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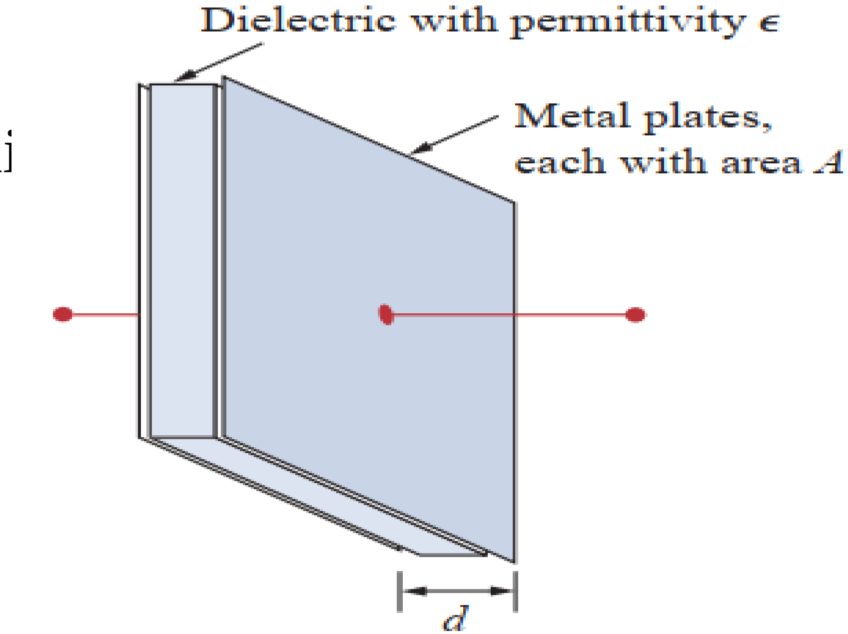
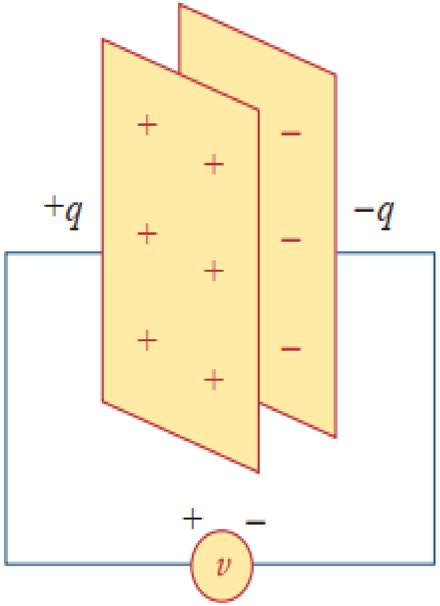
Capacitors

- A capacitor is a passive element
- Capacitor stores energy in its electric field.
- A capacitor consists of two conducting plates separated by an insulator (or dielectric).
- Plates e dielectric may be air, cerami



Capacitors



- When a voltage source is connected to the capacitor, the source deposits a positive charge q on one plate.
- A negative charge deposits on the other plate. The capacitor store the electric charge.
- The amount of charge stored, represented by q, is directly proportional to the applied voltage so that

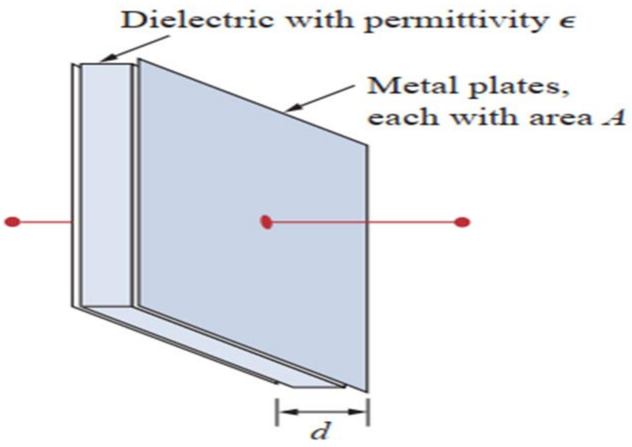
$$q = Cv$$

- where C, the constant of proportionality, is known as the capacitance of the capacitor. The unit of capacitance is the farad (F)

Capacitors

- Capacitance depends on the physical dimensions of the capacitor.

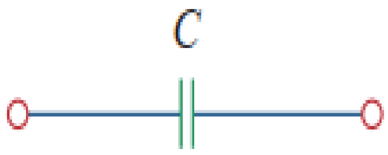
$$C = \frac{\epsilon A}{d}$$



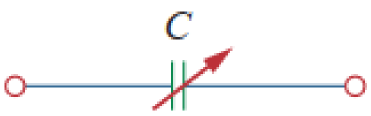
where A is the surface area of each plate,

d is the distance between the plates,

ε is the permittivity of the dielectric material between the plates



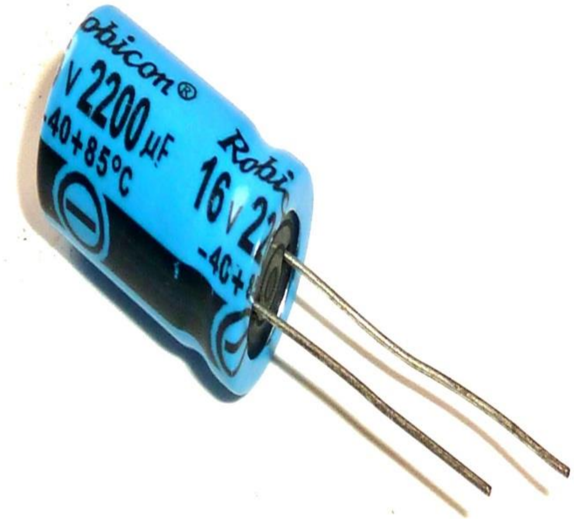
Fixed Capacitor



Variable Capacitor

Capacitors


Xuansn
102
15KV
High Voltage
Ceramic Capacitor
REACH


2200µF
-40+85°C
16v


EPCOS


CBB6-1
2µF±5%
450V.AC 50/60Hz
DB C P0
40/070/21
EN60252
CQC
E162459

Capacitors

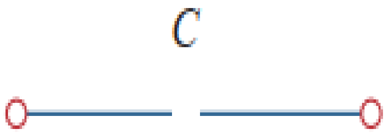
- Current-voltage relationship for a capacitor is

$$i = C\frac{dv}{dt}$$

- Energy stored in the capacitor is

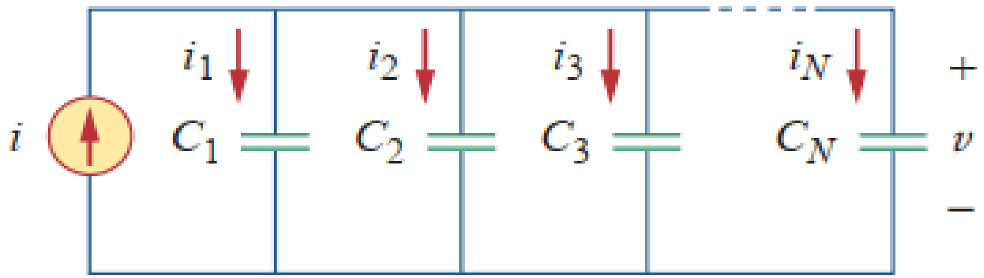
$$w = \frac{1}{2}Cv^2$$

- A capacitor is an open circuit to dc



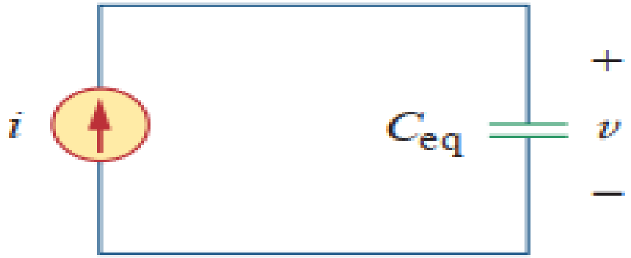
- The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.

