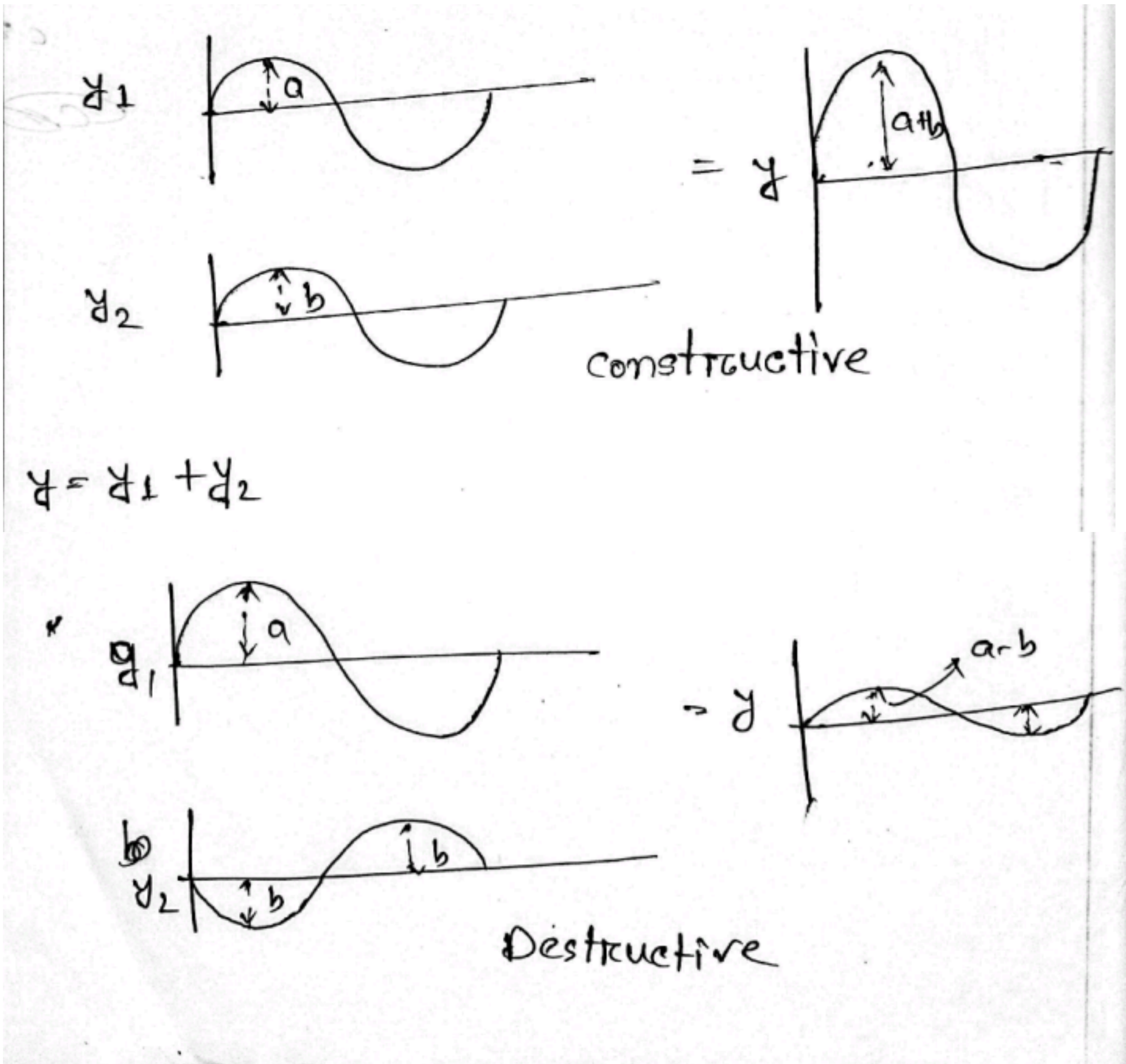
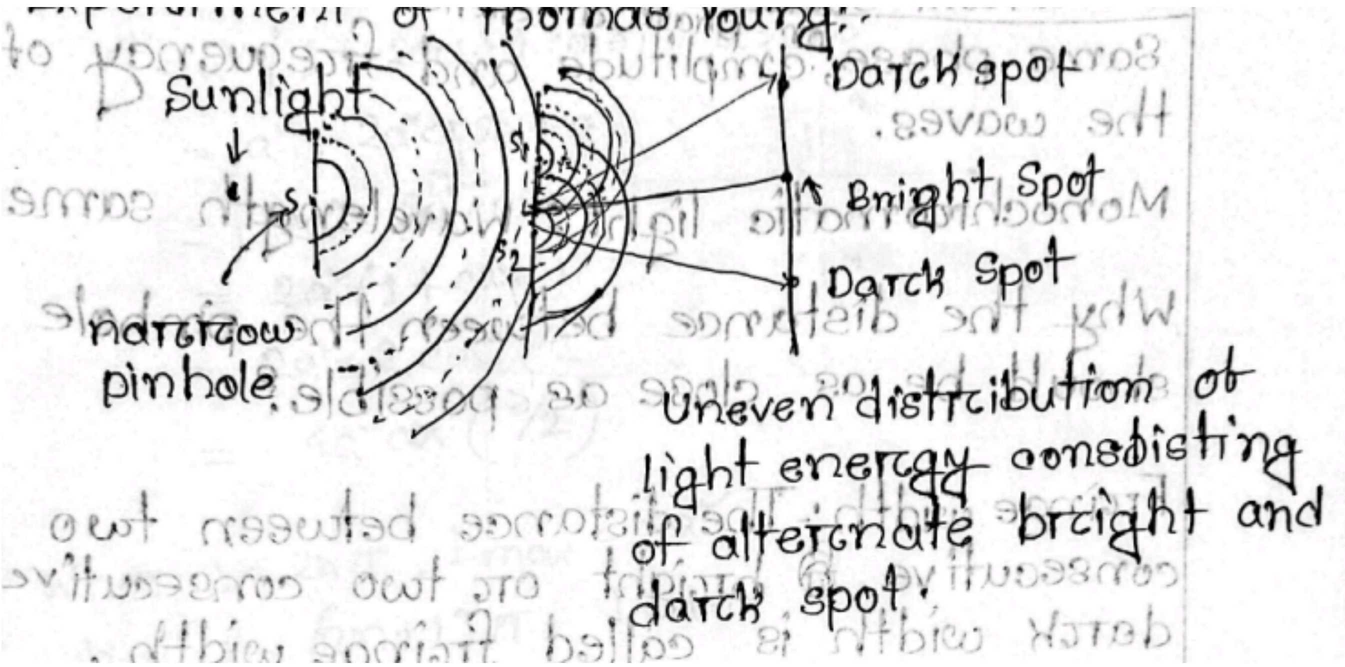
Superposition of Wave

