Active Filter

Text Books

1. Electronic Devices and Circuit Theory

by R Boylestad and L Nashelsky

***2.* Op-Amps and Linear Integrated Circuits**

by Ramakant A. Gayakwad

3. Microelectronic Circuits Analysis and Design

by Muhammad H. Rashid

4. Electronic Principles 7th Edition

by Albert Malvino, David Bates

***5.* Operational Amplifiers & Linear Integrated Circuits: Theory and Application**

by James M. Fiore

Passive Filters

- Consists of passive elements like:
  - Resistor,
  - Capacitor and
  - Inductor

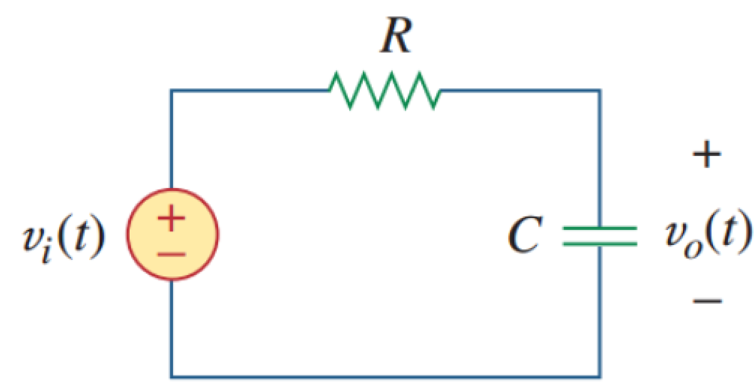
- Filters can be classified as :-
  1. Low Pass Filter
  2. High Pass Filter
  3. Band Pass Filter
  4. Band Stop Filter (Band Reject/Eliminate Filter)

Passive Low Pass Filter (LPF)

- LPF ideally allows lower frequencies and attenuates higher frequencies.
- One of the simplest form of LPF
  - Transfer Function:

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$



  - H(0) = 1 and H(∞) = 0 ⇒ Filter is LFP

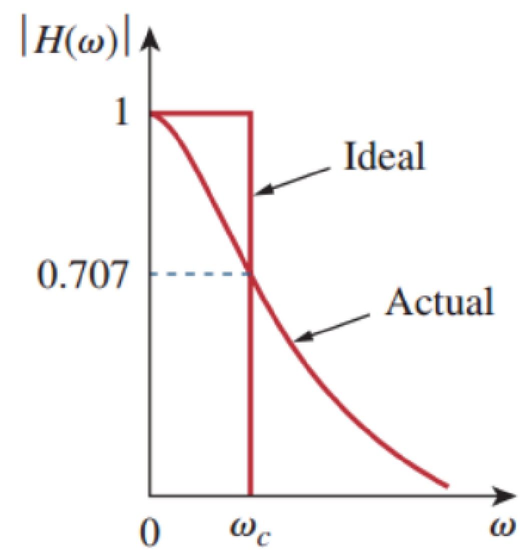
Passive Low Pass Filter (LPF)

- ω_c is the cut-off frequency.
  - It is a frequency at which $|H(\omega)|$ drops to 70.07% of $|H(\omega)|_{\max}$ or becomes $\frac{1}{\sqrt{2}}$ of $|H(\omega)|_{\max}$.

- So, here, ω_c can be calculated as:

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_c = \frac{1}{RC}$$



Passive High Pass Filter (HPF)

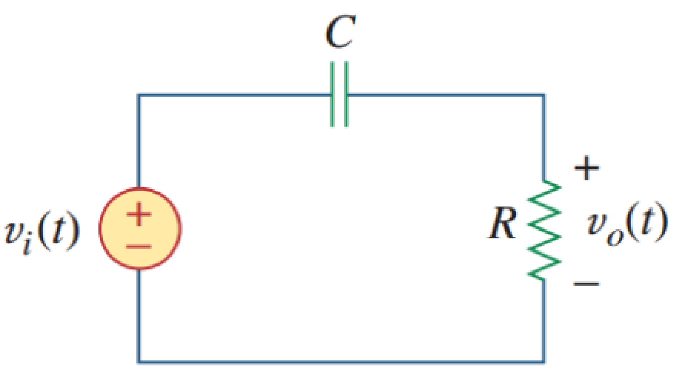
- Ideally, HPF attenuates lower frequencies and allows higher frequencies.
- One of the simplest form of HPF
  - Transfer Function:

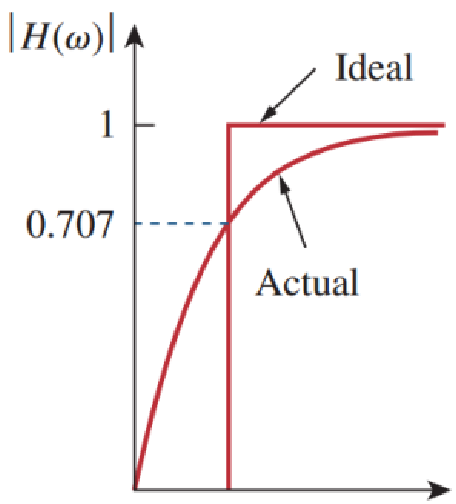
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

  - H(0) = 0 and H(∞) = 1 ⇒ Filter is HFP

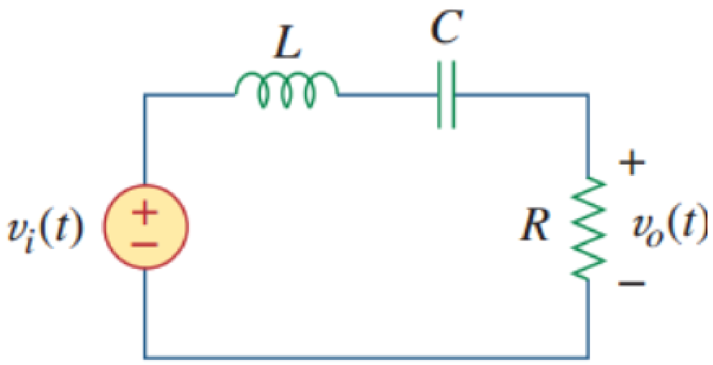
$$\omega_c = \frac{1}{RC}$$





Passive Band Pass Filter (BPF)

- BPF allows frequencies of a particular range and eliminates other frequencies.
- Typical example of BPF
- Transfer function :



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

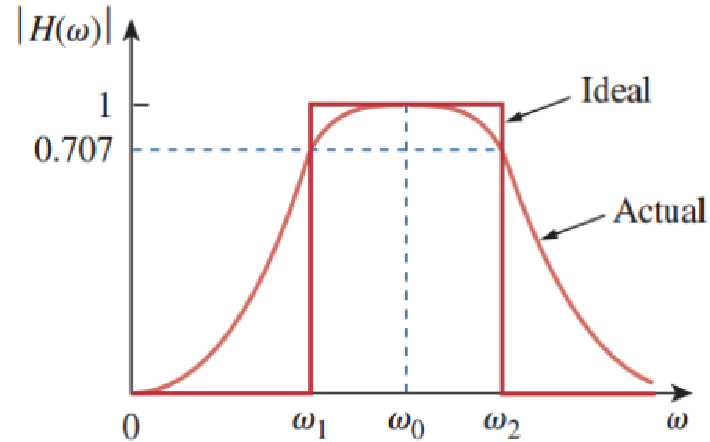
- Here, H(0) = 0 and H(∞) = 0
- How it is BPF ?
- Resonance Frequency, ω_0 !!!!!
- $Z_{eq} = R \Rightarrow$ Filter allows ω_0 means it is a BPF

Passive Band Pass Filter (BPF)

- Here, ω_1 and ω_2 are half power frequencies i.e. power dissipated is half of the maximum power.

