Data Structure

Preliminaries

Instructors:
Md Nazrul Islam Mondal
Department of Computer Science & Engineering
Rajshahi University of Engineering & Technology
Rajshahi-6204

Outline

- Mathematical Notations and Functions
- Algorithm Notation
- Control Structure
- Complexity of Algorithm
- Asymptotic Notations
- Sub-algorithm
- Variables, Data Type

Mathematical Notations and Functions

Mathematical Notations and Functions (Floor and Ceiling)

- Let x is a real number,

$\lfloor x \rfloor$ called floor of x

$\lceil x \rceil$ called ceiling of x

- If x is itself integer, then

$$\lfloor x \rfloor = \lceil x \rceil$$

- Otherwise,

$$\lfloor x \rfloor + 1 = \lceil x \rceil$$

Example:

$$\lfloor 3.4 \rfloor = 3 \text{ and } \lceil 3.4 \rceil = 4$$

$$\lfloor 3.4 \rfloor + 1 = 4 = \lceil 3.4 \rceil$$

Mathematical Notations and Functions (Modular Arithmetic)

- Let k be an **integer** and M be a **positive integer**,

$$k(\text{mod} M)$$

(readk modulo M)

$$Example:$$

$$25\,(\text{mod } 7) = 4$$

$$25(\text{mod } 5) = 0$$

$$-12(\text{mod } 7) = 2$$

Mathematical Notations and Functions (Integer and Absolute value)

- Let x be any real number, the integer value of x (written as INT(x)) converts x into an integer by deleting (truncation) the functional part of the number.

$$INT(3.4) = 3, \mathrm{INT}(-8.5) = -8.5, \mathrm{INT}(\sqrt{2}) = 1$$

- The absolute value of the real number x (written as ABS(x) or |x|) is defined as the greate of x or −x.

$$|8.5| = 8.5, |-8.5| = 8.5$$

$$ABS(-4) = 4$$

Mathematical Notations and Functions (Summation)

$$\sum_{i=0}^{n} a_i = a_1 + a_2 + a_3 + \dots + a_n$$

$$\sum_{i=1}^{5} i = 1 + 2 + 3 + 4 + 5 = 15$$

Mathematical Notations and Functions (Factorial)

$$n! = 1.2.3........(n-2)(n-1)n$$

$$4! = 1.2.3.4 = 24$$

Mathematical Notations and Functions (Permutation)

- A **permutation** of a set of n elements is an arrangement of the elements in a given order.

- For **example**, the permutation of the set consisting of the elements a, b, c are as follows:

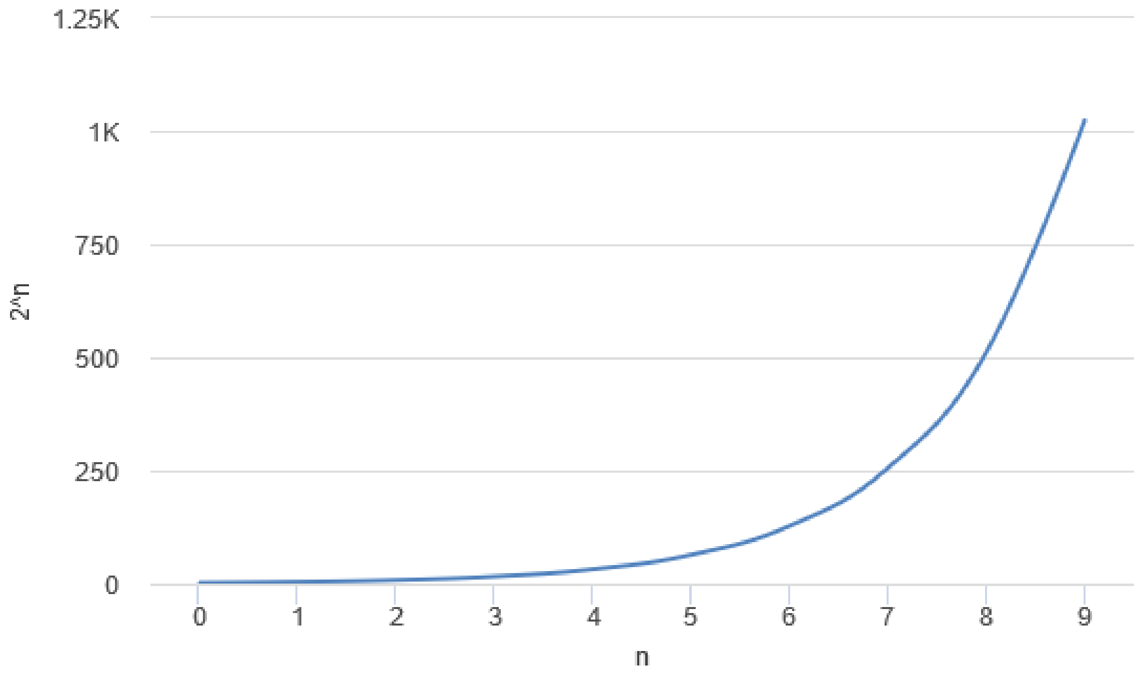
abc, acb, bac, bca, cab, cba

Mathematical Notations and Functions
(Exponents and Logarithm)

m be positive integer, $a^m = a.a.a....a$(m times) m be zero, $a^0 = 1$

m be negative integer, $m = -n$, $a^{-n} = \frac{1}{a^n}$

Mathematical Notations and Functions (Exponents and Logarithm)



Mathematical Notations and Functions
(Exponents and Logarithm)

$$\log_b x \quad input \rightarrow x \quad y$$

$$= \log_b x \quad x = b^y$$

$$objective \rightarrow find \quad y$$

Mathematical Notations and Functions (Exponents and Logarithm)

$$\log_2 64$$

$$input \rightarrow 64$$

$$y = \log_2 64$$

$$64 = 2^y$$

$$objective \rightarrow \mathrm{y} = 6$$

Mathematical Notations and Functions (Exponents and Logarithm)

- programA.c and programB.c solve same problem.
- Let, 64 integers are used as input of programA.c and programB.c i.e. $n = 64$
- Let, the complexity of programA.c is $2^n = 2^{64}$
- Let, the complexity of programB.c is $\log_2 n = \log_2 64 = 6$

Algorithm Notation

Algorithm Notations

Algorithm 2.1:

(Largest Element in Array) **LARGE(DATA, N, LOC, MAX)**

A nonempty array **DATA** with **N** numerical values is given. This algorithm finds the location **LOC** and the value **MAX** of the largest element of **DATA**.

Step 1. [initialize.] **Set K:=1, LOC:=1 and MAX:=DATA[1].**

Step 2. [Increment counter.] Set K:=K+1

Step 3. [Test counter.] If K>N then: **Write**: LOC, MAX and **Exit**
[End of If structure]

Step 4. [Compare and update.] **If** MAX<DATA[K] **then**: Set LOC:=K and MAX:=DATA[K].
[End of If structure]

Step 5. [Repeat loop.] **Go to Step 2.**

Algorithm Notations (Identifying Number)

Algorithm 2.1:

(Largest Element in Array) **LARGE(DATA, N, LOC, MAX)**

A nonempty array **DATA** with **N** numerical values is given. This algorithm finds the location **LOC** and the value **MAX** of the largest element of **DATA**.

Step 1. [initialize.] **Set K:=1, LOC:=1 and MAX:=DATA[1].**

Step 2. [Increment counter.] Set K:=K+1

Step 3. [Test counter]. If K>N then:

Write: LOC, MAX and **Exit**

Step 4. [Compare and update.] **If** MAX<DATA[K] **then:**

Set LOC:=K and MAX:=DATA[K].

Step 5. [Repeat

Identifying Number:
Each algorithm is assigned an identifying number

Example
Refers to the first algorithm in Chapter 2

Algorithm Notations (Steps, Control and Exit)

Algorithm 2.1:

(Largest Element in Array) **LARGE(DATA, N, LOC, MAX)**

A nonempty array **DATA** with **N** numerical values is given. This algorithm finds the location **LOC** and the value **MAX** of the largest element of **DATA**.

Step 1. [initialize.] **Set K:=1, LOC:=1 and MAX:=DATA[1].**
Step 2. [Increment counter.] Set K:=K+1
Step 3. [Test counter.] If K>N then:
  Write: LOC, MAX and **Exit**
Step 4. [Compare and update.] **If** MAX<DATA[K] **then**:
  Set LOC:=K and MAX:=DATA[K]
Step 5. [Repeat loop.] **Go to Step 2.**

1. Step: statements
2. Control: Loop and branch
3. Exit: Complete the lgorithm

Algorithm Notations (Comments)

Algorithm 2.1:

(Largest Element in Array) **LARGE(DATA, N, LOC, MAX)**
A nonempty array **DATA** with **N** numerical values is given. This algorithm finds the location **LOC** and the value **MAX** of the largest element of **DATA**.