When a medium is disturbed simultaneously by more than one wave then the instantaneous resultant displacement at every point at every instance is the algebraic sum of the displacement of the medium by each each individual wave



Experiment of Thomas Young



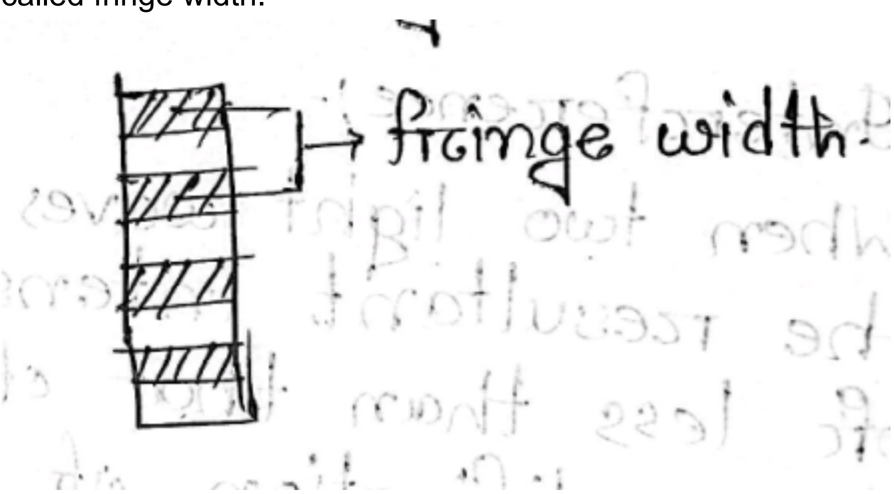
Interference

When two light waves superimposed, then the resultant intensity will be greater or less than that of a single wave. The modification of light energy due to superposition of light in this way is called interference of light.
In constructive interference, the intensity of light increases
In destructive interference, the intensity of light decreases.

Coherent Sources: Same phase or phase difference, amplitude and frequency of the waves

Monochromatic Light: Wavelength same

**Fringe width: The distance between two consecutive bright or two consecutive dark width is called fringe width.



$$y_1 = a \sin \omega t$$

$$y_2 = a \sin(\omega t + \delta)$$

$$y = y_1 + y_2$$

$$= a\sin\omega t + a\sin(\omega t + \delta)$$

$$= a\sin\omega t + a\sin\omega t\cos\delta + a\cos\omega t\sin\delta$$

$$= a\sin\omega t(1 + \cos\delta) + a\cos\omega t\sin\delta$$

Let,

$$a(1 + \cos\delta) = A\cos\theta$$

$$a\sin\delta = A\sin\theta$$

$$= A\sin\omega t\cos\theta + A\cos\omega t\sin\theta$$

$$= A\sin(\omega t + \theta)$$

which is a form of wave motion

Now,

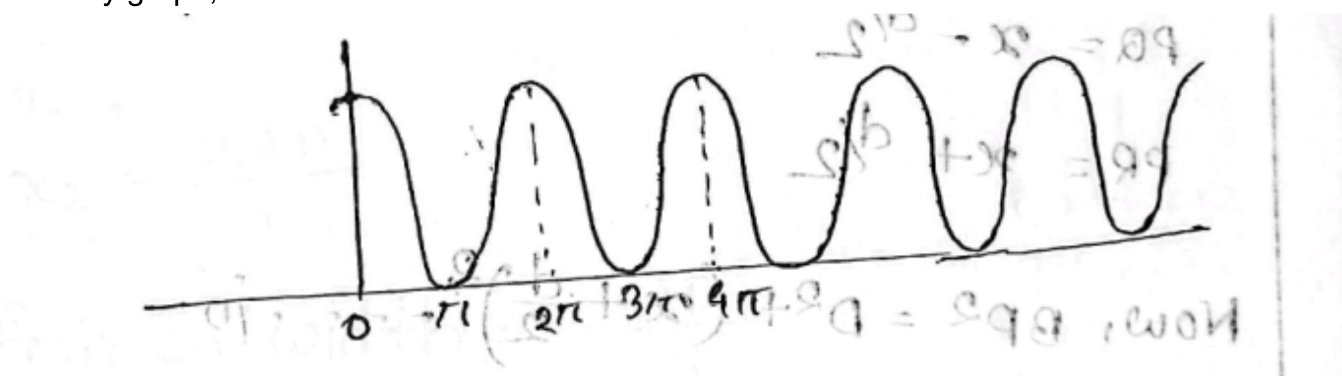
$$A^2 = a^2(1 + 2\cos\delta + \cos^2\delta) + a^2\sin^2\delta$$

