Word Rotator's Distance: Decomposing Vectors Gives Better Representations

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Abstract

One key principle for assessing semantic similarity between texts is to measure the degree of semantic overlap of them by considering word-by-word alignment. However, alignment-based approaches are inferior to the generic sentence vectors in terms of performance. We hypothesize that the reason for the inferiority of alignment-based methods is due to the fact that they do not distinguish word importance and word meaning. To solve this, we propose to separate word importance and word meaning by decomposing word vectors into their norm and direction, then compute the alignment-based similarity with the help of earth mover's distance. We call the method word rotator's distance (WRD) because direction vectors are aligned by rotation on the unit hypersphere. In addition, to incorporate the advance of cutting edge additive sentence encoders, we propose to re-decompose such sentence vectors into word vectors and use them as inputs to WRD. Empirically, the proposed method outperforms current methods considering the word-by-word alignment including word mover's distance (Kusner et al., 2015) with a big difference; moreover, our method outperforms state-of-the-art additive sentence encoders on the most competitive dataset, STSbenchmark.

1 Introduction

Measuring the *semantic textual similarity (STS)* between texts is a fundamental task in natural language processing (NLP) with applications such as loss function (Wieting et al., 2019) and evaluation metric in text generation (Zhao et al., 2019; Zhang et al., 2019).

One key principle for STS is to measure the degree of semantic overlap of two texts. While early approaches measured Jaccard similarity between bag-of-words, more recent work softly computes

word-by-word alignment (Sultan et al., 2014, 2015), with the help of earth mover's distance (Kusner et al., 2015; Clark et al., 2019; Zhao et al., 2019), an attention mechanism (Zhang et al., 2019), or fuzzy sets (Zhelezniak et al., 2019). However, these methods are empirically inferior to generic sentence vectors obtained by additive sentence encoders, which combine pre-trained word embeddings (Mu and Viswanath, 2018; Arora et al., 2017; Ethayarajh, 2018; Wieting et al., 2015; Wieting and Gimpel, 2018).

We hypothesize that one reason for the inferiority of alignment-based methods is due to the fact that they use *word importance* and *word meaning* simultaneously without distinguishing them. Here, we propose to separate word importance and word meaning by decomposing word vectors into their *norm* and *direction*, and then to compute word-alignment-based similarity using this decomposed representation. The alignment of normalized word vectors corresponds to a rotation on the unit hypersphere, we then call our method *word rotator's distance* (*WRD*).

Moreover, in order to incorporate the advance of the latest additive sentence encoder into our method, we propose to re-decompose the generated sentence vector into word vectors, and then use them as inputs to WRD. In experiments, on the most competitive STS-benchmark dataset, our methods outperform state-of-the-art methods based on additive sentence encoder.

In summary, our main contributions are:

- We propose a new alignment-based textual similarity measure considering the norm and direction of the word vector separately.
- We propose a new word vector transformation mechanism with the help of recent additive sentence encoders.
- We confirm that the proposed methods show empirically high performance in several STS tasks.

2 Task and Notation

Semantic textual similarity (STS) is the task of measuring the degree of semantic equivalence between two sentences (Agirre et al., 2012). For example, the sentences "Two boys on a couch are playing video games." and "Two boys are playing a video game." are mostly equivalent while the sentences "The woman is playing the violin." and "The young lady enjoys listening to the guitar." are not equivalent but on the same topic (Agirre et al., 2013).

What makes the problem difficult is the diversity of linguistic expressions; we humans can use various vocabulary and word order to express the same meaning linguistically, which makes assessing textual similarity a challenging task.

Evaluation Metric. The semantic similarity of two texts is typically annotated on a six-level scale, ranging from zero (no equivalence, different topic) to five (equivalence). Given gold scores, system predictions are evaluated by their Pearson correlation r. Hence, systems are only required to predict relative similarity rather than absolute scores.

No Supervision. We focus on *unsupervised* STS following (Arora et al., 2017; Ethayarajh, 2018); that is, we require only pre-trained word vectors, and do not use any supervision in the form of training data, external resources (e.g. paraphrase corpora), or data for related tasks (e.g. data for natural language inference).

Notation. We represent a sentence pair consisting of sentence s of length n and s' of length n' as sets of words

$$s = \{w_1, \dots, w_n\}, \ s' = \{w'_1, \dots, w'_{n'}\}.$$
 (1)

Bold face w_i denotes the word vector corresponding to word w_i . For more details on word vectors we used in this paper, see Appendix A.

Let $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$ denote the standard inner product and the Euclidean norm

$$\langle \boldsymbol{w}, \boldsymbol{w}' \rangle := \boldsymbol{w}^{\top} \boldsymbol{w}', \quad \|\boldsymbol{w}\| := \sqrt{\langle \boldsymbol{w}, \boldsymbol{w} \rangle}.$$
 (2)

3 Background: Earth Mover's Distance

Our proposal utilizes earth mover's distance (EMD) to compare two (weighted) set of word vectors. In this section, we introduce EMD and one of its instances, word mover's distance.

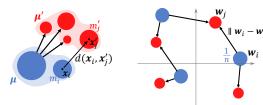


Figure 1: Figure 2: Earth Mover's Distance. Word Mover's Distance.

3.1 Earth Mover's Distance

Intuitively, *earth mover's distance* (*EMD*)¹ (Villani, 2009; Santambrogio, 2015; Peyré and Cuturi, 2019) is the minimum cost required to turn one pile (objects that have been placed little by little in various places) into the other pile. Formally, EMD takes the following inputs (Figure 1) ².

1. Two **probability distributions** μ (initial arrangement) and μ' (final arrangement):

$$\boldsymbol{\mu} = \left\{ (\boldsymbol{x}_i, \, m_i) \right\}_{i=1}^n, \, \boldsymbol{\mu}' = \left\{ (\boldsymbol{x}_j', \, m_j') \right\}_{j=1}^{n'}. \tag{3}$$

This means that each distribution μ consists of points (locations) (x_1, x_2, \dots) with corresponding mass (weights) (m_1, m_2, \dots) . $\sum_i m_i = 1$.

In Figure 1, each point is represented by a circle, where its position represents the vector x_i , x'_j and its magnitude represents the weight m_i , m'_i .