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$



- Bandwidth of BPF = ω_2 - ω_1
- Quality Factor,

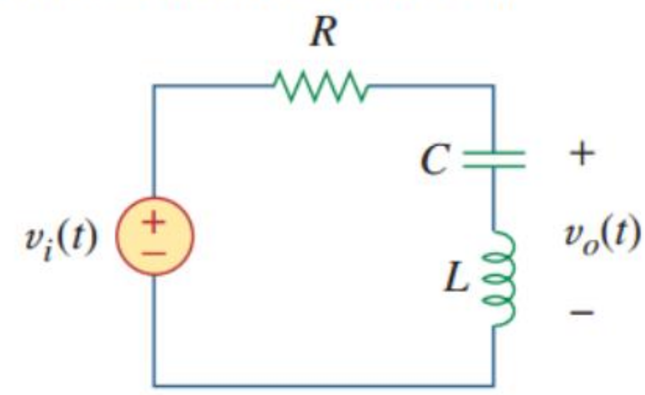
$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

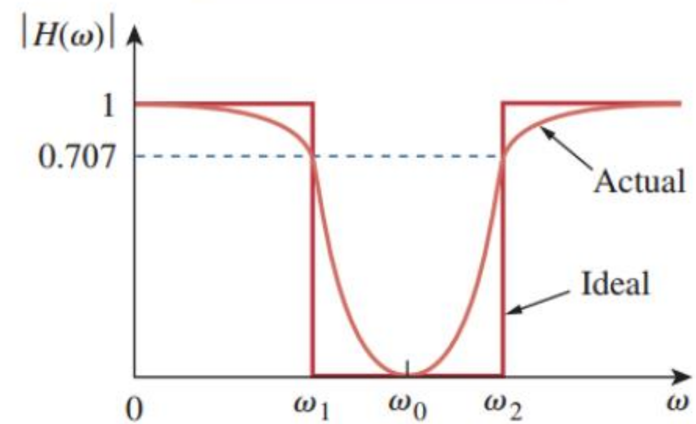
Where $\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\omega_1 \omega_1}$

Passive Band Stop Filter

- It rejects a particular range of frequencies and allows rest of the frequencies.
- Example of band stop filter :
  - Transfer Function

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$





- Here, H(0) = 1 and H(∞) = 1.
- But at resonance frequency,
  $v_0 = 0 \Rightarrow$ Filters does not allow ω_0

Electric Filters

It is an electronic circuit used to separate out signals of different frequencies.

Classification of Filters:

- **Analog or Digital**
  - **Analog filters process analog signal**
  - **Digital filters Process analog signal digitally**
- **Passive or Active**
  - **Passive elements: RLC**
  - **Active Elements: BJT/ Op-amp**
- **Audio (AF) or Radio Frequency (RF)**
  - **AF filter Elements: RC**
  - **RF filter Elements: LC/ Crystal**

Active Filters

Benefits of Active Filters over Passive Filters:

- **Gain and frequency adjustment flexibility**
- **No loading Problem**
- **Cost**

Most commonly used Active Filters:

- **High Pass**
- **Low Pass**
- **Band Pass or Notch**
- **Band reject**
- **All-Pass Filter**

