State of Matter

State of matter can be classified as follows

Matter

- Solid
- Fluid
    - Liquid
    - Gas

Properties of Solid

- Incompressible
    - Closely packed atom/molecule
    - Interatomic distance is very less.
- Inflexible
- Have mechanical strength
- Have specific shape, mass and volume

There are broadly 3 types of solid found in nature

- Crystalline Solid
- Poly-crystalline Solid
- Non-crystalline Solid / Amorphous

Crystalline Solid

- The constituents (atoms, molecules, or ions) are arrange in an orderly manner in a definite geometric shape.
    - Regular arrangement of constituent atom
    - Definite regular geometry
- Long range order
- **Lattice:** Regular and periodic arrangement of points in space
- **Basis:** Atom or group of atoms
- Lattice + Basis = Crystal Structure
- Example -> Diamond, Salt, most of the metals

Poly-crystalline Solid

- Occur as powder resemble amorphous but properties are just like crystalline solid.
- Arrangement amorphous
- Looks like amorphous
- Properties like crystalline
- Example -> Sand, Ceramic

Non-crystalline / Amorphous

- Constituents (atoms, ions, or molecules) are not arranged in a regular manner.
    - No regular geometry
- Have no electrical conductivity
- Melting point not fixed
- Example -> Glass, Rubber

| Crystalline	Non-crystalline / Amorphous
Have defined geometric shape	Does not have defined shape
Symmetry	No symmetry
Sharp melting point	Exponential melting point
Anisotropic (direction dependent)	Isotropic (Direction independent)
Shows conductivity	Does not show conductivity
Table salt, diamond, mostly metals	Rubbers, glass

Unit Cell

Unit cell is the smallest repeating unit that forms the total symmetry of a crystal.

There are certain properties of an unit cell which are called the **Lattice Parameter**. The lattice parameters are -

- Axial Length (a, b, c)

- Axial Angle (α, β, γ)
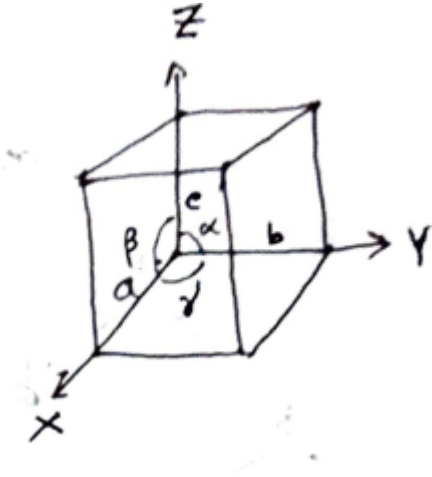
Lattice parameter refers to the physical dimension of unit cell.

Different lattice parameters can form different kind of crystal or unit cell. Some are given below:

Crystal System	Lattice Parameter
Triclinic	$\alpha \neq \beta \neq \gamma \neq 90°$ $a \neq b \neq c$
Monoclinic	$\alpha = \beta = 90° \neq \gamma$ $a \neq b \neq c$
Orthorhombic	$\alpha = \beta = \gamma = 90°$ $a \neq b \neq c$
Cubic	$\alpha = \beta = \gamma = 90°$ $a = b = c$
Trigonal	$\alpha = \beta = \gamma \neq 90°$ $a = b = c$
Tetragonal	$\alpha = \beta = \gamma = 90°$ $a = b \neq c$
Hexagonal	$\alpha = \beta = 90°, \gamma = 120°$ $a = b \neq c$

Unit cell can be broadly divided into 2 categories.

- Primitive (simple): Lattice points lie only at corners
  - Effective number of lattice point = 1
- Non-primitive: Lattice points lie at corners and other locations such as inside or surface.
  - Effective number of lattice point > 1.
  - This type of unit cell can be broadly classified as following categories

- Body Centered: Lattice point at every corner and at the center of the body
- Face Centered: Lattice point at every corner and at the center of the each 6 surfaces
- Base Centered: Lattice point at every corner and at the center of 2 parallel surface.

