%d",&n,&m);
111         for(i=0; i<m; i++) {
112             scanf("%d%d%d",&b[i].u,&b[i].v,&b[i].lo);
113         }
114         printf(" Case_#%d: ",ca);
115         solve();
116     }
117 }

```

### 9.1.10 全局最小割

```

1  using namespace std;
2  #define inf 1000000000
3  bool visit[502],com[502];
4  int map[502][502],W[502],s,t;
5  int maxadj(int N,int V) {
6      int CUT;
7      memset(visit,0,sizeof(visit));
8      memset(W,0,sizeof(W));
9      for(int i=0; i<N; i++) {
10         int Num=0,Max=-inf;
11         for(int j=0; j<V; j++)
12             if(!com[j]&&!visit[j]&&W[j]>Max) {
13                 Max=W[j];
14                 Num=j;
15             }
16         visit[Num]=true;
17         s=t;
18         t=Num;
19         CUT=W[t];
20         for(int j=0; j<V; j++)
21             if(!com[j]&&!visit[j])W[j]+=map[Num][j];
22     }
23     return CUT;
24 }
25 int stoer(int V) {
26     int Mincut=inf;
27     int N=V;
28     memset(com,0,sizeof(com));
29     for(int i=0; i<V-1; i++) {
30         int Cut;
31         s=0,t=0;
32         Cut=maxadj(N,V);
33         N--;
34         if(Cut<Mincut)Mincut=Cut;
35         com[t]=true;
36         for(int j=0; j<V; j++)
37             if(!com[j]) {
38                 map[j][s]+=map[j][t];
39                 map[s][j]+=map[t][j];

```

```

40     }
41 }
42 return Mincut;
43 }

```

### 9.1.11 最小树型图

```

1  #include<cstdio>
2  #include<cstring>
3  #include<cstdlib>
4  #include<cmath>
5  #include<algorithm>
6  using namespace std;
7  const int V=1200;
8  const int En=2100000;
9  struct Elf {
10     int u,v,len;
11 } b[En];
12 const int oo=1000000000;
13 int ret;
14 int N,M,Root;//点数，边数，根，默认从0开始
15 int id[V],pre[V],cnt,vis[V];
16 int in[V];
17 bool TreeMST() {
18     ret=0;
19     int i,u,v;
20     while(1) {
21         for(i=0; i<N; i++)
22             in[i]=oo;
23         memset(pre,-1,sizeof(pre));
24         for(i=0; i<M; i++) {
25             u=b[i].u;
26             v=b[i].v;
27             if(b[i].len<in[v]&&u!=v) {
28                 pre[v]=u;
29                 in[v]=b[i].len;
30             }
31         }
32         for(i=0; i<N; i++) {
33             if(i==Root)continue;
34             if(pre[i]==-1)return false;
35         }
36         in[Root]=0;
37         cnt=0;
38         memset(id,-1,sizeof(id));
39         memset(vis,-1,sizeof(vis));
40         for(i=0; i<N; i++) {
41             ret+=in[i];
42             v=i;
43             while(vis[v]!=i&&id[v]==-1&&v!=Root) {
44                 vis[v]=i;
45                 v=pre[v];
46             }
47             if(v!=Root&&id[v]==-1) {
48                 for(u=pre[v]; u!=v; u=pre[u])
49                     id[u]=cnt;
50                 id[v]=cnt++;
51             }
52         }
53         if(cnt==0)return true;
54         for(i=0; i<N; i++)
55             if(id[i]==-1)id[i]=cnt++;
56         for(i=0; i<M; i++) {

```

```
57     v=b[i].v;
58     b[i].u=id[b[i].u];
59     b[i].v=id[b[i].v];
60     if(b[i].u!=b[i].v)
61         b[i].len-=in[v];
62 }
63 N=cnt;
64 Root=id[Root];
65 }
66 return true;
67 }
```