

5 - ODE

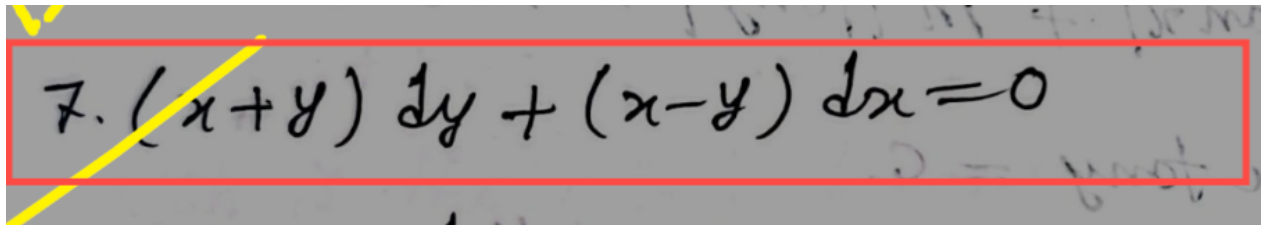
Differential form



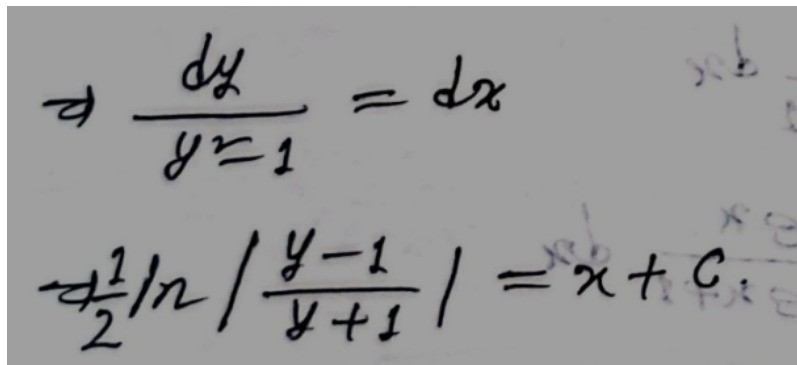
4. Find the differential equation whose solution $y^2 = 4a(x+a)$.

variable separation

1. direct
2. let $x + y = t$



7. $(x+y) dy + (x-y) dx = 0$

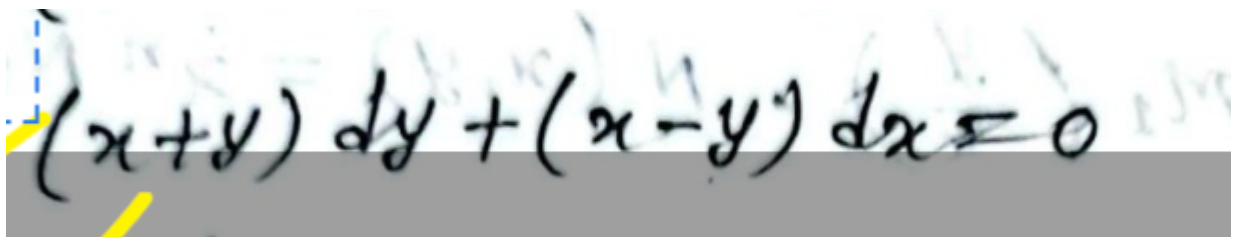


$\Rightarrow \frac{dy}{y-1} = dx$
 $\Rightarrow \frac{1}{2} \ln \left| \frac{y-1}{y+1} \right| = x + C$

- the formula

Homogenous

1. always $y = vx$



$(x+y) dy + (x-y) dx = 0$

Reducible to homogenous

$$\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$$

- case - 1: $\frac{a_1}{a_2} = \frac{b_1}{b_2}$: let $a_1x + b_1y = v$ then variable separation.
- case - 2 : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: $x = X + h$ & $y = Y + k$ then homogenous.

Exact

1. $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then solution :

$$\int M dx + \int N[\text{without } x \text{ term}] dy = c$$

Reducible to exact

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

1. $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ then Integrating Factor = $e^{\int f(x)dx}$.
2. $\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = g(y)$ then Integrating Factor = $e^{\int g(y)dy}$.

eikhane ja diye vag dibo and RHS e ja asbe sob 2nd part er sapekkhe... jemn

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x) \text{ eikhane vag geche } N \text{ diye and RHS e } f(x).$$

- just integrating factor

2. solve the differential equation 1. $(x^2 + y^2 + x)dx + xy dy = 0$.

2. $(y^4 + 2x)dx + (xy^3 + 2y^4 - 4x)dy = 0$.

First order linear differential equation

1. $\frac{dy}{dx} + P(x) \cdot y = Q(x)$
2. $\frac{dx}{dy} + P(y) \cdot x = Q(y)$

- for (1) the integrating factor = $e^{\int P(x)dx}$

- for (2) the integrating factor = $e^{\int P(y)dy}$

Then the solution will be :

$$1. \quad y \cdot I.F. = \int Q(x) \cdot I.F. + c$$

$$2. \quad x \cdot I.F. = \int Q(y) \cdot I.F. + c$$

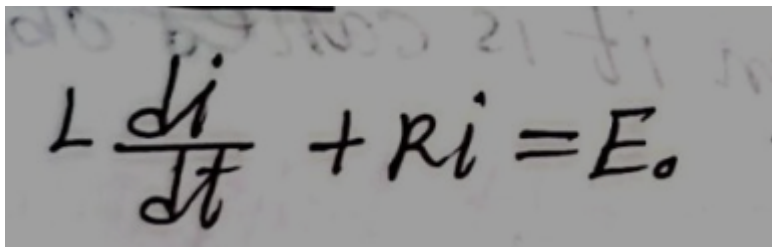
Reducible to linear (Bernouli equation)

$$\frac{dy}{dx} + P(x) \cdot y = Q(x) \cdot y^n$$

Let, $v = y^{1-n}$ then convert the equation to $\frac{dv}{dx}$ then linear.