Step 1. [initialize.] **Set K:=1, LOC:=1 and MAX:=DATA[1].**
Step 2. [Increment counter.] Set K:=K+1
Step 3. [Test counter.] If K>N then:
Write: LOC, MAX and **Exit**
Step 4. [Compare and update.] **If** MAX<DATA[K] **then**:
Set LOC:=K and MAX:=DATA[K].
Step 5. [Repeat loop.] **Go to Step 2.**

Comm

Algorithm Notations (Variable Names)

Algorithm 2.1:

(Largest Element in Array) **LARGE(DATA, N, LOC, MAX)**
A nonempty array **DATA** with **N** numerical values is given. This algorithm finds the location **LOC** and the value **MAX** of the largest element of **DATA**.

Step 1. [initialize.] **Set K:=1, LOC:=1 and MAX:=DATA[1].**
Step 2. [Increment counter.] Set K:=K+1
Step 3. [Test counter.] If K>N then:
    Write: LOC, MAX and **Exit**
Step 4. [Compare and update.] **If** MAX<DATA[K] **then:**
    Set LOC:=K and MAX:=DATA[K].
Step 5. [Repeat loop.] **Go to Step 2.**

Variable Name: Us
Capital Letter

Algorithm Notations (Assignment Statement)

Algorithm 2.1:

(Largest Element in Array) **LARGE(DATA, N, LOC, MAX)**

A nonempty array **DATA** with **N** numerical values is given. This algorithm finds the location **LOC** and the value **MAX** of the largest element of **DATA**.

```
Step 1. [initialize.] Set K:=1, LOC:=1 and MAX:=DATA[1].
Step 2. [Increment counter.] Set K:=K+1
Step 3. [Test counter.] If K>N then:
        Write: LOC, MAX and Exit
Step 4. [Compare and update.] If MAX<DATA[K] then:
        Set LOC:=K and MAX:=DATA[K].
Step 5. [Repeat loop.] Go to Step 2.
```

Assign
State

Algorithm Notations (Input and Output)

Algorithm 2.1:

(Largest Element in Array) **LARGE(DATA, N, LOC, MAX)**

A nonempty array **DATA** with **N** numerical values is given. This algorithm finds the location **LOC** and the value **MAX** of the largest element of **DATA**.

Step 1. [initialize.] **Set K:=1, LOC:=1 and MAX:=DATA[1].**
Step 2. [Increment counter.] Set K:=K+1
Step 3. [Test counter.] If K>N then:
  Write: LOC, MAX and **Exit**
Step 4. [Compare and update.] **If** MAX<DATA[K] **then**:
  Set LOC:=K and MAX:=DATA[K].
Step 5. [Repeat loop.] **Go to Step 2.**

Output Input

Read: x

Algorithm Notations (Procedures)

Algorithm 2.1:

(Largest Element in Array) **LARGE(DATA, N, LOC, MAX)**

A nonempty array **DATA** with **N** numerical values is given. This algorithm finds the location **LOC** and the value **MAX** of the largest element of **DATA**.

Step 1. [initialize.] **Set K:=1, LOC:=1 and MAX:=DATA[1].**

Step 2. [Increment counter.] Set K:=K+1

Step 3. [Test counter.] If K>N then:

  Write: LOC, MAX and **Exit**

Step 4. [Compare and update.] **If** MAX<DATA[K] **then:**

  Set LOC:=K and MAX:=DATA[K].