2. The **ground metric** (transportation cost function) *d*:

$$d \colon \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}. \tag{4}$$

The d determines the distance between two points (the transportation cost per unit amount), $d(x_i, x_i')$.

Optimization. The EMD between μ and μ' is then defined via the following optimization prob-

¹ In this paper, following the convention, we use the term earth mover's distance (EMD) in the sense of optimal transport cost by the Kantrovich's formulation, which is also called the 1-Wasserstein distance.

² Strictly speaking, Equation (3) is $\mu = \sum_{i=1}^n m_i \delta[x_i]$, where the Dirac delta function describes a discrete probability measure. We omit delta for notational simplicity.

lem

$$EMD(\boldsymbol{\mu}, \boldsymbol{\mu}'; d) := \min_{\boldsymbol{T} \in \mathbb{R}_{\geq 0}^{n \times n'}} \sum_{i,j} \boldsymbol{T}_{ij} d(\boldsymbol{x}_i, \boldsymbol{x}'_j) \quad (5)$$

s.t.
$$T1 = (m_1, \dots, m_n)^{\top},$$
 (6)

$$\boldsymbol{T}^{\top} \mathbb{1} = (m'_1, \dots, m'_{n'})^{\top}. \tag{7}$$

Here, the matrix $T \in \mathbb{R}^{n \times n'}_{\geq 0}$ denotes a transportation plan, in which each element T_{ij} denotes the transport of mass from x_i to x'_j . To summarize, $\mathrm{EMD}(\mu,\mu';d)$ is the cost of the best transportation plan between the two distributions μ and μ' on metric space (\mathbb{R}^d,d) .

Side Effect – Alignment. Under the above optimization, if the locations x_i in μ and x_j' in μ' are close (the transportation cost $d(x_i, x_j')$ is small), they are likely to be *aligned* (T_{ij} might be assigned a large value). In this way, EMD implicitly aligns the points of two discrete distributions. In fact, EMD is actively used as an alignment tool in various domains (Wang et al., 2013; Solomon et al., 2016). This is a reason why we adopt EMD as a key technology for the STS problem.

3.2 Word Mover's Distance

Word mover's distance (WMD) (Kusner et al., 2015) is a measure of the dissimilarity between texts, a pioneering work that introduced EMD into the NLP community. Our study is also strongly inspired by this work. We introduce WMD as a preparation for the proposed method (Figure 2).

After removing stopwords, Kusner et al. regard each sentence s as uniformly weighted distribution μ_s consisting of word vectors (bag-of-word-vector distribution)

$$\mu_s := \left\{ (\boldsymbol{w}_i, \frac{1}{n}) \right\}_{i=1}^n, \ \mu_{s'} := \left\{ (\boldsymbol{w}'_j, \frac{1}{n'}) \right\}_{j=1}^{n'}.$$
 (8)

In Figure 2, each circle represents each word, where its position represents the vector w_i , w'_j , and its magnitude represents the weights 1/n, 1/n'. Next, they uses Euclidean distance as a dissimilarity measure between word vectors

$$d_{\mathbf{E}}(\boldsymbol{w}_i, \boldsymbol{w}_i') := \|\boldsymbol{w}_i - \boldsymbol{w}_i'\|. \tag{9}$$

Then, WMD is defined as the EMD between such two distributions

$$WMD(s, s') := EMD(\boldsymbol{\mu}_s, \boldsymbol{\mu}_{s'}; d_E). \tag{10}$$

This formulation is intuitive, and its empirical effects have been confirmed. However, WMD has the problem of mixing the importance and meaning of words. This issue will be discussed in detail in the next section.

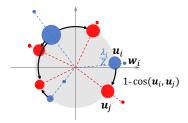


Figure 3: Word Rotator's Distance.

4 Word Rotator's Distance

Here and subsequently, λ and u denotes the norm and the direction vector of word vector w

$$\lambda := \|\boldsymbol{w}\|, \quad \boldsymbol{u} := \frac{\boldsymbol{w}}{\|\boldsymbol{w}\|}. \tag{11}$$

4.1 Norm and Direction of Word Vectors

Our idea is to use the *norm* of the pre-trained word vector as the importance weight of the word, and the *direction* of the word vector to measure the similarity to another word.

Norm as Importance Weight

It is well known that a very simple sentence vector—the average of the vector of the words in it—has achieved remarkable results in STS and a lot of downstream tasks (Mitchell and Lapata, 2010; Mikolov et al., 2013; Wieting et al., 2016). Similar results have been reported for contextualized word vectors (Perone et al., 2018; Ma et al., 2019).

$$s_{\text{ADD}} = \sum_{w_i \in s} w_i, \quad s'_{\text{ADD}} = \sum_{w'_j \in s'} w'_j$$
 (12)

$$sim(s, s') = cos(\boldsymbol{s}_{ADD}, \boldsymbol{s}'_{ADD})$$
 (13)

Henceforth, we call such averaging method **additive composition**. Equation 13 may appear to treat each word vector equally. However, several studies confirmed that the norm (size) of word vectors varies greatly (Schakel and Wilson, 2015; Arefyev et al., 2018); the word vectors that make up such a sentence vector include long one and short one (Table 1). In conclusion, the word vector with large norm (length) will be dominant in the resulting sentence vector. The usefulness of additive composition suggests that *the norm of each word vector will function as the importance of the word*.

Our hypothesis can be supported by simple experiments. Let us consider another additive composition that excludes the effect of weighting by norm

$$s_{ ext{ADD NORMALIZED}} = \sum_{w_i \in s} \frac{w_i}{\lambda_i} = \sum_{w_i \in s} u_i.$$
 (14)

	the	next	conference	in	Seattle
GloVe word2vec fastText	1.07	5.11 2.06 2.89	6.63 2.84 4.20	5.09 1.33 2.49	6.33 3.02 4.41
ELMo BERT-large		36.3 22.2	39.1 23.0	29.6 20.4	34.2 22.6

Table 1: The norm of the word vector ||w|| of each word w (each column) for each pre-trained word embedding model (each row). In each line, the two largest values are shown in **boldface**, and the two smallest values are shown in underlined.