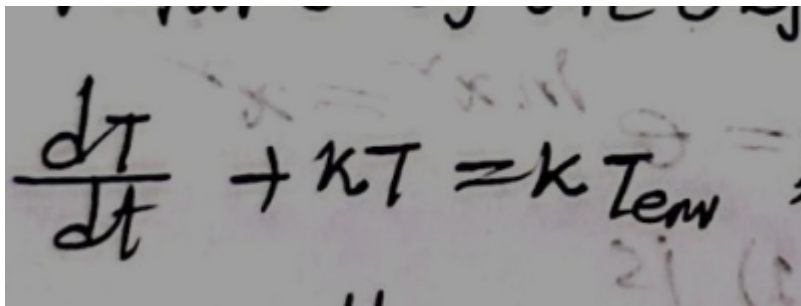
Application of 1st order linear equation

- Electrical Circuit: (equation for LR circuit)



$$L \frac{di}{dt} + Ri = E_0$$

- Newton's law of cooling:



$$\frac{dT}{dt} + kT = kT_{env}$$

- Population growth with limited resources
- Radioactive decay

Homogenous linear differential equations of order n with constant coefficient

$$a_0 D^n + a_1 D^{n-1} + \dots + a_n y = 0 \quad [D = \frac{dy}{dx}]$$

now let $D = m$ and convert the equation:

$$a_0 m^n + a_1 m^{n-1} + \dots + a_n = 0$$

then find the roots

- case - 1 : All roots are distinct.

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} + \dots + C_n e^{m_n x}$$

- case - 2 : All roots are real and same.

$$y = (C_1 + C_2 x + C_3 x^2 + \dots + C_n x^n) e^{m x}$$

- case - 3 : if the roots are complex number. $(a \pm ib)$

$$y = e^{ax} (C_1 \cos bx + C_2 \sin bx)$$

Non-homogenous linear equation with constant coefficient

Complementary function:

$L(y) = f(x)$ where $L(y)$ is linear operator and $f(x)$ is a known function. The complementary function is the general solution of the associated homogenous equation. i.e. $L(y) = 0$.

Partial Integral:

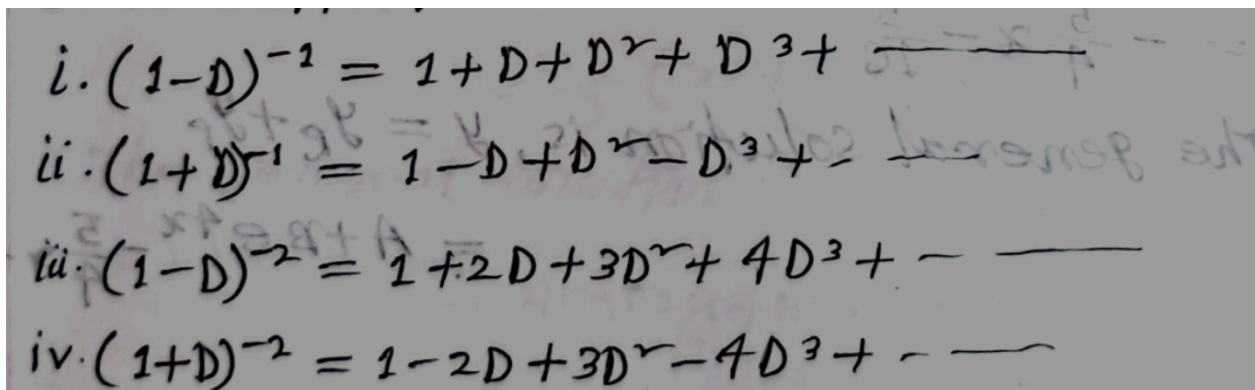
The partial integral is any one specific solution of the full non-homogenous equation.

the complete solution of the equation is :

$$y = C.F. + P.I.$$

- Rule - 01: $y_p = \frac{1}{D-a} \cdot x^m$
eikhane common niye eke $\frac{1}{-a} (1 - \frac{D}{a})^{-1} \cdot x^m$ convert korte hobe

then :



The image shows four handwritten equations for the inverse of differential operators:

- i. $(1-D)^{-1} = 1 + D + D^2 + D^3 + \dots$
- ii. $(1+D)^{-1} = 1 - D + D^2 - D^3 + \dots$
- iii. $(1-D)^{-2} = 1 + 2D + 3D^2 + 4D^3 + \dots$
- iv. $(1+D)^{-2} = 1 - 2D + 3D^2 - 4D^3 + \dots$

- Rule - 02: $y_p = \frac{1}{f(D)-a} \cdot e^{ax}$

eibar $f(a) \neq 0$ houya porjonto opore x diye multiply and niche derivative korte hobe r check korte hobe 0 hoi kina. Jekhane 0 hobe na oitai je value pabo oitar equation holo PI

- Rule - 03: $y_p = \frac{1}{f(D)-a} \cdot \sin(ax+b)/\cos(ax+b)$

eikhane shurute D^2 er jaigai $-a^2$ er value bosate hobe...

[$a = 2$ then $-a^2 = -4$]

then jodi 0 na ase tahole oitai PI. r 0 asle opore x multiply then niche derivative. then niche D^2 nai tai niche $(a+b)(a-b) = a^2 - b^2$ er moto banate hobe opore niche multiply kore. then abr check diye dekhte hobe niche 0 kina. 0 na hole opore vangaye niye ja derivative ase korte hobe.. then oitai ans.