Step 5. [Repeat loop.] **Go to Step 2.**

Procedure Example Proced 4.3

Control Structures

Control Structures

- Sequence Logic or Sequential Flow
- Selection Logic or Conditional Flow
- Iteration Logic or Repetitive Flow

Control Structures (Sequence Logic)

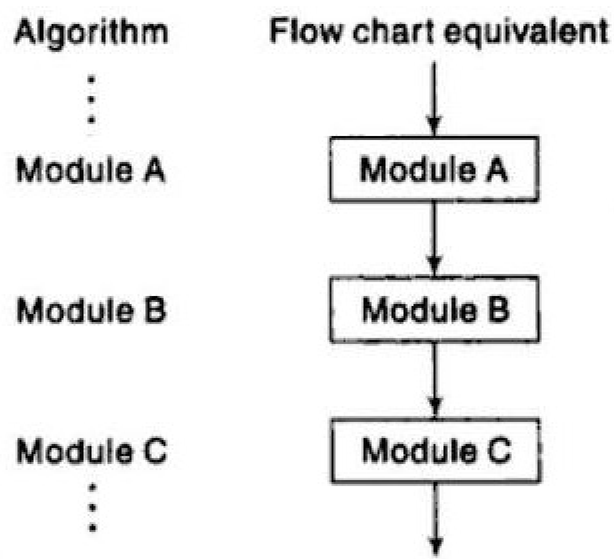
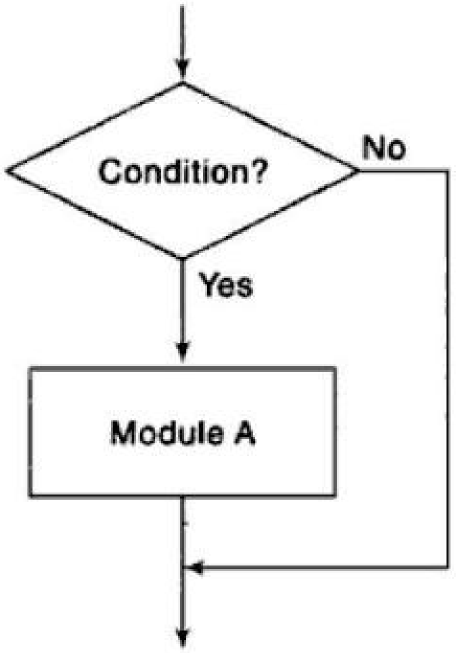


Fig. 2.3 Sequence Logic

Control Structures (Selection Logic)

1. Single Alternative:

```
If condition, then:
     [Module A]
[End of If structure]
```

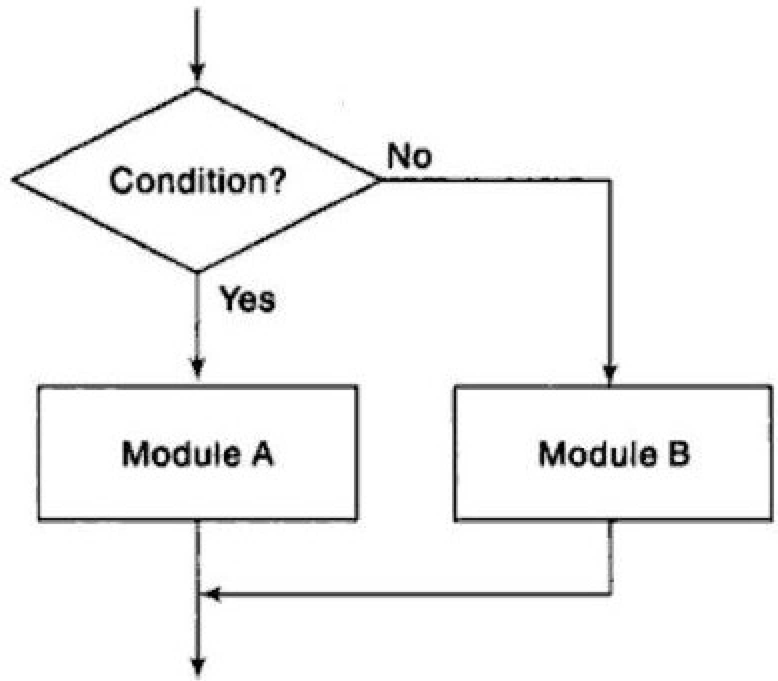


(a) Single alternative.

Control Structures (Selection Logic)

1. Double Alternative:

```
If condition, then:
     [Module A]
Else:
     [Module B]
[End of If structure]
```



(b) Double alternative.

