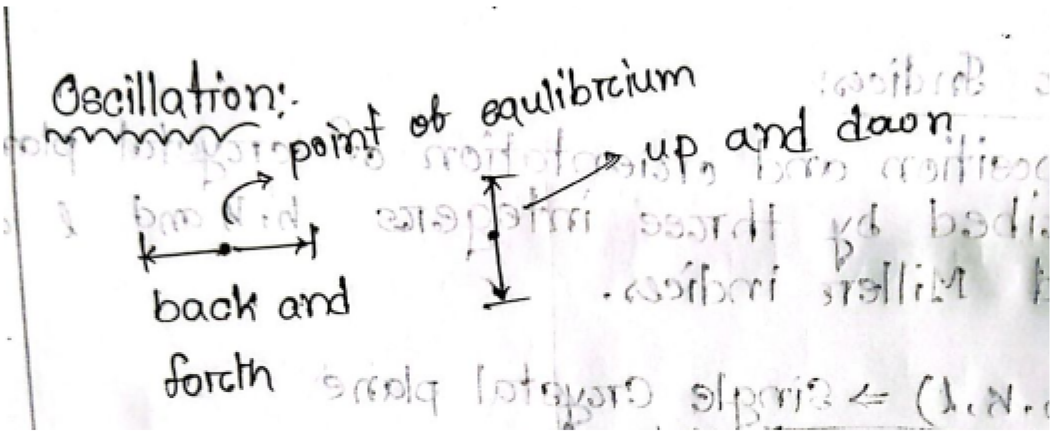
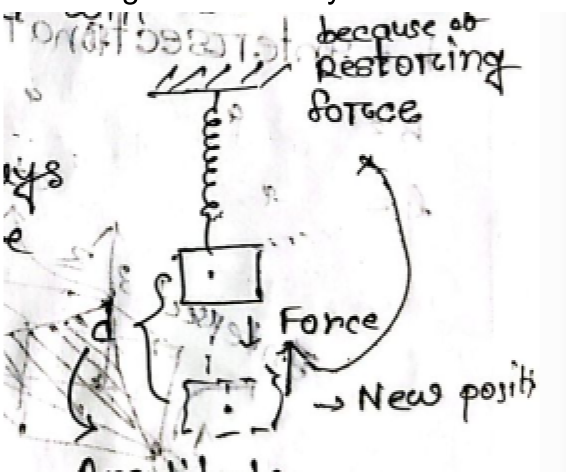
If millions of oscillators oscillate periodically then wave if formed

Oscillation:



- Periodic $\rightarrow$ Time period will be same
- Amplitude same
- Restoring force is always directed towards the main position or point of equilibrium



If all the conditions are fulfilled, then the motion will be considered as simple harmonic motion.

Derivation:

$$F \propto y$$

$$\Rightarrow F = -ky$$

where, k = Spring Constant

$$m\ddot{y} + ky = 0$$

$$m \times \frac{d^2y}{dt^2} + ky = 0$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \ldots (i)$$

where, ω is the angular velocity and $\omega = \sqrt{\frac{k}{m}}$
This is the differential equation of simple harmonic motion

Have to solve the equation to see the nature of the displacement
multiply $2\frac{dy}{dt}$ on both sides of the equation we get

$$2\frac{dy}{dt}\ddot{y} = -\omega^2 y \times 2\frac{dy}{dt}$$

$$\Rightarrow \left(\frac{dy}{dt}\right)^2 = -\omega^2 y^2 + c, \text{[Integrating both sides we get]}$$