Capacitors

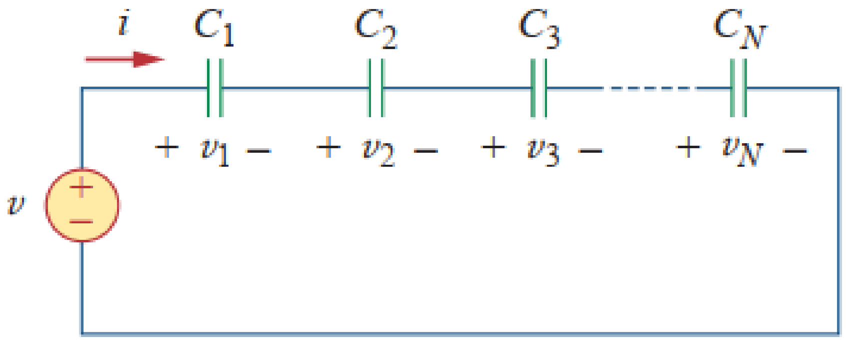


Equivalent capacitor C_{eq} of the parallel connected capacitor is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$



Capacitors

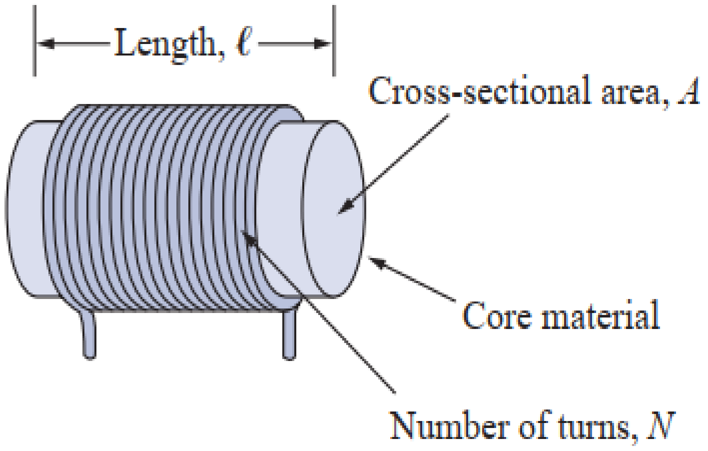


Equivalent capacitor C_{eq} of the series connected capacitor is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

Inductor

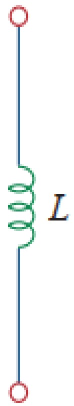
- Inductor is a passive element.
- Inductor store energy in its magnetic field.
- An inductor consists of a coil of conducting wire.
- voltage across the inductor is directly proportional to the time rate of change of current



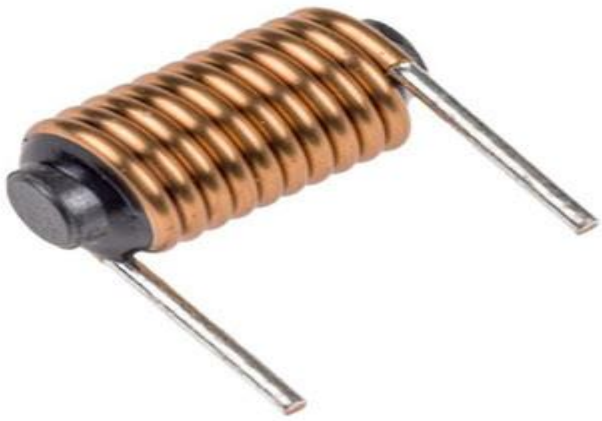
$$v = L\frac{di}{dt}$$

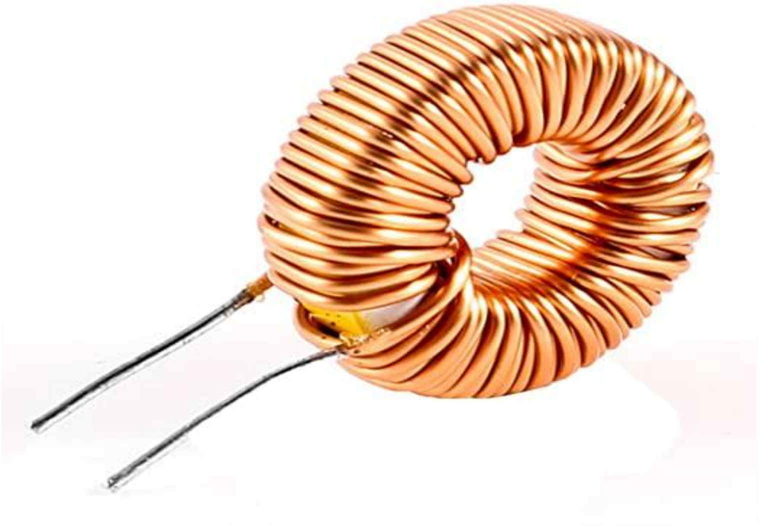
- where L is the constant of proportionality called the inductance of the inductor. The unit of inductance is the henry (H).

Inductor



Symbols of Inductor



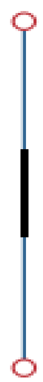


Inductor

- Energy stored in inductor is

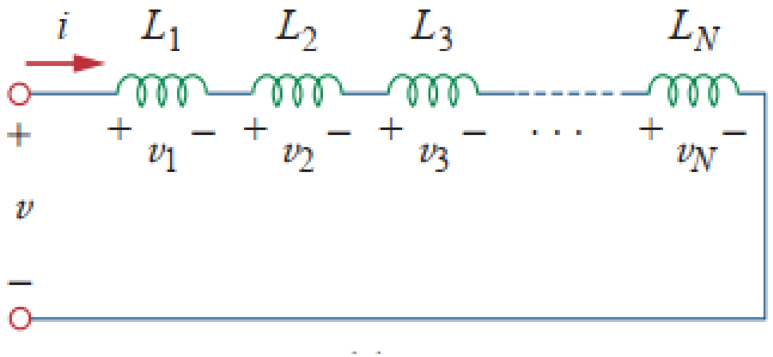
$$w = \frac{1}{2}Li^2$$

- An inductor acts like a short circuit to dc



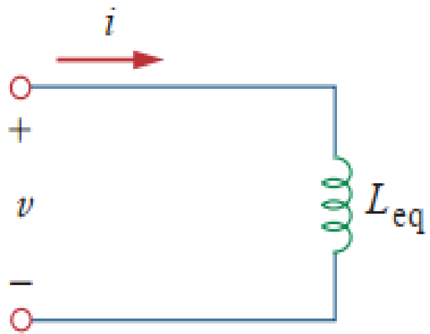
- Ideal inductor does not dissipate energy. The energy stored in it can be returned at a later time.