Number of Lattice Point and Packing Factor

$$N = \frac{N_c}{8} + N_i + \frac{N_f}{2}$$

Here,

N = Effective Lattice Point

N_c = Number of Lattice Point at Corner

N_i = Number of Lattice Point inside

N_f = Number of Lattice Point at Surface

$$\text{Density of Crystal Matter} = \frac{\text{mass per unit cell}}{\text{volume of unit cell}}$$

$$= \frac{\text{number of atom} \times \text{mass of one atom}}{V}$$

$$\rho = \frac{n \times \frac{M}{N_A}}{V}$$

$$\rho = \frac{nM}{N_A V}$$

Packing Factor: Packing factor indicates how atoms are closely packed in an unit cell

$$\text{packing factor} = \frac{\text{Volume of all atoms in unit cell}}{\text{Volume of unit cell}}$$

$$P.F = \frac{n \times \frac{4}{3}\pi r^3}{V}$$

Packing Factor of a Simple Cubic Unit Cell

In a simple cubic structure, $a = 2r$ where,

a = length of a side of the cubic structure

r = radius of an atom
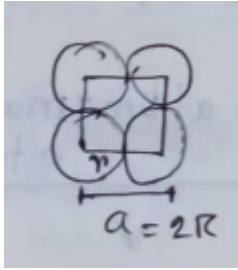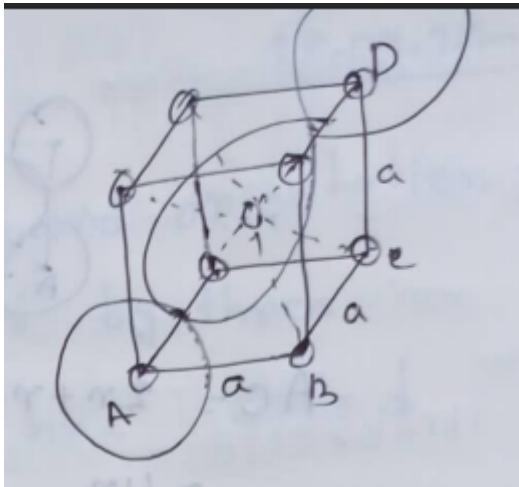

and number of effective lattice is,

$$N = \frac{N_C}{8} = \frac{8}{8} = 1$$

$$(PF)_{SC} = \frac{N \times \frac{4}{3}\pi \times \left(\frac{a}{2}\right)^3}{a^3}$$

$$= \frac{4\pi}{24} = 0.52 \times 100\% = 52\%$$

$$\therefore \text{Empty Space} = 48\%$$

Packing Factor of a Body Centered Cubic Unit Cell

In this figure,

$$AD = r + 2r + r = 4r$$

again,

$$AD^2 = AB^2 + BC^2 + CD^2 = 3a^2$$

$$\therefore 4r = \sqrt{3}a$$

$$r = \frac{\sqrt{3}}{4}a$$

Number of effective lattice point would be,

$$N = \frac{N_c}{8} + N_i = \frac{8}{8} + 1 = 2$$

Thus the packing factor will be,

$$(PF)_{BCC} = \frac{N \times \frac{4}{3}\pi \times \frac{3\sqrt{3}}{64} \times a^3}{a^3} = 68\%$$

$$\text{Empty Space} = 32\%$$

We can come to the conclusion that Body Centered Cubic unit cells are more closely packed than simple cubic unit cell.

Packing Factor of a Face Centered Cubic Unit Cell
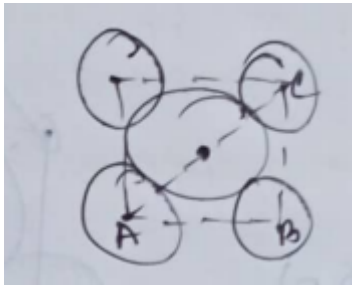
Surface of an FCC unit cell

$$AC = r + 2r + r = 4r$$

again,

$$AC^2 = AB^2 + BC^2 = 2a^2$$

$$\therefore 4r = \sqrt{2}a$$

$$r = \frac{\sqrt{2}}{4}a$$

Number of effective lattice point,

$$N = \frac{N_c}{8} + \frac{N_f}{2} = \frac{8}{8} + \frac{6}{2} = 4$$

Thus the packing factor will be

$$(PF)_{FCC} = \frac{N \times \frac{4}{3}\pi \times \left(\frac{\sqrt{2}a}{4}\right)^3}{a^3} = 74\%$$

$$\text{Empty Space} = 26\%$$

There, Face Centered Cubic is the most closely pact structure among the structure we discussed so far.