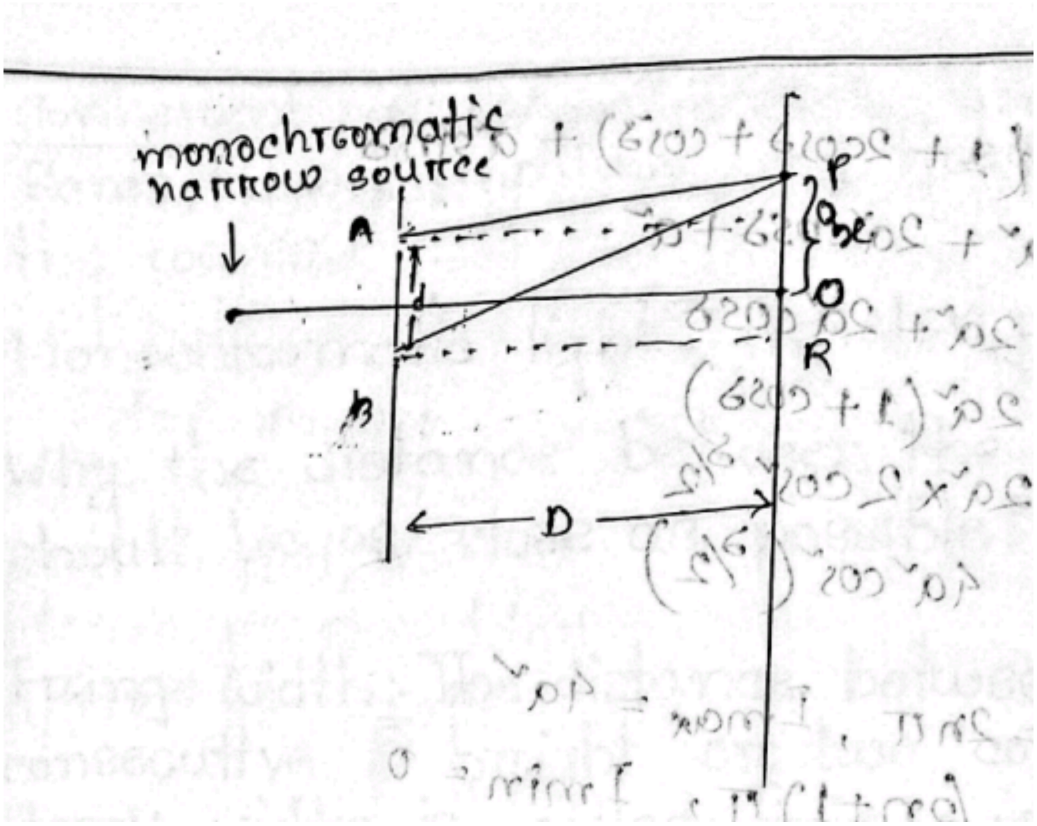
$$= 2a^2(1 + \cos\delta)$$

$$= 4a^2\cos^2\frac{\delta}{2}$$

when, $\delta = 2n\pi$, $I_{max} = 4a^2$
When, $\delta = (2n+1)\pi$, $I_{min} = 0$

Intensity graph,





Here, D is the distance between the coherent sources and the screen, and d is the distance between two slits. and x is the distance from central bright.