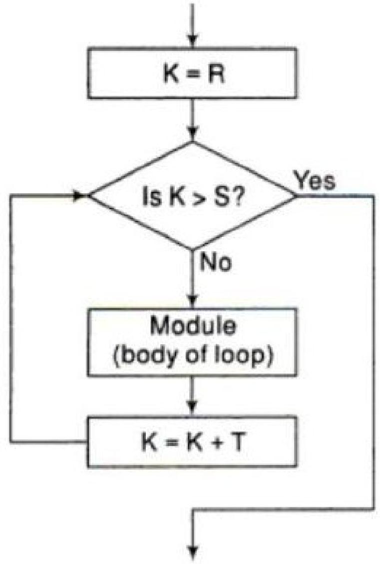
Control Structures (Selection Logic)

1. Multiple Alternative:

```
If condition(1), then:
     [Module A1]
Else If condition(2), then:
     [Module A2]
     .
     .
     .
Else If condition(M), then:
     [Module AM]  Else:
     [Module B]
[End of If structure]
```

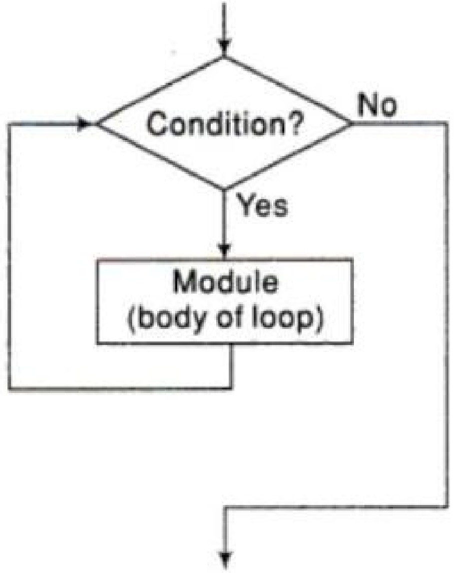
Control Structures (Iteration Logic)

```
Repeat for K = R to S by T:
          [Module]
[End of loop]
```



(a) Repeat-For structure.

```
Repeat while condition:
         [Module]  [End
of loop]
```



(a) Repeat-While structure.

Complexity of Algorithm

Complexity of Algorithm

- Let, M is an algorithm.
- The **time** and **space** used by M are the two measures for
  the efficiency of M.
- Time
  - Measured by counting the **number of key operations** – in sorting and searching algorithm, for example, the **number of comparis**ons.
- Space
  - Measured by counting the maximum of memory needed by the algorithm.

Complexity of Algorithm

- The complexity of M is **the function f(n)** which gives the running time and/or storage space requirement of the algorithm in terms of the size n of the input data.
- Frequently, space complexity = **Cn**
- **The term "complexity " shall refer to the running time of the algorithm.**

Complexity of Algorithm

- Find the complexity function f(n) for certain cases-
  - **Best Case**
    - The minimum possible value of f(n) for any possible input
  - **Worst Case**
    - The maximum possible value of f(n) for any possible input
  - **Average Case**
    - The expected value of f(n)

Complexity of Algorithm

- **Average Case**:
  - The analysis of the average case assume a certain **probabilistic distribution** for the input data.
  - Suppose the numbers n_1, n_2, ..., n_k occur with respective **probabilities** p_1, p_2, ..., p_k. The the ***expectation or average value*** **E** is given by

$$E = n_1 p_1 + n_2 p_2 + \dots + n_k p_k$$

Complexity of Algorithm (Linear Search)

Algorithm 2.4: (Linear Search) A linear array DATA with N elements and a specific ITEM of information are given. This algorithm finds the location LOC of ITEM in the array DATA or sets LOC = 0.

1. [Initialize] Set K := 1 and LOC := 0.
2. Repeat Steps 3 and 4 while LOC = 0 and K ≤ N.
3. If ITEM = DATA[K], then: Set LOC: = K.
4. Set K := K + 1. [Increments counter.]

   [End of Step 2 loop.]
5. [Successful?]

   If LOC = 0, then:

   Write: ITEM is not in the array DATA.

   Else:

   Write: LOC is the location of ITEM.

   [End of If structure.]
6. Exit.

Complexity of Algorithm (Linear Search)

- The complexity of the search algorithm is given by the number of C of comparisons between ITEM and DATA[K].
- **Worst Case**:
  - Clearly, the worst case occures when ITEM is the last element in the array DATA or is not there at all.