Crystal Plane & Miller Indices

Crystal plane are some imaginary plane inside a crystal in which large concentration of atom are present.

Crystal plane is defined as some imaginary plane in which the atom concentration is large.

The position and orientation of a crystal plane is describe be three integers h, k, and l are called Miller Indices

$(h\ k\ l)$ -> Refers to single plane
$\{h\ k\ l\}$ -> Refers to family of planes
$[h\ k\ l]$ -> Refers to direction of $(h\ k\ l)$ plane
$< h\ k\ l >$ -> Family of direction

Example of drawing the plane (2 3 1)

We first take reciprocal of (2 3 1)

Then we go half of the total length on X-axis, one third on the Y-axis and one full on the Z-axis. We create a plane by connecting those points as per the following diagram
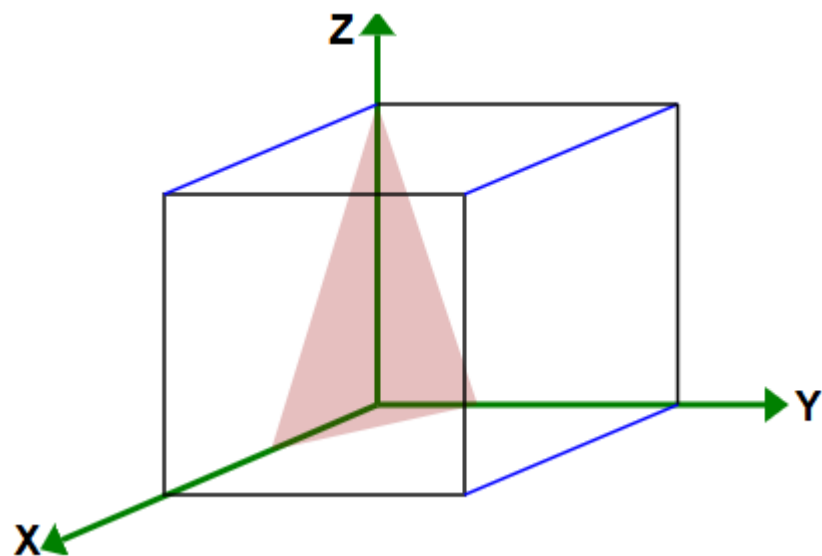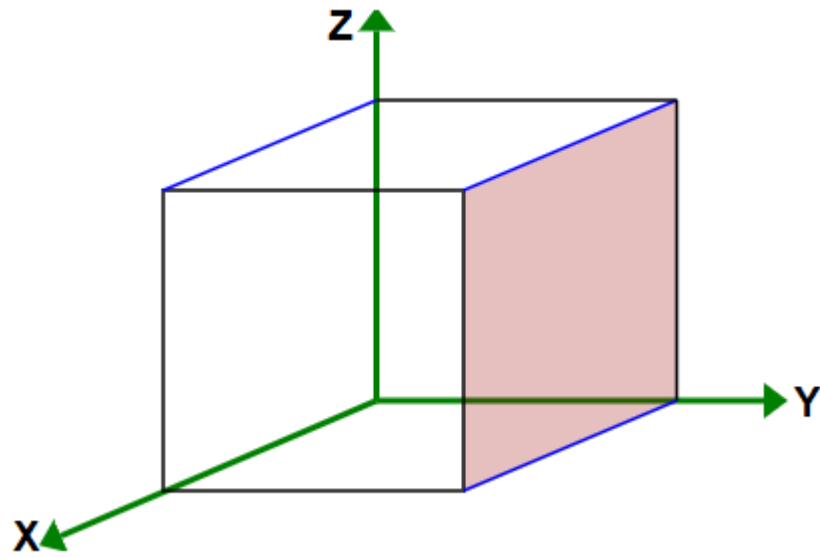
Some more Miller's plane

