Md. Abdul Malek

Assistant Professor, Dept. of Electrical & Electronic Engineering (EEE)

Rajshahi University of Engineering & Technology (RUET)

Nodal Analysis

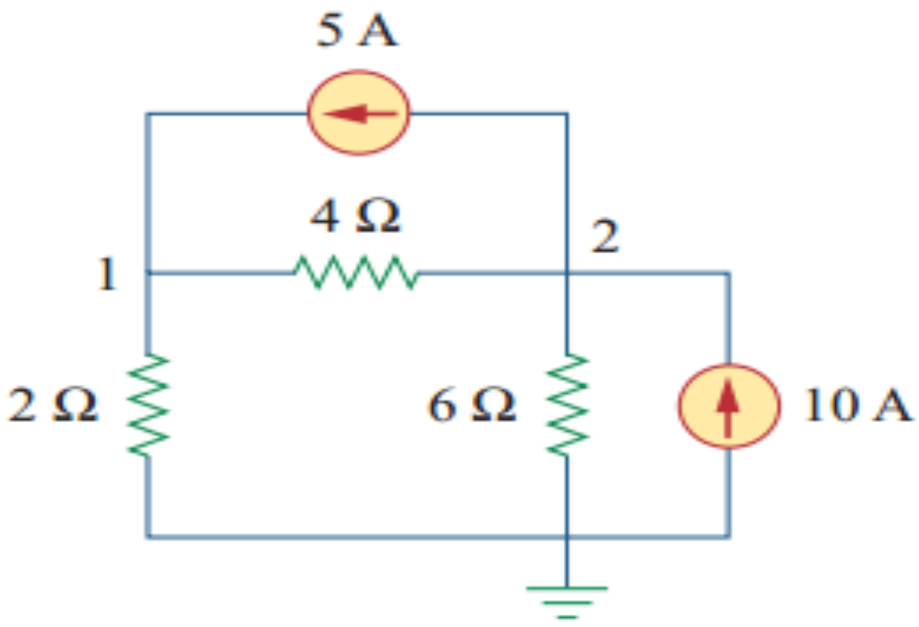
Nodal analysis provides a procedure for analyzing circuits using node voltages as the circuit variables.

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages v_1, $v_2, \ldots, v_{n-1}$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Nodal Analysis

Problem: Calculate the node voltages in the circuit.