$$OQ = OR = \frac{d}{2}$$

$$PQ = x - \frac{d}{2}$$

$$PR = x + \frac{d}{2}$$

Now,

$$BP^2 = D^2 + \left(x + \frac{d}{2}\right)^2 \ldots (i)$$

$$AP^2 = D^2 + \left(x - \frac{d}{2}\right)^2 \ldots (ii)$$

Now the two rays are BP and AP and the path difference between them should be,

$$\Delta x = BP - AP$$

$(i) - (ii)$

$$BP^2 - AP^2 = 2xd$$

$$BP - AP = \frac{2xd}{BP + AP}$$

$$BP - AP = \frac{2xd}{2D} = \frac{xd}{D}$$

$$\therefore \Delta x = \frac{xd}{D}$$

The condition for bright fringe, $\Delta x = n\lambda$

$$\therefore \Delta x = \frac{xd}{D}$$

$$\Rightarrow n\lambda = \frac{xd}{D}$$

$$\Rightarrow x = \frac{n\lambda D}{d}$$

When, $n = 1$

$$x_1 = \frac{\lambda D}{d}$$

$n = 2$,

$$x_2 = \frac{2\lambda D}{d}$$

$$\therefore \text{fringe width}, \beta = x_2 - x_1 = \frac{\lambda D}{d}$$

For dark, $\Delta x = (2n+1)\frac{\lambda}{2}$

$$\therefore (2n+1)\frac{\lambda}{2} = \frac{xd}{D}$$

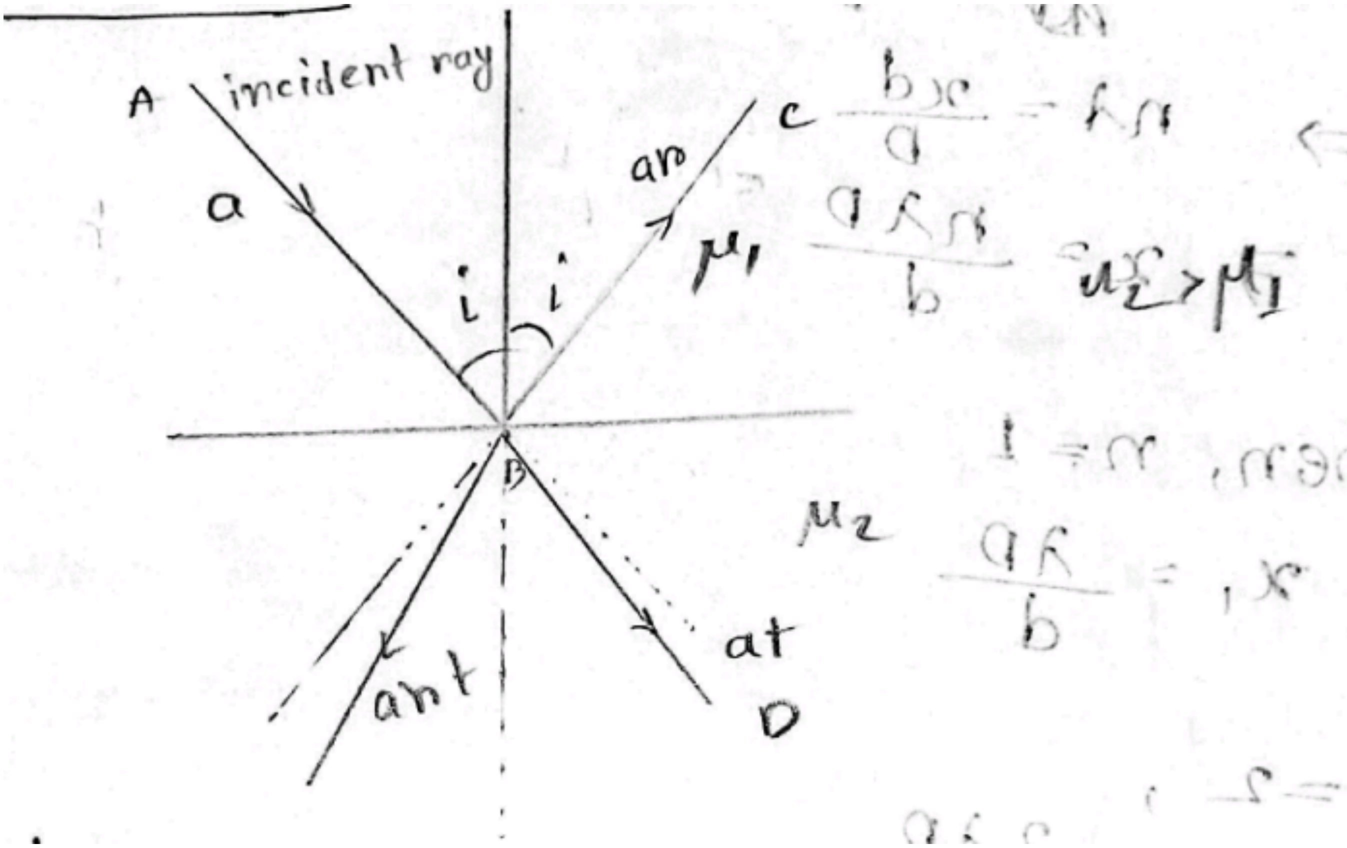
$$x = \frac{(2n+1)\lambda D}{2d}$$

These are the conditions for dark and bright fringe

Thin Film Interference

Thin film is a layer of transparent material with very low thickness (in nanometre or Armstrong)