(1 0 0)

$$(1 \ 0 \ 0)$$

$$\left(\frac{1}{1} \quad \frac{1}{0} \quad \frac{1}{0} \right)$$

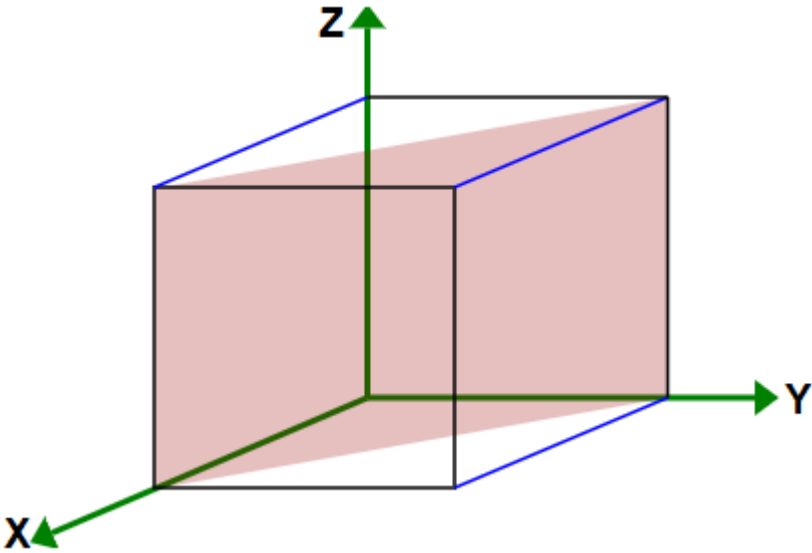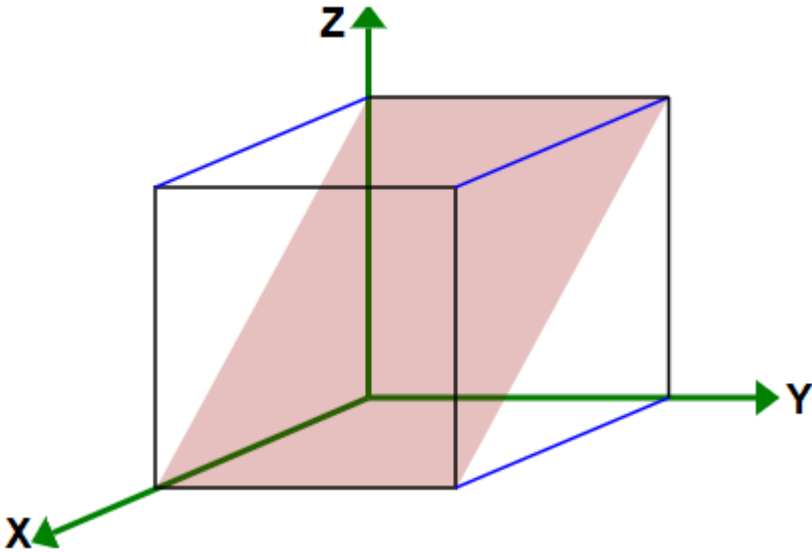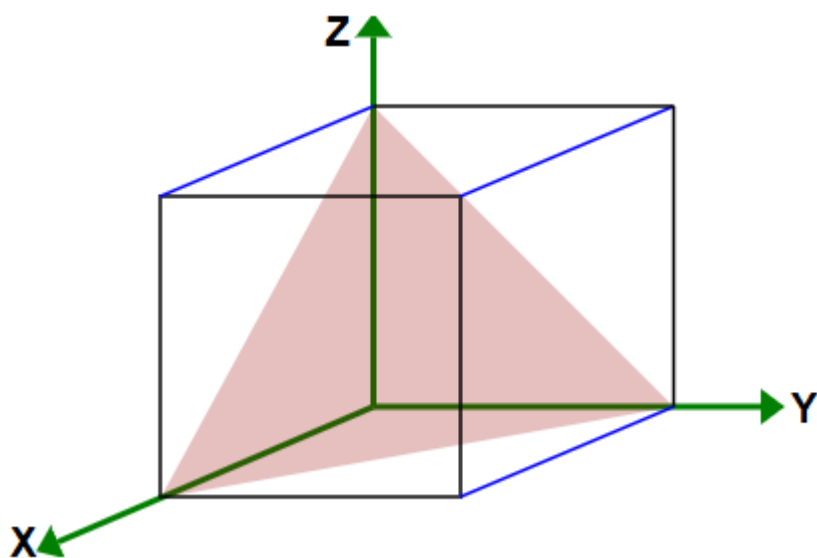$$\begin{array}{ccc} (1 & \infty & \infty) \\ \downarrow & \downarrow & \downarrow \\ x & y & z \end{array}$$
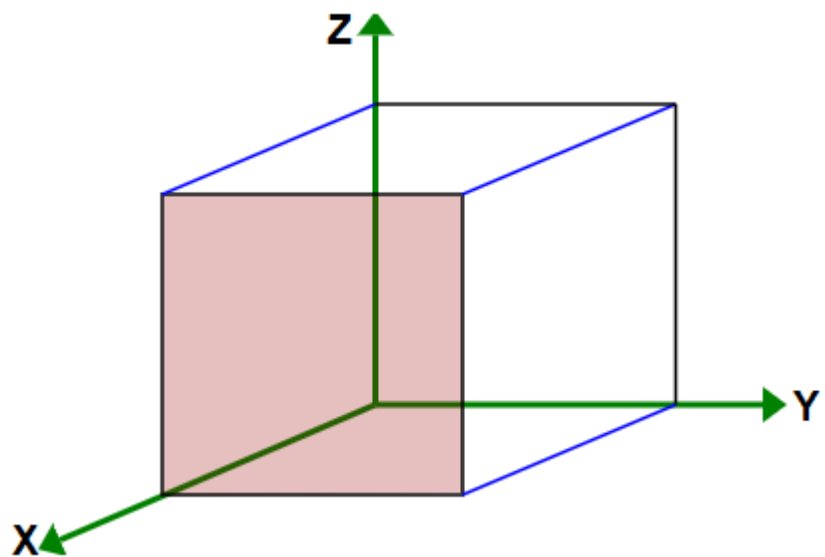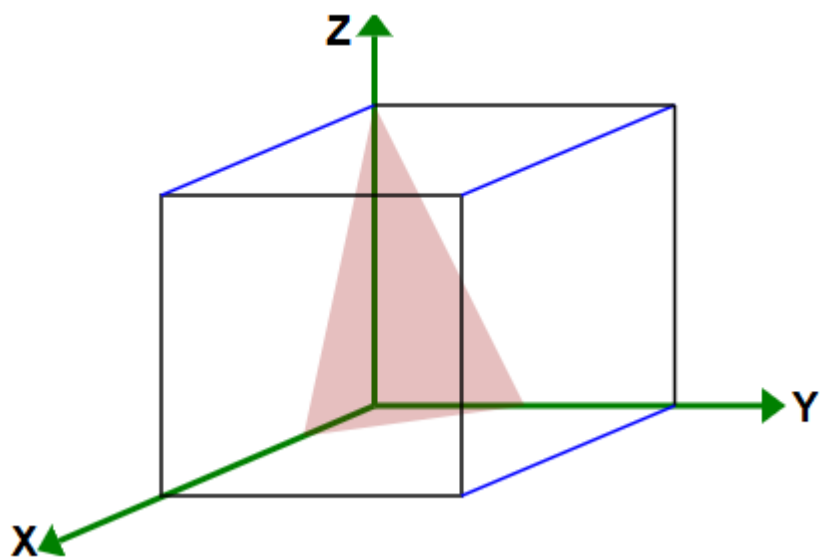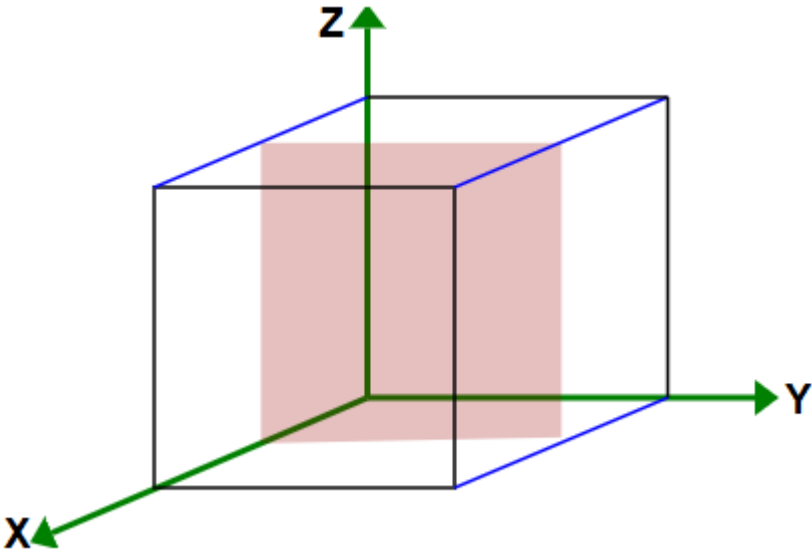
(0 0 1)
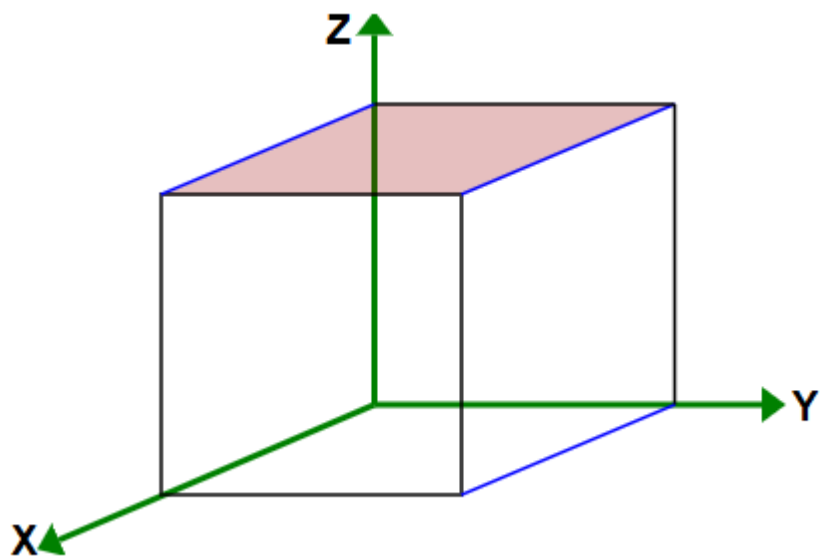
(0 1 0)