Frequency Response of Active Filters

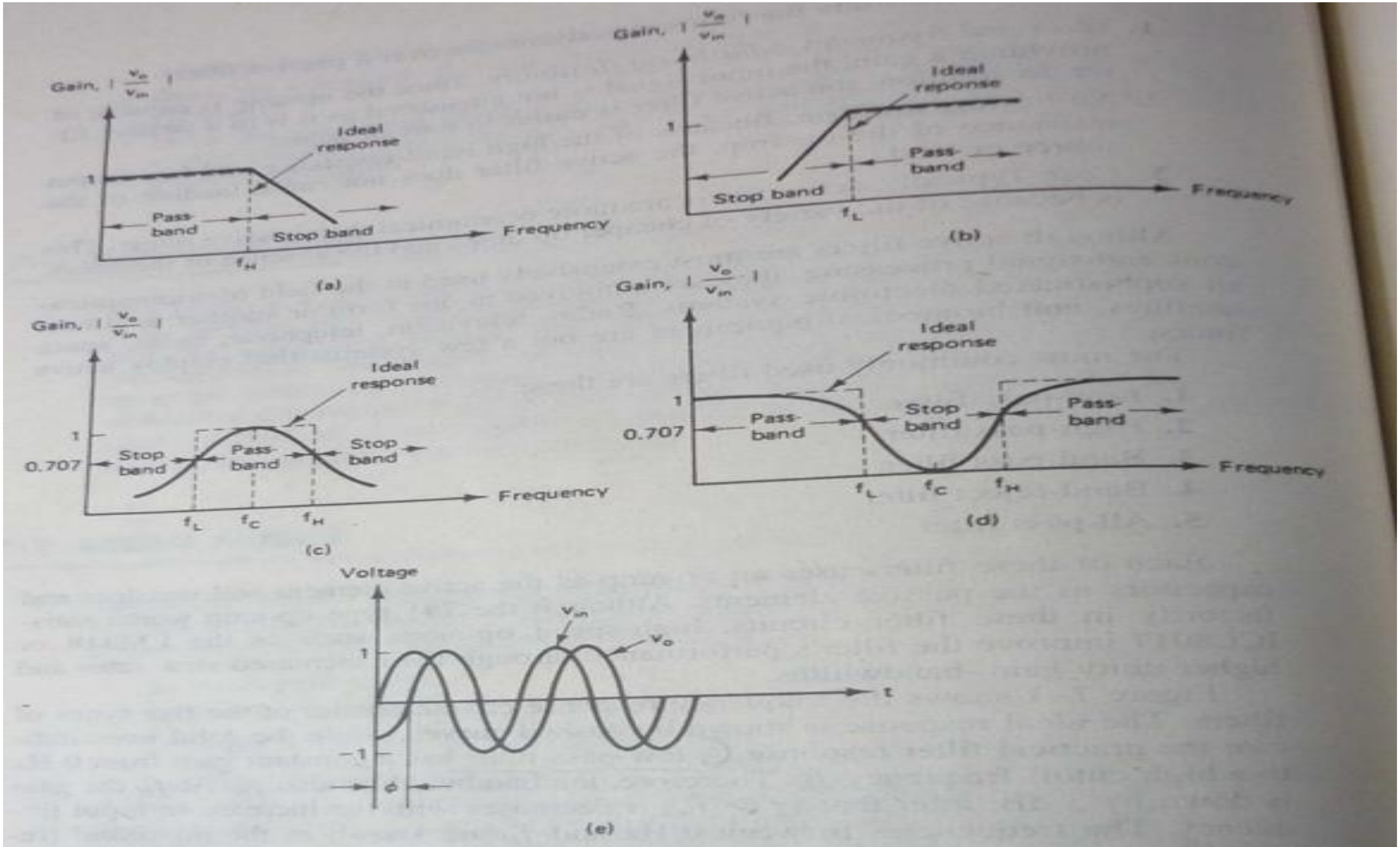
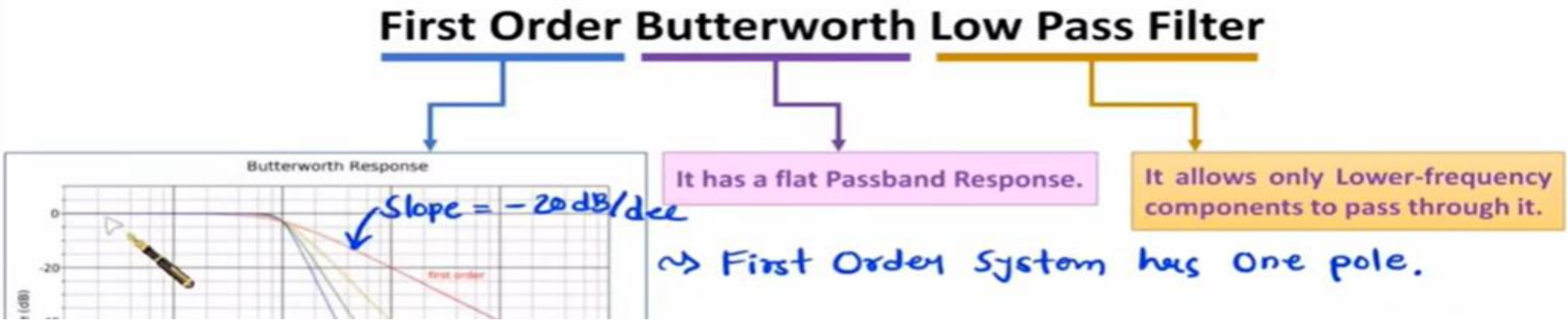


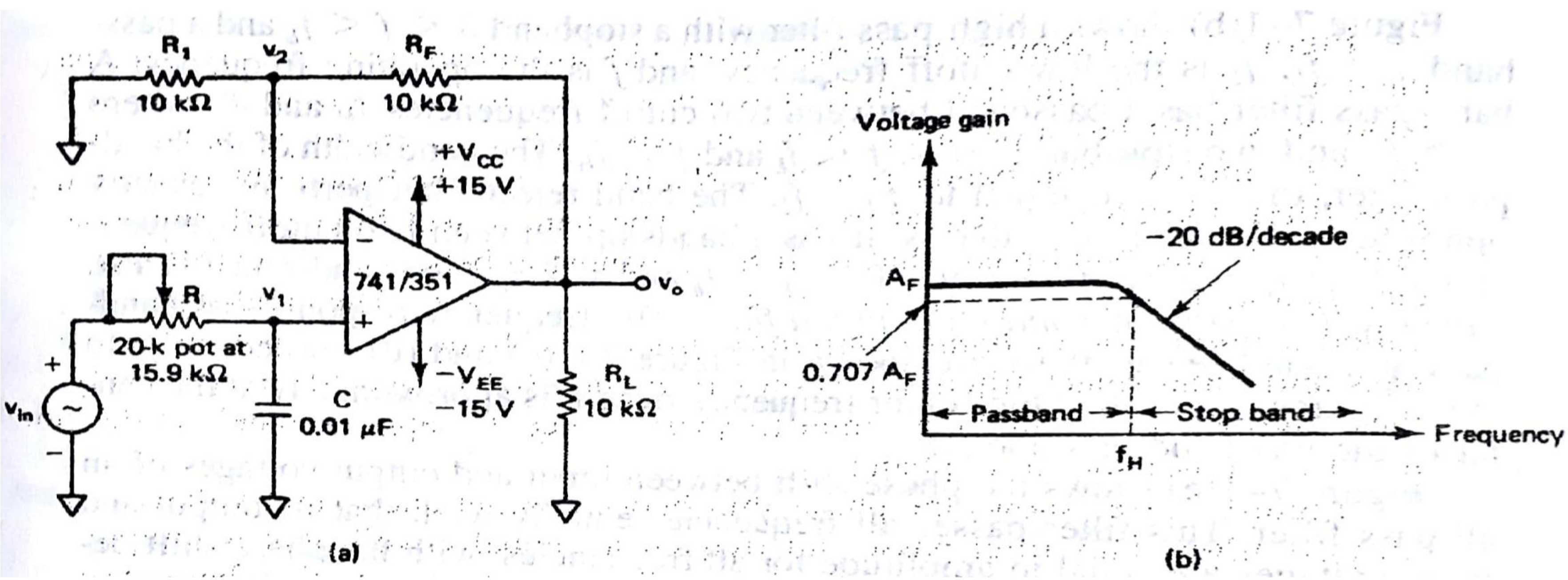
FIGURE 7–1 Frequency response of the major active filters. (a) Low pass. (b) High pass. (c) Band pass. (d) Band reject. (e) Phase shift between input and output voltages of an all-pass filter.

Chapter 7, Page 268 of **Op-Amps and Linear Integrated Circuits** *by Ramakant A. Gayakwad*

First Order Low Pass Butterworth Filter



Poles and zeros are properties of the transfer function, and in general, solutions that make the function tend to zero are called, well, zeros, and the roots that make the function tend towards its maximum function are called poles (see last optical slide at the end).