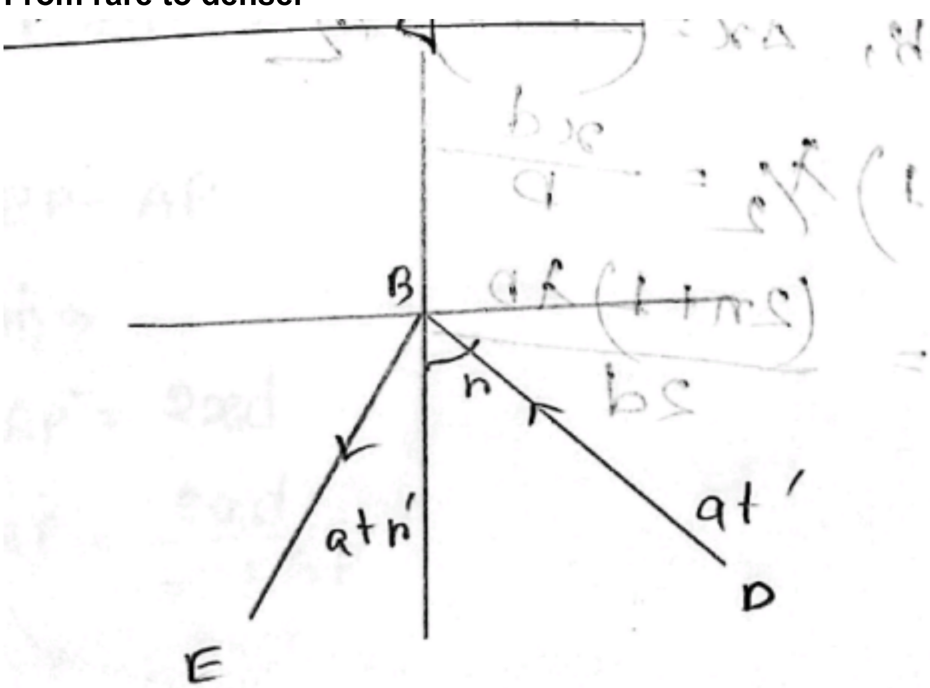
Law of Stocks



Let, number fraction of light was absorbed,

$$r + t = 1$$

From rare to denser



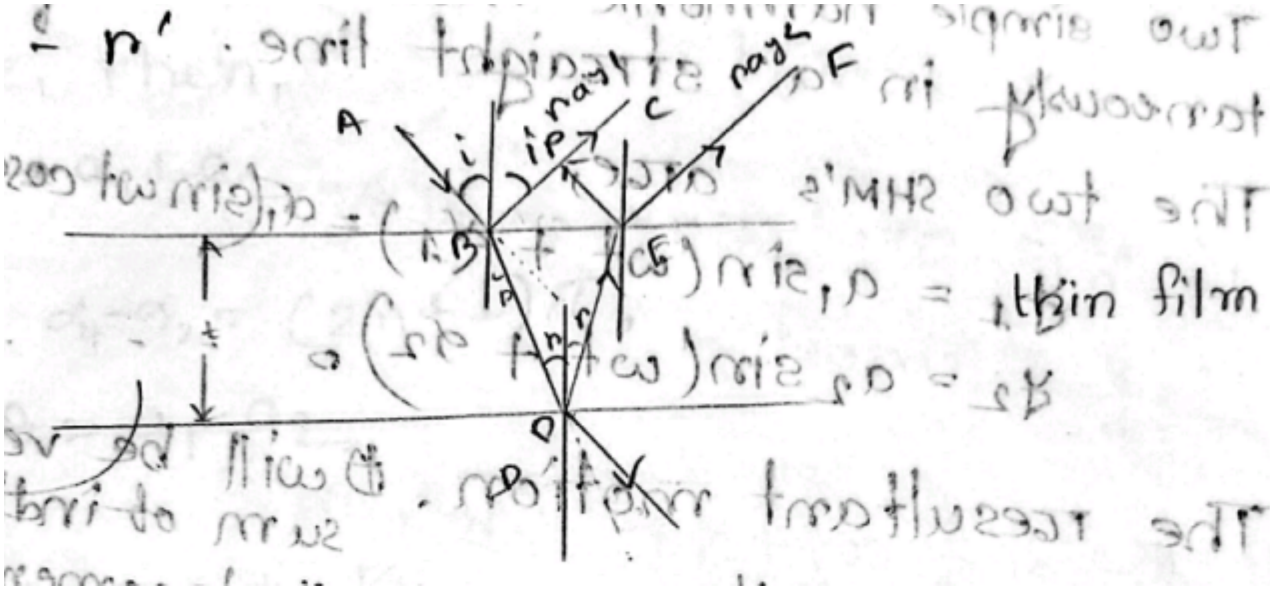
$$art + atr' = 0$$

$$at(r + r') = 0$$

$$r + r' = 0$$

$$r = -r'$$

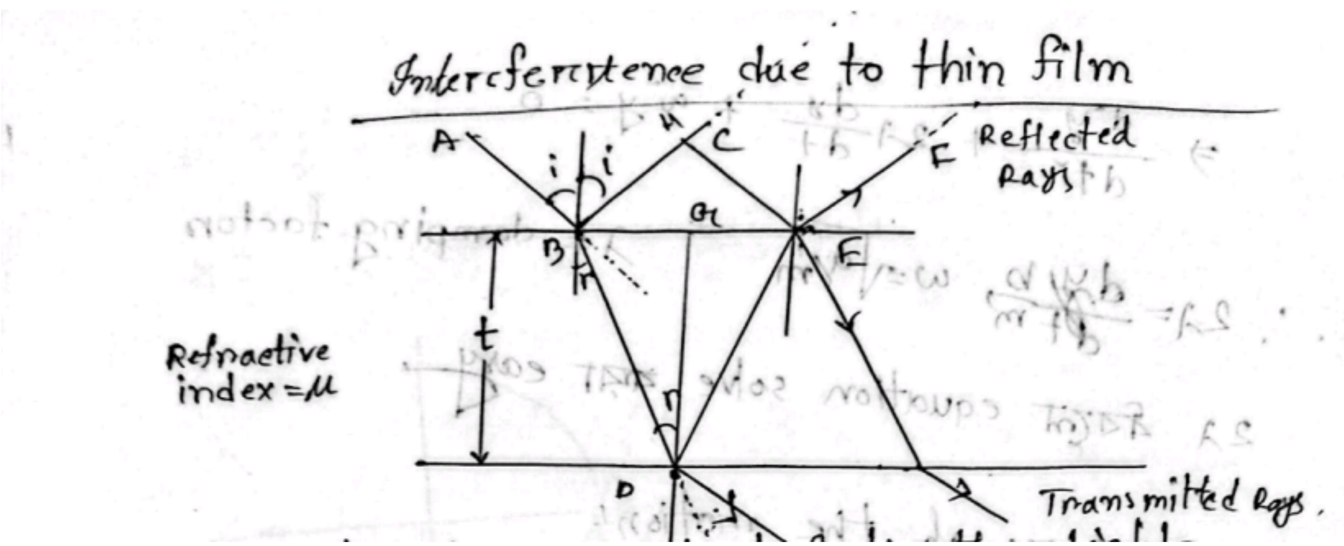
Stock's law indicates when light travels from a less dense medium into a denser medium and reflects back from the interface, it undergoes a phase shift of π radian.



$$\Delta x = \mu(BD + DE) - BP$$

For actual path difference $\frac{\lambda}{2}$ should be added of subtracted according to Stock law.