At, $y = a$, $v = 0$
So,

$$\frac{dy}{dt} = 0$$

$$\therefore 0^2 = -\omega^2 a^2 + c$$

$$c = \omega^2 a^2$$

$$\therefore \left(\frac{dy}{dt}\right)^2 = -\omega^2 y^2 + \omega^2 a^2$$

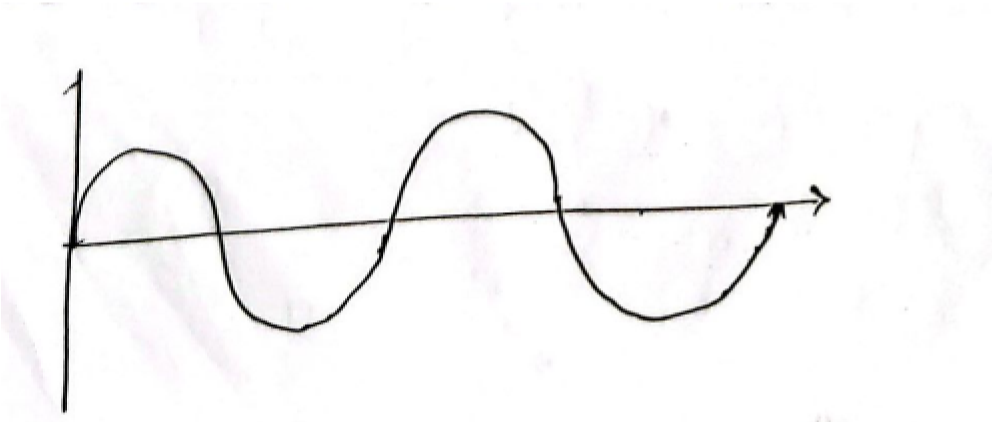
$$\Rightarrow \frac{dy}{dt} = \pm\omega\sqrt{a^2 - y^2}$$

$$\Rightarrow \int \frac{dy}{\sqrt{a^2 - y^2}} = \int \omega dt$$

$$\Rightarrow \sin^{-1}\frac{y}{a} = \omega t + \phi$$

$$\therefore y = a\sin(\omega t + \phi) \ldots (ii)$$

This is the solution of equation (i)
One and only and the best solution of the differential equation of simple harmonic motion



Further dissecting equation (ii)

$$y = a\sin\omega t\cos\phi + a\cos\omega t\sin\phi$$

$$\Rightarrow a\cos\phi\sin\omega t + a\sin\phi\cos\omega t$$

$$= A\sin\omega t + B\cos\omega t$$

In special cases either A or B may be zero.

$$y_1 = A\sin\omega t$$

$$y_2 = B\sin\omega t$$

This is also the solution of the differential equation
$y = y_1 + y_2$ satisfies the differential equation