Inductor

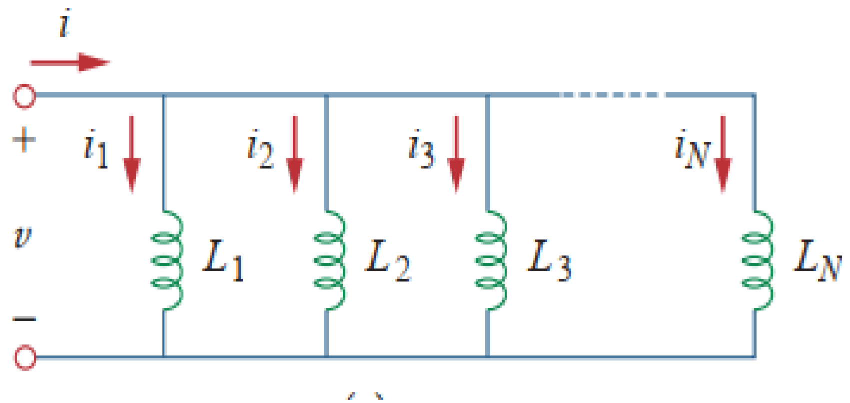


- The equivalent inductance of series-connected inductors is

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

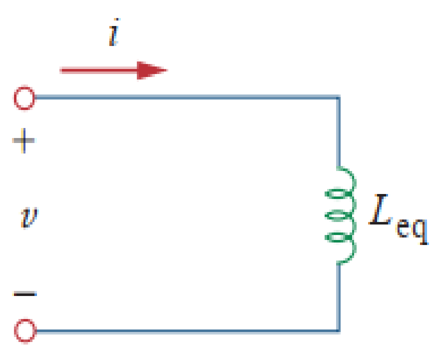


Inductor



- The equivalent inductance of parallel-connected inductors is

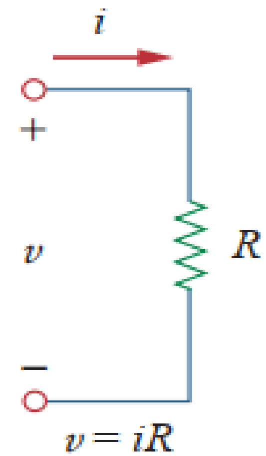
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$



Phasor Relationships for Resistor

Assume, current through a resistor R is

$$i = I_m \cos(\omega t + \phi),$$



voltage across the resistor is

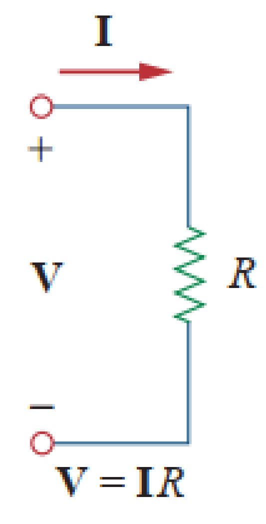
$$v = iR = RI_m \cos(\omega t + \phi)$$

Phasor form of this voltage is

$$\mathbf{V} = RI_m \angle \phi$$

Phasor representation of the current is

$$\mathbf{I} = I_m \angle \phi$$

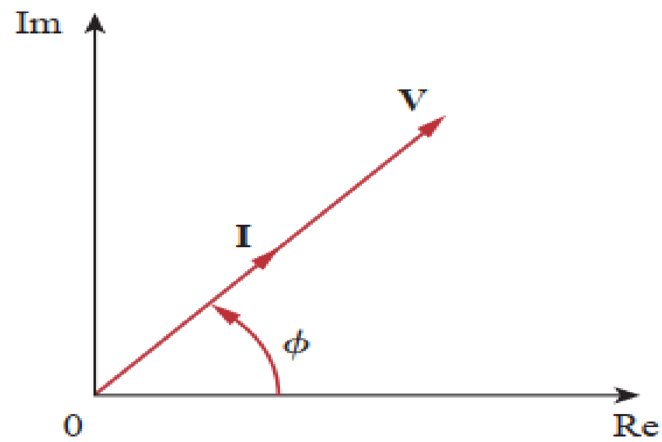


Voltage-current relationship in phasor form is

$$\mathbf{V} = R\mathbf{I}$$

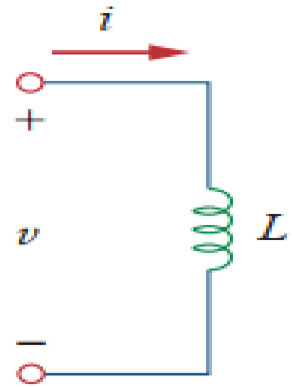
Phasor Relationships for Resistor

Voltage-current relation for the resistor in the phasor domain is shown following figure



Voltage and current are in phase.

Phasor Relationships for Inductor



$$v = L\frac{di}{dt}$$

Assume the current through the inductor is

$$i = I_m \cos(\omega t + \phi),$$

Voltage across the inductor is

$$\begin{aligned} v &= L\frac{di}{dt} \\ &= -\omega L I_m \sin(\omega t + \phi) \\ &= \omega L I_m \cos(\omega t + \phi + 90^\circ) \\ &= \omega L I_m \underline{/\phi + 90^\circ} \end{aligned}$$

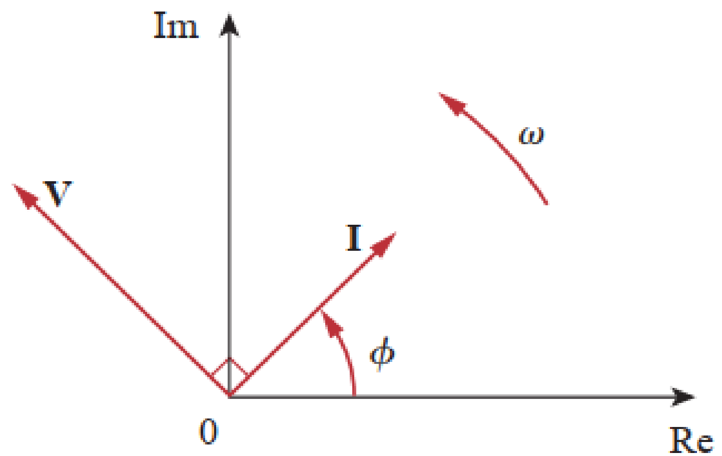
$$-\sin A = \cos(A + 90^\circ).$$

$$\mathbf{V} = j\omega L\mathbf{I}$$

$$I_m \underline{/\phi} = \mathbf{I},$$

Phasor Relationships for Inductor

Phasor diagram for inductor is



Phase angle between voltage and current $= (\varphi+90°) - \varphi = 90°$

Voltage leads current by 90°

Current lags voltage by 90°

Phasor Relationships for Inductor

Problem: The voltage $v = 12\cos(60t + 45^\circ)$ is applied to a 0.1-H inductor. Find the steady-state current through the inductor.

Solution:

$$\omega = 60 \text{ rad/s} \qquad \mathbf{V} = 12\underline{/45^\circ} \text{ V}.$$

For the inductor $\mathbf{V} = j\omega L\mathbf{I}$,

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{12\underline{/45^\circ}}{j60 \times 0.1} = \frac{12\underline{/45^\circ}}{6\underline{/90^\circ}} = 2\underline{/-45^\circ} \text{ A}$$