Nodal Analysis

Solution: Assign node voltages

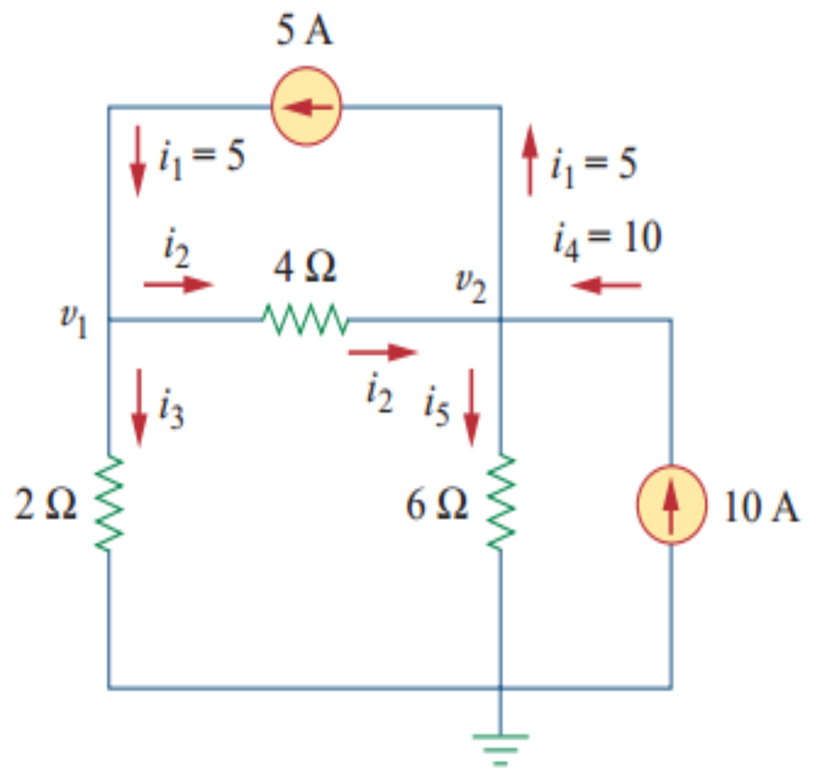
Labeling of the currents is arbitrary

At node 1, applying KCL

$$i_1 = i_2 + i_3$$

$$5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

$$3v_1 - v_2 = 20.....(i)$$



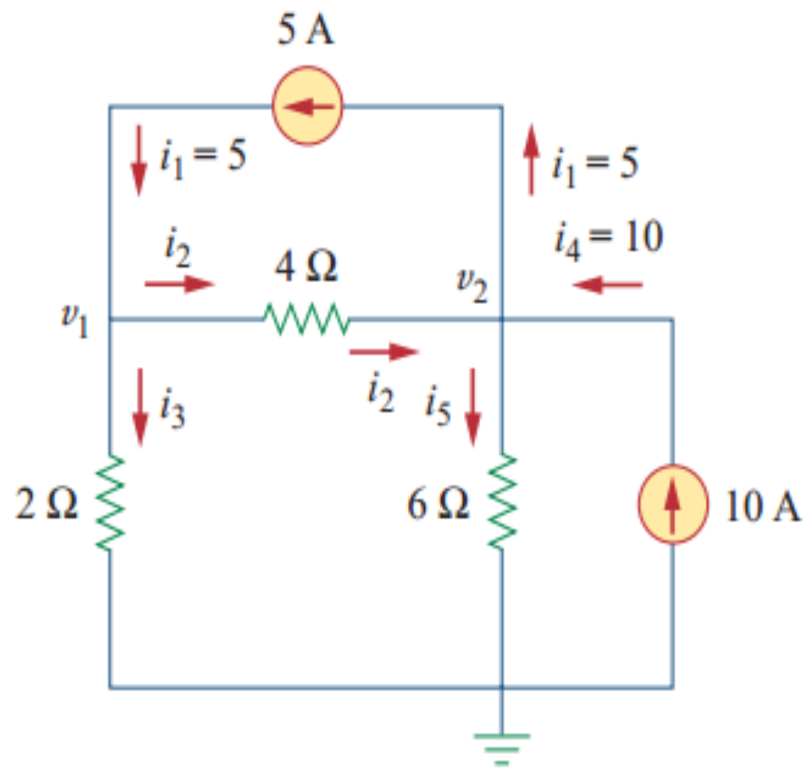
Nodal Analysis

At node 2, applying KCL

$$i_2 + i_4 = i_1 + i_5$$

$$\frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

$$\frac{v_1 - v_2 + 40}{4} = \frac{30 + v_2}{6}$$



$$-3v_1 + 5v_2 = 60.....(ii)$$

Nodal Analysis

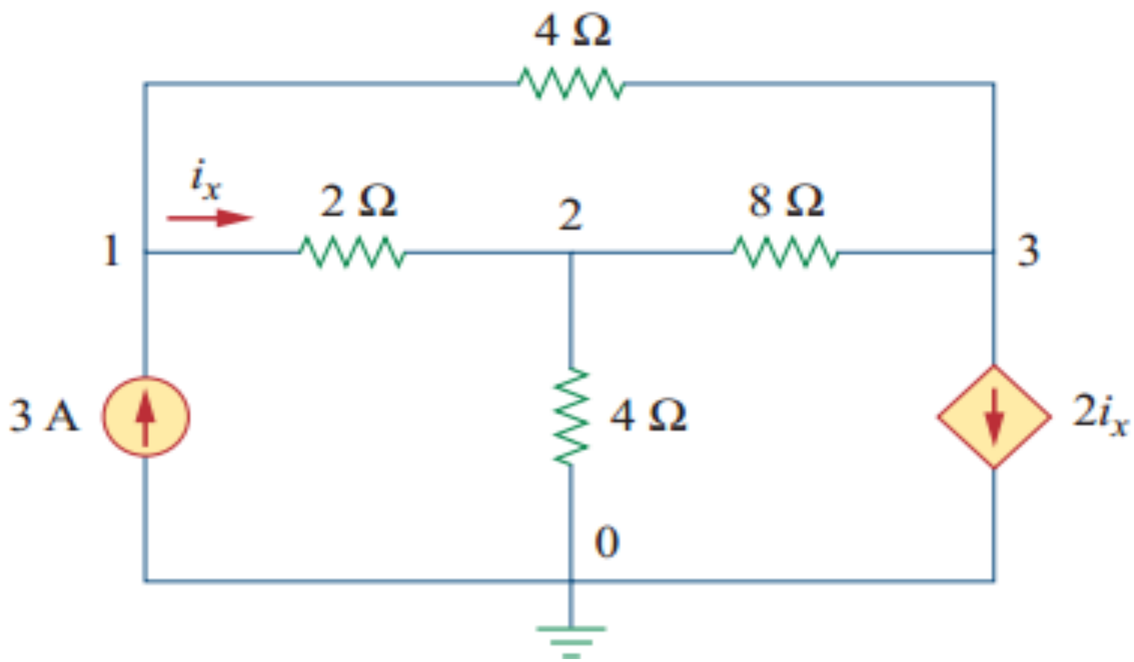
After solving equation (i) and (ii), we get,

$$v_1 = 13.333\ V$$

$$v_2 = 20\ V$$

Nodal Analysis

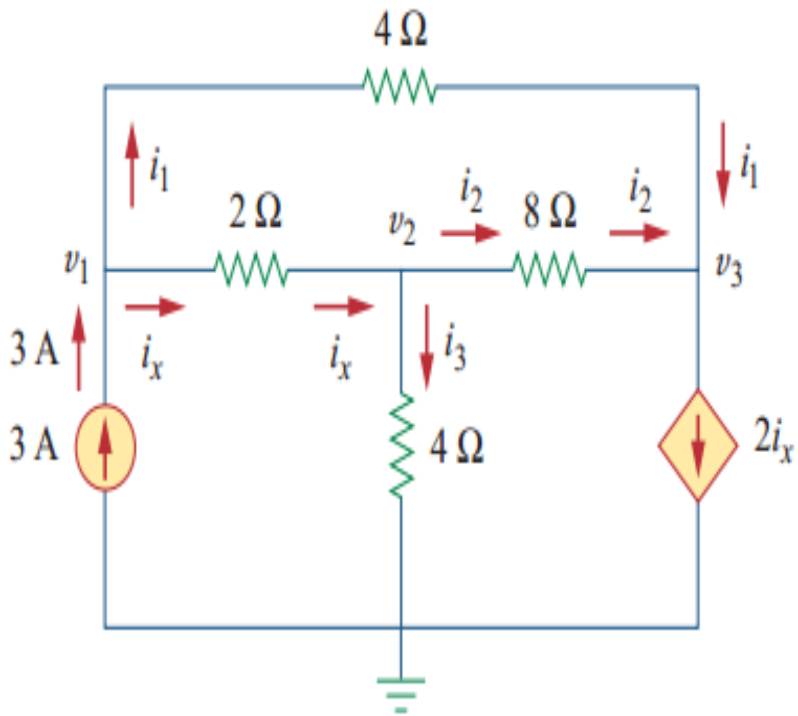
Problem: Determine the voltages at the nodes.