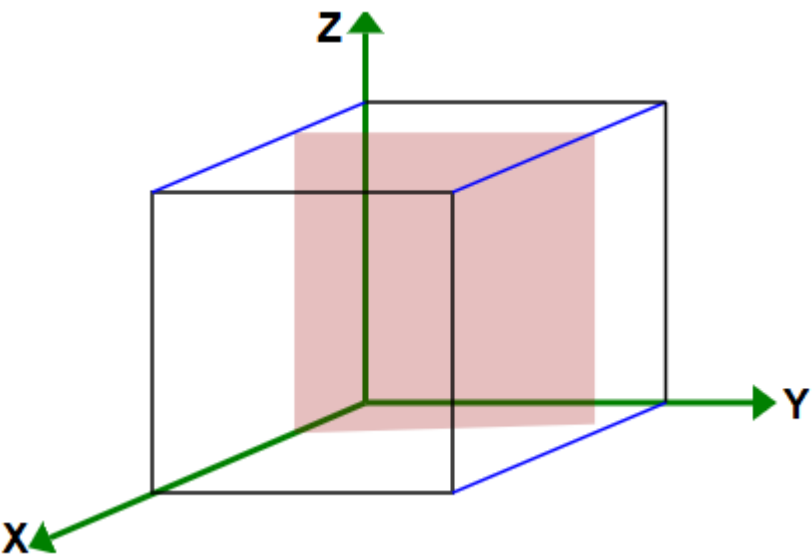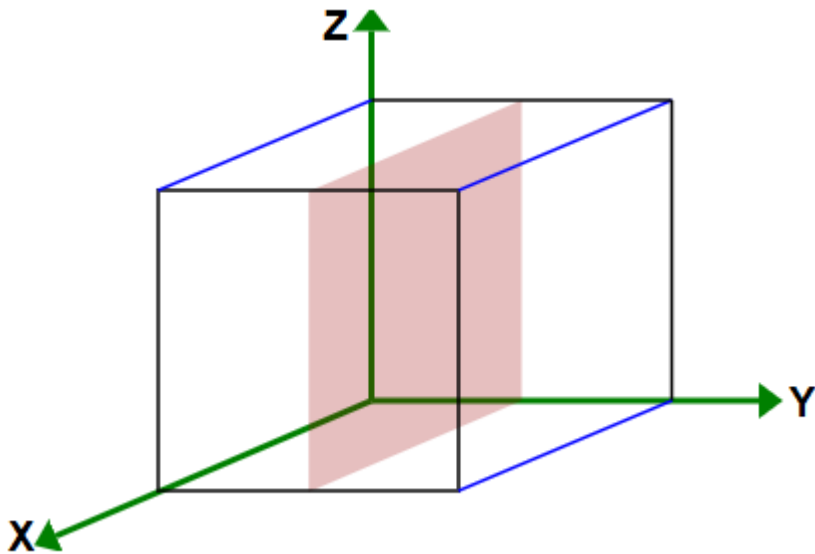(1 1 0)