	ADD	ADD NORMALIZED
GloVe	54.16	46.25
word2vec	72.43	63.20
fastText	70.40	56.31
ELMo	63.22	57.96
BERT-large	65.76	64.04

Table 2: **Pearson's** $r \times 100$ between predicted scores, computed by cosine, and gold scores for each word vector (each row) and each methods (each column). The STS-B dataset (dev) is used.

Table 2 shows the experimental results of the STS-benchmark task by using two kind of sentence vectors (Eq. 13, 14). The results show that ignoring the norm of word vectors will result in consistently worse performance. This indicates that the norm of the word vector plays a role of the importance weight of the word.

Direction as Lexical Meaning

We use the cosine of word vectors for computing word similarity, which is the most conventional and successful way. Note that cosine similarity only considers directions, ignoring their norm

$$\cos(\boldsymbol{w}, \boldsymbol{w}') = \frac{\langle \boldsymbol{w}, \boldsymbol{w}' \rangle}{\lambda \lambda'} = \langle \boldsymbol{u}, \boldsymbol{u}' \rangle.$$
 (15)

As a natural interpretation, *the direction of the word vector* is considered to expresses the meaning of the word; the norm does not matter.

Although the cosine similarity has been widely adopted, several studies have used Euclidean distance (Kusner et al., 2015; Zhao et al., 2019) and dot product (Zhang et al., 2019; Zhelezniak et al., 2019) to compute word similarity. So the question is that which is the most suitable method for computing word similarity? Cosine similarity that ignores the norm? Dot product or Euclidean distance that is inevitably affected by the norm? To compare

	GloVe			word2vec			
	cos	L2	DOT	cos	L2	DOT	
MEN	80.49	73.36	80.79	78.20	62.31	74.46	
MTurk287	69.18	60.87	69.50	68.37	49.43	66.6	
MC30	78.81	75.22	76.77	78.87	69.88	76.57	
RW	47.28	40.37	45.64	53.39	31.70	48.66	
RG65	76.90	70.75	77.79	76.17	71.30	72.58	
SCWS	62.96	55.87	61.94	44.19	32.24	43.28	
SimLex999	40.84	35.16	38.99	44.19	32.24	43.28	
WS353-REL	68.75	49.74	72.35	61.40	40.74	56.65	
WS353-SIM	79.57	69.03	79.54	77.39	55.82	74.89	

Table 3: **Spearman's** $\rho \times 100$ between predicted scores and gold scores for each word similarity task (each row) and each word vector and similarity measure (each column). The best result and results where the difference from the best < 0.5 in each task and each word vector are in **bold**.

the three fundamental operations, we conducted several word similarity tasks (for more details, see Appendix D). Table 3 shows that using cosine similarity (ignoring the norm of word vectors) yields consistently higher correlation with human evaluation than using dot product or Euclidean distance which make use of the norm. This indicates that the direction of word vector encodes the meaning; the norm does not matter.

4.2 Problems with Word Mover's Distance

From the above, we can see that there are two problems with WMD.

Weighting of Words. The EMD can consider the weights of each point (each word vector) via a probability mass (3), and the importance of each word vector is encoded in the norm (Section 4.1). Nevertheless, WMD ignores the norm and uniformly weights each word vector (8).

Computing Similarity between Words. EMD can consider the dissimilarity between points (word vectors) via a ground metric (4), and the semantic similarity of word can be measured by cosine similarity (Section 4.1). Nevertheless, WMD uses Euclidean distance that *mixes* importance and meaning (9). For example, for word-vector pairs with similar meanings but with significantly different concreteness or importance, such as the "animal" vector and the "Persian cat" vector, the similarity is estimated to be low.

4.3 Word Rotator's Distance

Here, we propose a very simple but powerful sentence similarity measure utilizing EMD, named word rotator's distance (WRD) (Figure 3). Our method brings the importance and lexical meaning of words implicitly encoded in word vectors into the world of earth mover's distance. The idea is that WRD uses the norm of each word vector as the probability (weight), and uses the direction vector for the distance (dissimilarity between words) of EMD.

Formally, we regard each sentence s as discrete distribution ν_s on hypersphere consisting of direction vectors weighted by norm (bag-of-direction-vector distribution)

$$\boldsymbol{\nu}_s := \left\{ (\boldsymbol{u}_i, \, \frac{\lambda_i}{Z}) \right\}_{i=1}^n \quad \left(Z := \sum_i \lambda_i \right)$$
 (16)

$$\boldsymbol{\nu}_{s'} := \left\{ (\boldsymbol{u}'_j, \, \frac{\lambda_j}{Z'}) \right\}_{j=1}^{n'} \quad \left(Z' := \sum_j \lambda'_j \right), \quad (17)$$

where Z and Z' are normalization constants for making the sum of weights 1. In Figure 3, each circle represents each word, where its position represents the vector \boldsymbol{u}_i , \boldsymbol{u}_j' , and its magnitude represents the weights λ_i/Z , λ_j'/Z' . In other words, Each sentence is represented as a discrete distribution on the unit hypersphere. Next, we use cosine distance as a ground metric

$$d_{\cos}(\boldsymbol{u}_i, \boldsymbol{u}_j') := 1 - \cos(\boldsymbol{u}_i, \boldsymbol{u}_j'). \tag{18}$$

That is, to align words (each word is represented as mass placed on the hypersphere), it takes a cost of rotation. Then, the word rotator's distance (WRD) between two sentences is defined as EMD between such two distributions:

$$WRD(s, s') := EMD(\boldsymbol{\nu}_s, \boldsymbol{\nu}_{s'}; d_{\cos}). \tag{19}$$

WRD first considers each sentence as a distribution of unit hypersphere and then aligns words (mass) by moving (rotating) them on the hypersphere.

This allows WRD, unlike WMD, to make the following appropriate correspondences between EMD and word vectors.

- Probability mass (weight of each point)
- \leftrightarrow Norm (**importance** of each word)
- Ground metric (distance between points)

 ⇔ Angle (dissimilarity between words)

Algorithm

The algorithm used in the actual computation of WRD is shown in Algorithm 1. For EMD compu-

Algorithm 1 Word Rotator's Distance (WRD)

Input: a pair of sentences $s = \{w_1, \dots, w_n\}$, $s' = \{w'_1, \dots, w'_{n'}\}$ Compute Mass $m_s \in \mathbb{R}^n$, $m_{s'} \in \mathbb{R}^{n'}$:

1: $Z \leftarrow \sum_{i=1}^n \|w_i\| \in \mathbb{R}$ 2: $Z' \leftarrow \sum_{j=1}^n \|w'_j\| \in \mathbb{R}$ 3: $m_s \leftarrow \frac{1}{Z}(\|w_1\|, \dots, \|w_n\|) \in \mathbb{R}^n$ 4: $m_{s'} \leftarrow \frac{1}{Z'}(\|w'_1\|, \dots, \|w'_{n'}\|) \in \mathbb{R}^{n'}$ Compute cost matrix $C \in \mathbb{R}^{n \times n'}$:

5: for $i \leftarrow 1$ to n, $j \leftarrow 1$ to n' do

6: $C_{ij} \leftarrow 1 - \cos(w_i, w'_j)$ 7: end for

Compute WRD by EMD:

8: WRD $(s, s') \leftarrow \text{EMD}(m_s, m_{s'}; C)$

tation, off-the-shelf libraries can be used³. Note that most EMD (optimal transport) libraries can be given two probabilities (mass) $m \in \mathbb{R}^n$, $m' \in \mathbb{R}^{n'}$ (weights) and a cost matrix $C \in \mathbb{R}^{n \times n'}$ with $C_{ij} = d(x_i, x_j')$ as inputs. They have the same information as the discrete distributions μ , μ' and a ground metric d. To ensure the reproducibility, the notation of Algorithm 1 follows this style.

5 Word Vector Converter

Output: WRD $(s, s') \in \mathbb{R}$

Recently, sophisticated additive sentence encoders have been proposed for creating better sentence vectors. In order to incorporate such cutting edge sentence vectors, we propose to re-decompose them into word vectors, then employ them as inputs to WRD.

First, in Section 5.1, we point out that such sentence encoders implicitly transform each input word vector. Next, in Section 5.2, we confirm that the transformed word vector is better than the original word vector both in terms of norm and direction. We expect that the better word vectors created in this way will further enhance WRD.

5.1 Word Vector Converter Induced by Sentence Encoder

Based on Arora's pioneering random-walk language model (Arora et al., 2016, 2017), several unsupervised sentence encoders have been proposed (Arora et al., 2017; Mu and Viswanath, 2018;

³In our experiments, we used the well-developed Python Optimal Transport (POT) library. In particular, the ot.emd() function in https://github.com/rflamary/POT/ was used.

Ethayarajh, 2018; Liu et al., 2019b,a). These encoders produce sentence vectors by combining pretrained word vectors, and the cosine similarity between such sentence vectors achieved high performance in STS tasks. These sentence encoders can be summarized as the following form

Encode(s) =
$$f_3\left(\frac{1}{n}\sum_{w\in s}\alpha_2(w)f_1(w)\right)$$
, (20)

where

- f_1 acts for "denoising" each word vector,
- α_2 acts for scaling each word vector, and
- f_3 acts for "denoising" each sentence vector after additive composition.

For the positioning of existing methods in Equation 20, see Appendix B. Here, especially because all the proposed denoising function f_3 is affine, Equation 20 can be rewritten as follows

$$\operatorname{Encode}(s) = \frac{1}{n} \sum_{w \in s} \widetilde{\boldsymbol{w}} \tag{21}$$

$$\widetilde{\boldsymbol{w}} = f_{\text{VC}}(\boldsymbol{w}) = f_3 \left(\alpha_2(\boldsymbol{w}) \cdot f_1(\boldsymbol{w}) \right).$$
 (22)

That is, these encoders first perform a transformation $f_{\rm VC}$ on each word vectors independently and then simply sum them up (additive composition!). In other words, certain type of unsupervised sentence encoders act as word vector converters (VC).

Specific Examples: Algorithm and Hyperparameters

Hereinafter, as specific examples, we consider

- All-but-the-top (Mu and Viswanath, 2018), Conceptor negation (Liu et al., 2019a), or dimension-wise Normalization (Arora et al., 2017) for f_1 ,
- SIF Weighting (Arora et al., 2017) for α_2 ,
- and common component Removal (Arora et al., 2017) for f_3 ,

which particularly high performance has been confirmed. For abbreviation, we write A, C, N, R, W for them. In addition, GloVe + VC(AWR) denotes word vectors converted from pre-trained GloVe by the Vector Converter f_{VC} (Eq. 22) induced by All-but-the-top, SIF Weighting, and common component **R**emoval.

Algorithm 2 summarizes the overall procedure of word vector converter $f_{\rm VC}$ (Eq. 22). When computing Algorithm 2, we set hyperparameters as $D_{\rm A} \, = \, 3, \, \alpha_{\rm C} \, = \, 2, \, a_{\rm W} \, = \, 10^{-3}, \, {\rm and} \, \, D_{\rm R} \, = \, 5,$ following Mu and Viswanath (2018), Liu et al. (2019a), Arora et al. (2017), and Ethayarajh (2018), Algorithm 2 Word Vector Converter (VC), induced from All-but-the-top (Mu and Viswanath, 2018) or Conceptor negation (Liu et al., 2019a), SIF Weighting (Arora et al., 2017), and common component Removal (Arora et al., 2017).

Input: pre-trained word vectors $\{w_1, \dots, w_{|\mathcal{V}|}\}$ $\subseteq \mathbb{R}^d$, sentences in interest $\{s_1, \dots, s_{|\mathcal{S}|}\}$, word unigram probability $\mathbb{P} \colon \mathcal{V} \to [0,1]$, and constants $D_{\rm A}$ (or $\alpha_{\rm C}$), $a_{\rm W}$, $D_{\rm R}$ Compute parameters of f_1 : · · · if using All-but-the-top:

1:
$$\overline{\boldsymbol{w}} \leftarrow \frac{1}{|\mathcal{V}|} \sum_{i=1}^{|\mathcal{V}|} \boldsymbol{w}_i \in \mathbb{R}^d$$

2: **for**
$$i \leftarrow 1$$
 to $|\mathcal{V}|$ **do**

3:
$$\overline{\boldsymbol{w}}_i \leftarrow \boldsymbol{w}_i - \overline{\boldsymbol{w}}$$

4: end for

5:
$$m{u}_1, \dots, m{u}_{D_{\mathbf{A}}} \leftarrow \mathrm{PCA}(\{\overline{m{w}}_1, \dots, \overline{m{w}}_{|\mathcal{V}|}\})$$
 $ightarrow$ top $D_{\mathbf{A}}$ singular vectors

6:
$$\boldsymbol{A}_1 \leftarrow \boldsymbol{I} - \sum_{j=1}^{D_{\mathrm{A}}} \boldsymbol{u}_j \boldsymbol{u}_j^{\top} \in \mathbb{R}^{d \times d}$$
7: $\boldsymbol{b}_1 \leftarrow \overline{\boldsymbol{w}} \in \mathbb{R}^d$

7:
$$oldsymbol{b}_1 \leftarrow \overline{oldsymbol{w}} \in \mathbb{R}^{\widehat{d}}$$

· · · else if using Conceptor negation:

8:
$$m{R} \leftarrow rac{1}{|\mathcal{V}|} \sum_{i=1}^{|\mathcal{V}|} m{w}_i m{w}_i^{ op} \in \mathbb{R}^{d imes d}$$

9:
$$C \leftarrow R(R + \alpha_{\text{C}}^{-2}I)^{-1} \in \mathbb{R}^{d \times d}$$

10:
$$A_1 \leftarrow I - C \in \mathbb{R}^{d \times d}$$

11:
$$\boldsymbol{b}_1 \leftarrow \mathbf{0} \in \mathbb{R}^d$$

Compute parameters of f_3 :

12: **for**
$$i \leftarrow 1$$
 to $|\mathcal{S}|$ **do**

13:
$$\boldsymbol{s}_i \leftarrow \sum_{w \in \boldsymbol{s}_i} \alpha_2(w) \boldsymbol{A}_1(\boldsymbol{w} - \boldsymbol{b}_1)$$

14: **end for**

15:
$$v_1, \dots, v_{D_{\mathbf{R}}} \leftarrow \operatorname{PCA}(\{s_1, \dots, s_{|\mathcal{S}|}\})$$
 $\Rightarrow \operatorname{top} D_{\mathbf{R}} \text{ singular vectors}$

16:
$$\boldsymbol{A}_{3} \leftarrow \boldsymbol{I} - \sum_{j=1}^{D_{\mathrm{R}}} \boldsymbol{v}_{j} \boldsymbol{v}_{j}^{\top} \in \mathbb{R}^{d \times d}$$

Convert word vectors:

17: **for** $i \leftarrow 1$ to $|\mathcal{V}|$ **do**

18:
$$\alpha_2(w) \leftarrow a_{\mathrm{W}}/(\mathbb{P}(w) + a_{\mathrm{W}})$$

19:
$$\widetilde{\boldsymbol{w}}_i \leftarrow \boldsymbol{A}_3(\alpha_2(w)\boldsymbol{A}_1(\boldsymbol{w}_i - \boldsymbol{b}_1))$$

20: end for

Output: Converted word vectors $\{\widetilde{\boldsymbol{w}}_1, \cdots, \widetilde{\boldsymbol{w}}_{|\mathcal{V}|}\}$

respectively, without tuning. We used the unigram probability \mathbb{P} of English words estimated with the massive scale corpora (Speer et al., 2018)⁴.

5.2 Norm and Direction

We expect that importance weight of w is "better" encoded in the norm of converted word vector $\widetilde{m{w}} = f_{
m VC}(m{w})$ than original pre-trained word vector w. To verify our hypothesis, we compare the additive composition of converted word vectors (Eq.

⁴https://github.com/LuminosoInsight/wordfreq

	ADD	ADD NORMALIZED
GloVe	54.16	46.25
GloVe + A	68.30	59.62
GloVe + AW	76.68	59.62
GloVe + VC (AWR)	79.13	63.60

Table 4: **Pearson's** $r \times 100$ between predicted scores and gold scores for each word vector (each row) and each methods (each column). The STS-B dataset (dev) is used. The best result in each row is in **bold**.

	GloVe	GloVe + A	GloVe + VC (AWR)
MEN	80.49	82.43	82.26
MTurk287	69.18	72.77	69.32
MC30	78.81	77.99	80.67
RW	47.28	54.75	54.34
RG65	76.90	75.18	76.89
SCWS	62.96	67.09	65.83
SimLex999	40.84	46.74	49.83
WS353-REL	68.75	70.73	72.35
WS353-SIM	79.57	80.97	79.32

Table 5: **Spearman's** $\rho \times 100$ between predicted scores (cosine similarity) and gold scores for each word similarity task (each row) and each word vector (each column). The best result and results where the difference from the best < 0.5 in each row are in **bold**. "GloVe + AW" is omitted from the table because W (SIF weighting, just scaling) alone does not change the direction vectors.

21) and similar method but ignoring norm (see Eq. 14). Table 4 shows that ignoring the norm of word vectors will result in consistently worse predictive performance. Even when the norm is ignored, the performance is improved by the sequence of transformation of word vectors. The reason for this might be the improvement of the direction vector.

Also, we expect that word meaning of w is "better" encoded in the direction vector of converted word vector $\widetilde{\boldsymbol{w}} = f_{\text{VC}}(\boldsymbol{w})$ than original pre-trained word vector \boldsymbol{w} . Table 5 demonstrates that the lexical meaning becomes accurately encoded by its direction as the transformation proceeds. This suggests that VC improves the performance of word vectors both in terms of the importance of words encoded in the norm and in terms of the meaning of words encoded in the direction vectors.

5.3 Word Rotator's Distance Powered by Word Vector Converter

The converted word vector $\tilde{\boldsymbol{w}} = f_{\text{VC}}(\boldsymbol{w})$ can be applied to WRD as it is. We believe that using $\{\tilde{\boldsymbol{w}}\}$ will improve the performance of WRD, be-

	WMD	WMD	WRD	WRD
Removing Stopwords		\checkmark		\checkmark
GloVe	62.56	71.34	64.66	71.13
GloVe + A	65.74	75.19	68.83	75.19
GloVe + AW	63.34	74.41	77.21	76.44
GloVe $+ A + SIF$ weights	76.81	76.56	-	-
GloVe + VC (AWR)	61.42	72.81	<u>79.20</u>	78.60
word2vec	67.26	72.41	71.05	73.19
word2vec + A	67.22	72.46	71.32	73.65
word2vec + AW	63.89	71.59	71.59	74.91
word2vec + A + SIF weights	74.70	73.98	-	-
word2vec + VC(AWR)	62.76	70.22	<u>77.07</u>	76.43
fastText	61.64	70.46	67.93	74.07
fastText + A	64.00	73.52	69.95	76.45
fastText + AW	61.15	72.63	78.26	77.64
fastText + A + SIF weights	75.50	75.06	-	-
fastText + VC(AWR)	59.78	71.27	<u>79.14</u>	78.62

Table 6: **Pearson's** $r \times 100$ between predicted scores and gold scores for each word vector (each row) and each methods (each column). The STS-B dataset (dev) is used. The best result and results where the difference from the best < 0.5 in each row are in **bold**, and the best result in each word vector is further **underlined**.

cause WRD depends on weights and word meaning encoded in norm and the direction vector.

6 Experiments

In this section, we experimentally verify the performance of the proposed method, word rotator's distance (**WRD**, see Section 4) and the word vector converters (**VC**, see Section 5). For experimental procedure and evaluation metric, see Section 2. For datasets, see Appendix C. For word vectors we used, see Appendix A.

First, we confirm how much the proposed methods improve predictive performance compared to baselines. Table 6 shows some positive experimental results.

- In almost all cases, the WRD shows higher predictive performance than WMD. The difference is more noticeable when not removing stopwords. This is probably because WRD can consider the difference in importance between words through the norm without relying on stopwords removal.
- As the word vector is transformed by VC, the performance of WRD improves steadily. This is because WRD can directly utilize the importance and meaning encoded in the norm and direction vector, the quality of which is enhanced by VC.
- One might think that the weight of SIF can be directly used as the probability for WMD com-

putation. "+ SIF weights" in the Table 6 denotes such computation. Even if WMD removes stopwords and uses SIF directly as a probability value, it does not reach the performance of WRD.

Next, we compare the performance of the proposed methods, **WRD** and **VC**, with various baselines, including recent alignment-based methods such as WMD (Kusner et al., 2015), BERTScore (Zhang et al., 2019), and Dyna-Max (Zhelezniak et al., 2019). The results are shown in Table 7. We summarized some major findings as follows.

- Among the methods that consider word-by-word alignment, WRD achieved the best performance. This is probably because other methods uses Euclidean distance (Kusner et al., 2015) or dot product (Zhang et al., 2019; Zhelezniak et al., 2019) as word similarity measures; thus they cannot distinguish the two types of information (weight and meaning), which each word vector holds them separately by dividing it into a norm and a direction vector.
- For some datasets, the proposed method outperforms existing state-of-the-art additive composition methods. This is surprising given that we are using the method optimized for additive composition without tuning. Perhaps considering word-by-word alignment is an inherently good hypothesis and we want to deepen this direction in the future.
- See Appendix E for full results including semisupervised settings.

7 Conclusion

To show the great potential of considering the word-by-word alignment when computing STS, we proposed a new textual similarity measure named word rotator's distance (WRD). WRD exploit word importance and word meaning, which are implicitly encoded in norm and direction of word vectors. Moreover, several unsupervised additive sentence encoders can be used as word vector converters (VC); converted word vectors further enhance WRD. Empirical results show that our methods achieved high performance with a significant difference from other word-by-word-alignment-based measures including word mover's distance (Kusner et al., 2015); moreover, our methods were comparable to the latest sentence encoders.

Future Work. Our method and most baseline methods express sentences as bag-of-word-vectors. EMD between structural data (Alvarez-Melis et al., 2018; Titouan et al., 2019) can directly extend our method and is considered suitable for handling language.

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⁵http://ixa2.si.ehu.es/stswiki/index.php/ STSbenchmark

	STS'12	STS'13	STS'14	STS'15	STS-B	Twitter	SICK-R
GloVe – Additive Composition							
${ m GloVe}^{\dagger}$	52.01	42.86	55.77	56.08	41.45	29.56	66.51
GloVe + WR [†] (Arora et al., 2017)	60.33	55.50	68.09	64.66	65.61	40.24	69.53
GloVe + UP (Ethayarajh, 2018)	64.9	63.6	74.4	76.1	71.5	-	73.0
GloVe – Considering Word-by-word Alignment							
WMD GloVe [†] (Kusner et al., 2015)	57.53	50.06	62.79	67.29	61.67	41.15	62.91
DynaMax GloVe (Zhelezniak et al., 2019)	58.2	53.9	65.1	70.9	-	-	-
BERTScore GloVe [†] (Zhang et al., 2019)	52.05	49.82	63.15	71.38	57.44	52.37	64.3
WRD GloVe + $VC(CWR)$ (ours)	63.60	61.10	71.50	76.25	75.21	48.75	67.57
WRD GloVe + VC(NWR) (ours)	64.24	59.97	72.29	<u>76.74</u>	74.62	<u>54.10</u>	67.90
word2vec – Additive Composition							
word2vec [†]	59.94	53.03	66.99	67.71	62.06	30.50	72.66
word2vec + WR [†] (Arora et al., 2017)	60.79	58.96	70.69	71.74	69.96	35.21	70.58
word2vec - Considering Word-by-word Alignment	t						
WMD word2vec [†] (Kusner et al., 2015)	57.87	49.62	64.69	70.00	66.80	34.54	62.71
DynaMax word2vec (Zhelezniak et al., 2019)	53.7	59.5	68.0	74.2	-	-	-
BERTScore word2vec [†] (Zhang et al., 2019)	46.23	45.8	57.04	67.23	44.62	26.25	60.98
WRD word2vec + VC(CWR) (ours)	62.58	59.75	69.91	74.46	74.14	38.90	67.44
WRD word2vec + $VC(NWR)$ (ours)	62.17	57.84	<u>70.78</u>	<u>74.80</u>	72.87	<u>42.42</u>	66.06
fastText – Additive Composition							
fastText [†]	59.03	52.31	66.22	67.98	58.92	51.22	70.24
fastText + WR [†] (Arora et al., 2017)	63.16	60.01	72.86	74.1	72.11	48.81	71.66
fastText – Considering Word-by-word Alignment							
WMD fastText [†] (Kusner et al., 2015)	57.12	50.65	63.23	67.60	61.52	39.43	62.37
DynaMax fastText (Zhelezniak et al., 2019)	60.9	60.3	69.5	76.6	-	-	-
BERTScore fastText [†] (Zhang et al., 2019)	46.36	49.59	60.74	71.81	48.14	48.19	63.76
WRD fastText + VC (CWR) (ours)	64.35	<u>61.16</u>	71.71	76.34	<u>76.29</u>	51.97	67.91
WRD fastText + VC(NWR) (ours)	64.33	60.72	<u>73.29</u>	<u>76.83</u>	76.02	<u>55.04</u>	67.66
Sent2Vec (Pagliardini et al., 2018)	-	-	-	-	75.5*	-	-
Skip-Thought [‡] (Kiros et al., 2015)	41	29	40	46	-	-	-
ELMo (All layers, 5.5B) [‡] (Peters et al., 2018)	55	53	63	68	-	-	-

Table 7: **Pearson's** $r \times 100$ between predicted scores and gold scores for each method (each row) and each dataset (each column). The best results in each dataset, word vector, and strategy for computing textual similarity ("Additive composition" or "Considering Word-by-word Alignment") is in **bold**; and the best results regardless of the strategy for computing textual similarity is further <u>underlined</u>. The results of our methods are *slanted*. Each row marked (†) is re-implemented by us. Each value marked (‡) is taken from Perone et al. (2018). Each value marked (*) is taken from STS Wiki⁵.

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	f_1 denoising word vectors	$lpha_2$ scaling	f_3 denoising sentence vectors
well-known heuristic	_	Stop Words Removal	_
well-known heuristic	_	IDF (Inverse Document Frequency)) —
Arora et al. (2017)	_	SIF (Smoothed Inverse Frequency)	Common Component Removal
Mu and Viswanath (2018) all-but-the-top	_	_
Ethayarajh (2018)	Dimension-wise Normalization	uSIF (Unsupervised SIF)	Piecewise Common Component Removal
Liu et al. (2019b)	Conceptor Negation	_	_
Liu et al. (2019a)	_	SIF	Conceptor Removal

Table 8: Unsupervised sentence encoders.

A Pre-trained Word Embeddings.

We used following pre-trained word embeddings in our experiments.

- GloVe trained with Common Crawl (Pennington et al., 2014)⁶
- word2vec trained with Google News (Mikolov et al., 2013)⁷
- fastText trained with Common Crawl (Bojanowski et al., 2017)⁸
- **PSL**, the ParagramSL-999 embeddings, trained with the PPDB paraphrase database (Wieting et al., 2015)⁹
- **ParaNMT** trained with ParaNMT-50, a large scale English-English paraphrase database (Wieting and Gimpel, 2018)¹⁰
- **ELMo** pre-trained with 1 Billion Word Benchmark, a corpus with approximately 30 million sentences (Chelba et al., 2014) (BiLSTM hidden size of 4096, output size of 512, and 2 highway layers) (Peters et al., 2018)¹¹
- **BERT-Large** pre-trained with the BooksCorpus (800M words) and English Wikipedia (2500M words) (uncased, 24 layers, hidden size of 1024, 16 self-attention heads, and 340M parameters) (Devlin et al., 2019)¹². We use the PyTorch implementation of BERT (Wolf et al., 2019)¹³.

B Unsupervised Sentence Encoders

For positioning of existing unsupervised sentence encoders in Equation 20, see Table 8.

C STS Datasets Used in Experiments

We used following STS datasets in our experiments.

- STS'12 (Agirre et al., 2012), STS'13 (Agirre et al., 2013), STS'14 (Agirre et al., 2014), and STS'15 (Agirre et al., 2015): semantic textual similarity shared tasks in SemEval
- STS-B: semantic textual similarity benchmark (Cer et al., 2017), which is the collection from SemEval STS tasks 2012–2017 (Agirre et al., 2012, 2013, 2014, 2015, 2016; Cer et al., 2017)
- Twitter: paraphrase and semantic similarity in twitter (PIT) task in SemEval 2015 (Xu et al., 2015)
- SICK-R: SemEval 2014 semantic relatedness task (Marelli et al., 2014)

Tokenization. In all the experiments, we first tokenized all the corpora other than the Twitter by StanfordNLP (Qi et al., 2018). The Twitter dataset has already tokenized by the organizer. We then lower cased all the corpora to conduct experiments under the same conditions with cased embeddings and non-cased embeddings.

```
6https://nlp.stanford.edu/projects/glove/
7https://code.google.com/archive/p/word2vec/
8https://fasttext.cc/docs/en/english-vectors.html
9http://www.cs.cmu.edu/~jwieting/
10https://github.com/kawine/usif
11https://allennlp.org/elmo
12https://github.com/google-research/bert
13https://github.com/huggingface/transformers
```

D Word Similarity Datasets Used in Experiments

We used following word similarity datasets in our preliminary experiments.

- MEN (Bruni et al., 2012)
- MTurk287 (Radinsky et al., 2011)
- MC30 (Miller and Charles, 1991)
- **RW** (Luong et al., 2013)
- **RG65** (Rubenstein and Goodenough, 1965)
- **SCWS** (Huang et al., 2012)
- SimLex999 (Hill et al., 2015)
- **WS353** (Finkelstein et al., 2002)

E Full Results of Comparative Experiments

See Table 9 for full results.

¹⁴http://ixa2.si.ehu.es/stswiki/index.php/STSbenchmark

	STS'12	STS'13	STS'14	STS'15	STS-B	Twitter	SICK-R
Unsupervised							
GloVe – Additive Composition	~~ 0.4	42.05		- - 00		•0 •	
GloVe [†]	52.01	42.86	55.77	56.08	41.45	29.56	66.51
GloVe + WR [†] (Arora et al., 2017) GloVe + UP (Ethayarajh, 2018)	60.33 64.9	55.50 63.6	68.09 74.4	64.66 76.1	65.61 71.5	40.24	69.53 73.0
GloVe – Considering Word-by-word Alignment	04.2	00.0	7-11-1	70.1	71.0		75.0
WMD GloVe [†] (Kusner et al., 2015)	57.53	50.06	62.79	67.29	61.67	41.15	62.91
DynaMax GloVe (Zhelezniak et al., 2019)	58.2	53.9	65.1	70.9	-	-	-
BERTScore GloVe [†] (Zhang et al., 2019)	52.05	49.82	63.15	71.38	57.44	52.37	64.3
WRD GloVe + VC(CWR) (ours) WRD GloVe + VC(NWR) (ours)	63.60 64.24	61.10 59.97	71.50 72.29	76.25 76.74	$\frac{75.21}{74.62}$	48.75 54.10	67.57 67.90
word2vec – Additive Composition							
word2vec [†]	59.94	53.03	66.99	67.71	62.06	30.50	72.66
word2vec + WR [†] (Arora et al., 2017)	60.79	58.96	70.69	71.74	69.96	35.21	$\frac{72.68}{70.58}$
word2vec - Considering Word-by-word Alignment							
WMD word2vec [†] (Kusner et al., 2015)	57.87	49.62	64.69	70.00	66.80	34.54	62.71
DynaMax word2vec (Zhelezniak et al., 2019)	53.7	59.5	68.0	74.2	-	-	-
BERTScore word2vec [†] (Zhang et al., 2019) WRD word2vec + VC(CWR) (ours)	46.23 62.58	45.8 59.75	57.04 69.91	67.23 74.46	44.62 74.14	26.25 38.90	60.98 67.44
WRD word2vec + VC(NWR) (ours)	$\frac{62.36}{62.17}$	57.73 57.84	70.78	74.40 74.80	$\frac{74.14}{72.87}$	<i>42.42</i>	66.06
fastText – Additive Composition							
fastText [†]	59.03	52.31	66.22	67.98	58.92	51.22	70.24
fastText + WR [†] (Arora et al., 2017)	63.16	60.01	72.86	74.1	72.11	48.81	71.66
fastText – Considering Word-by-word Alignment							
WMD fastText [†] (Kusner et al., 2015)	57.12	50.65	63.23	67.60	61.52	39.43	62.37
DynaMax fastText (Zhelezniak et al., 2019)	60.9	60.3	69.5	76.6	-	-	-
BERTScore fastText [†] (Zhang et al., 2019) WRD fastText + VC(CWR) (ours)	46.36 64.35	49.59 61.16	60.74 71.71	71.81 76.34	48.14 76.29	48.19 <i>51.97</i>	63.76 67.91
WRD fastText + VC(NWR) (ours)	$\frac{64.33}{64.33}$	$\frac{61.10}{60.72}$	73.29	76.83	$\frac{76.25}{76.02}$	<i>55.04</i>	67.66
Sent2Vec (Pagliardini et al., 2018)	_	_			75.5*		
Skip-Thought [‡] (Kiros et al., 2015)	41	29	40	46	-	-	-
ELMo (All layers, 5.5B) [‡] (Peters et al., 2018)	55	53	63	68	-	-	-
Semi-supervised							
PPDB supervision – Additive Composition							
PSL [†] (Wieting et al., 2016)	54.02	47.42	60.51	61.11	48.62	36.57	66.29
PSL + WR [†] (Arora et al., 2017)	64.60	62.56	73.56	74.92	72.86	44.53	71.14
PSL + UP (Ethayarajh, 2018) PPDB supervision – Considering Word-by-word Alignment	<u>65.8</u>	<u>65.2</u>	<u>75.9</u>	<u>77.6</u>	74.8	-	<u>72.3</u>
WMD PSL [†] (Kusner et al., 2015)	57.93	51.84	65.44	69.96	64.95	45.1	63.01
DynaMax PSL (Zhelezniak et al., 2019)	58.2	54.3	66.2	72.4	-	-	-
BERTScore PSL [†] (Zhang et al., 2019)	54.52	48.56	64.14	71.35	61.19	47.02	66.09
WRD PSL + VC(CWR) (ours)	64.96	62.10	72.85	76.55	75.40	48.44	66.90
WRD PSL + VC(NWR) (ours)	65.13	62.53	74.00	77.11	<u>75.57</u>	52.44	67.33
ParaNMT supervision – Additive Composition	66.65	C1 07	76.07	50.50	70.5 0	40.22	745
ParaNMT [†] (Wieting and Gimpel, 2018) ParaNMT + WR [†] (Arora et al., 2017)	66.67	61.87	76.87	79.58	$\frac{79.59}{77.61}$	49.22	74.7
ParaNMT + WR' (Arora et al., 2017) ParaNMT + UP (Ethayarajh, 2018)	66.02 68.3	63.67 66.1	75.40 78.4	77.63 79.0	77.61 79.5	34.57	72.59 73.5
ParaNMT supervision – Considering Word-by-word Alignment	t = ====	<u> </u>					
WMD ParaNMT [†] (Kusner et al., 2015)	59.68	52.30	67.62	69.52	64.56	46.19	64.61
DynaMax ParaNMT (Zhelezniak et al., 2019)	66.0	65.7	75.9	80.1	-	-	-
BERTScore ParaNMT [†] (Zhang et al., 2019) WRD ParaNMT + VC(CWR) (ours)	50.24 65.52	53.41 <i>62.62</i>	63.17 75.05	71.84 78.26	49.70 77.89	35.03 44.68	64.33 69.28
WRD ParaNMT + VC(CWR) (ours) WRD ParaNMT + VC(NWR) (ours)	65.96	62.62 63.31	75.03 76.13	78.26 78.85	77.89 78.66	52.31	69.28 69.71
SNLI supervision							
USE (Transformer) [‡] (Cer et al., 2018)	61	64	71	74	_	_	_
InferSent [‡] (Conneau et al., 2017)	61	56	68	71	75.8*	_	_
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Table 9: **Pearson's** $r \times 100$ between predicted scores and gold scores for each method (each row) and each dataset (each column). The best results in each dataset, word vector, and strategy for computing textual similarity ("Additive composition" or "Considering Word-by-word Alignment") is in **bold**; and the best results regardless of the strategy for computing textual similarity is further <u>underlined</u>. The results of our methods are *slanted*. Each row marked (†) is re-implemented by us. Each value marked (‡) is taken from Perone et al. (2018). Each value marked (*) is taken from STS Wiki¹⁴.