Interference Due to Thin Film



Geometrical path $\rightarrow$ Actual path the light travels. Let the length of geometrical path be x

Optical path $\rightarrow$ If a ray goes through a medium whose refractive index is mu then optical path is μx

$$\Delta x = \mu(BD + DE) - BH$$

Now,

$$D = t$$

where, t is the thickness of the thin film

$$BD = DE$$

$$BD + DE = 2BD$$

$$\therefore \cos r = \frac{D}{BD}$$

$$BD = \frac{t}{\cos r}$$

Again,

$$\sin i = \frac{BH}{BE}$$

$$BE = 2B$$

$$\tan r = \frac{B}{D}$$

$$B = D \times \tan r = t \times \tan r$$

$$\therefore BH = 2B \times \sin i = 2B \times \mu \times \sin r$$

$$= 2\mu B \sin r$$

$$= 2\mu t \sin r \tan r = 2\mu t \frac{\sin^2 r}{\cos r}$$

$$BD + DE = 2BD$$

$$BD = \frac{t}{\cos r}$$

$$\Delta x = \frac{2\mu t}{\cos r} - BH$$

$$= \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$= \frac{2\mu t(1 - \sin^2 r)}{\cos r} = 2\mu t \cos r$$

According to Stokes' Law,

$$\Delta x = 2\mu t \cos r \pm \frac{\lambda}{2}$$

For bright, $\Delta x = n\lambda$

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = (2n + 1)\frac{\lambda}{2}$$

For Dark, $\Delta x = (2n + 1)\frac{\lambda}{2}$

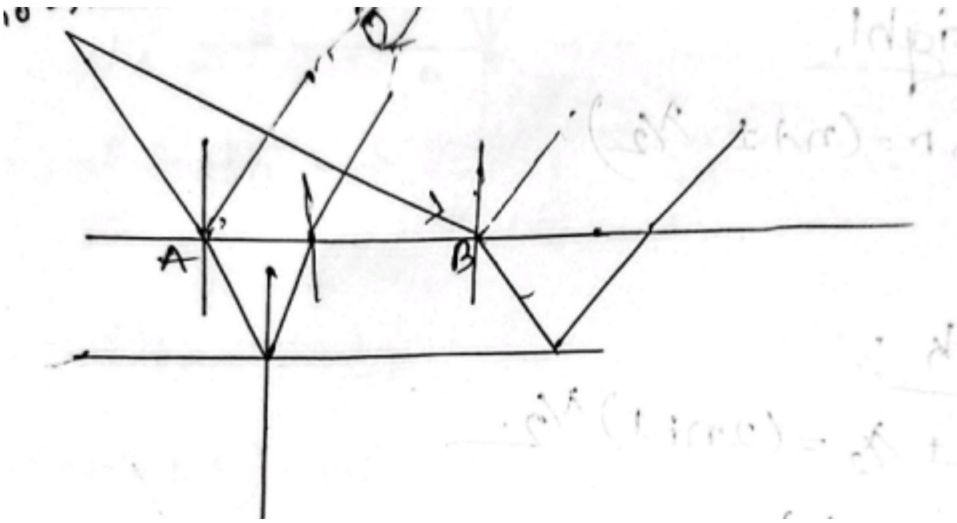
$$2\mu t\cos r+\frac{\lambda}{2}=(2n+1)\frac{\lambda}{2}$$

$$2\mu t\cos r=n\lambda$$

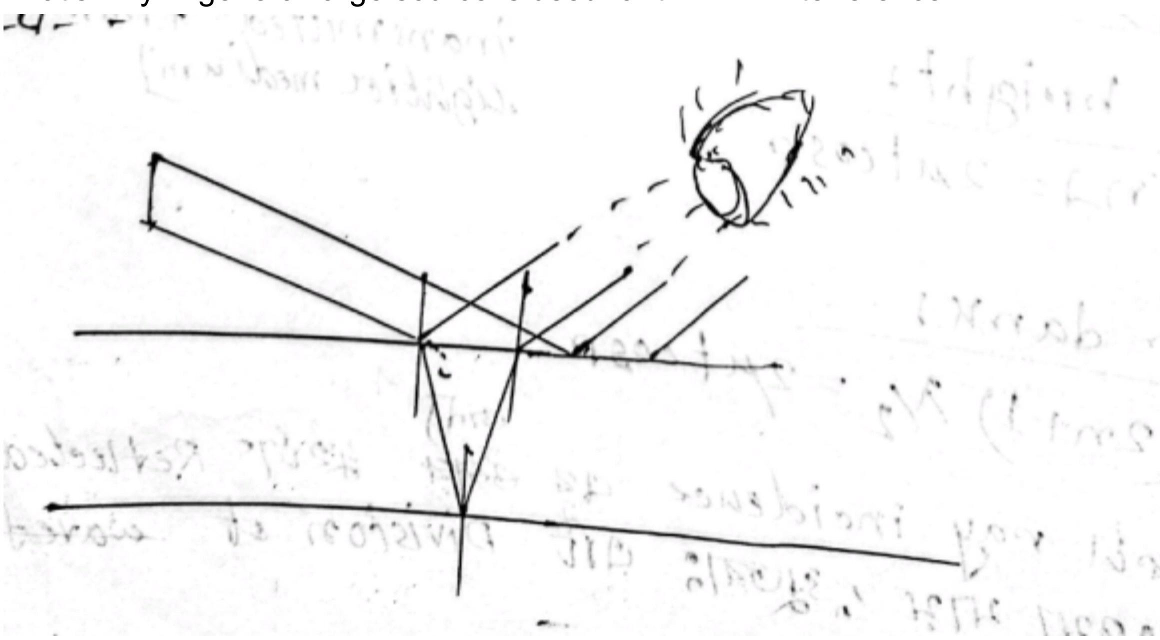
For transmitted rays the extra $\frac{\lambda}{2}$ will not be be considered because there is no phase change as both rays transmitted from denser to lighter medium. For a ray incidence in thin film, we get to reflected ray, therefore it is called the division of amplitude