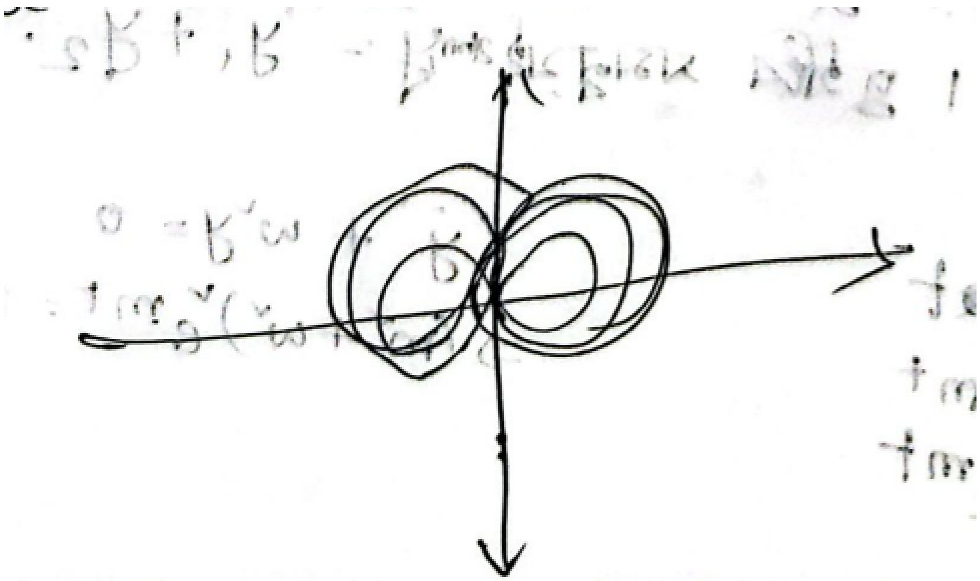
There is another form of the equation

$$y(t) = \text{Real}[Ae^{i(\omega t+\phi)}]$$

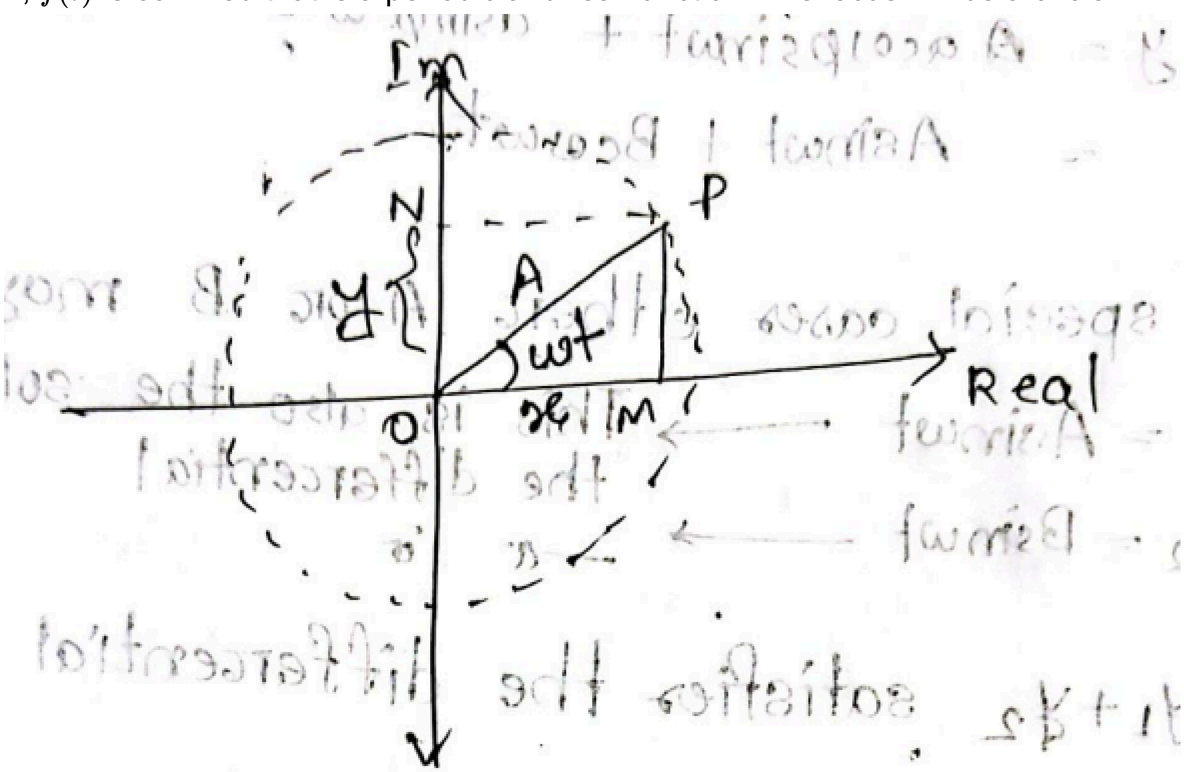
where, $i = \sqrt{-1}$

$$y(t) = \text{Real}[(A\cos(\omega t+\phi) + if(t))]$$

where, $f(t)$ is a real function. If $f(t)$ is arbitrary, plotting the $y(t)$ we will find the locus will be mysterious.



If, $f(t)$ is confined that is a periodic and real function. The locus will be a circle.



The projection will rotate across the circumference.

$$x = a\cos \omega t = y_1$$

$y = a\sin \omega t = y_2$

If a particle revolves in a circle in same angular velocity then we get two simple harmonic motion, combiningt

$y = e^{mt}$

$\dot y = m e^{mt}$

$$\ddot y = m^2e^{mt}$$

$$eknowthat,$$

$$\ddot y + \omega ^2y = 0$$

$$\Rightarrow (m^2 + \omega ^2)e^{mt} = 0$$

$$as\$e^{mt} \neq 0\$weget,$$

$$m^2 + \omega ^2 = 0$$

$$\therefore m = \pm i \omega$$

$$Thetwosolutionsare,$$

$$y_1 = Ae^{i \omega t}$$

$$y_2 = Be^{-i\omega t}$$

$$Botharethesolutionofthedifferentialequationofsimpleharmonicmotion. Now, differentiating\$e^{i\omega t}\$$$

$$\dot y = \ldots e^{i \omega t}$$

$$\ddot y = \ldots e^{i \omega t}$$

$$\dddot y = \ldots e^{i \omega t}$$

$$Thisisthesimpleharmonicmotioncharacteristic. Thiscannotbekilledbydifferentiation. So, itiscompar$$

$$y = A \sin \omega t + B \cos \omega t$$

$$y_1 = A \sin \omega t$$

$$y_2 = B \cos \omega t$$

$$y = Ae^{\pm i \omega t}$$

$$[[Pastedimage20251019171102.png]]$$

$$v = \frac{dy}{dt}$$

$$a = \frac{d^2y}{dt^2}$$

Examples of SHM: Spring, AC circuit, atomic vibration, electro-magnetic wave ($\overrightarrow{E}$ and $\c

y = l\theta

\Rightarrow \dot y = l \frac{d\theta}{dt}

\Rightarrow \ddot y = l \frac{d^2\theta}{dt^2}

m\overrightarrow{a} = m \ddot y = ml\frac{d^2\theta}{dt^2}

$$From Newton's second law of motion,$$

ml\frac{d^2\theta}{dt^2} = -mg\theta

ml\frac{d^2\theta}{dt^2} + mg\theta = 0 \ldots(i)

\Rightarrow \frac{d^2\theta}{dt^2} + \omega^2\theta = 0

\omega = \sqrt{\frac gl}

$$This equation looks very similar to the differential equation of SHM Solution of the equation is,$$

\theta = \theta_o\sin(\omega t + \phi)

This indicates that the motion is oscillatory ## LC Circuit ![[Pasted image 20251019172107.png]] At first capa

V = \frac QC

$$The e.m.f. developed in the inductor dew to the change of current through it will be$$

E = L\frac{di}{dt}

$$Considering it a mechanical hindrance, there will be a negative sign$$

\frac QC = -L\frac{di}{dt}

L\frac{d^2Q}{dt^2} + \frac QC = 0 \ldots(i)

$$ecangetthisalsobyapplyingKVLtothecircuit$$

\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0

\frac{d^2Q}{dt^2} + \omega ^2 Q = 0

$$where,$$

$$\omega^2 = \frac 1{LC}$$

Thisequationistheequationofthemotionofchargebetweenthecapacitorandtheinductorwhichisverysim

$$Q = Q_o\sin(\omega t + \phi)$$

[[Pastedimage20251019172829.png]]Thenatureofthecharge, currentandvoltagewillbeoscillatoryIfthec

$$\frac{d^2Q}{dt^2} + \omega^2Q + iR = 0$$

[[Pastedimage20251019173002.png]]Fornon − conservativeforceaffectingthesystem, wehavetoaddall

$$\therefore E = K.E. + P.E.$$

$$= \frac 12 mv^2 + P.E.$$

$$= \frac 12 m \left\{ a \omega \cos(\omega t + \theta) \right\}^2 + \frac 12 k y^2$$

$$= \frac 12 ma^2\omega^2\cos^2(\omega t + \theta) + \frac 12ka^2\sin^2{(\omega t + \theta)}$$

$$= \frac 12 ka^2\cos^2(\omega t + \theta) + \frac 12 ka^2\sin^2(\omega t + \theta)$$

$$= \frac 12 ka^2,\ \omega = \sqrt{\frac km}$$

$$= 2\pi^2n^2a^2m$$

where, $n$isfrequency, andthetermisconstant[[Pastedimage20251019174111.png]]AverageKineticen

$$\frac 1T\int_o^T\frac 12mv^2dt = \frac 14ka^2$$

Similarly average potential energy is also $\frac 14 ka^2$ # Composition of SHM Two simple harmonic motio

$$y_1 = a_1\sin(\omega t + \alpha_1) = a_1(\sin\omega t\cos\alpha_1 + \cos \omega t \sin \alpha_1)$$

$$y_2 = a_2\sin(\omega t + \alpha_2)$$

Theresultantmotionwillbevectorsumofindividualdisplacement

$$y = y_1 + y_2$$

$$= (a_1\cos\alpha_1 + a_2\cos\alpha_2)\sin\omega t + (a_1\sin\alpha_1 + a_2\sin\alpha_2)\cos\omega t$$

$$= A \cos \phi \sin \omega t + A \sin \phi \cos \omega t$$

here,

$$A\cos\phi = a_1\cos\alpha_1 + a_2\cos\alpha_2 \ldots(i)$$

$$A\sin\phi = a_1\sin\alpha_1 + a_2\sin\alpha_2 \ldots(ii)$$

Resultantmotionwillbe,

$$y = A\sin(\omega t + \phi)$$

hichistheequationofSHMmotionsothenatureofmotionwillalsobeoscillatory. Howevertheamplitudew

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(\alpha_1 - \alpha_2)}$$

If, \$$\alpha_1 = \alpha_2 = \alpha$\$*thatmeanstwovibrationsisinsamephase, then*

$$A = a_1 + a_2$$

if, \$$\alpha_1 - \alpha_2 = (2n = 1)\pi$\$*then,*

$$A = a_1 - a_2$$

![[Pasted image 20251019175735.png]] ## Composition of two perpendicular SHM motion Let,

$$x = a \sin(\omega t + \phi)$$

$$y = b \sin \omega t$$

$$\therefore \frac xa = \sin\omega t \cos \phi+ \cos \omega t \sin \phi$$

$$\therefore \frac yb = \sin \omega t$$

$$\therefore \frac xa = \frac yb\cos\phi + \sqrt{1 - \frac{y^2}{b^2}}\sin \phi$$

$$\Rightarrow \left(\frac xa - \frac yb \cos \phi \right)^2 = \left(1 - \frac{y^2}{b^2}\right)\sin^2\phi$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos \phi = \sin^2\phi$$

Itsageneralequationofconic. Theshapewilldependupon\$phi\$and\$a\$and\$b\$If, \$$\phi = 0, 2\pi, 4\pi, \ldots, 2n\pi$\$

$$\left(\frac xa - \frac yb \right)^2 = 0$$

$$\Rightarrow y = \pm \frac ba x$$

Itrepresentstheequationofapairofcoincidentstraightlinespassingthroughtheorigin. [[Pastedimage20

$$\therefore y = \mp \frac ba x$$

[[Pastedimage20251019180624.png]]If, $\phi = \frac{\pi}{2}$then,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Equationofasymmetricalellipse, if, $a = b$, thenitwillbeanequationofcircle. [[Pastedimage2025101911

$$x = a\sin(2\omega t + \phi)$$

$$y = b\sin\omega t \Rightarrow \frac yb = \sin \omega t$$

$$\frac xa = 2\sin\omega t\cos\omega t \cos\phi + (1-2\sin^2\omega t)\sin\phi$$

$$\Rightarrow \frac xa = 2\frac yb \sqrt{1- \frac{y^2}{b^2}} \cos \phi + (1 - 2\frac{y^2}{b^2}) \sin \phi$$

$$\left(\frac xa - \sin \phi\right)^2 = 4\frac{y^4}{b^4}\sin^2\phi + 2\left(\frac xa - \sin \phi\right)\cdot2\frac{y^2}{b^2}\sin^2\phi = \frac{4y^2}{b^2}\cos^2\phi\left(1 - \frac{y^2}{b^2}\right)$$

$$\Rightarrow \left(\frac xa - \sin \phi\right)^2 + \frac{4y^2}{b^2}\left(\frac{y^2}{b^2} + \frac xa \sin \phi - 1\right) = 0$$

Equationofacurvehavingtwoloopshen, $\phi = 0$

$$\frac{x^2}{y^2} + \frac{4y^2}{b^2}\left(\frac{y^2}{b^2} - 1\right) = 0$$

Itwilldisplaythefigureofeight[[Pastedimage20251019182112.png]]Example Thetableofmicrowaverot

$$y^2 = -\frac{b^2}{2a}(x - a)$$

which represents a parabola ![[Pasted image 20251019182315.png]] # Damped Oscillator Differential equatio

$$\frac{d^2y}{dt^2} + 2\lambda\frac{dy}{dt} + \omega^2y = 0\ldots(i)$$

Solutionfortheequation(auxiliarysolution)

$$y = Ae^{kt}$$

$$\dot y = Ake^{kt}$$

$$\ddot y = Ak^2e^{kt}$$

Puttingthevaluesin(i)

$$(k^2 + 2\lambda k + \omega ^2)Ae^{kt}$$

If, $Aekt \neq 0$then,

$$k^2 + 2\lambda k + \omega ^2 = 0$$

$$\Rightarrow k = \frac{-2\lambda \pm \sqrt{4 \lambda^2 - 4\omega^2}}2$$

$$\therefore k = -\lambda \pm\sqrt{\lambda ^2 - \omega ^2}$$

$$k_1 = -\lambda + \sqrt{\lambda ^2 - \omega ^2}$$

$$k_2 = -\lambda - \sqrt{\lambda ^2 - \omega ^2}$$

$\therefore$ twosolutionsare,

$$y_1 = A_1e^{(-\lambda + \sqrt{\lambda^2 - \omega^2})t} + A_2e^{(-\lambda - \sqrt{\lambda^2 - \omega^2})t}$$

Thisisthegeneralsolutionoftheequationofthedampedoscillator. where,

$$y_1 = A_1e^{(-\lambda + \sqrt{\lambda^2 - \omega^2})t} + A_2e^{(-\lambda - \sqrt{\lambda^2 - \omega^2})t}$$

A_1, A_2isstillunknown, asweustassumeditThevalueofA_1andA_2canbedeterminedbyapplyingbo

$$y_1 = A_1e^{(-\lambda + \sqrt{\lambda^2 - \omega^2})t} + A_2e^{(-\lambda - \sqrt{\lambda^2 - \omega^2})t}$$

at$t = 0$

$$a_o = A_1 + A_2\ldots(2)$$

Here,

$$\frac{dy}{dt} = 0$$

$$\frac {dy}{dt} = A_1(-\lambda + \sqrt{\lambda^2 - \omega^2}) + A_2(-\lambda - \sqrt{\lambda^2 - \omega^2}) = 0$$

$$A_1 - A_2 = \frac{\lambda a_o}{\sqrt{\lambda^2 - \omega^2}} \ldots(3)$$

$(2) + (3)$

$$2A_1 = a_0 + \frac{\lambda a_o}{\sqrt{\lambda^2 - \omega^2}}$$

$$A_1 = \frac 12a_0\left(1 + \frac{\lambda}{\sqrt{\lambda^2 - \omega^2}}\right)$$