(1 0 1)

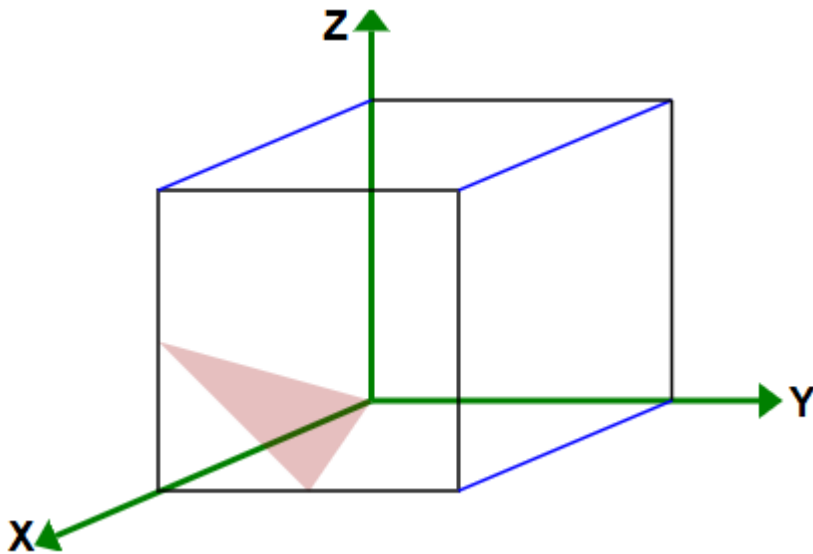(1 1 1)

(3 2 1)

(2 0 0)

(3 0 0)

(0 2 0)

Sometimes h, k or l can be represented as negative integer or given a bar ($\bar{1}$) like this above them, which generally indicates change in origin.

In general the origin is at $(0, 0, 0)$ but if we use negative integer on x axis the origin become $(x, 0, 0)$ where x is the length of the plane along with x-axis. For example:

$(\bar{1}, 2, 2)$

Here the origin is on the lower left point. Similarly negative indices can be applied to y-axis and z-axis

Direction of Miller's plane

To find the direction of Miller's plane we first need to consider the origin of the lattice point (considering negative indices as well). After that we need to divide every index by $max(h, k, l)$.

We need to find the specific point after that and draw a line from the origin and we will have our intended direction. For example

[2 1 0] will be transformed to $\frac{2}{2}$, $\frac{1}{2}$, and $\frac{0}{2}$ as 2 is the maximum between h, k, and l. then we will draw a line from the origin as per the following diagram.