And as for Youngs' Double Slit experiment, division of waveform is observed.

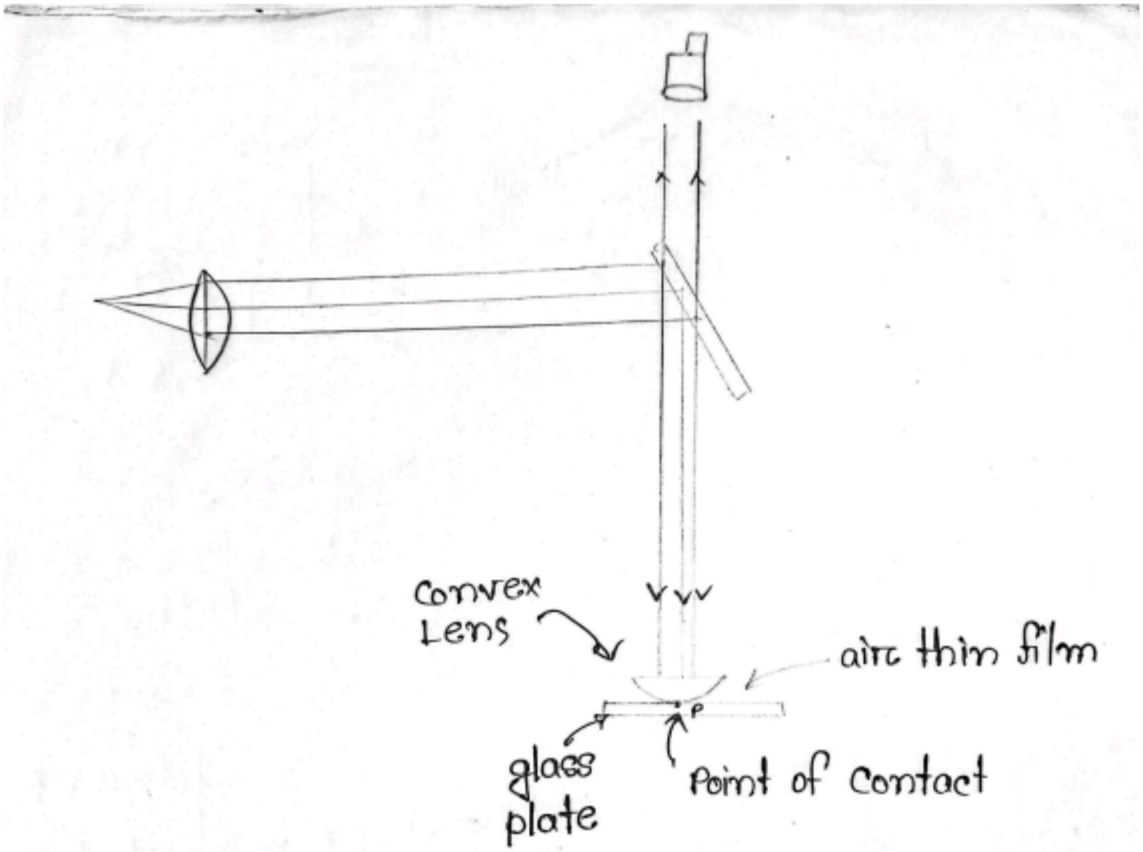
For thin film observation, if narrow-source is used, it creates limitation of visibility range as only, single fringed could be seen at once.