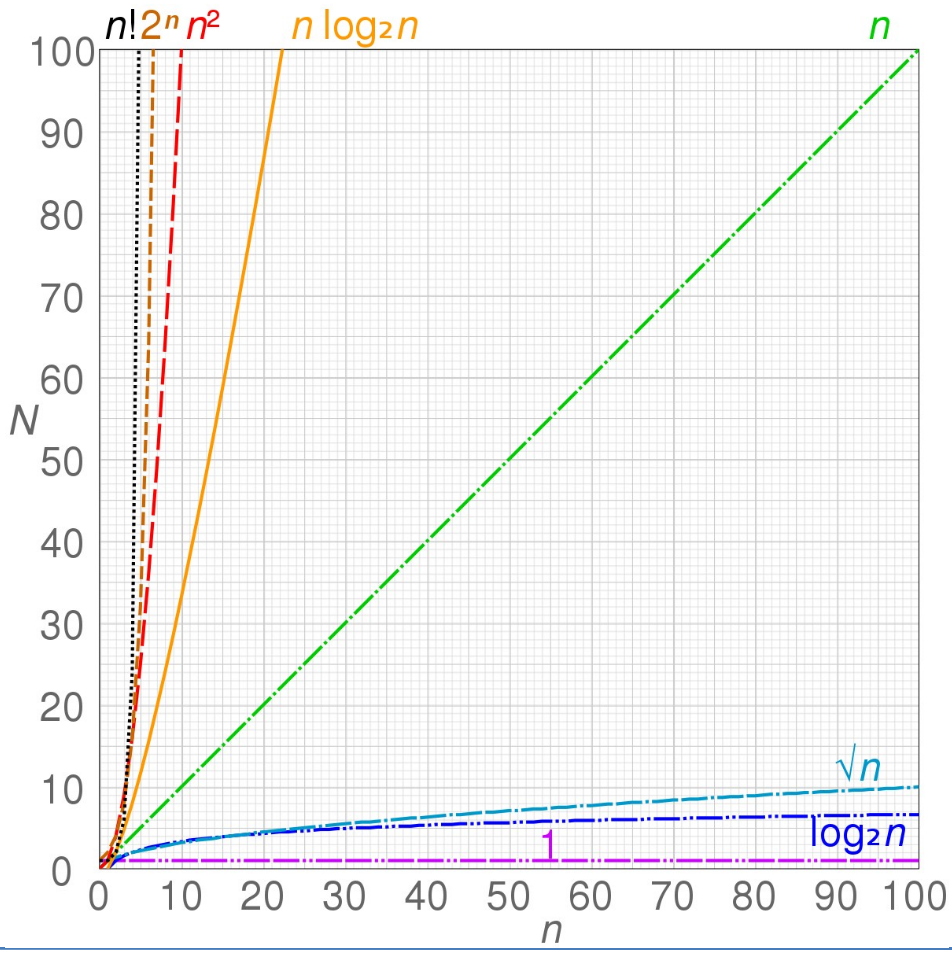
$$C(n) = n$$

Complexity of Algorithm (Linear Search)

- **Average Case**:
  - Assume that ITEM does appear in DATA
  - Accordingly the number of comparisons can be any of the numbers 1, 2, 3, . . . , n
  - Each number occurs with probability p = 1/n

$$C(n) = 1.\frac{1}{n} + 2.\frac{1}{n} + 3.\frac{1}{n} + \ldots + n.\frac{1}{n} = \frac{1}{n}(1 + 2 + 3 + \ldots + n) = \frac{1}{n}\,\frac{n(n+1)}{2} = \frac{n+1}{2}$$

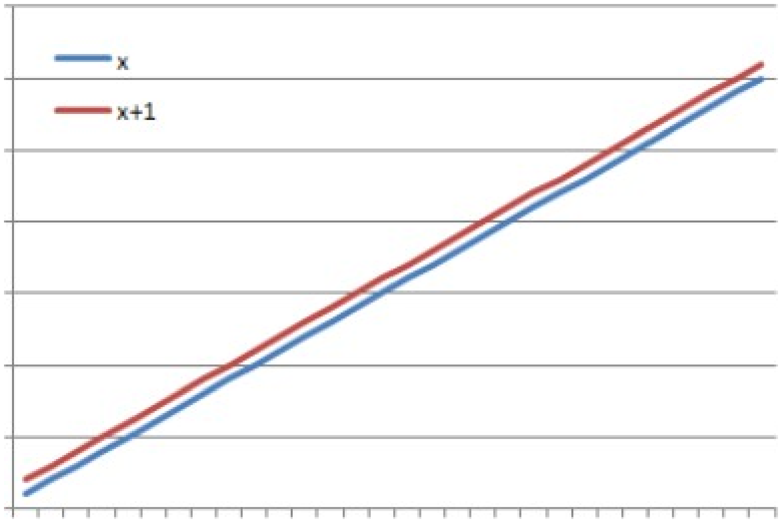
Complexity of Algorithm



Asymptotic Notations

Asymptotic Notations

- **Asymptotic**:
  - It means a line that continually approaches a given curve but does not meet it at any finite distance.
  - **Example**: x is asymptotic with x + 1 as shown in graph.
  - Asymptotic may also be defined as a way to **describe the behavior of functions in the limit or without bounds**



Asymptotic Notations

- Big-Oh Notation (O)
- Big-Omega Notation (Ω)
- Big-Theta Notation (Θ)
- Little-Oh Notation (o)

Asymptotic Notations

- Suppose f(n) and g(n) are **positive functions** with the property that
  - f(n) is **bounded** by some multiple of g(n) for almost all n.

Asymptotic Notations (Big-Oh Notation)

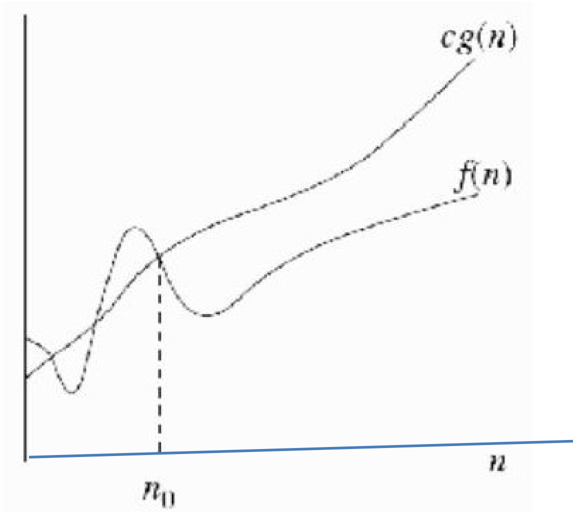
- Suppose there exist a positive integer $\mathbf{n_0}$ and a positive number **M** such that, for all $\mathbf{n \geq n_0}$, we have

$$|f(n)| \leq M|g(n)|$$

- Then we may **write**

$$f(n) = O(g(n))$$

- Which is **read "f(n) is of order g(n)"**



Asymptotic Notations (Big-Oh Notation)

- **Example**:
  - Given, $f(n) = n^2+50n$
  - **We can write,** $n^2+50n \leq n^2+50n^2$ **when** $n \geq 0$
  - $\mathbf{n_0 = 0}$
  - We have, $n^2 + 50n \leq 51n^2$ where $n \geq n_0$
  - $\mathbf{M = 51, g(n) = n^2}$
  - We can write, $f(n) = O(g(n))$ i.e. $\mathbf{f(n) = O(n^2)}$

- **Upper Bound**

Asymptotic Notations (Big-Omega Notation)

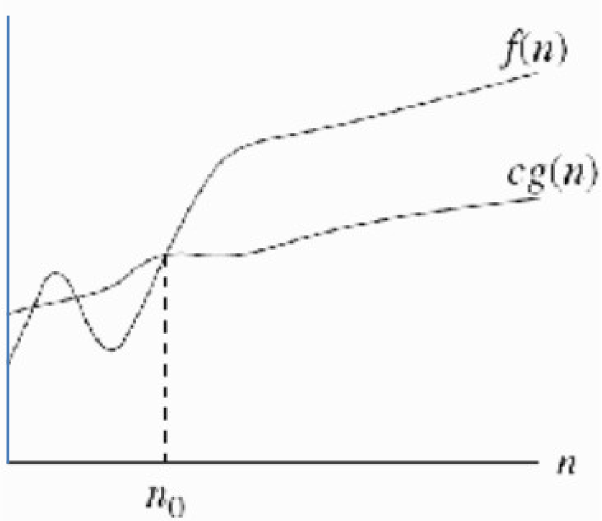
- Suppose there exist a positive integer $\mathbf{n_0}$ and a positive number **M** such that, for all $\mathbf{n \geq n_0}$, we have

$$|f(n)| \geq M|g(n)|$$

- Then we may **write**

$$f(n) = \Omega(g(n))$$

- Which is **read "f of n is omega of g of n"**



Asymptotic Notations (Big-Omega Notation)