Current in time domain

$$i(t) = 2\cos(60t - 45^\circ) \text{ A}$$

Phasor Relationships for Capacitor

Assume the voltage across capacitor is

$$v(t) = V_m \cos(\omega t + \phi)$$

Current through the capacitor is

$$i = C\frac{dv}{dt}$$

$$= -\omega C V_m \sin(\omega t + \varphi)$$

$$= \omega C V_m \cos(\omega t + \varphi + 90°) \qquad -\sin A = \cos(A + 90°).$$

$$= \omega C V_m \underline{/\phi + 90°}$$

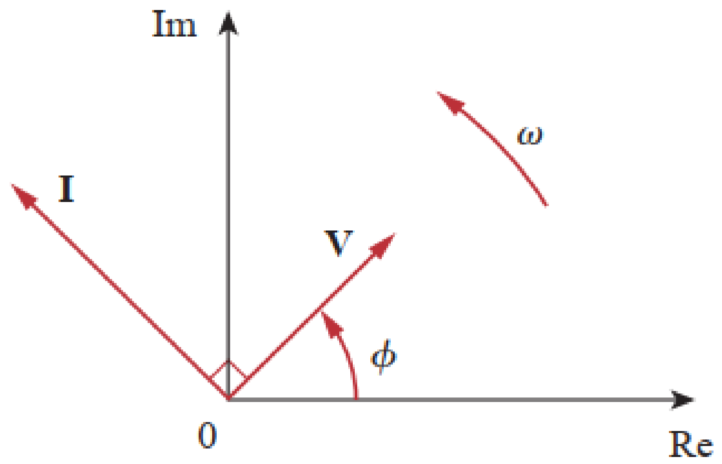
$$\mathbf{I} = j\omega C \mathbf{V}$$

Phase angle between voltage and current $= \varphi - (\varphi + 90°) = -90°$

Voltage lags current by 90°

Phasor Relationships for Capacitor

Phasor diagram for capacitor is



Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

Impedance

Voltage-current relations for the three passive elements are

$$\mathbf{V} = R\mathbf{I}, \qquad \mathbf{V} = j\omega L\mathbf{I} \qquad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

resistor Inductor Capacitor

These equations may be written in terms of the ratio of the phasor voltage to the phasor current as

$$\frac{\mathbf{V}}{\mathbf{I}} = R, \qquad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L, \qquad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}$$

Ohm's law in phasor form for any type of element as

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{or} \quad \mathbf{V} = \mathbf{Z}\mathbf{I}$$

where Z is a frequency-dependent quantity known as impedance, measured in ohms

Impedance

The impedance Z of a circuit is the ratio of the phasor voltage V to the phasor current I

Units of impedance is ohms (Ω).

The impedances of resistor is

$$\frac{\mathbf{V}}{\mathbf{I}} = R$$

$$\mathbf{Z} = R$$

The impedances of inductor is

$$\frac{\mathbf{V}}{\mathbf{I}} = j\omega L$$

$$\mathbf{Z} = j\omega L$$

The impedances of capacitor is

$$\frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}$$

$$\mathbf{Z} = \frac{1}{j\omega C}$$

Impedance

Impedance may be expressed in rectangular form as

$$\mathbf{Z} = R + jX$$

where $R = \text{Re}\,\mathbf{Z}$ is the *resistance*

$X = \text{Im}\,\mathbf{Z}$ is the *reactance*

Impedance is inductive when X is positive or capacitive when X is negative

Impedance $Z=R+jX$ is said to be inductive or lagging since current lags voltage.

Impedance $Z=R-jX$ is said to be capacitive or leading since current leads voltage.

Impedance

The impedance may also be expressed in polar form as

$$\mathbf{Z} = |\mathbf{Z}| \underline{/\theta}$$

$$\mathbf{Z} = R + jX = |\mathbf{Z}| \underline{/\theta}$$

where

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}, \qquad \theta = \tan^{-1}\frac{X}{R}$$

and

$$R = |\mathbf{Z}|\cos\theta, \qquad X = |\mathbf{Z}|\sin\theta$$

Impedance

The admittance **Y** is the reciprocal of impedance.

The admittance **Y** of an element (or a circuit) is the ratio of the phasor current through it to the phasor voltage across it

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

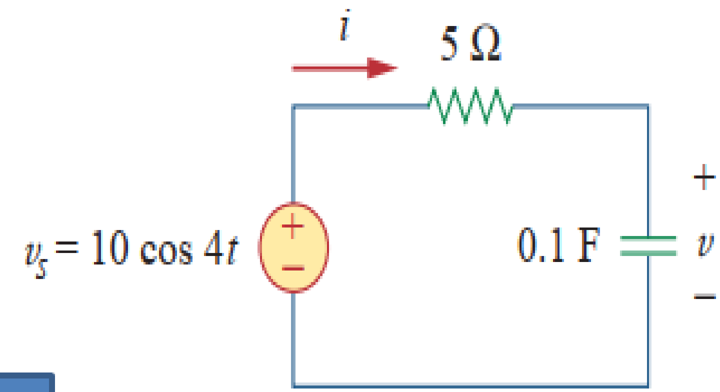
Wh

sus

Un

Impedance

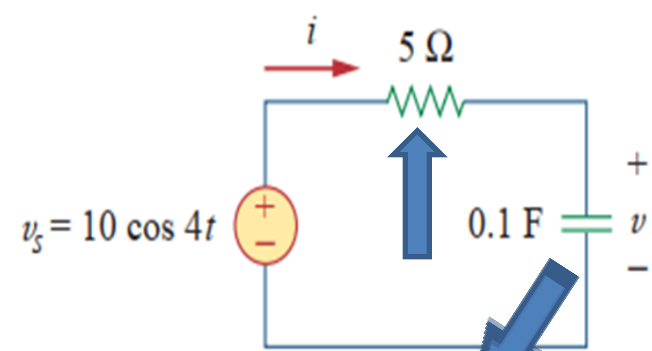
Find $v(t)$ and $i(t)$ in the circuit



Solution: $\omega=4$ $\quad v_s = 10\cos 4t$

Phase angle 0

$\mathbf{V}_s = 10\angle 0^\circ$ V $\quad v_s = 10\cos 4t$



The impedance is

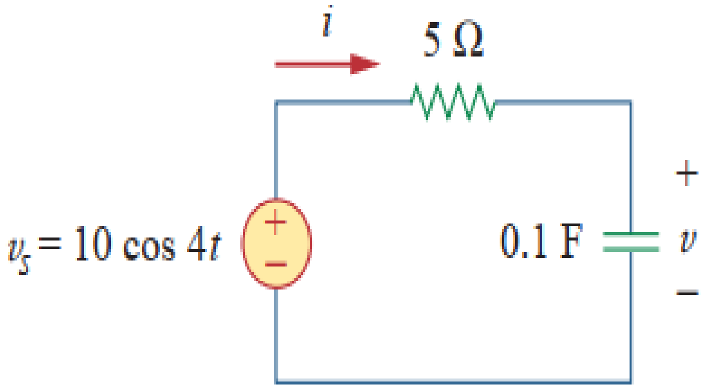
$$\mathbf{Z} = 5 + \frac{1}{j\omega C}$$

$$= 5 + \frac{1}{j4 \times 0.1}$$

$$= 5 - j2.5\ \Omega$$

$$\frac{1}{j\omega C}$$

Impedance



Solution: $\omega = 4$ $\quad \mathbf{V}_s = 10\angle 0^\circ \text{ V}$

Current $\mathbf{I} = \dfrac{\mathbf{V}_s}{\mathbf{Z}}$

$$= \frac{10\angle 0^\circ}{5 - j2.5}$$

$$= 1.6 + j0.8 = 1.789\angle 26.57^\circ \text{ A}$$

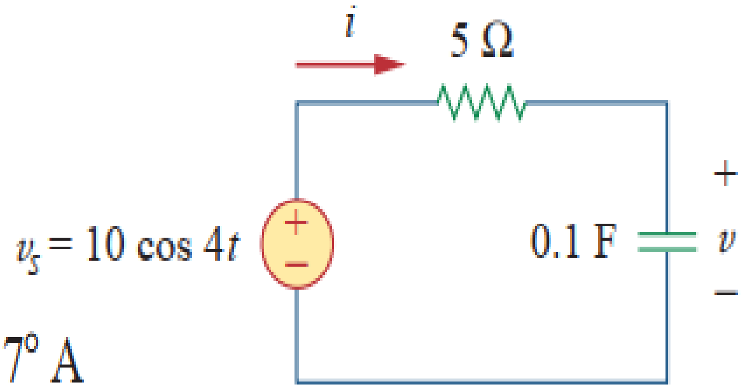
Voltage across the capacitor is

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = \frac{\mathbf{I}}{j\omega C}$$

$$= \frac{1.789\angle 26.57^\circ}{j4 \times 0.1} = 4.47\angle -63.43^\circ \text{ V}$$