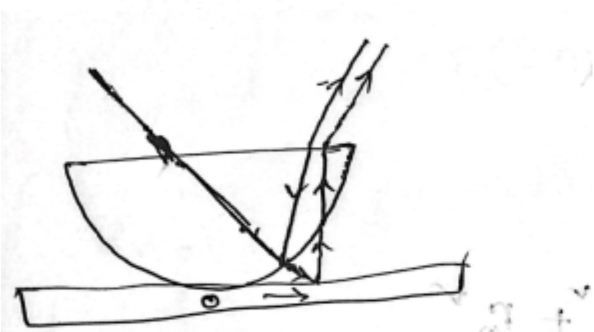


That's why in general large source is used for thin film interference



Newton's Ring

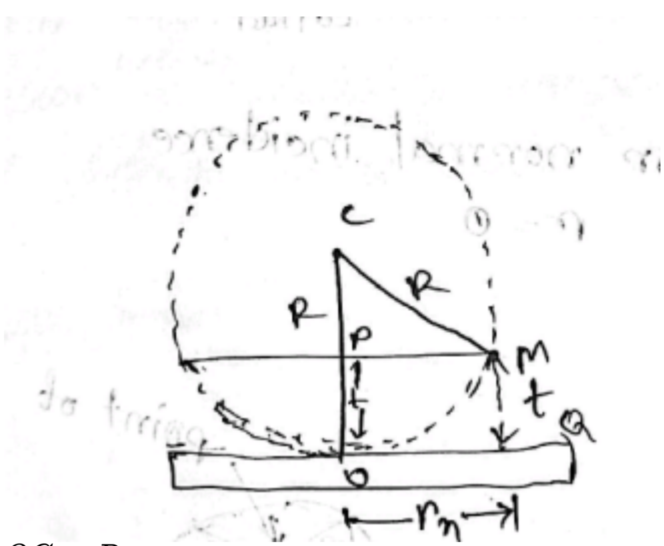




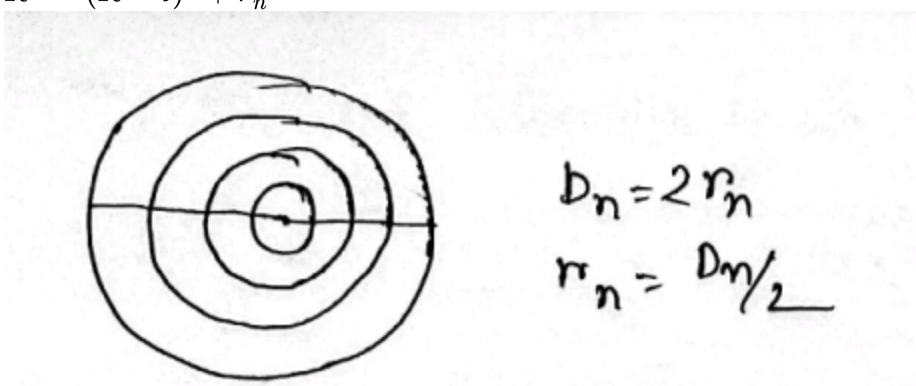
We know that for bright, $\Delta x = n\lambda$
and for dark = $\Delta x = (2n+1)\frac{\lambda}{2}$
At $t = 0$, (thickness)

$$\Delta x = 2\mu t \cos r \pm \frac{\lambda}{2} = \frac{\lambda}{2}$$

Which represents dark that is why center ring is dark.



$OC = R$
$MQ = t = OP$
$CP = OC - OP = R - t$
$OQ = r_n$
at ΔPCM,
$R^2 = (R-t)^2 + r_n^2$



$$R^2 = (R-t)^2 + r_n^2$$

$$\Rightarrow r_n^2 = 2Tt - t^2$$

$$r_n^2 = 2Rt, [R >> t]$$

$$\frac{D_n^2}{4} = 2Rt$$

$$t = \frac{D_n^2}{8R}$$

For air thin film, $\mu = 1$, $r = 0$
for bright,

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

$$2t = (2n+1)\frac{\lambda}{2}$$

$$2 \times \frac{D_n^2}{8R} = (2n+1)\frac{\lambda}{2}$$

$$D_n^2 = 4R(2n+1)\frac{\lambda}{2} \ldots (i)$$

Similarly,

$$D_{n+p}^2 = 4R[2(n+p)+1]\frac{\lambda}{2} \ldots (ii)$$

$$(ii) - (i)$$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

Obtained by solving simply for λ

This also can be easily obtained by considering dark fringe and that is one more easier in the last part. However the answer will be the same.

If we are given a liquid instead of air thin film, and is asked to calculate it's refractive index.

$$2\mu t \cos r = m\lambda$$

Let, $r = 0$, and μ = refractive index of the liquid

$$\Rightarrow 2\mu t = m\lambda$$

$$2t = \frac{m\lambda}{\mu}$$

$$2 \times \frac{D_m^2}{8R} = \frac{m\lambda}{\mu}$$

$$D_m^2 = \frac{4m\lambda R}{\mu}$$

Similarly,

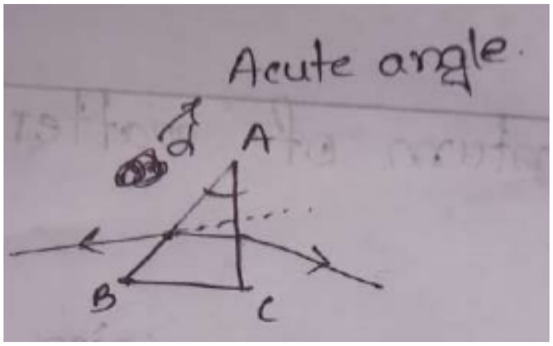
$$D_{m+p}^2 = \frac{4(m+p)\lambda R}{\mu}$$

$$\therefore [D_{m+P}^2 - D_m^2]_{liuid} = \frac{4p\lambda R}{\mu}$$

Now, $4p\lambda R$, represents the difference between the diameters of an air thin film when the $\mu = 1$

$$\therefore \mu = \frac{[D_{m+p}^2 - D_m^2]_{air}}{[D_{m+P}^2 - D_m^2]_{liuid}}$$

Fresnel's Bi-prism



Here, AB and AB are refracting face.

$$BC \to \text{ase}$$

After adjusting two acute angle prism: