Belief reliability: A new metrics for products' reliability

Article *in* Fuzzy Optimization and Decision Making · March 2013

DOI: 10.1007/s10700-012-9138-5

3 authors:

Zhiguo Zeng
CentraleSupélec

Meilin Wen
Beihang University (BUAA)

Rui Kang
Beihang University (BUAA)

Fuzzy Optim Decis Making (2013) 12:15–27
DOI 10.1007/s10700-012-9138-5

Belief reliability: a new metrics for products' reliability

Zhiguo Zeng · Meilin Wen · Rui Kang

Published online: 5 October 2012

Abstract Traditional reliability metrics are based on probability measures. However, in engineering practices, failure data are often so scarce that traditional metrics cannot be obtained. Furthermore, in many applications, premises of applying these metrics are violated frequently. Thus, this paper will give some new reliability metrics which can evaluate products' reliability with few failure data. Firstly, the new metrics are defined based on uncertainty theory and then, numerical evaluation methods for them are presented. Furthermore, a numerical algorithm based on the fault tree is developed in order to evaluate systems' reliability in the context of defined metrics. Finally, the proposed metrics and evaluation methods are illustrated with some case studies.

Keywords Uncertainty theory · Reliability · Fault tree

1 Introduction

Reliability is an important property of products, defined as the ability that a component or system will perform a required function for a given period of time under

Z. Zeng · R. Kang
School of Reliability and Systems Engineering, Beihang University, Beijing 100191, China

Z. Zeng
e-mail: zengzhiguo@dse.buaa.edu.cn

R. Kang
e-mail: kangrui@buaa.edu.cn

M. Wen (✉)
State Key Laboratory of Virtual Reality Technology and Systems and School of Reliability and Systems Engineering, Beihang University, Beijing 100191, China
e-mail: wenmeilin@buaa.edu.cn

stated operating conditions (Ebeling 2010). Reliability metrics are defined to measure products' reliability and the process to obtain them is regarded as reliability evaluation. Traditional reliability metrics are defined on the basis of probability theory and the evaluation of them is based on statistical inferences of failure data (Meeker and Escobar 1998).

Though traditional reliability metrics have achieved great success, there are scenarios where they do not function well. Cai et al. (1991) summarized that three premises must be satisfied so that probability measures can make sense: 1. Precisely defined events; 2. Probabilistic repetitiveness in the collected data; and 3. Large sample size. However, the three premises are widely violated in numerous engineering practices. Moreover, since products' reliability continues to grow, it is often impossible to get enough failure data under restricted time and expense constraints (Meeker and Hamada 1995). Thus, challenges for traditional reliability metrics are becoming more and more severe. One way to cope with these challeges is to conduct accelerated or degradation testing for pseudo life data (Nelson 1990). Many products, however, do not fail even under accelerated conditions. Reliability evaluation of such highly reliable products requires new methods and metrics.

One possible solution might be the comprehensive evaluation. There are numerous reliability tasks in a product's life cycle, such as Design for Reliability, FMECA, Simulation Tests, etc. In fact, it is these tasks that decide the reliability of a product. If they are performed effectively, high reliability can be believed even without information from life tests. In the comprehensive evaluation, the effects of these tasks are assessed by domain experts and the reliability evaluation is conducted accordingly. However, experts' judgments will lead to human uncertainty and under these situations, probability theory might yield counterintuitive results (Liu 2012). Thus, probability-based reliability metrics are inappropriate for the comprehensive evaluation. Uncertainty theory proposed by Liu (2007) and refined by Liu (2011) is considered to be a reasonable complementation of probability theory under these settings. Therefore, this paper will develop some new reliability metrics based on uncertainty theory and discuss how to apply them to evaluate the reliability of highly reliable systems.

Uncertainty theory is a branch of axiomatic mathematics dealing with human uncertainty. Since inaugurated by Liu (2007), it has been widely used by many scholars in various areas to model human uncertainty (Chen and Liu 2010; Tan and Tang 2006), etc.. Its applications in reliability were started by Liu (2010), in which reliability index was defined as a measure of systems' reliability and some simple system reliability models were discussed, such as series and parallel models. Wang (2010) introduced Liu's definition of reliability index into structural reliability analysis.

In this paper, previous work is extended to cope with challenges in reliability evaluation of highly reliable systems. The rest of the paper is organized as follows: Sect. 2 introduces some preliminaries on uncertainty theory. In Sect. 3, the needs for reliability metrics based on uncertainty theory are discussed. Section 4 gives the definitions as well as numerical evaluation methods for the proposed metrics. In Sect. 5, evaluation of systems' reliability based on the proposed metrics is discussed and a numerical evaluation method based on fault trees is presented. Some examples are also given as an illustration to the proposed method.

2 Preliminaries

Uncertainty theory is a branch of axiomatic mathematics founded by Liu (2007) and refined by Liu (2011). Let Γ be a nonempty set, and $\mathcal{L}$ a σ -algebra over Γ. Each element Λ in $\mathcal{L}$ is called an event. An uncertain measure is a set function $\mathcal{M}$ from $\mathcal{L}$ to [0, 1] satisfying the following axioms (Liu 2007):

Axiom 1 (*Normality Axiom*) $\mathcal{M}\{\Gamma\} = 1$ for the univeral set Γ.

Axiom 2 (*Duality Axiom*) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.

Axiom 3 (*Subadditivity Axiom*) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots,$

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

The triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space. A product uncertain measure was defined by Liu (2009) in order to obtain an uncertain measure of a compound event, thus producing the fourth axiom of uncertainty theory:

Axiom 4 (*Product Axiom*) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \ldots$ The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigvee_{k=1}^{\infty} \mathcal{M}\{\Lambda_k\}$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \ldots$, respectively.

An uncertain variable (Liu 2007) is a measurable function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., for any Borel set B of real numbers, the set $\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$ is an event.

The uncertain variables $\xi_1, \xi_2, \ldots, \xi_m$ are said to be independent (Liu 2009) if

$$\mathcal{M}\left\{\bigcap_{i=1}^{m} (\xi_i \in B_i)\right\} = \bigvee_{i=1}^{m} \mathcal{M}\{\xi_i \in B_i\}$$

for any Borel sets $B_1, B_2, \ldots, B_m$ of real numbers.

In practice, an uncertain variable is described by the uncertainty distribution, defined by Liu (2007) as $\Phi(x) = \mathcal{M}\{\xi \leq x\}, \forall x \in \Re$.

An uncertainty distribution is said to be regular (Liu 2011) if its inverse function Φ^{-1} exists and is unique for each $\alpha \in (0, 1)$.

The sufficient and necessary conditions that a function is an uncertainty distribution are proved by Peng and Iwamura (2010): A function $\Phi : \Re \to [0, 1]$ is an uncertainty distribution if and only if it is a monotone increasing function except $\Phi(x) \equiv 0$ and $\Phi(x) \equiv 1$.

The expected value of an uncertain variable ξ is defined by Liu (2007) as an average value of the uncertain variable in the sense of uncertain measure, i.e.,

$$\mathrm{E}[\xi] = \int_{0}^{\infty} (1 - \Phi(x))\, \mathrm{d}x - \int_{-\infty}^{0} \Phi(x) \mathrm{d}x$$

provided that at least one of the two integrals is finite. Liu (2011) proved if the uncertainty distribution Φ was regular, then the expected value can be obtained from the inverse uncertainty distribution Φ^{-1} via

$$\mathrm{E}[\xi] = \int_{0}^{1} \Phi^{-1}(\alpha) \mathrm{d}\alpha. \tag{1}$$

The variance of an uncertain variable ξ is defined by Liu (2007) as an indication of an uncertain variable's degree of spread. Let ξ be an uncertain variable with finite expected value e. Then the variance of ξ is $V[\xi] = \mathrm{E}[(\xi - e)^2]$. In order to obtain variance from uncertainty distribution, Liu (2011) stipulated

$$\begin{aligned} V[\xi] &= \int_{0}^{+\infty} \left(1 - \Phi(e + \sqrt{x}) + \Phi(e - \sqrt{x})\right) \mathrm{d}x \\ &= 2 \int_{0}^{+\infty} x\, (1 - \Phi(e + x) + \Phi(e - x))\, \mathrm{d}x. \end{aligned} \tag{2}$$

Liu (2011) developed operation laws for uncertain variables so that the distribution of functions of uncertain variables can be achieved. Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_n$, then $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ is an uncertain variable with inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f\left(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)\right). \tag{3}$$

3 Needs for new reliability metrics

As stated before, reliability evaluation of highly reliable products is a challenge for reliability engineers. Consider a product whose life expectancy is 11,000 h. Such high reliability makes it impossible to obtain failure data even under accelerated conditions. Thus, evaluation methods based on failure data are not applicable for these products.

On the other hand, there is a lot of information about products' reliability other than failure data. For example, many reliability tasks are conducted in products' life cycle. If these tasks are performed effectively, high reliability can be believed even without conducting life tests. The effects of these reliability tasks can be evaluated by domain experts and a comprehensive evaluation of reliability can be conducted accordingly.

Results from the comprehensive evaluation reflect our belief degrees of the products' reliability. However, as Liu asserts (Liu 2012), it is inappropriate to regard the belief degree as probability. Let's take a series system comprising of 30 components as an example.

Suppose all components in the system have the same life distributions

$$\mathcal{N}(11000, 950^2)\text{(in hours)}$$

and are independent from each other. The reliability of the system at 8,000 h can be easily obtained, which is,

$$R_s = \left(1 - \Phi\left(\frac{8000 - 11000}{950}\right)\right)^{30} = 0.9764. \tag{4}$$

However, in reality, the real distribution is unknown to us and has to be estimated by domain experts. Since human beings usually overweight unlikely events (Tversky and Kahneman 1986), the estimated distribution might have larger variance the real one. Assume that the estimated life distribution is

$$\mathcal{N}(11000, 2850^2) \text{ (in hours)}.$$

Then the estimated reliability of the system at 8,000 h will be

$$R_s = \left(1 - \Phi\left(\frac{8000 - 11000}{2850}\right)\right)^{30} = 0.0087. \tag{5}$$

Results in Eqs. (4) and (5) are at opposite poles. This fact demonstrates that sticking to probability-based metrics in a comprehensive reliability evaluation might lead to an unacceptable result. Thus, this paper develops a set of new metrics and discusses the evaluation of system's reliability based on them.

4 The new reliability metrics

4.1 Definitions

Definition 1 (*Reliability index*) (Liu 2010) Assume a system contains uncertain variables $\xi_1, \xi_1, \ldots, \xi_n$, and there is a function R such that the system is working if and only if $R(\xi_1, \xi_1, \ldots, \xi_n) \geq 0$. Then the reliability index is

$$Reliability = \mathcal{M}\{R(\xi_1, \xi_1, \ldots, \xi_n) \geq 0\}. \tag{6}$$

Remark 1 To avoid confusions with probability-based reliability index, the reliability index in Definition 1 is regarded as Belief Reliability (R_B) in this paper. Often life is assumed to be an uncertain variable T with uncertainty distribution $\Phi(t)$, so Eq. (6) becomes

$$R_B(t) = \mathcal{M}\{T > t\} = 1 - \mathcal{M}\{T \leq t\} = 1 - \Phi(t). \tag{7}$$

Remark 2 Equation (7) indicates that belief reliability is a time-varying function (often decreasing). Thus, it is regarded as belief reliability function and the processes to obtain this function are called time-dependent analyses. In some applications, the focus will be the value of $R_B(t)$ at a given period $t = t_0$. These applications are regarded as time-static analyses. Both the time-static and time-dependent analyses will be discussed in detail in Sect. 5.

Definition 2 (*Belief Reliable Life*, $BL(\alpha)$) Assume that products' life is an uncertain variable T with belief reliability function $R_B(t)$ and uncertainty distribution $\Phi(t)$. Let α be a real number from (0, 1). The belief reliable life $BL(\alpha)$ is

$$BL(\alpha) = \sup\{t | R_B(t) \geq \alpha\}. \tag{8}$$

Theorem 1 *Let* $\Phi(t)$ *be a regular uncertainty distribution with inverse uncertainty distribution* $\Phi^{-1}(\alpha)$. *Then*

$$BL(\alpha) = \Phi^{-1}(1 - \alpha). \tag{9}$$

Proof It can be easily proved since a regular uncertainty distribution $\Phi(t)$ is strictly increasing at each point t with $0 < \Phi(t) < 1$. □

Definition 3 (*Mean Time to Failure*, $MTTF_B$) Assume products' life is an uncertain variable T with belief reliability function $R_B(t)$. The mean time to failure $MTTF_B$ is defined by

$$MTTF_B = \mathrm{E}[T] = \int_0^\infty R_B(t)\mathrm{d}t. \tag{10}$$

Definition 4 (*Variance of Life*, VL_B) Assume that products' life is an uncertain variable T and its mean time to failure is $MTTF_B$. The variance of life VL_B is defined by

$$VL_B = \mathrm{E}[(T - MTTF_B)^2]. \tag{11}$$

Comments So far the metrics, namely $R_B(t)$, $BL(\alpha)$, $MTTF_B$ and VL_B, have been defined. Based on uncertainty theory, these metrics are intended to characterize reliability when few or none failure data can be achieved so that domain experts are relied on to evaluate reliability.

Belief reliability $R_B(t)$ is a function over time and it reflects experts' personal belief degrees of products' reliability. Belief reliable life $BL(\alpha)$ is the longest duration that a product can last with its R_B greater than a given level α. Since it is in time scale, it has more straightforward meaning than $R_B(t)$. Mean time to failure $MTTF_B$ and variance of life VL_B characterize the location and variation of life distribution, respectively.

It should be noted that since the life distribution is achieved by expert's experimental data rather than actual failure data, the defined metrics might change their values with the knowledge of experts and the information they get about products' reliability. Generally speaking, the more experienced experts are, and the more information they can get about products' reliability, the more accurate these metrics will be.

4.2 Numerical evaluation methods

In practice, it is often difficult to obtain the proposed metrics analytically and numerical evaluation methods are needed. In this section, numerical methods are provided for obtaining $MTTF_B$ and VL_B from $BL(\alpha)$, respectively. Note that all uncertainty distributions discussed in this section are supposed to be regular.

Theorem 2 *(*Numerical Evaluation of $MTTF_B$*) Let T be an uncertain variable with regular uncertainty distribution $\Phi(t)$ and belief reliable life $BL(\alpha)$. Then*

$$MTTF_B = \int_0^1 BL(\alpha)\mathrm{d}\alpha. \tag{12}$$

Proof From Theorem 1, we have

$$\int_0^1 BL(\alpha)\mathrm{d}\alpha = \int_0^1 \Phi^{-1}(1-\alpha)\mathrm{d}\alpha = \int_0^1 \Phi^{-1}(\alpha)\mathrm{d}\alpha = MTTF_B.$$

□

Theorem 3 (Numerical Evaluation of VL_B) *Let T be an uncertain variable with regular uncertainty distribution $\Phi(t)$ and belief reliable life $BL(\alpha)$. Then*

$$VL_B = 2\left[\int_0^{R_B(MTTF_B)} (MTTF_B - BL(\alpha))\alpha\mathrm{d}BL(\alpha) + \int_{R_B(MTTF_B)}^1 (BL(\alpha) - MTTF_B)(1-\alpha)\mathrm{d}BL(\alpha)\right].$$

Proof The theorem can be proved from Eq. (2) by the method of variable transformation. □

In practice, the integrals in the two theorems are often conducted numerically by computers, which allow computationally flexible evaluations of $MTTF_B$ and VL_B.

5 Systems' belief reliability

5.1 Time-static systems

A time-static system is a system whose reliability does not change over time and can be viewed as a special case of a time-dependent system at a specified $t = t_0$. Although most real systems are time-dependent, time-static analyses can help us understand the relationship between components' failures and system's failures.

The systems to be discussed are assumed to be binary-state. Therefore their corresponding time-static systems are Boolean systems. Liu (2010) had developed evaluation methods for Boolean systems through the structural function. In engineering practices, structural functions are often obtained through fault trees and fault trees are more familiar to engineers than structural functions. A typical fault tree is illustrated in Fig. 1 and readers might refer to Ebeling (2010) for details on the fault tree. The rest of this section will discuss quantitative fault tree analyses in the context of belief reliability. Quantitative analyses of a fault tree involve two steps:

1. Enumerate all minimum cut sets;
2. Calculate the system's reliability based on the minimum cut sets.

A cut set is a set of components whose failures interrupt all connections between input and output ends and cause an entire system to fail. A minimal cut set is the smallest combination of components which will cause the system's failure if they all

Fig. 1 A typical fault tree

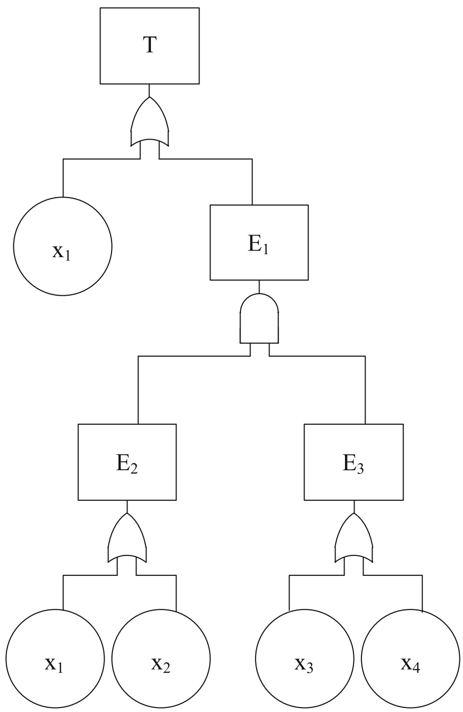

Table 1 The implementation of MOCUS

Step 1	Step 2	Step 3	Step 4
x_1	x_1	x_1	x_1
E_1	$\mathrm{E}_2, \mathrm{E}_3$	x_1, E_3	x_1, x_3
		x_2, E_3	x_1, x_4
			x_2, x_3
			x_2, x_4

fail. There are numerous algorithms for enumerating minimum cut sets from a fault tree. Among them, MOCUS by Fussell and Vesely (1972) is an efficient and widely used one and will be applied in this paper. An implementation of MOCUS to the fault tree in Fig. 1 is demonstrated blow.

- Construct an empty table.
- List all output events of an AND gate in a single row of the table, while list each of the output events in individual rows for an OR gate, as shown in Table 1. Then the last column of Table 1 lists all cut sets.
- Discard the cut sets which include other cut sets.

Since both $\{x_1, x_3\}$ and $\{x_1, x_4\}$ include $\{x_1\}$, the minimum cut sets are

$$\{x_1\}, \{x_2, x_3\}, \{x_2, x_4\}.$$

Once the minimum cut sets are decided, the belief reliability of the system can be obtained from them, as stated in Theorem 5.

Theorem 4 (Liu 2011) *Assume that $\xi_1, \xi_2, \ldots, \xi_n$ are independent Boolean uncertain variables, i.e., $\xi_i = 1$ with uncertainty measure a_i for $i = 1, 2, \ldots, n$. Then the minimum $\xi = \xi_1 \wedge \xi_2 \wedge \cdots \wedge \xi_n$ is a Boolean uncertain variable such that*

$$\mathcal{M}\{\xi = 1\} = a_1 \wedge a_2 \wedge \cdots \wedge a_n.$$

The maximum $\eta = \xi_1 \vee \xi_2 \vee \cdots \vee \xi_n$ is a Boolean uncertain variable such that

$$\mathcal{M}\{\eta = 1\} = a_1 \vee a_2 \vee \cdots \vee a_n.$$

Theorem 5 *Assume that a system comprises of n components and each component is represented by an uncertain Boolean variable ξ_i such that $\mathcal{M}\{\xi_i = 1\} = R_i$, where $R_i, i = 1, 2, \ldots, n$ are the reliabilities of the components. Suppose the system has m minimum cut sets $C_1, C_2, \ldots, C_m$. If the failures of the components are independent from each other, the system's reliability will be*

$$R_S = \bigwedge_{i=1}^{m} \left\{ \bigvee_{j \in C_i} R_j \right\}. \tag{13}$$

Proof Let a Boolean uncertain variable ξ denote the state of the system (The system is working when $\xi = 1$). Since the system has m minimum cut sets $C_1, C_2, \ldots, C_m$,

$$\xi = \bigwedge_{i=1}^{m} \eta_i$$

where $\eta_i = \bigvee_{j \in C_i} \xi_j,\ i = 1, 2, \ldots, m$. From Theorem 4,

$$\mathcal{M}\{\eta_i = 1\} = \bigvee_{j \in C_i} \mathcal{M}\{\xi_j = 1\} = \bigvee_{j \in C_i} R_j.$$

Thus,

$$R_S = \mathcal{M}\{\xi = 1\} = \bigwedge_{i=1}^{m} \mathcal{M}\{\eta_i = 1\} = \bigwedge_{i=1}^{m} \left\{ \bigvee_{j \in C_i} R_j \right\}.$$

□

Algorithm 1 is a combination of MOCUS and Theorem 5, which tells how to conduct belief reliability evaluation for a time-static fault tree.
Algorithm 1 (Belief Reliability Evaluation for a Time-Static Fault Tree):

Step 1: Enumerate all minimum cut sets:
Loop from Top to Bottom:
if Gate == AND
Increase the elements in each cut set;
if Gate == OR
Increase the number of the cut sets.
Discard the cut sets which include other cut sets.
Step 2: Obtain system reliability from Theorem 5.
End.

Example 1 Consider a time-static system whose fault tree is given in Fig. 1. In the fault tree, top event T denotes the system's failure and x_1, x_2, x_3, x_4 denote the failures of the four components, respectively. Reliabilities of the components are

$$R_1 = 0.85,\ \ R_2 = 0.92,\ \ R_3 = 0.95,\ \ R_4 = 0.90.$$

Algorithm 1 is applied to evaluate the system's reliability.

Step 1: The minimum cut sets are

$$\{x_1\}, \{x_2, x_3\}, \{x_2, x_4\}.$$

Table 2 Life distributions of the components

Component	Life distribution
x_1	$\mathcal{L}(450, 550)$
x_2	$\mathcal{N}(500, 10)$
x_3	$\mathcal{Z}(450, 500, 550)$
x_4	$\mathcal{N}(500, 20)$

Step 2: From Theorem 5,

$$R_S = \min\{R_1, \max\{R_2, R_3\}, \max\{R_2, R_4\}\} = 0.85.$$

5.2 Time-dependent systems

In this section, results obtained in Sect. 5.1 will be used to evaluate a time-dependent system.

Theorem 6 (Evaluation of Time-Dependent Systems) *Let S be a time-dependent system whose time-static fault tree T has m minimum cut sets $C_1, C_2, \ldots, C_m$. Assume the system comprises of n components $X_1, X_2, \ldots, X_n$ and their belief reliable life functions are $BL_1(\alpha), BL_2(\alpha), \ldots, BL_n(\alpha),\ 0 < \alpha < 1$, respectively. Then the system's belief reliable life will be*

$$BL_S(\alpha) = \bigwedge_{i=1}^{m}\left\{\bigvee_{j\in C_i} BL_j(\alpha)\right\}. \tag{14}$$

Proof Let T denote the system's life and t_i denote the life of component X_i. Since T can be obtained from the lives of elements in each minimum cut set by

$$T = \bigwedge_{i=1}^{m}\left\{\bigvee_{j\in C_i} t_j\right\}$$

and $f(t_1, t_2, \ldots, t_n) = \bigwedge_{i=1}^{m}\left\{\bigvee_{j\in C_i} t_j\right\}$ is a strictly increasing function with respect to $t_1, t_2, \ldots, t_n$, according to the operation law,

$$\Phi_{\mathrm{T}}^{-1}(\alpha) = \bigwedge_{i=1}^{m}\left\{\bigvee_{j\in C_i} \Phi_{\mathrm{j}}^{-1}(\alpha)\right\}.$$

Thus Eq. (14) follows immediately from Theorem 1. □

Example 2 Consider a time-dependent system whose time-static fault tree is shown in Fig. 1. Life distributions of the components are listed in Table 2. Decide the belief reliability function of the system and the belief reliable life for $\alpha = 0.9$.