$Similarly,$

$$A_1 = \frac 12a_0\left(1 - \frac{\lambda}{\sqrt{\lambda^2 - \omega^2}}\right)$$

$Here, therecanbethreephenomenon$

$$\lambda > \omega$$

$$\lambda = \omega$$

$$\lambda < \omega$$

Overdamping $\lambda > \omega$ Here, $\sqrt{\lambda^2 - \omega^2}$ is a real quantity. Both terms on th

$$y = (A_1 + A_2)e^{-\lambda t}$$

The nature of the displacement will be non-oscillatory, the amplitude will decrease to zero in a very short time.

$$\therefore -\sqrt{\omega^2 - \lambda^2} = ig$$

$$y = e^{-\lambda t}[A_1e^{-gt} + A_2e^{-igt}]$$

$$= e^{-\lambda t}[(A_1 + A_2)\cos gt + i(A_1 - A_2)\sin gt]$$

$$e^{-\lambda t}C_o\sin(gt + \phi)$$

$[[Pastedimage20251019184657.png]]Itwillbecomezeroat\$t = \infty\$, becauseitdecaysexponentially. Exam$

$$y = a_oe^{-\lambda t}\sin(gt + \phi)$$

$Relaxationtime \ Foralowdampedoscillator, thetimerequirefortheamplitudetodecayto\$\frac{1}{e}\$ - thofitsi$

$$a = a_oe^{-\lambda t}$$

$$\Rightarrow \frac {a_o}e = a_oe^{-\lambda \tau}$$

$$\tau = \frac 1 \lambda$$

$Energyofalowdampedoscillatoreknow, Totalenergy = KE + PEeknow, \$E \propto a^2\$As, theamplitudedec$

$$\frac 1T \int_o^T\frac 14 mv^2dt = \frac 1ama_o^2e^{-2\lambda t}g^2$$

here, g*istheangularfrequencyforlowdampedoscillatorAveragepotentialenergy*

$$\frac 1T\int_0^TFdydt = \frac 14ma_o^2e^{-2\lambda t}g^2$$

$\therefore$ *averageenergy*

$$E = \frac 12m_o^2e^{-2\lambda t}g^2$$

Decayrateof E = $E_oe^{-2\lambda t}$

$$g = \omega^2 - \lambda^2$$

Timeperiod = $\frac{g}{2\pi}$ *Powerdissipation*(P)

$$P = -\frac{dE}{dt} = 2 \lambda E$$

Thelossofenergywillappearinthebornofheat, *therateofenergydecayisfasterthantherateofamplituded*

$$Q = 2\pi \frac{\text{Energy Stored}}{\text{Energy lost per period}}$$

$$= 2\pi \cdot \frac{E}{\frac PT}$$

$$= 2\pi \frac{E}{2\lambda E / \frac {2\pi} g} = \frac g{2\lambda}$$

$g \rightarrow$ *Frequencyofalow* $-$ *dampedoscillator*. *Lessthedampingthebatterwillbethequalityoftheoscillatc*

$$a_n = a_oe^{\lambda\frac T4}$$

$$a_{n + 1} = a_oe^{-\lambda\frac{3T}4}$$

\therefore \frac{a*n}{a*{n + 1}} = e^{\lambda\frac T2} = \text{constant} = d

$$\ln d = \frac{\lambda T}2$$

Force Vibration The differential equation of damped oscillator

$$\frac{d^2y}{dt^2} + 2\lambda\frac{dy}{dt} + \omega^2y = 0$$

Iftheexternalforceaddedisperiodicthentheoscillationwillbeperiodictoo. *Theoscillatorwilloscillatetillt*

$$F = F_o\sin \phi t$$

Frequency, $f = \frac{\phi}{2\pi}$ [[*Pastedimage*20251019190530.*png*]] *TransientState* $\rightarrow$ *iscalledshakinginmec*

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ay = F_o\sin \phi t$$

$$here, \$b\$ = dampingconstant\$a\$ = springconstant$$

$$\ddot y + \frac bm \dot y + \frac am y = \frac{F_o}m\sin \phi t$$

$$\Rightarrow \ddot y + 2\lambda\dot y + \omega^2 y = f_o\sin \phi t \ldots(i)$$

$$The frequency of a damping factor decreases over time. If we want to continue it, we have to apply an externa$$

$$y = A\sin(\phi t - \theta)$$

$$\dot y = Ap\cos(\phi t - \theta)$$

$$\ddot y = -Ap^2\sin(\phi t - \theta)$$

$$\$(i) \Rightarrow \$$$

$$-A\phi^2\sin(\phi t - \theta) + 2\lambda A\phi\cos(\phi t - \theta) + \omega^2A\sin(\phi t - \theta) = f_o\sin(\phi t - \theta)\cos \theta + f_o\cos(\phi t - \theta)\sin \theta$$

$$\Rightarrow \ddot y + 2\lambda\dot y + \omega^2 y = f_o\sin\phi t$$

$$Equation the coefficient we get,$$

$$A(\omega^2 - \phi^2) = f_0\cos\theta \ldots(2)$$

$$2\lambda A \phi = f_o\sin\theta \ldots(3)$$

$$Solving these we get,$$

$$A = \frac{f_o}{\sqrt{(\omega^2 - \phi^2)^2 + 4\lambda^2\phi^2}}$$

$$y_p = A = \frac{f_o}{\sqrt{(\omega^2 - \phi^2)^2 + 4\lambda^2\phi^2}}\sin(\phi t - \theta)$$

$$In the absence of applied force, the corresponding homogeneous equation,$$

$$\ddot y + 2 \lambda \dot y + \omega ^2y = 0$$

$$So, there are three solutions for low damping,$$

$$y_c = a_oe^{-\lambda t}\sin(gt + \phi)$$

$$The general solution will be the combination of the solution of the two linear equation$$

$$y = y_c + y_p$$

\Rightarrow y = a_oe^{-\lambda t}\sin(\phi t + \theta) + A\sin(\phi t - \theta)

$The first term is for transient part and the second term is for steady part [[Pasted image 20251019194314.pn$

A = \frac{f_o}{\sqrt{(\omega ^2 - \phi ^2)^2 + 4 \lambda^2 \phi^2}}

$\$A\$ will be maximum when the denominator is minimum$

\frac{d}{d\phi}\sqrt{(\omega ^2 - \phi ^2)^2 + 4 \lambda^2 \phi^2} = 0

\phi = \sqrt{\omega^2 - 2\lambda^2}

$It is also called the resonance condition, the frequency at which, the amplitude of a forced oscillator is maxir$

A_{max} = \frac{f_o}{2\lambda\sqrt{\omega^2 - \lambda^2}}

$If, there is no damping, that is \$\lambda = 0\$, \$A_{max}\$ tends to become infinity, this, however never happens. Bec$

A_{max} = \frac{f_o}{2 \lambda \omega}

![[Pasted image 20251019195042.png]] The sharpness of resonance depends on the value of λ # Wav

y = a \sin \omega t \ldots(i)

$The equation of motion of another particle at p$

y - \sin(\omega t - \phi)

$here, \$\phi\$ is the phase difference between the two particles e know that,$

\lambda \rightarrow 2\pi

x \rightarrow \frac{2\pi}{\lambda}x = \phi

\therefore a\sin\left(\omega t - \frac {2\pi}\lambda x\right)

a\sin (\omega t - kx)

a\sin \lambda(\omega t - kx)

$These are the equation plane progressive wave. Because it travels forward and it's diameter increases over t$

(\omega t - kx) \rightarrow \text{constant phase}

\therefore \frac d{dt}(\omega t - kx) = 0

$$\Rightarrow \omega - k\frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = \frac{\omega}k = \text{phase velocity} = n\lambda = \phi = \text{wave velocity (sound wave)}$$

$$\dot u = \frac{du}{dt} = \frac{2\pi av}{\lambda}\cos\frac{2 \pi}x (vt - x) \rightarrow \text{particle velocity}$$

$$\overrightarrow a \ddot y = \frac{-4\pi v^2}{\lambda^2}\sin\frac{2\pi}\lambda (vt - x) = -\frac{4\pi^2v^2}{\lambda^2}y$$

avetravelsbyeithercompressionorexpansion.Thisiscalledstrain,Strain,

$$\frac {dy}{dx} = \frac{-2\pi a}{\lambda}\cos\frac{2\pi}\lambda(vt - x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{4\pi^2}{\lambda ^2}a\sin \frac{2\pi}\lambda(vt - x) \rightarrow \text{curvature}$$