Impedance

Solution:



Converting **I** and **V** to the time domain,

$$i(t) = 1.789\cos(4t + 26.57^\circ)\ \mathrm{A} \quad \Leftarrow \quad 1.789\angle 26.57^\circ\ \mathrm{A}$$

$$\mathbf{V}_s = 10\angle 0^\circ\ \mathrm{V} \quad \Leftarrow \quad v_s = 10\cos 4t$$

$$v(t) = 4.47\cos(4t - 63.43^\circ)\ \mathrm{V} \quad \Leftarrow \quad 4.47\angle -63.43^\circ\ \mathrm{V}$$

Circuit Theorem

Kirchhoff's voltage law in phasor form

$$\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n = 0$$

Kirchhoff's current law in phasor form

$$\mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n = 0$$

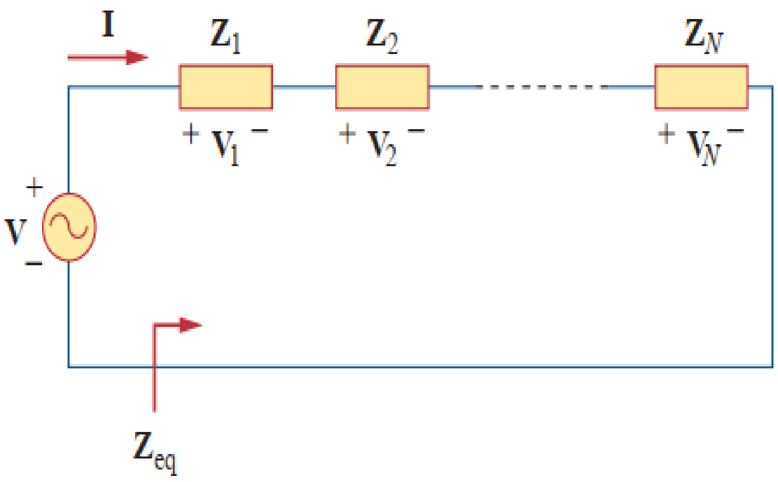
Ohm's law in phasor form

$$\mathbf{V} = \mathbf{Z}\mathbf{I}$$

Impedance Combinations

The same current I flows through the impedances

Applying KVL around the loop,



$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_N$$

$$= \mathbf{I}(\mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N)$$

The equivalent impedance at the input terminals is

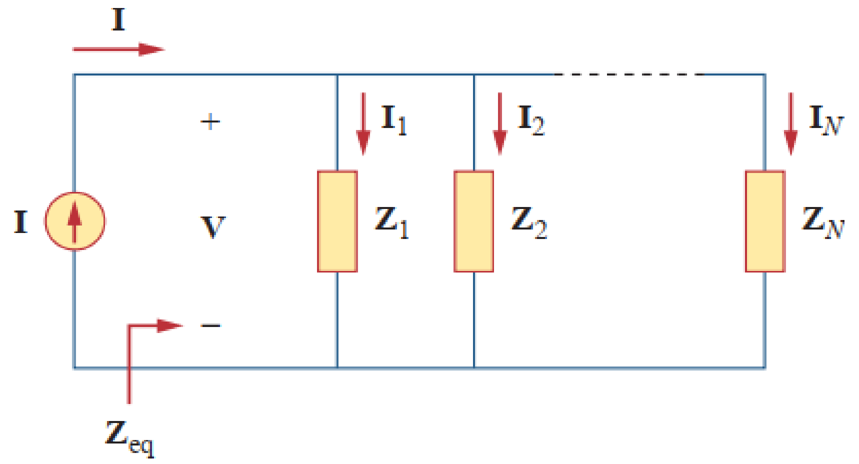
$$\mathbf{Z}_{eq} = \frac{\mathbf{V}}{\mathbf{I}} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N$$

$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N$$

Impedance Combinations

The voltage across each impedance is the same.

Applying KCL at the top node,



$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_N$$

$$= \mathbf{V}\left(\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}\right)$$

The equivalent impedance is,

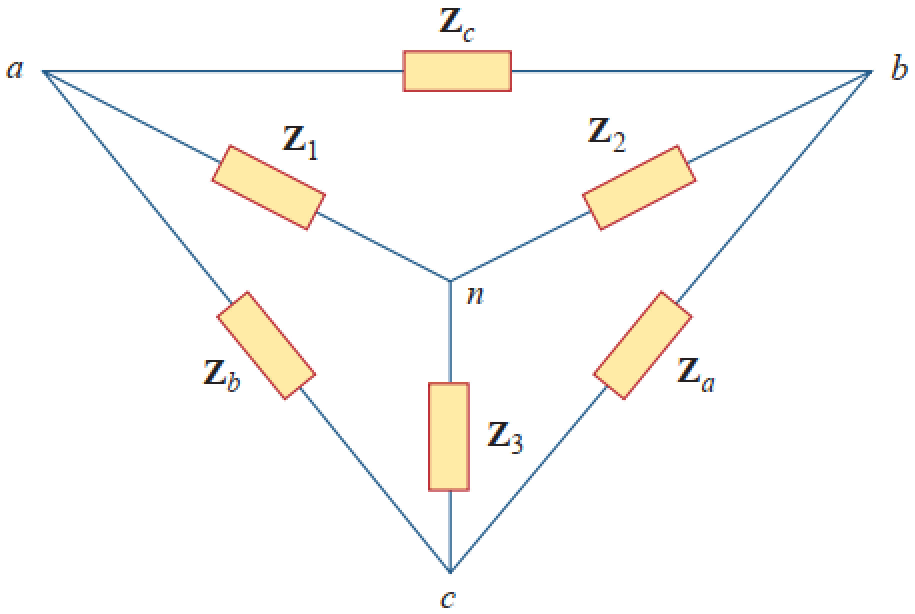
$$\frac{1}{\mathbf{Z}_{eq}} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}$$

Equivalent admittance is,

$$\mathbf{Y}_{eq} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_N$$

Impedance Combinations

Delta-to-wye and wye-to-delta transformations for impedance is same as resistive circuit.



Y-Δ conversion:

$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

Δ-Y conversion:

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_a\mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

Impedance Combinations

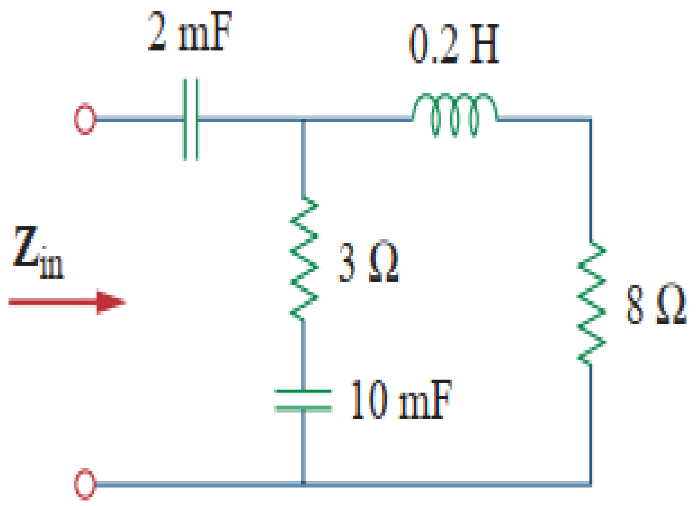
A delta or wye circuit is said to be balanced if it has equal impedances in all three branches.

When a Δ-Y circuit is balanced, the equation may write

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \quad \text{or} \quad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta$$

Impedance Combinations

Problem: Find the input impedance of the circuit. Assume that the circuit operates at $\omega=50$ rad/s



Solution: We know,

Impedance of inductor is

$$Z_L = j\omega L$$

Impedance of capacitor is

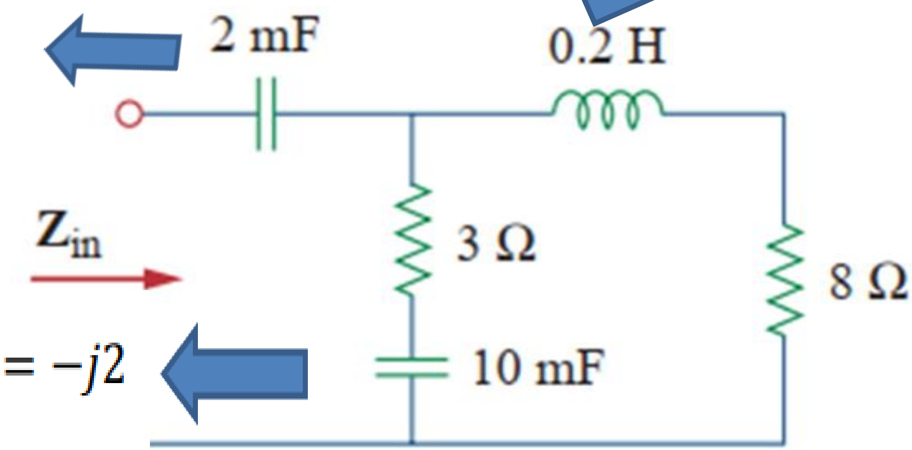
$$Z_C = \frac{1}{j\omega C}$$

Impedance Combinations

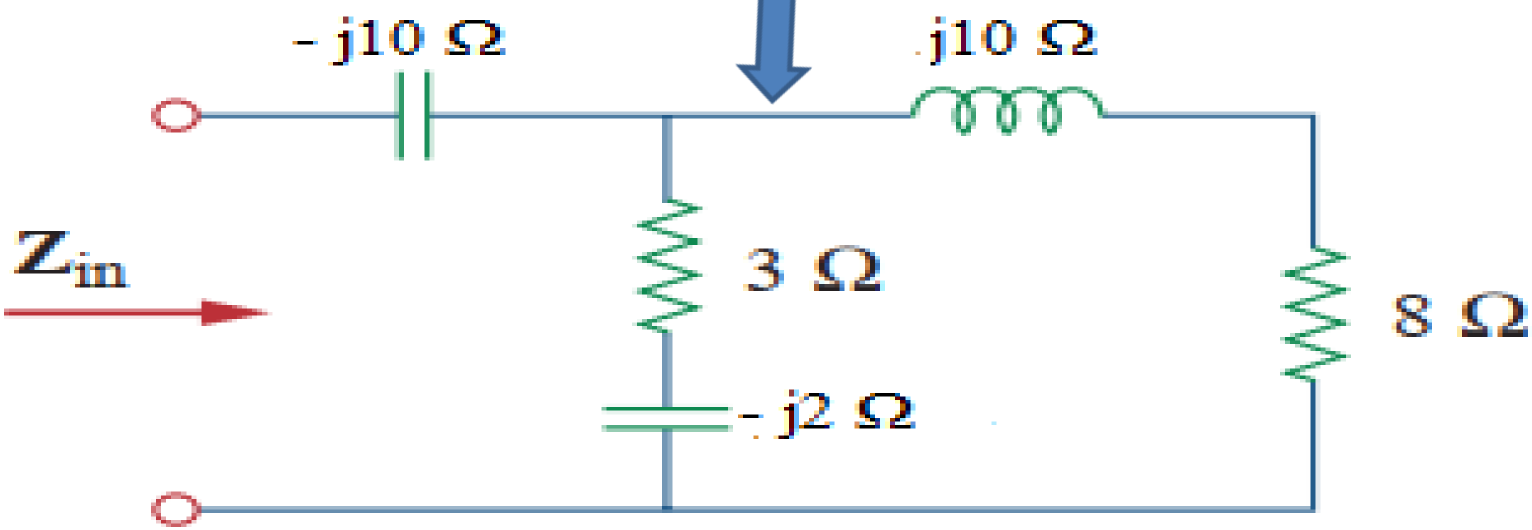
Solution:

$$Z_L = j\omega L = j*50*0.2 = j10$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j*50*0.002} = -j10$$

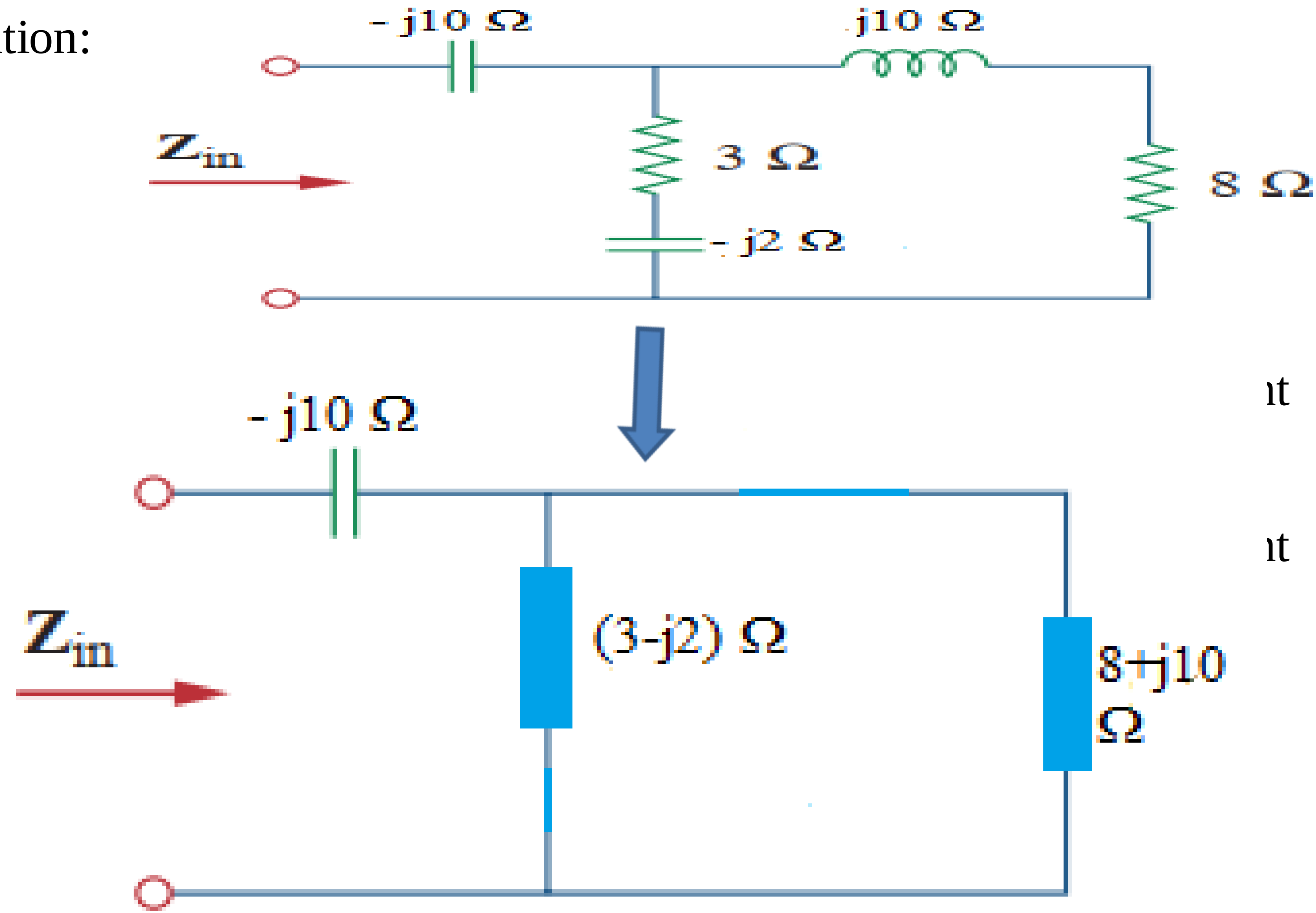


$$Z_C = \frac{1}{j\omega C} = \frac{1}{j*50*0.01} = -j2$$



Impedance Combinations

Solution:



8Ω ... nt

imp

3Ω ... nt

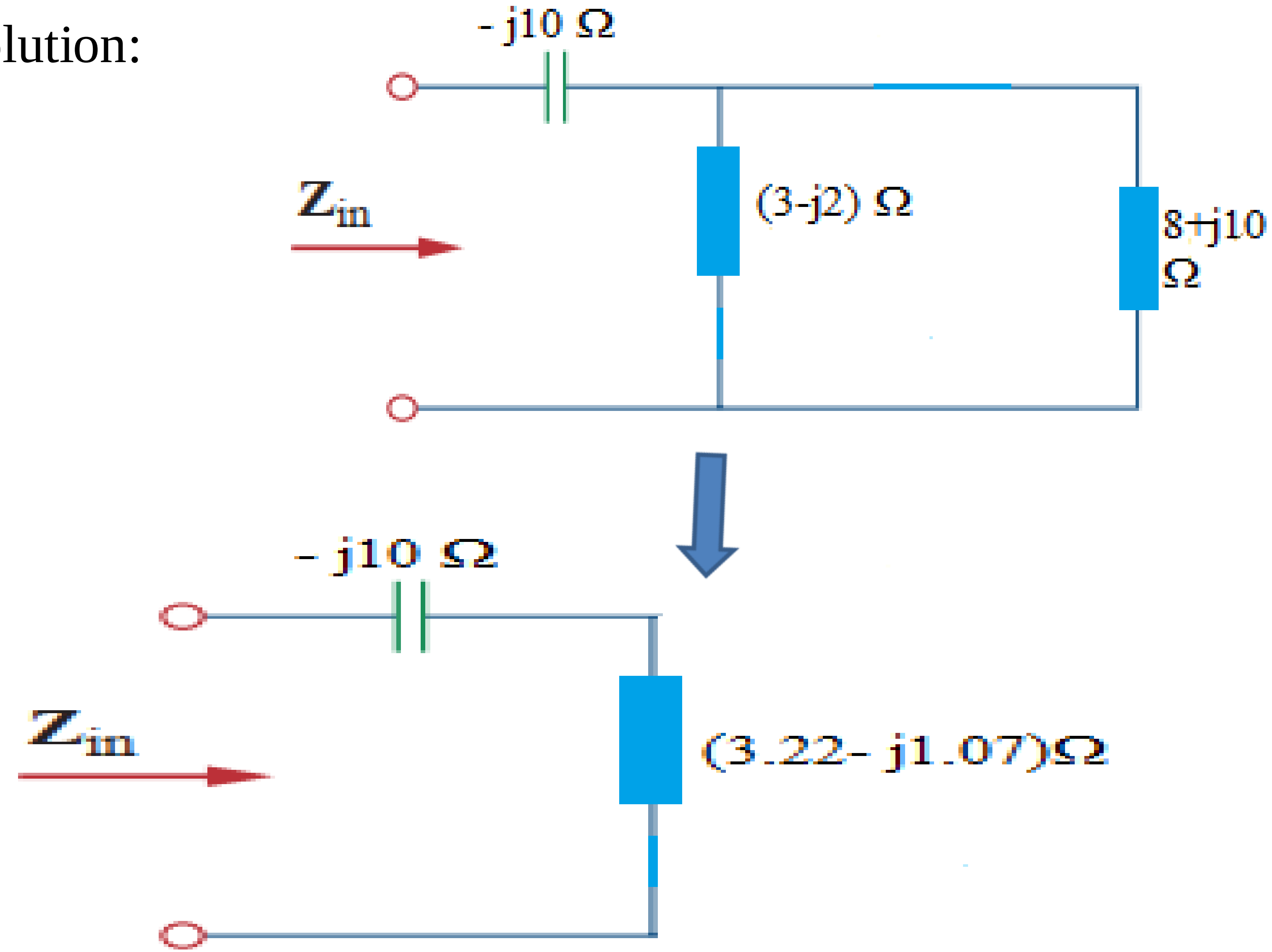
imp

Impedance Combinations

Solution:

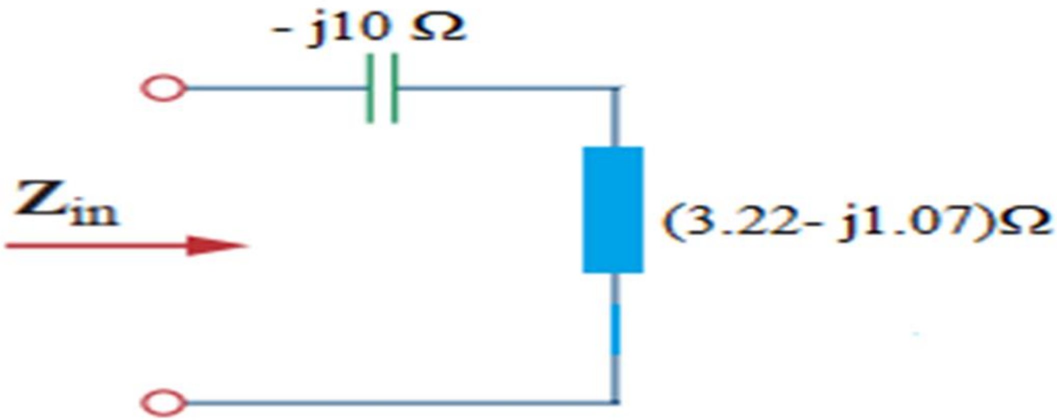
(8

ar



Impedance Combinations

Solution:



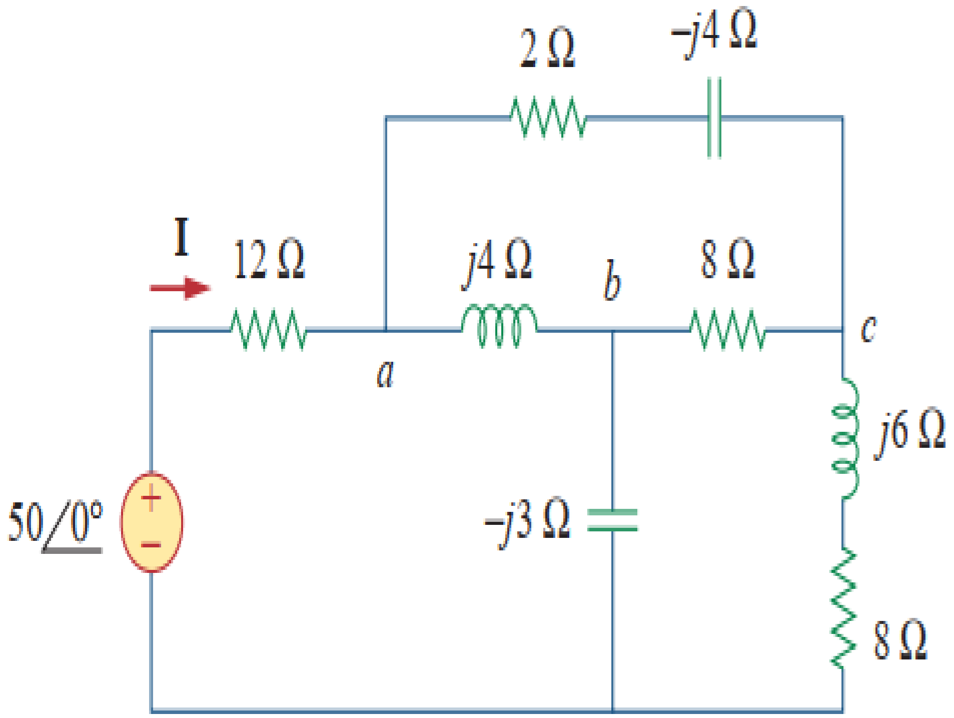
-j10 Ω impedance is connected series with (3.22-j1.07) Ω impedance and their equivalent impedance is

$$-j10 + 3.22 - j1.07\ \Omega$$

$$\mathbf{Z}_{in} = 3.22 - j11.07\ \Omega$$

Impedance Combinations

Problem: Find current I in the circuit.



Impedance Combinations

Solution: The delta network

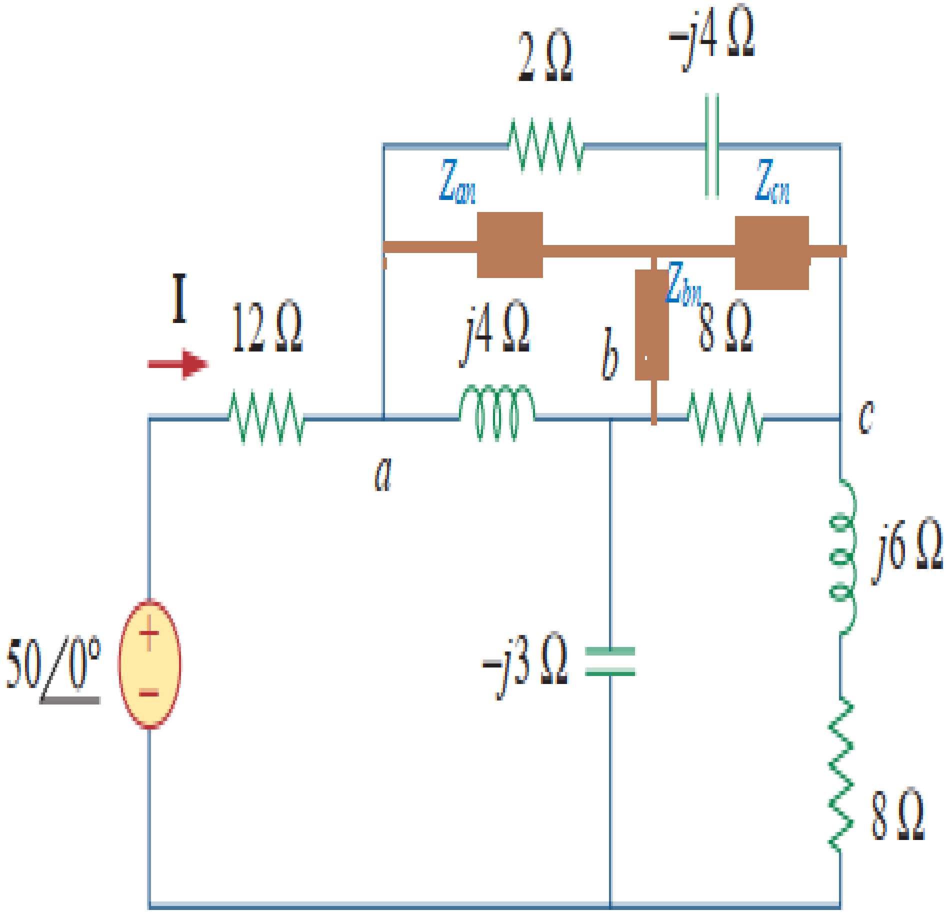
connecte $\mathbf{Z}_1 = \dfrac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$

can be converted tothe Y network.

$$\mathbf{Z}_{an} = \frac{j4(2-j4)}{j4+2-j4+8} = (1.6+j0.8)\,\Omega$$

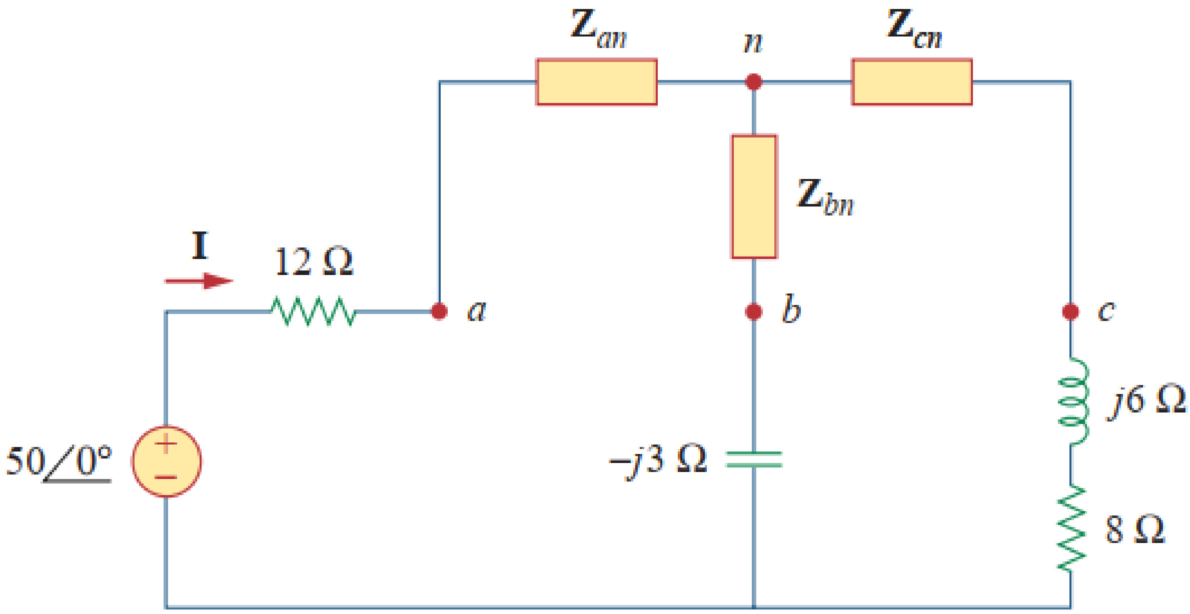
$$\mathbf{Z}_{bn} = \frac{j4(8)}{10} = j3.2\,\Omega,$$

$$\mathbf{Z}_{cn} = \frac{8(2-j4)}{10} = (1.6-j3.2)\,\Omega$$



Impedance Combinations

Solution:



$$\mathbf{Z} = 12 + \mathbf{Z}_{an} + (\mathbf{Z}_{bn} - j3) \| (\mathbf{Z}_{cn} + j6 + 8)$$

$$= 12 + 1.6 + j0.8 + (j0.2) \| (9.6 + j2.8)$$

$$= 13.6 + j1 = 13.64\angle 4.204^\circ\ \Omega$$

Impedance Combinations

Solution:

Current,

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\angle 0^\circ}{13.64\angle 4.204^\circ} = 3.666\angle -4.204^\circ \text{ A}$$

Assignment

Example: 9.11

Practice Problem: 9.8, 9.9, 9.10, 9.11, 9.12

Thank You