Nodal Analysis

Solution: Assign node voltages and labeling the current

At node 1, applying KCL



$$3 = i_1 + i_x$$

$$3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

$$3 = \frac{v_1 - v_3 + 2v_1 - 2v_2}{4}$$

$$3v_1 - 2v_2 - v_3 = 12.....(i)$$

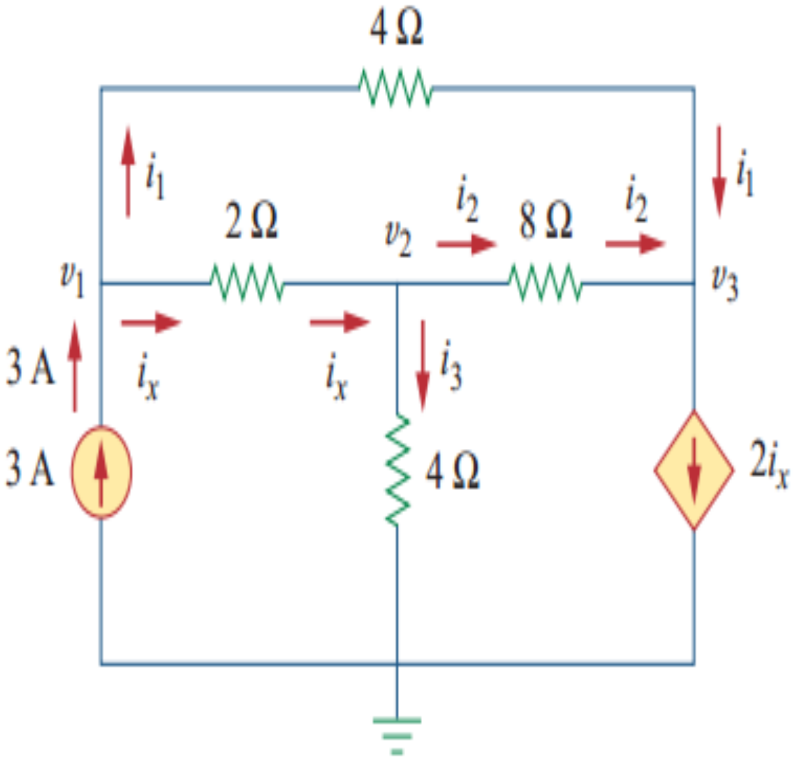
Nodal Analysis

At node 2, applying KCL

$$i_x = i_2 + i_3$$

$$\frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

$$-4v_1 + 7v_2 - v_3 = 0.....(ii)$$



Nodal Analysis

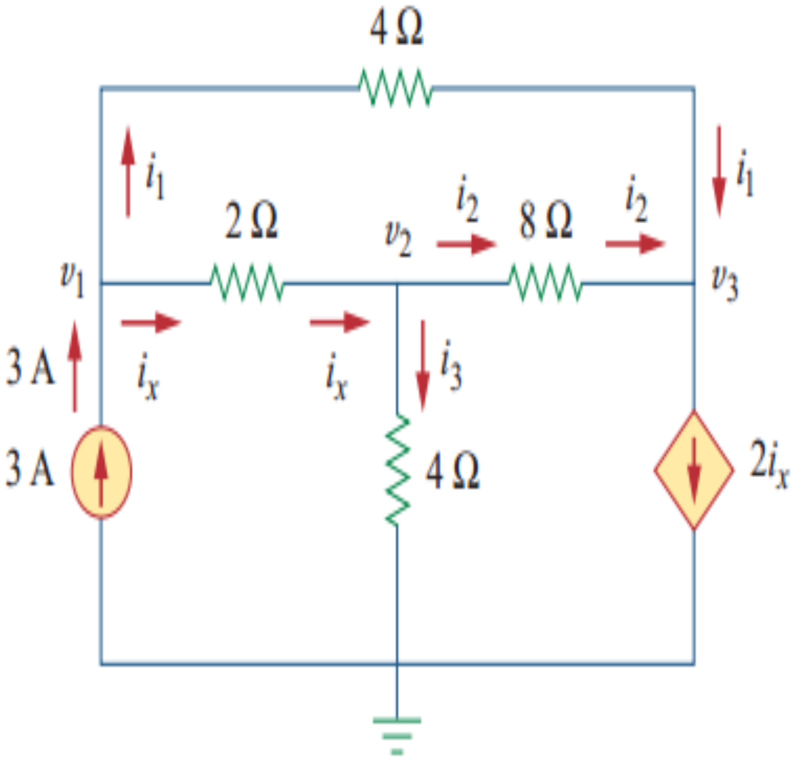
At node 3, applying KCL

$$i_1 + i_2 = 2i_x$$