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First Order Low Pass Butterworth Filter

$$v_1 = \frac{-jX_c}{R - jX_c} v_{in} = \frac{\dfrac{1}{j2\pi fC}}{R + \dfrac{1}{j2\pi fC}} v_{in} = \frac{1}{1 + j2\pi fRC} v_{in}$$

$$v_1 = \frac{1}{1 + j2\pi fRC} v_{in}$$

$$f_H = \frac{1}{2\pi RC}$$

$$v_0 = \left(1 + \frac{R_F}{R_1}\right) v_1 = \left(1 + \frac{R_F}{R_1}\right) \frac{1}{1 + j2\pi fRC} v_{in}$$

$$v_0 = \frac{A}{1 + j2\pi fRC} v_{in}$$

First Order Low Pass Butterworth Filter

$$v_0 = \frac{A}{1 + j2\pi fRC} v_{in}$$

$$A = \left(1 + \frac{R_F}{R_1}\right)$$

$$f_H = \frac{1}{2\pi RC}$$

$$v_0 = \frac{A}{1 + j\left(f / f_H\right)} v_{in}$$

$$f_H = \frac{1}{2\pi RC}$$

$$\left|\frac{v_0}{v_{in}}\right| = \frac{A}{\sqrt{1 + \left(f / f_H\right)^2}}$$

$$\phi = -\tan^{-1}\left(f / f_H\right)$$

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First Order Low Pass Butterworth Filter

Filter Response

$$f < f_H, \left|\frac{v_0}{v_{in}}\right| \cong A$$

$$f_H = \frac{1}{2\pi RC}$$

$$f = f_H, \left|\frac{v_0}{v_{in}}\right| = \frac{A}{\sqrt{2}} = 0.707A$$

$$f > f_H, \left|\frac{v_0}{v_{in}}\right| < A$$

First Order Low Pass Butterworth Filter

Filter Design

Filter Design Steps

$$f_H = \frac{1}{2\pi RC}$$

1. Choose f_H
2. Select C ≤ 1μF
3. Calculate R

$$R = \frac{1}{2\pi f_H C}$$

Select R_1 and R_F depending on A_F

$$A_F = \left(1 + \frac{R_F}{R_1}\right)$$

First Order Low Pass Butterworth Filter

Filter Design

Design a low pass filter at a cutoff frequency of 1kHz with a passband gain of 2.

$$f_H = \frac{1}{2\pi RC}$$

1. **f_H= 1kHz**

2. Let C= 0.01 μF

$$R = \frac{1}{2\pi f_H C} = \frac{1}{2\pi.10^3.10^{-8}} = 15.9k\Omega$$

Use 20 $k\Omega$ potentiometer

Let $R_F = R_1 = 10k\Omega$

First Order Low Pass Butterworth Filter

Frequency Scaling

Frequency Scaling is the process to convert an original cutoff frequency f_H to a new cutoff frequency f_H'

$$f_H = \frac{1}{2\pi RC}$$

<u>Process</u>

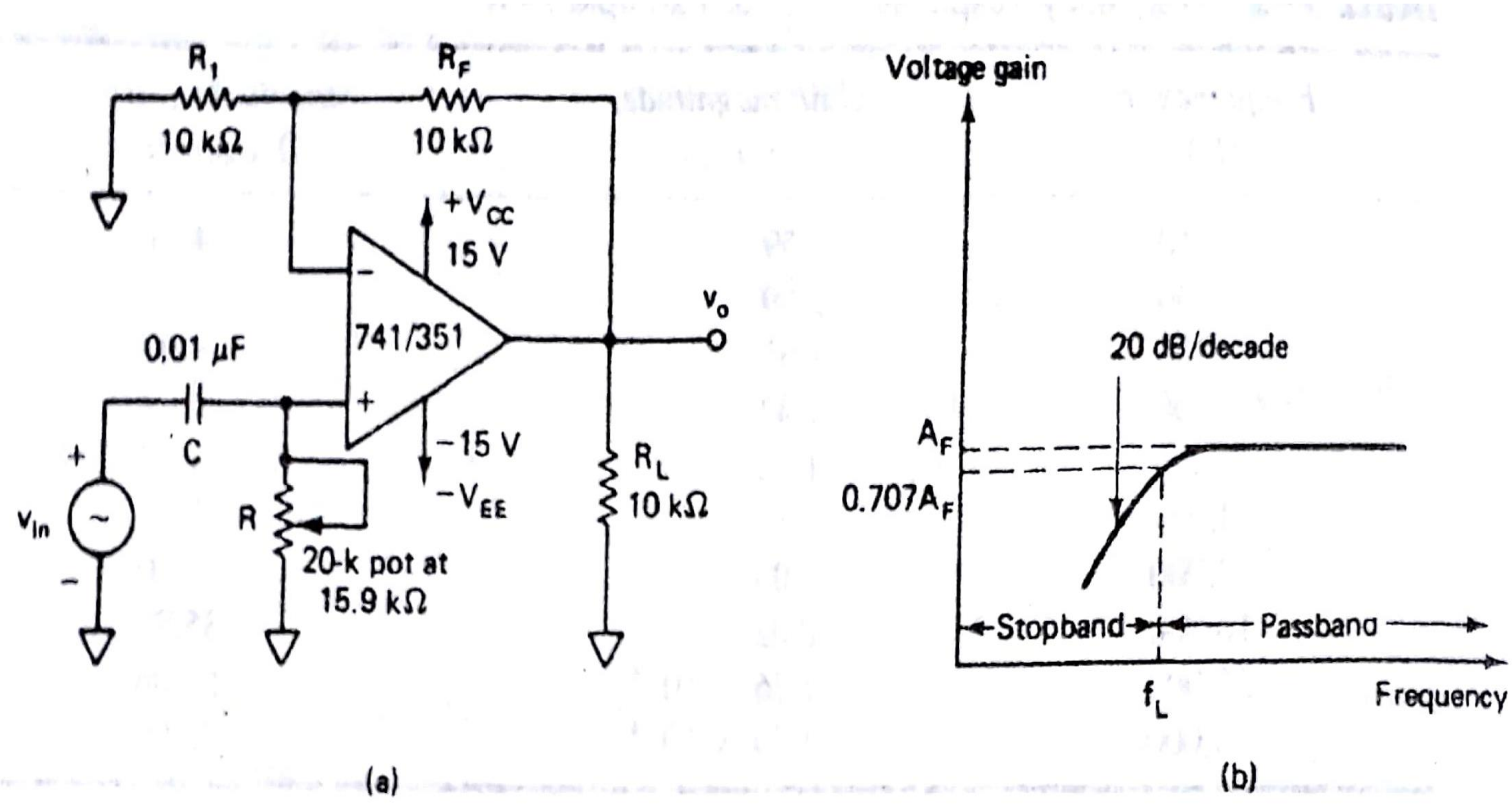
1.Change only R or C

2. Multiply old R by the factor F to get new R

$$F = \frac{Original\ Cutoff\ Frequency}{New\ Cutoff\ Frequency}$$

- **Example 7.2 to 7.3 are needed to be solved.**

First Order High Pass Butterworth Filter



$$v_1 = \frac{R}{R - jX_c} v_{in} = \frac{R}{R + \frac{1}{j2\pi fC}} v_{in} = \frac{j2\pi fRC}{1 + j2\pi fRC} v_{in}$$

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First Order High Pass Butterworth Filter

$$v_o = \left(1 + \frac{R_F}{R_1}\right) v_1$$

$$v_o = A_F \frac{j2\pi fRC}{1 + j2\pi fRC} v_{in}$$

$$v_o = A_F \frac{j(f / f_L)}{1 + j(f / f_L)} v_{in}$$

$$f_H = \frac{1}{2\pi RC}$$

$$f_L = \frac{1}{2\pi RC}$$

$$\left|\frac{v_o}{v_{in}}\right| = A_F \frac{(f / f_L)}{\sqrt{1 + (f / f_L)^2}}$$

- **Example 7.5 is needed to be solved.**

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Band pass filters

A band-pass filter has a passband between two cutoff frequencies f_H and f_L such that $f_H > f_L$. Any input frequency outside this passband is attenuated.