[[Pastedimage20251019201255.png]]

$$\frac{\frac{d^2y}{dt^2}}{\frac{d^2y}{dx^2}} = v^2$$

$$\Rightarrow \frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$

Differentialequationofwavemotioninonedimensionalcase EnergyofPlaneProgressiveave enow,

$$K.E = \frac 12 mu^2$$

where$u$isparticlevelocityecan'tdeterminetheK.Eforallparticlessotheaverage

$$K.E. = \frac 12 \rho u^2$$

$$= \frac 12 \rho \frac{4\pi^2a^2v^2}{\lambda^2}\cos^2(vt - x)$$

AverageP.E

$$P.E = \int Fdy = \int\rho ady = = \frac 12 \rho \frac{4\pi^2a^2v^2}{\lambda^2}\sin^2(vt - x)$$

Averagetotalenergy

$$PE + KE = 2 \pi^2 \frac{v^2}{\lambda^2}a^2 \rho = 2\pi^2n^2a^2\rho$$

where, n is the frequency of the wave The energy of plane progressive wave is constant, It's independent of $

$$y_1 = a\sin\frac{2\pi}{\lambda}(vt - x)$$

Reflectedwave,

$$y_2 = -a\sin\frac{2\pi}\lambda(vt + x)$$

Equationofprogressivewave,

$$y = a\sin(\omega t - kx) = a\sin \frac {2\pi}\lambda(vt - x)$$

Resultantwave,

$$y = y_1 + y_2;$$

eknow,

$$\sin C + \sin D = 2\cos\frac{C+D}2 \sin\frac{C-D}2$$

$$\therefore y = -2a\sin \frac {2\pi x}{\lambda} \cos \frac{2 \pi vt}{\lambda}$$

Amplitude,

$$A = \therefore -2a\sin \frac {2\pi x}{\lambda}$$

whichisafunctionof$x$orpositionLet,

$$\frac{dy}{dt} = u = \text{particle velocity}$$

$$\therefore u = \frac{4 \pi a v}\lambda \sin \frac{2\pi}\lambda x \sin\frac{2\pi vt}\lambda$$

$$\therefore \overrightarrow a = \frac{8 \pi^2 a v^2}{\lambda^2} \sin\frac{2\pi}{\lambda}x\cos{\frac{2\pi v t}{\lambda}}$$

Strain,

$$\frac {dy}{dx} = -\frac{4 \pi a}{\lambda} \cos\frac{2\pi}{\lambda} x\cos{\frac{2\pi v t}{\lambda}}$$

hen, $\sin\frac{2\pi}{\lambda}x = 0$, $\cos\frac{2\pi}{\lambda}x = \pm 1$ $y = 0$, $u = 0$, $a = 0$, $\frac{dy}{dx}$ = maximumItispossibleonlywh

$$k = -\frac{p}{dy/dx}$$

$$P = -k\frac{dy}{dx}$$

$$k = \frac{\text{stress}}{\text{strain}}$$

herestrainisvolumestrain, eknow,

$$V = \sqrt {\frac k \rho}$$

$$\Rightarrow P = v^2\rho\frac{dy}{dx}$$

$$= v^2rho \frac{4\pi a}{\lambda}\cos\frac{2\pi}\lambda x \cos\frac{2 \pi vt}{\lambda}$$

$$\Rightarrow p = \rho \times \cos \frac{2 \pi v t}{\lambda}$$

where$p$ispressureorkdoneperunit$Pudt$Rateofenergytransfer

$$\int_o^T\frac{Pudt}T$$

which evaluates to 0 Therefore, no energy will transfer, energy will be redistributed Examples: Microwave Ove

$$y_1 = A \sin \omega_1t$$

$$y_2 = A\sin\omega_2t$$

if,$n_1 - n_2 \leq 10Hz$Resultantdisplacement

$$y = y_1 + y_2 = 2a\sin\frac{(\omega_1 + \omega_2)t}{2}\cos\frac{(\omega_1 - \omega_2)t}{2}$$

$$B\sin\frac{(\omega_1 + \omega_2)t}{2}$$

where,

$$B = 2a\cos\frac{(\omega_1 - \omega_2)t}{2} \rightarrow \text{weak}$$

Theremainingpartafter$B$iscalledhardsoundorstrongsound[[Pastedimage20251019211352.png]]Per

$$\cos 2\pi\left(\frac{n_1 - n_2}2 \right)t = \pm1$$

where,

$$t = \frac n{n_1 - n_2}$$

$$\text{beat period} = \frac 1{n_1 - n_2}$$

$$\text{beat frequency} = n_1 - n_2$$

Forsoft,

$$\cos 2\pi\left(\frac{n_1 - n_2}2 \right)t = 0$$

$$\therefore t = \frac{2n + 1}{2(n_1 - n_2)}$$

$Beatperiodandfrequencyremainsthesame. Applications \quad Musicalinstruments, Sirens, RADARtodet$