$$\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = 2\frac{v_1 - v_2}{2}$$

$$\frac{2v_1 - 2v_3 + v_2 - v_3}{8} = v_1 - v_2$$

$$6v_1 - 7v_2 + v_3 = 0.....(iii)$$



Nodal Analysis

After solving equation (i), (ii) and (iii), we get,

$$v_1 = 4.8\ V$$

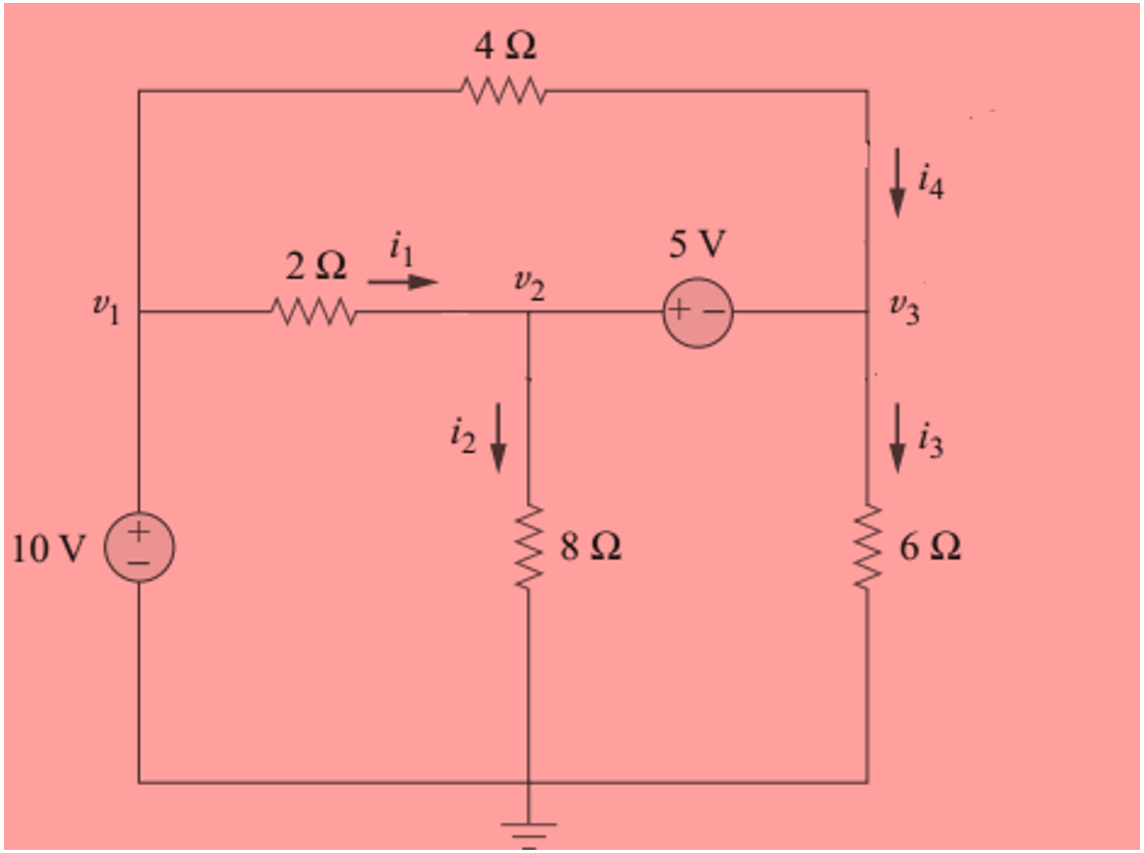
$$v_2 = 2.4\ V$$

$$v_3 = -2.4\ V$$

Nodal Analysis with Voltage Sources

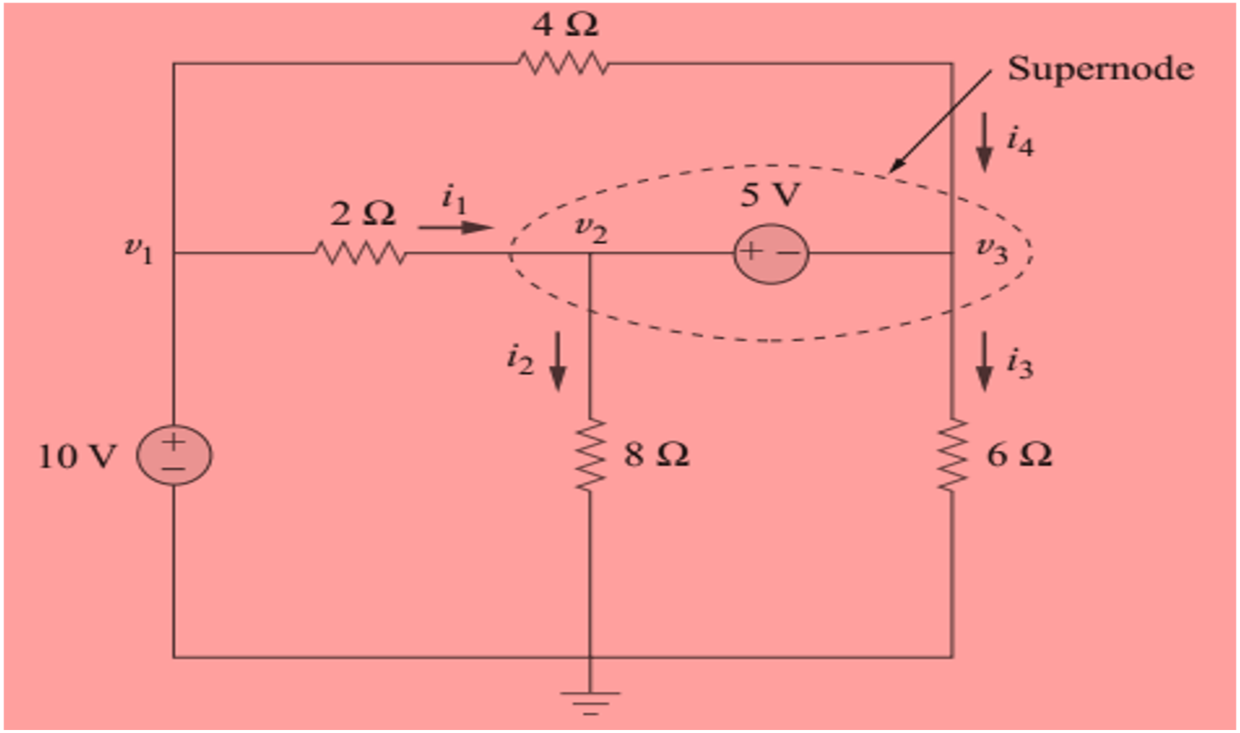
If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source.

$$v_1 = 10\ V$$



Nodal Analysis with Voltage Sources