- **Example**:
  - Given, $f(n) = n^2+50n$
  - **We can write,** $n^2+50n \geq n^2$ **when** $n \geq 0$
  - $\mathbf{n_0 = 0}$
  - We have, $n^2 +50n \geq n^2$ where $n \geq n_0$
  - $\mathbf{M = 1, g(n) = n^2}$
  - We can write, $f(n) = \Omega(g(n))$ i.e. $\mathbf{f(n) = \Omega(n^2)}$

- **Lower Bound**

Asymptotic Notations (Big-Theta Notation)

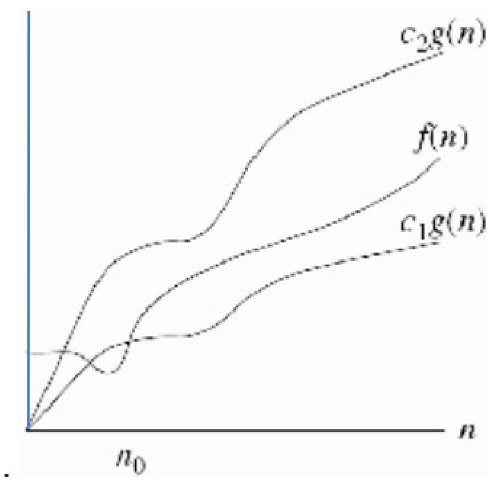
- Suppose there exist a positive integer $\mathbf{n_0}$ and two positive number $\mathbf{c_1}$ **and** $\mathbf{c_2}$ such that, for all $\mathbf{n \geq n_0}$, we have

$$c_1|g(n)| \leq |f(n)| \leq c_2|g(n)|$$

- Then we may **write**

$$f(n) = \Theta(g(n))$$

- Which is **read "f of n is theta of g of n"**



Asymptotic Notations (Big-Omega Notation)

- **Example**:
  - Given, $f(n) = n^2+50n$
  - **We can write,** $n^2+50n \geq n^2$ **when** $n \geq 0$
  - $\mathbf{c_1 = 1, n_0 = 0}$
  - **We can write,** $n^2+50n \leq n^2+50n^2$ **when** $n \geq 0$
  - $\mathbf{c_2 = 51, n_0 = 0}$
  - **We can write,** $n^2 \leq n^2+50n \leq 51n^2$ **when** $n \geq 0$
  - $\mathbf{g(n)} = n^2$
  - We can write, $f(n) = \Theta(g(n))$ i.e. $\mathbf{f(n) = \Theta(n^2)}$

- **Both upper and lower Bound**

Asymptotic Notations (Little-Oh Notation)

- Iff $f(n) = O(g(n))$ and $f(n) \neq \Omega(g(n))$
- Then we may **write**

$$f(n) = o(g(n))$$

- Which is **read "f of n is little oh of g of n"**

Asymptotic Notations (Little-Oh Notation)

- **Example**:
  - Given, $f(n) = 18n+9$
  - **We can write,** $18n+9 \le 18n+9n$ **when** $n \ge 0$
  - **So $f(n) = O(n)$**
  - **We can write,** $18n+9 \ge 18n$ **when** $n \ge 0$
  - **So $f(n) = \Omega(n)$**
  - **So $f(n) \ne o(n)$**

But

  - **We can write,** $18n+9 \le 27n^2$ **when** $n \ge 1$
  - **So $f(n) = O(n^2)$**
  - **But $f(n) \ne \Omega(n^2)$**
  - **So $f(n) = o(n)$**

Sub-Algorithm

Sub-Algorithm

- A subalgorithm is a complete and **independently** defined algorithm module which is used (or invoked or called) by some **main algorithm** or by some **other subalgorithm**.

Variables, Data Type

Variables, Data Type

- Four Data Type:
  - » Character
  - » Real (or floating point)
  - » Integer (or fixed point)
  - » Logical
- Local and global variables

END