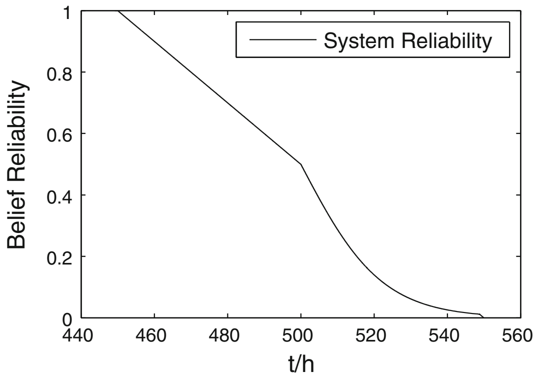

Fig. 2 Evaluation results

The three minimum cut sets are $\{x_1\}$, $\{x_2, x_3\}$, and $\{x_2, x_4\}$. So the system's belief reliable life $BL_S(\alpha)$ is obtained from Eq. (14) and the belief reliability function of the system is demonstrated in Fig. 2. For $\alpha = 0.9$, $BL(\alpha) = 460$ (h). Mean time to failure $MTTF_B$ and variance of life VL_B are achieved from Theorems 2 and 3, respectively.

$$MTTF_B = 494.97 \text{ (h)}, \; VL_B = 576.82 \text{ (h}^2\text{)}.$$

6 Conclusion

This paper focused on the application of uncertainty theory in reliability evaluation of systems with high reliability and long life.

1. Some new reliability metrics based on uncertainty theory were defined to characterize systems' reliability from different aspects.
2. Numerical methods for obtaining the defined metrics $MTTF_B$ and VL_B from belief reliable life $BL(\alpha)$ were proposed.
3. Fault tree analysis methods in the context of belief reliability were developed for numerical evaluation of complex systems.

References

Cai, K., Wen, C., & Zhang, M. (1991). Fuzzy variables as a basis for a theory of fuzzy reliability in the possibility context. *Fuzzy Sets And Systems, 42*(2), 145–172.

Chen, X., & Liu, B. (2010). Existence and uniqueness theorem for uncertain differential equations. *Fuzzy Optimization and Decision Making, 9*(1), 69–81.

Ebeling, C. (2010). *An introduction to reliability and maintainability engineering*. Long Grove: Waveland Press.

Fussell, J., & Vesely, W. (1972). New methodology for obtaining cut sets for fault trees. *Transactions of American Nuclear Society, 15*(1), 262–269.

Liu, B. (2007). *Uncertainty theory* (2nd ed.). Berlin: Springer.

Liu, B. (2009). Some research problems in uncertainty theory. *Journal of Uncertain Systems, 3*(1), 3–10.

Liu, B. (2010). Uncertain risk analysis and uncertain reliability analysis. *Journal of Uncertain Systems, 4*(3), 163–170.

Liu, B. (2011). *Uncertainty theory: a branch of mathematics for modeling human uncertainty*. Berlin: Springer.

Liu, B. (2012). Why is there a need for uncertainty theory?. *Journal of Uncertain Systems, 6*(1), 3–10.

Meeker, W., & Escobar, L. (1998). *Statistical methods for reliability data*. New York: Wiley.

Meeker, W., & Hamada, M. (1995). Statistical tools for the rapid development and evaluation of high-reliability products. *IEEE Transactions on Reliability, 44*(2), 187–198.

Nelson, W. (1990). *Accelerated testing: statistical models, test plans and data analyses*. New York: Wiley.

Peng, Z., & Iwamura, K. (2010). A sufficient and necessary condition of uncertainty distribution. *Journal of Interdisciplinary Mathematics, 3*(1), 277–285.

Tan, M., & Tang, X. (2006). The further study of safety stock under uncertain environment. *Fuzzy Optimization and Decision Making, 5*(2), 193–202.

Tversky, A., & Kahneman, D. (1986). Rational choice and the framing of decisions. *Journal of Business, 59*, 251–278.

Wang, Z. (2010). Structural reliability analysis using uncertainty theory. In *Proceedings of the first international conference on uncertainty theory* (pp. 166–170), Urumchi.