If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalized node or supernode; we apply both KCL and KVL to determine the node voltages.



Nodal Analysis with Voltage Sources

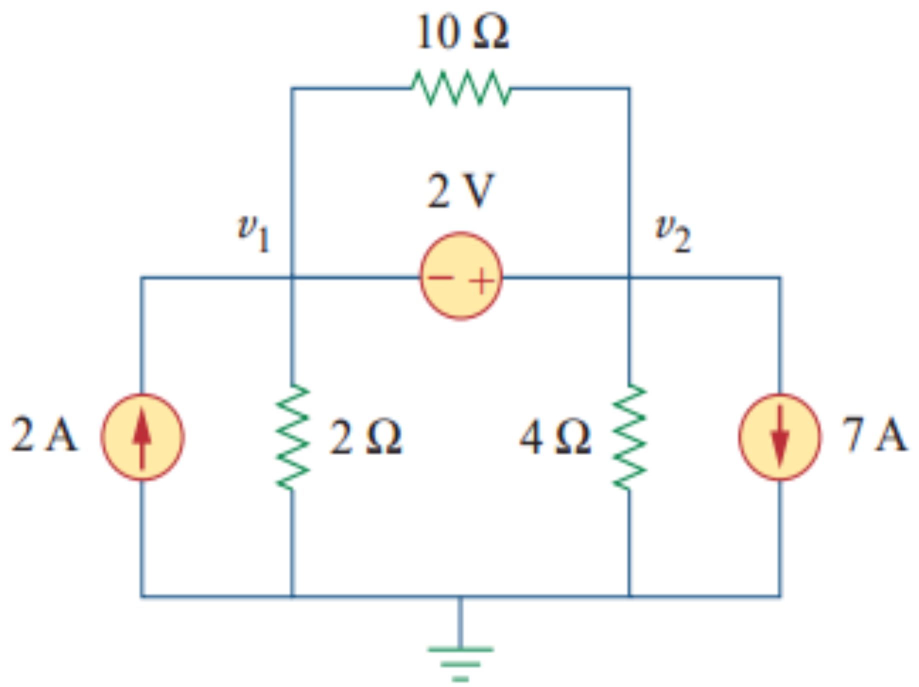
A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

A supernode has no voltage of its own.

A supernode requires the application of both KCL and KVL

Nodal Analysis with Voltage Sources

Determine the voltages at the nodes.



Nodal Analysis with Voltage Sources

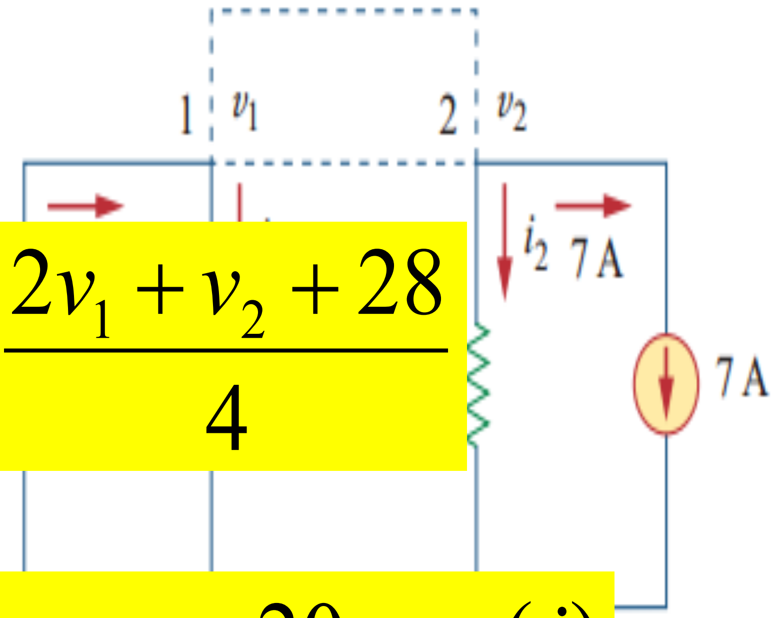
Solution: The supernode contains the 2V source, nodes 1 and 2, and the 10Ω resistor.

Labeling of the currents is arbitrary

Applying KCL at supernode,

$$2 = i_1 + i_2 + 7$$

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7$$



$$2 = \frac{2v_1 + v_2 + 28}{4}$$

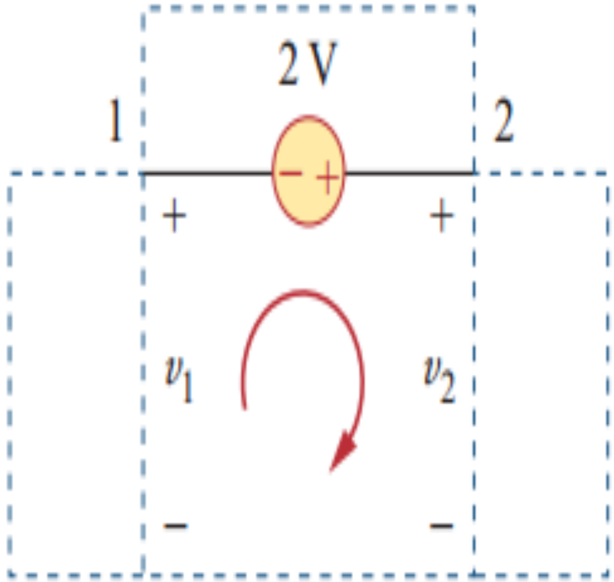
$$2v_1 + v_2 = -20 \ldots\ldots (i)$$

Nodal Analysis with Voltage Sources

To get the relationship between v_1 and v_2 we apply KVL

$$-v_1 - 2 + v_2 = 0$$

$$-v_1 + v_2 = 2......(ii)$$



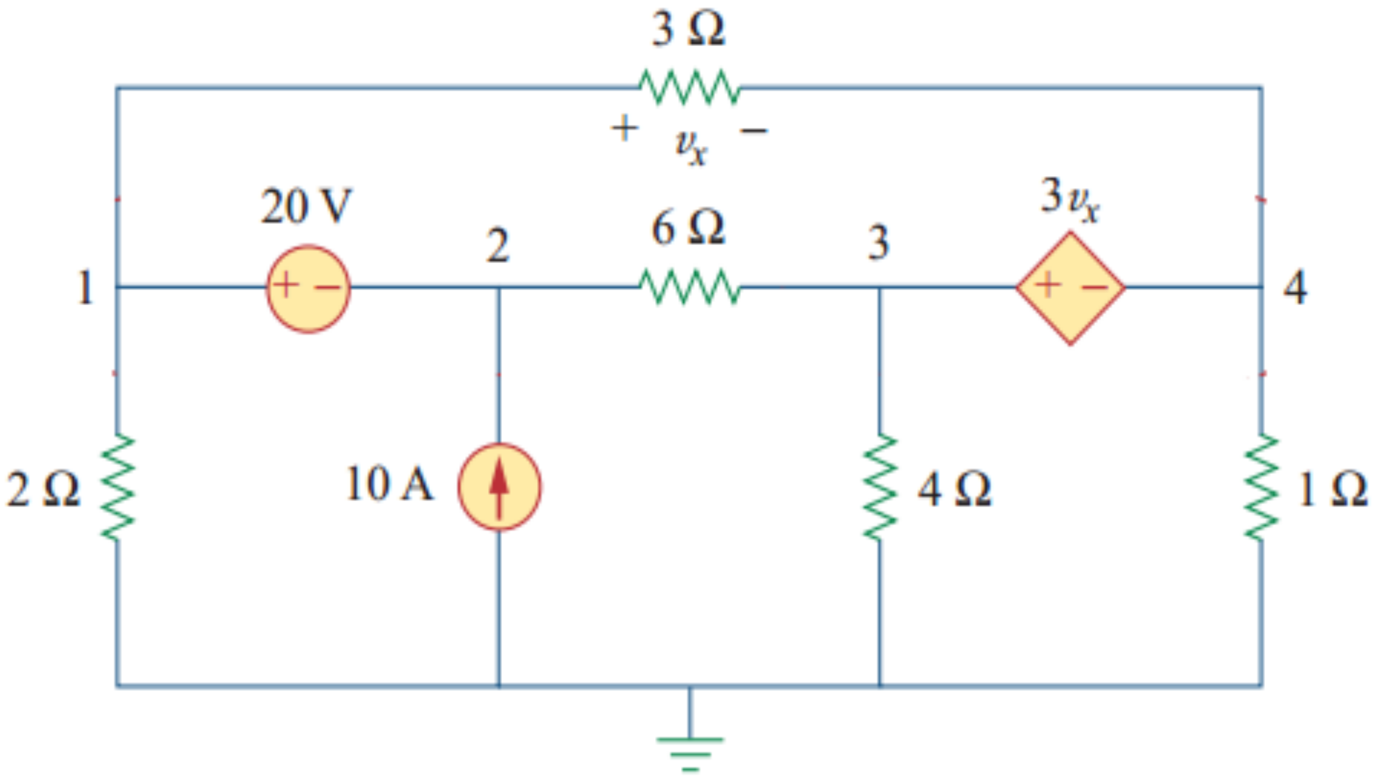
After solving equation (i) and (ii), we get,

$$v_1 = -7.333\,V$$

$$v_2 = -5.333\,V$$

Nodal Analysis with Voltage Sources

Problem: Find the node voltages

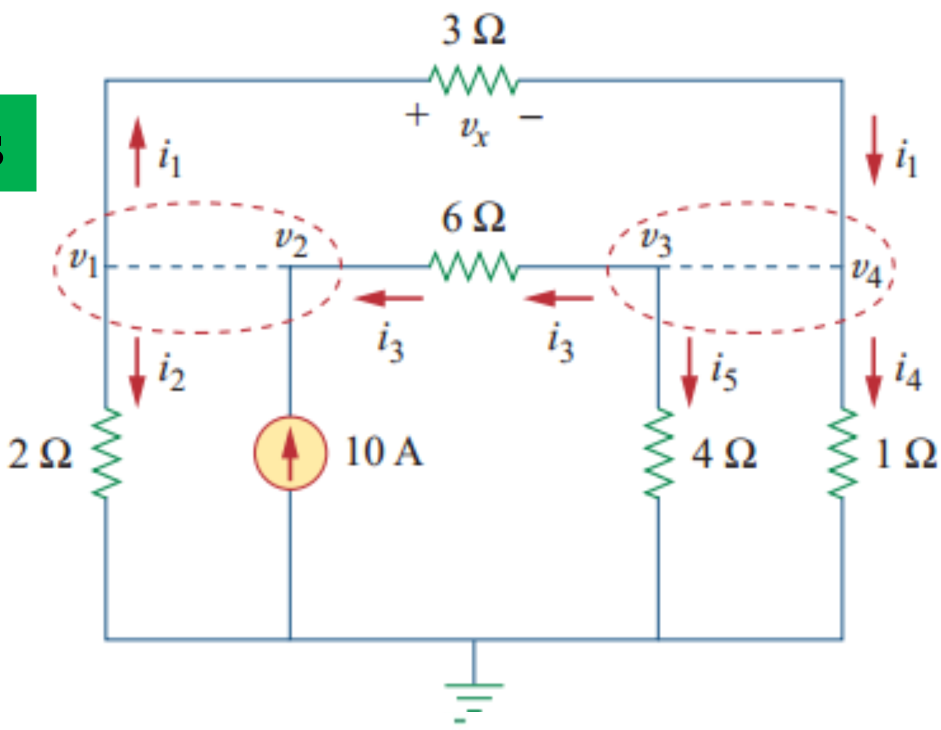


Nodal Analysis with Voltage Sources

Solution: Nodes 1 and 2 form a supernode

Nodes 3 and 4 form another supernode

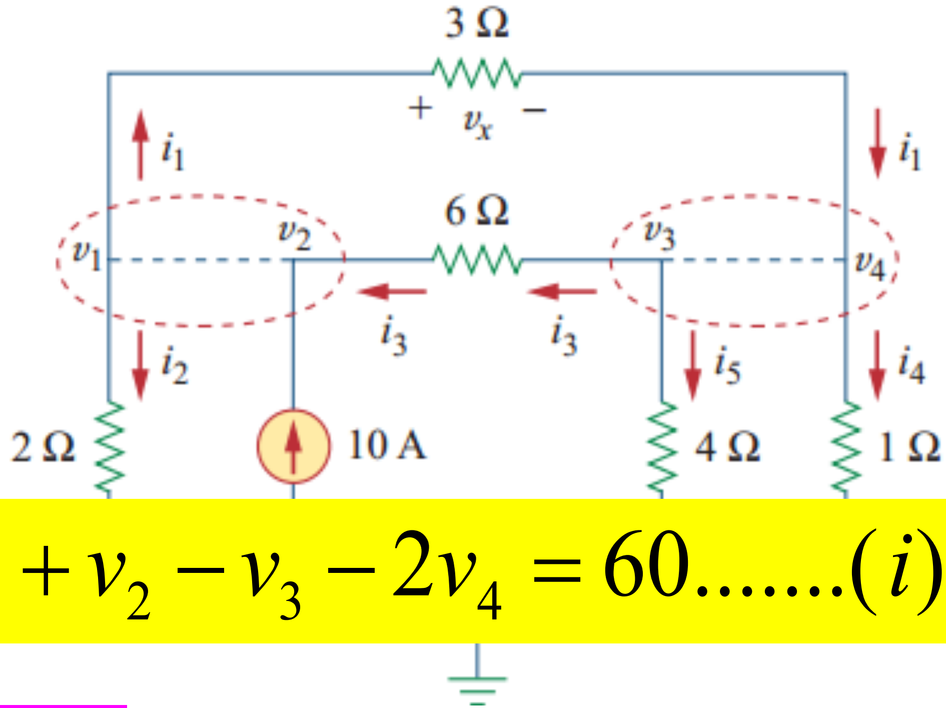
Labeling the direction of currents



Nodal Analysis with Voltage Sources

Applying KCL at supernode 1-2,

$$i_3 + 10 = i_1 + i_2$$



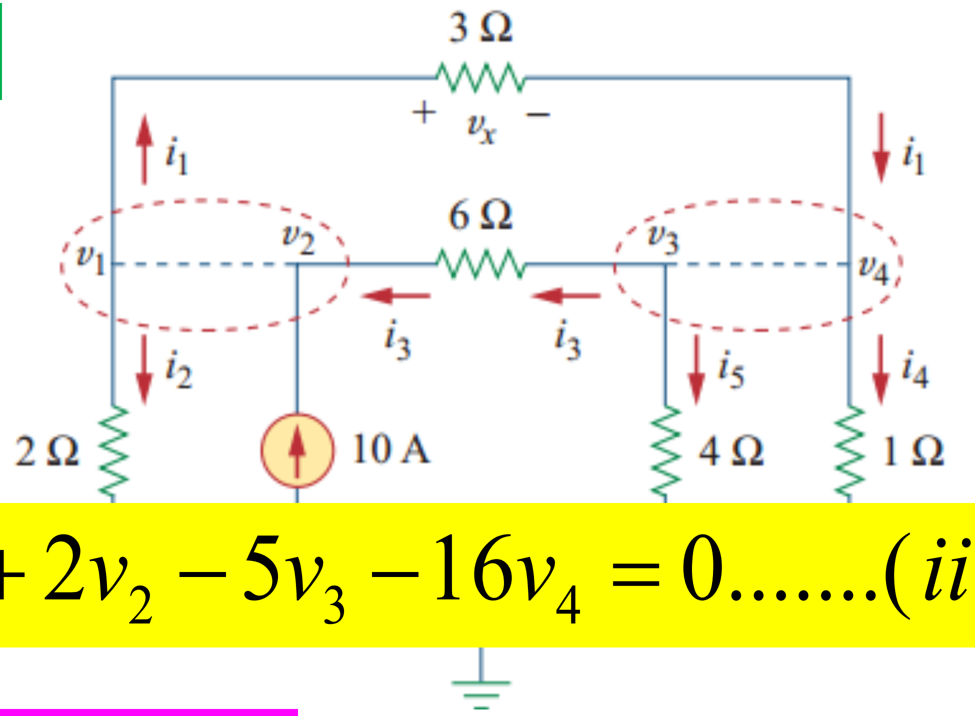
$$5v_1 + v_2 - v_3 - 2v_4 = 60 ....(i)$$

$$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1 - 0}{2}$$

Nodal Analysis with Voltage Sources

Applying KCL at supernode 3-4,

$$i_1 = i_3 + i_4 + i_5$$



$$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0.......(ii)$$

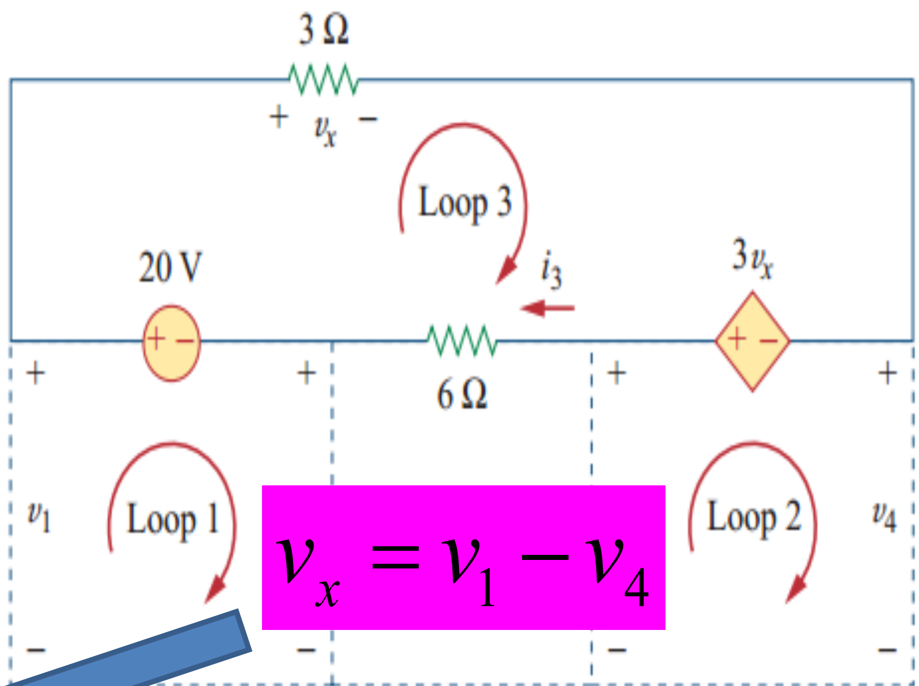
$$\frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4 - 0}{1} + \frac{v_3 - 0}{4}$$

Nodal Analysis with Voltage Sources

Apply KVL to loop 1,

$$-v_1 + 20 + v_2 = 0$$

$$v_1 - v_2 = 20....(iii)$$



$$v_x = v_1 - v_4$$

Apply KVL to loop 2,

$$-v_3 + 3v_x + v_4 = 0$$

$$3v_1 - v_3 - 2v_4 = 0.......(iv)$$

Nodal Analysis with Voltage Sources

After solving equation (i), (ii), (iii) and (iv), we get,

$$v_1 = 26.67\,V$$

$$v_3 = 173.33\,V$$

$$v_2 = 6.667\,V$$

$$v_4 = -46.67\,V$$

Thank You