5 - ODE

Differential form

- 4. Find the differential equation whose solution $y^2=4a(x+a)$.

variable separation

1. direct
2. let $x+y=t$

- 7. $(x+y)\,dy+(x-y)\,dx=0$
- the formula
  $$\Rightarrow \frac{dy}{y^2-1}=dx$$
  $$\Rightarrow \frac{1}{2}\ln\left|\frac{y-1}{y+1}\right|=x+c$$

Homogenous

1. always $y=vx$

- $(x+y)\,dy+(x-y)\,dx=0$

Reducible to homogenous

$$\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$$

- case - 1: $\frac{a_1}{a_2} = \frac{b_1}{b_2}$: let $a_1x + b_1y = v$ then variable separation.
- case - 2 : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: $x = X + h$ & $y = Y + k$ then homogenous.

Exact

1. $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then solution :

$$\int M\,dx + \int N[\text{without x term}]\,dy = c$$

Reducible to exact

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

1. $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ then Integrating Factor = $e^{\int f(x)dx}$.
2. $\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = g(y)$ then Integrating Factor = $e^{\int g(y)dx}$.
   eikhane ja diye vag dibo and RHS e ja asbe sob 2nd part er sapekkhe... jemn $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ eikhane vag geche N diye and RHS e $f(x)$.

- just integrating factor

1. Solve the differential equation 1. $(x^2+y^2+x)\,dx + xy\,dy = 0$.

2. $(y^4+2x)\,dx + (xy^3+2y^4-4x)\,dy = 0$

First order linear differential equation

1. $$\frac{dy}{dx} + P(x) \cdot y = Q(x)$$

2. $$\frac{dx}{dy} + P(y) \cdot x = Q(y)$$

- for (1) the integrating factor = $e^{\int P(x)dx}$

- for (2) the integrating factor = $e^{\int P(y)dy}$
  Then the solution will be :

1. $$y \cdot I.F. = \int Q(x) \cdot I.F. + c$$

2. $$x \cdot I.F. = \int Q(y) \cdot I.F. + c$$

Reducible to linear (Bernouli equation)

$$\frac{dy}{dx} + P(x) \cdot y = Q(x) \cdot y^n$$

Let, $v = y^{1-n}$ then convert the equation to $\frac{dv}{dx}$ then linear.

Application of 1st order linear equation

- Electrical Circuit: (equation for LR circuit)

$$L\frac{di}{dt} + Ri = E_0$$

- Newton's law of cooling:

$$\frac{dT}{dt} + KT = kT_{env}$$

- Population growth with limited resources
- Radioactive decay

Homogenous linear differential equations of order n with constant coefficient

$$a_0D^n + a_1D^n - 1 + \ldots.. a_ny = 0 \quad [D = \frac{dy}{dx}]$$

now let $D = m$ and convert the equation:

$$a_0 m^n + a_1 m^{n-1} + \ldots + a_n = 0$$

then find the roots

- case - 1 : All roots are distinct.

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \cdots + c_n e^{m_n x}$$

- case - 2 : All roots are real and same.

$$y = (c_1 + c_2 x + c_3 x^2 + \cdots + c_n x^n) e^{mx}$$

- case - 3 : if the roots are complex number. $(a \pm ib)$

$$y = e^{ax} (c_1 \cos bx + c_2 \sin bx)$$

Non-homogenous linear equation with constant coefficient

Complementary function:

$L(y) = f(x)$ where $L(y)$ is linear operator and $f(x)$ is a known function. The complementary function is the general solution of the associated homogenous equation. i.e. $L(y) = 0$.

Partial Integral:

The partial integral is any one specific solution of the full non-homogenous equation.

the complete solution of the equation is :

$$y = C.F. + P.I.$$

- Rule - 01: $y_p = \frac{1}{D-a} \cdot x^m$
  eikhane common niye eke $\frac{1}{-a}(1 - \frac{D}{a})^{-1} \cdot x^m$ convert korte hobe

then :

i. $(1-D)^{-1} = 1 + D + D^2 + D^3 + \cdots$

ii. $(1+D)^{-1} = 1 - D + D^2 - D^3 + \cdots$

iii. $(1-D)^{-2} = 1 + 2D + 3D^2 + 4D^3 + \cdots$

iv. $(1+D)^{-2} = 1 - 2D + 3D^2 - 4D^3 + \cdots$

- Rule - 02: $y_p = \frac{1}{f(D)-a} \cdot e^{ax}$
  eibar $f(a) \neq 0$ houya porjonto opore x diye multiply and niche derivative korte hobe r check korte hobe 0 hoi kina. Jekhane 0 hobe na oitai je value pabo oitar equation holo PI
- Rule - 03: $y_p = \frac{1}{f(D)-a} \cdot sin(ax+b)/cos(ax+b)$
  eikhane shurute D^2 er jaigai $-a^2$ er value bosate hobe...
  $[a = 2 \; then \; -a^2 = -4]$
  then jodi 0 na ase tahole oitaii PI. r 0 asle opore x multiply then niche derivative. then niche D^2 nai tai niche $(a+b)(a-b) = a^2 - b^2$ er moto banate hobe opore niche multiply kore. then abr check diye dekhte hobe niche 0 kina. 0 na hole opore vangaye niye ja derivative ase korte hobe.. then oitaii ans.