Basically, there are two types of band-pass filters: (1) wide band pass, and (2) narrow band pass. Unfortunately, there is no set dividing line between the two. However, we will define a filter as wide band pass if its *figure of merit* or *quality factor* $Q < 10$. On the other hand, if $Q > 10$, we will call the filter a narrow band-pass filter. Thus Q is a measure of selectivity, meaning the higher the value of Q, the more selective is the filter or the narrower its bandwidth (BW). The relationship between Q, the 3-dB bandwidth, and the center frequency f_C is given by

$$Q = \frac{f_C}{\text{BW}} = \frac{f_C}{f_H - f_L}$$

For the wide band-pass filter the center frequency f_C can be defined as

$$f_C = \sqrt{f_H f_L} \tag{7-9b}$$

where f_H = high cutoff frequency (Hz)

f_L = low cutoff frequency of the wide band-pass filter (Hz)

In a narrow band-pass filter, the output voltage peaks at the center frequency.

Wide band pass filters

- ✓ Cascading high pass, low pass
- ✓ The order of the band pass filter depends on the order of the high pass and low pass filter sections

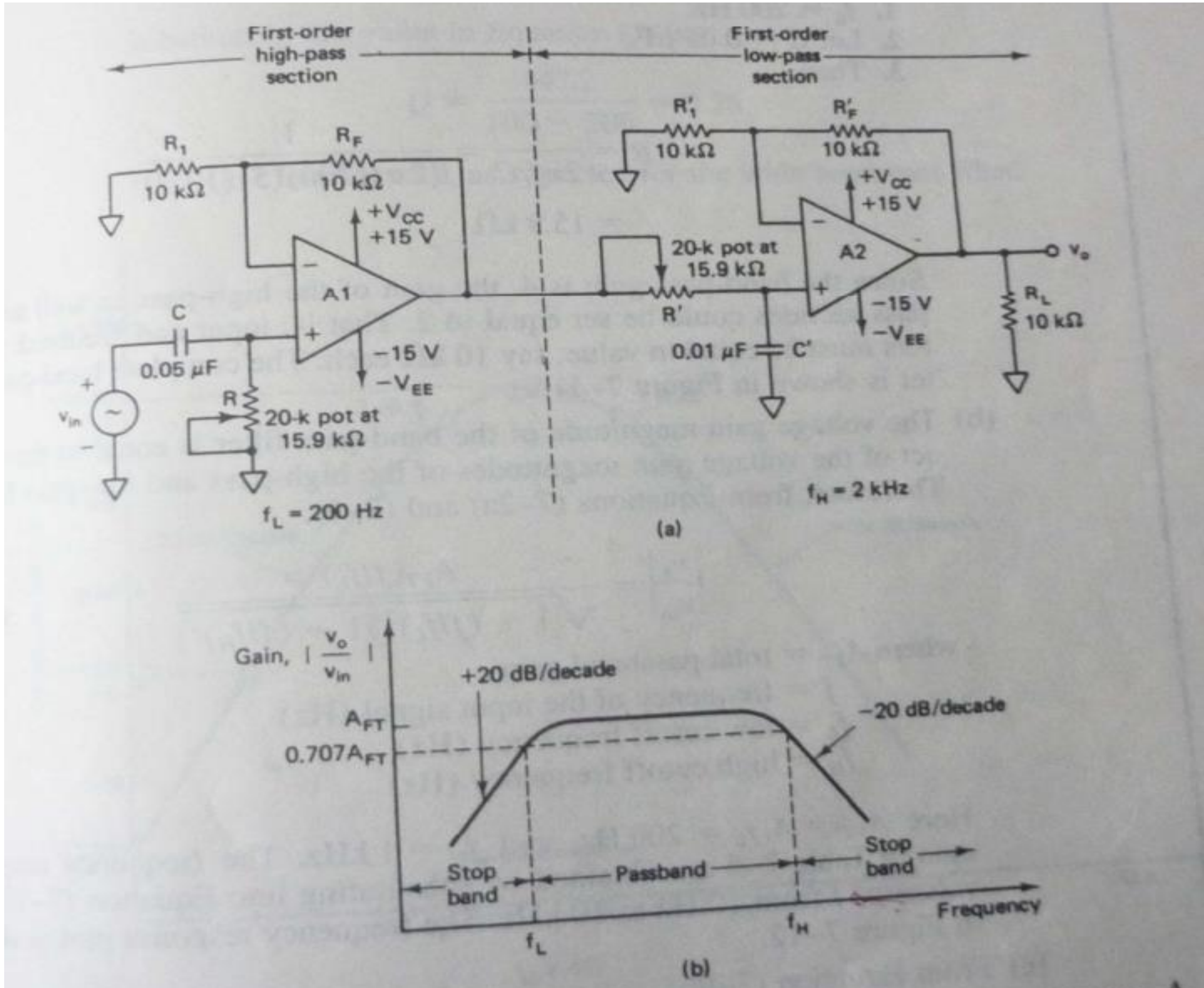
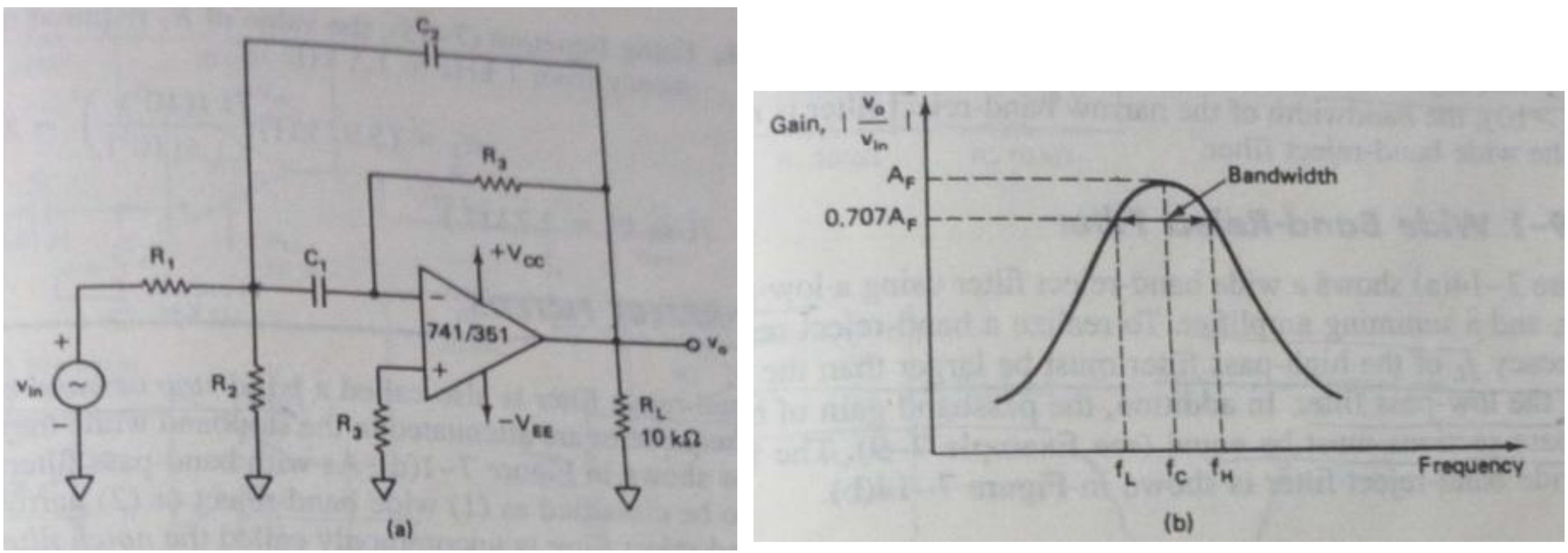


FIGURE 7-11 (a) +20 dB/decade-wide band-pass filter. (b) Its frequency response. A

Narrow band pass filters

Compared to all other filters, this filter is unique in the following respects:

1. It has two feedback paths, hence the name multiple-feedback filter
2. The Op-amp is used in the inverting mode

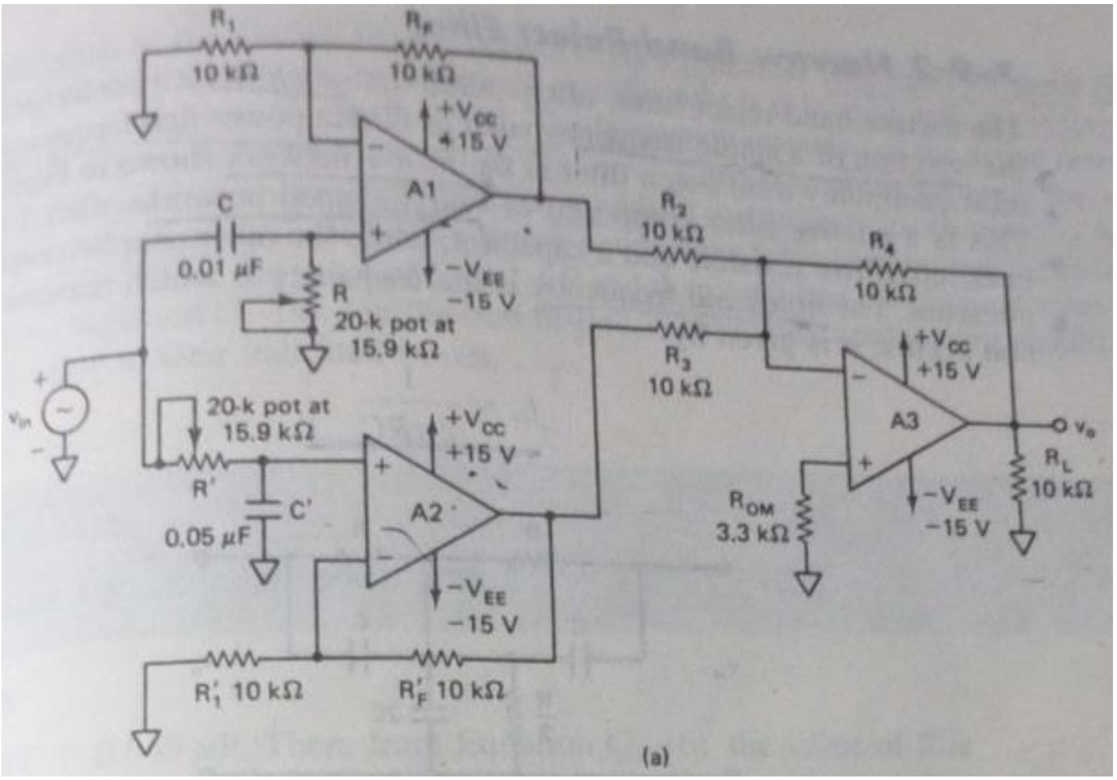


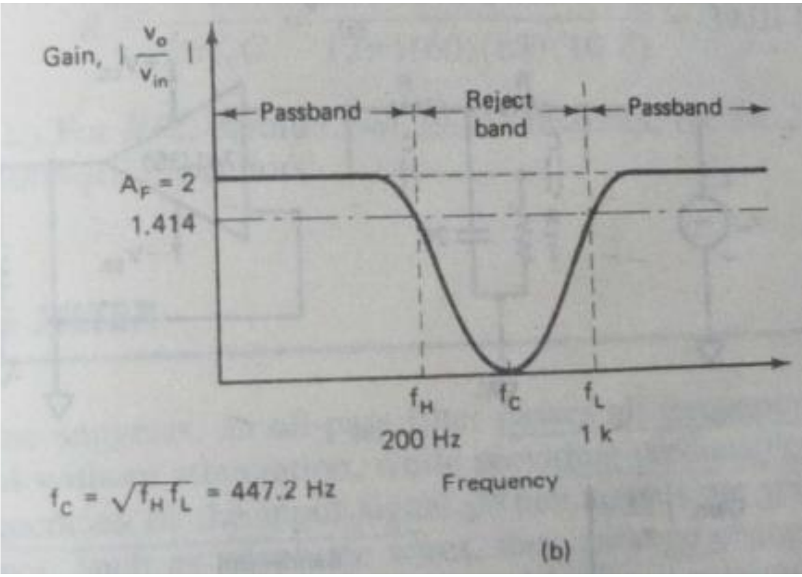
IGURE 7–13 (a) Multiple-feedback narrow band-pass filter. (b) Its frequency response.

Band reject filters

- The band-reject filter is also called a band-stop or band elimination filter.
- In this filter, frequencies are attenuated in the stopband while they are passed outside this band.
- It can be classified as (1) wide band-reject (2) narrow band-reject (also called as notch filter)

Wide band reject filter





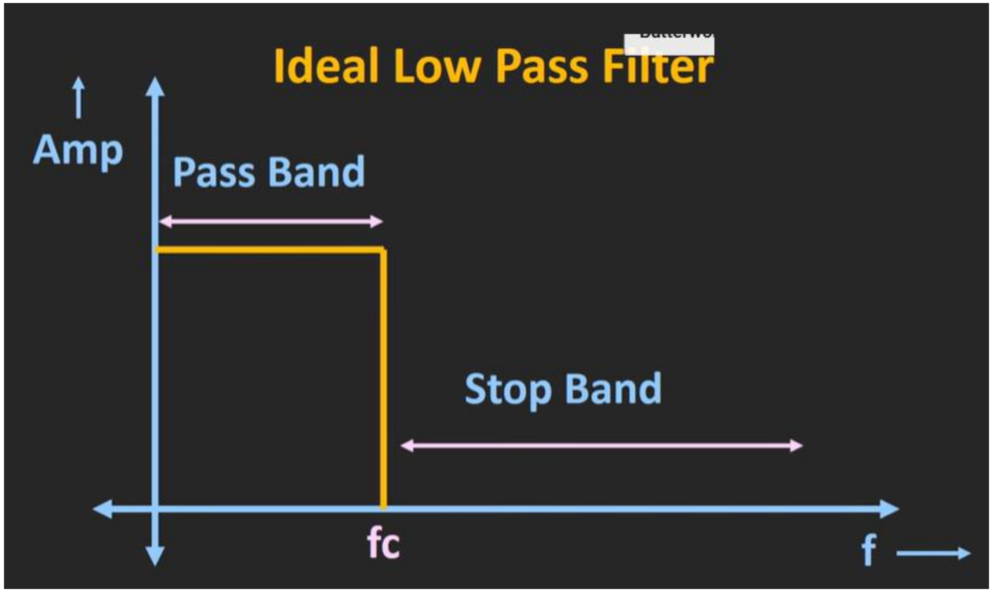
- High pass, low pass filters and a summing amplifier

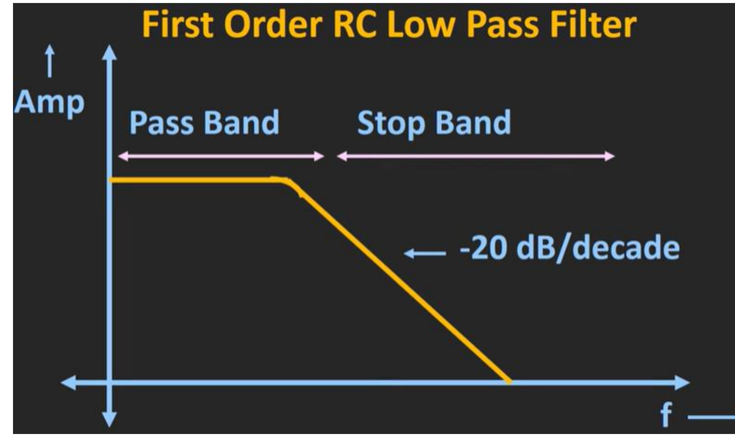
- To realize band reject response, lower cutoff *f* of high pass filter must be grater than the high cutoff *f* of low pass filter

Please solve Examples and Exercise problems of related topics

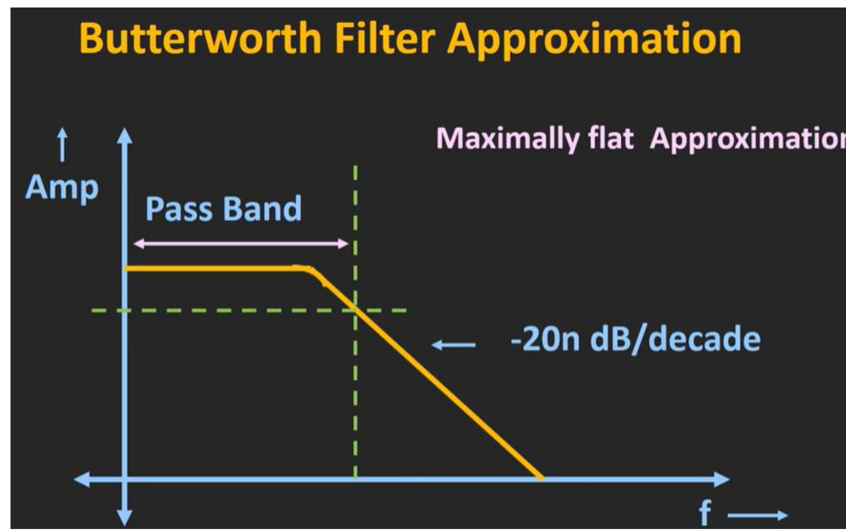
Practice yourself and send me your feedback, if any.

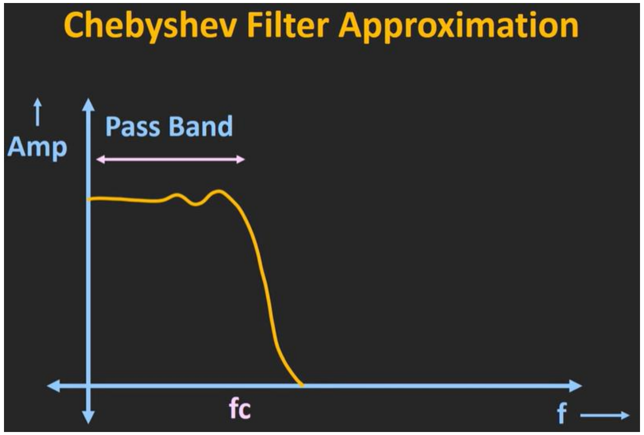
Next slides are Optional just for understand

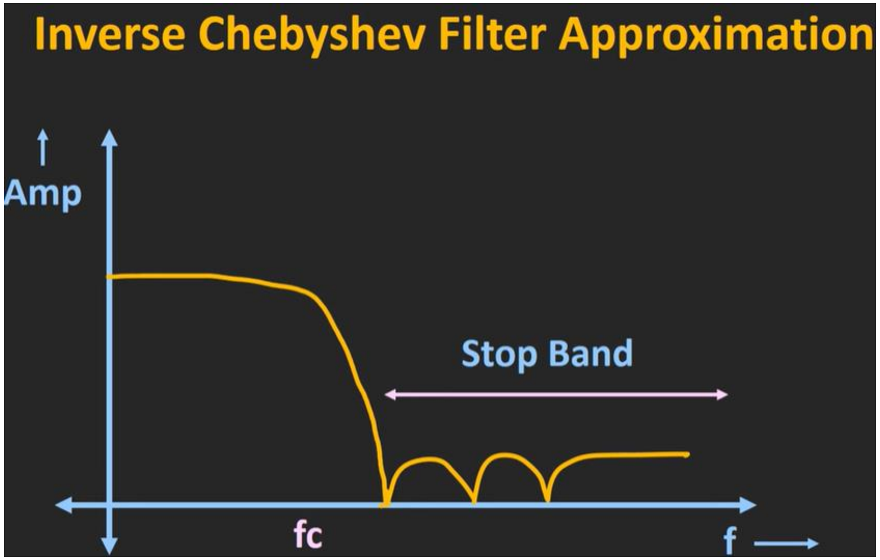

Ideal Low Pass Filter
Amp
Pass Band
Stop Band
fc
f

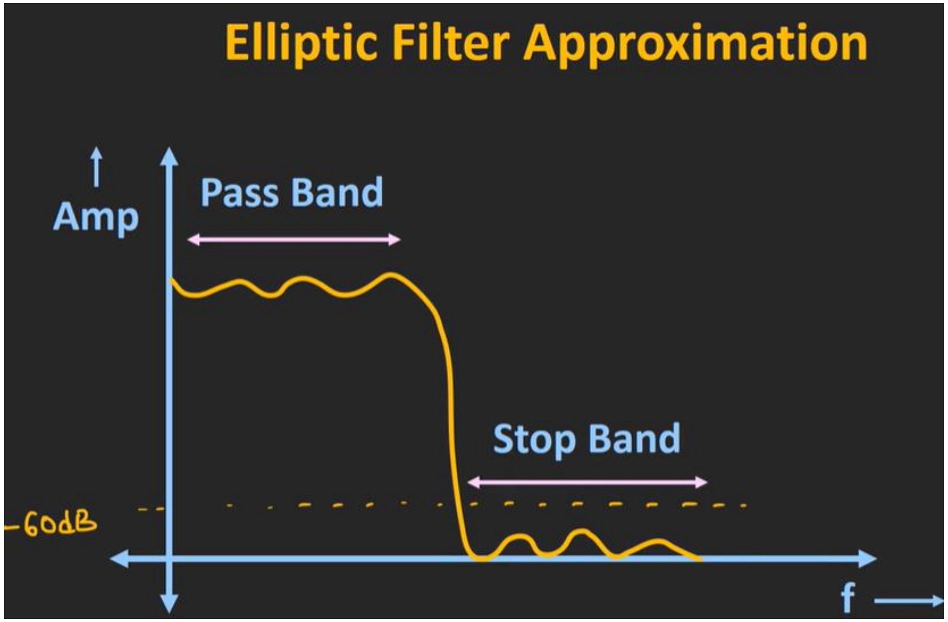

First Order RC Low Pass Filter
Amp
Pass Band
Stop Band
-20 dB/decade
f

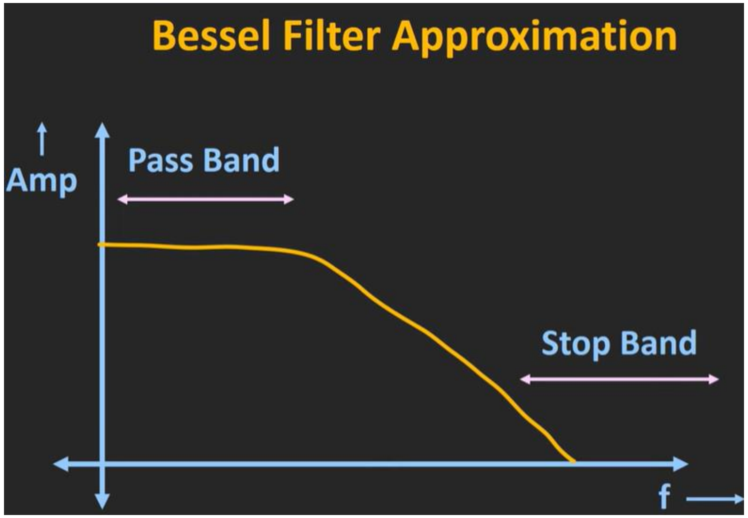

Filter Approximation
Butterworth Filter Approximation
Chebyshev Filter Approximation
Inverse Chebyshev Filter Approximation
Elliptic Filter Approximation
Bessel Filter Approximation


Butterworth Filter Approximation
Maximally flat Approximation
Amp
Pass Band
-20n dB/decade
f


Chebyshev Filter Approximation
Amp
Pass Band
fc
f


Inverse Chebyshev Filter Approximation
Amp
Stop Band
fc
f


Elliptic Filter Approximation
Amp
Pass Band
Stop Band
-60dB
f


Bessel Filter Approximation
Amp
Pass Band
Stop Band
f

Comparison of Filter Approximations

Type	Passband	Stopband	Roll-off	Step Response
Butterworth	Flat	Monotonic	Good	Good
Chebyshev	Rippled	Monotonic	Very Good	Poor
Inverse Chebyshev	Flat	Rippled	Very Good	Good
Elliptic	Rippled	Rippled	Best	Poor
Bessel	Flat	Monotonic	Poor	Best

1 System Poles and Zeros

The transfer function provides a basis for determining important system response characteristics without solving the complete differential equation. As defined, the transfer function is a rational function in the complex variable $s = \sigma + j\omega$, that is

$$H(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0} \tag{1}$$

It is often convenient to factor the polynomials in the numerator and denominator, and to write the transfer function in terms of those factors:

$$H(s) = \frac{N(s)}{D(s)} = K\frac{(s-z_1)(s-z_2)\ldots(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_{n-1})(s-p_n)}, \tag{2}$$

where the numerator and denominator polynomials, $N(s)$ and $D(s)$, have real coefficients defined by the system's differential equation and $K = b_m/a_n$. As written in Eq. (2) the z_i's are the roots of the equation

$$N(s) = 0, \tag{3}$$

and are defined to be the system *zeros*, and the p_i's are the roots of the equation

$$D(s) = 0, \tag{4}$$

and are defined to be the system *poles*. In Eq. (2) the factors in the numerator and denominator

■ Example

A linear system is described by the differential equation

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 2\frac{du}{dt} + 1.$$

Find the system poles and zeros.

Solution: From the differential equation the transfer function is

$$H(s) = \frac{2s+1}{s^2+5s+6}. \tag{5}$$

which may be written in factored form

$$\begin{aligned} H(s) &= \frac{1}{2}\frac{s+1/2}{(s+3)(s+2)} \\ &= \frac{1}{2}\frac{s-(-1/2)}{(s-(-3))(s-(-2))}. \end{aligned} \tag{6}$$

The system therefore has a single real zero at $s = -1/2$, and a pair of real poles at $s = -3$ and $s = -2$.